Online Quantum Field Theory Course

Exercises 3

Week from Sep/05/04: Reading Zee’s book: pp 30-60, try to solve the exercises
Reading my script: Sects. 3.1-3.3 (including)

(1) **Symmetries and Conservation laws**

Relates to: my script Sect. 3.3. For my taste, the nutshell, Zee chose to represent Noether’s theorem (it comes a little later in his book than this week’s suggested reading) is too small. It should be more of the size of a coconut than a peanut! :-) I tried my best in Sect. 3.3. We are concerned with classical field theory for the moment. So here are some exercises:

(a) Derive the structure of “infinitesimal Poincaré transformations”:

\[ \delta x^\mu = \delta \omega^\mu_{\nu} x^\nu + \delta a^\mu. \]  

Show from the fact that, for \( \delta a^\mu \), the Minkowski product of two vectors \( x \) and \( y \) must be unchanged the relation

\[ \delta \omega^\rho_{\nu} := g^\rho_{\mu} \delta \omega^\mu_{\nu} = -\delta \omega^\nu_{\rho}. \]  

(b) Consider a massive vector field (describing, e.g., a \( \rho \) meson with a mass of about 770 MeV). Its free Lagrangian reads

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A^\mu A^\nu \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]  

From this, derive the equations of motion from the Hamilton principle of least action.

(c) Consider infinitesimal Poincaré transformations:

\[ x' = x + \delta x, \quad A'_\mu(x') = A^\mu(x) + \delta \omega^\mu_{\nu} A^\nu(x), \]  

where \( \delta x \) is given by (1).

With help Sect. 3.3 in my script show explicitly that the action of invariant under these transformations. Why is this calculation more or less “superfluous”?

(d) Derive the 10 “Noether currents” and the corresponding “Noether charges”, leading to the energy-momentum tensor (total energy and momentum of the field) and the angular-momentum-centre-of-mass tensor (total angular momentum and centre-of-mass of the field).

(e) Look at the limit \( m \rightarrow 0 \), describing then massless bosons with helicities 1 and \(-1\) like the photon. Is the expression for the energy-momentum tensor gauge invariant? How about the total energy and momentum?

**Hint:** This is a little bit cumbersome index shuffeling etc., but I promise you, it’s worth the effort! It’s one of the most beautiful and aesthetic parts of physics: Symmetries rule the natural laws!