Resonances in the medium

Hendrik van Hees

Goethe-Universität Frankfurt and FIAS

July 16, 2013

based on
HvH, PRD 65, 025010 (2001); PhD thesis 2000
Ivanov, Knoll, Voskresensky, NPA 657, 413 (1999); NPA 672, 313 (2000)
Outline

1. Warm-up: Resonance in quantum mechanics
2. Φ-derivable approximations
3. Transport equations
4. Summary
What’s a resonance?

- quantum mechanics 101: Particle in a potential pot
- wave packet with energy around transmission-resonance peak
- nearly no reflection
What’s a resonance?

- Quantum mechanics 101: Particle in a potential pot
- Wave packet with energy around resonance peak
- Nearly no reflection; stays a while in pot

\[ t = 0.00 \text{ a.u.} \]
Schwinger-Keldysh real-time formalism

- calculate **expectation values** of observables
- statistical operator defines state at **initial time**, \( t_i \) \( \Rightarrow \) “in-in formalism”
- time evolution

\[
\langle O \rangle (t) = \text{Tr} \left[ \hat{\rho}(t_i) \mathcal{T}_a \left\{ \exp \left[ +i \int_{t_i}^{t} dt' \mathcal{H}_I(t') \right] \right\} \right]
\]

- anti time-ordered

\[
O_I(t) \mathcal{T}_c \left\{ \exp \left[ -i \int_{t_i}^{t} dt' \mathcal{H}_I(t') \right] \right\}
\]

- time-ordered

- Schwinger-Keldysh real-time contour:

\[
\mathcal{C} = \mathcal{K}_1 + \mathcal{K}_2
\]
Baym’s $\Phi$ functional

- write generating functional for Green’s functions as path integral
- introduce local and bilocal sources

$$Z[J, K] = N \int D\phi \exp \left[ iS[\phi] + i \left\{ J_1 \phi_1 \right\}_1 + \left\{ \frac{i}{2} K_{12} \phi_1 \phi_2 \right\}_{12} \right]$$

- generating functional for connected Green’s functions

$$W[J, K] = -i \ln Z[J, K]$$

- functional Legendre transform

$$\Gamma[\varphi, G] = W[J, K] - \left\{ \varphi_1 J_1 \right\}_1 - \frac{1}{2} \left\{ (\varphi_1 \varphi_2 + i G_{12}) K_{12} \right\}_{12}$$

- loop expansion

$$\Gamma[\varphi, G] = S_0[\varphi] + \frac{i}{2} \text{Tr} \ln(-i G^{-1}) + \frac{i}{2} \left\{ D_{12}^{-1} (G_{12} - D_{12}) \right\}_{12}$$

$$+ \Phi[\varphi, G] \leftarrow \text{all closed 2PI interaction diagrams}$$

$$D_{12}^{-1} = -\Box - m^2$$
Baym’s $\Phi$ functional

- equations of motion
  \[
  \frac{\delta \Gamma}{\delta \varphi_1} = -J_1 - \{K_{12} \varphi_2\}_2 \overset{!}{=} 0, \quad \frac{\delta \Gamma}{\delta G_{12}} = -\frac{i}{2} K_{12} \overset{!}{=} 0
  \]

- mean field
  \[
  -\square \varphi - m^2 \varphi := j = -\frac{\delta \Phi}{\delta \varphi}
  \]

- “full” propagator $G \Rightarrow$ Dyson equation:
  \[
  -i(D_{12}^{-1} - G_{12}^{-1}) := -i\Sigma = 2 \frac{\delta \Phi}{\delta G_{21}}
  \]

- retarded Green’s function for homogeneous system in momentum space
  \[
  G_{\text{ret}}(p) = \frac{1}{p^2 - m^2 - \Sigma_{\text{ret}}(p)}
  \]

- spectral function
  \[
  A(p) = -2 \text{Im} G_{\text{ret}}(p) = -2 \frac{\text{Im} \Sigma_{\text{ret}}(p)}{[p^2 - m^2 - \text{Re} \Sigma_{\text{ret}}(p)]^2 + [\text{Im} \Sigma_{\text{ret}}(p)]^2}
  \]
Properties of $\Phi$-derivable approximations

- Truncations of $\Phi$ functional $\Rightarrow$ $\Phi$-derivable approximations

\[ i\Phi = \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \end{array} + \begin{array}{c} \text{diagram 4} \\ \text{diagram 5} \end{array} + \frac{1}{2} \begin{array}{c} \text{diagram 6} \\ \text{diagram 7} \end{array} + \frac{1}{2} \begin{array}{c} \text{diagram 8} \end{array} + \frac{1}{3} \cdots \]

\[ -ij = \begin{array}{c} \text{diagram 9} \\ \text{diagram 10} \end{array} + \begin{array}{c} \text{diagram 11} \end{array} + \begin{array}{c} \text{diagram 12} \end{array} + \cdots \]

\[ -i\Sigma = \begin{array}{c} \text{diagram 13} \\ \text{diagram 14} \end{array} + \begin{array}{c} \text{diagram 15} \end{array} + \begin{array}{c} \text{diagram 16} \end{array} + \begin{array}{c} \text{diagram 17} \end{array} + \cdots \]

- Conservation laws for expectation values of conserved quantities
- In thermal equilibrium $i\Gamma = \ln Z$
- Thermodynamic consistency: bulk properties like pressure, energy, entropy in accordance with dynamics
- Same result from partition sum as from Green’s functions!
- “$\Phi$ derivability” sufficient and necessary scheme!
Transport equations

- start from $\Phi$-derivable Dyson equation for Green’s function

\[
(\Box_1 - \Box_2)D^{12}(x_1, x_2) = \int \mathcal{C} \, dx_3 [\Sigma(x_1, x_3)D(x_3, x_2^2) - D(x_1^1, x_3)\Sigma(x_3, x_2^2)] = \text{Coll}(x_1^1, x_2^2)
\]

- assume smallness of space-time gradients in “collective macroscopic” variable $R = (x_1 + x_2)/2$

- Wigner transform of any two-point function, $F$

\[
F(x_1, x_2) = \int \frac{d^4p}{(2\pi)^4} \exp[-ip \cdot (x_1 - x_2)] \tilde{F}\left(\frac{x_1 + x_2}{2}, p\right).
\]

- assume space-time gradients wrt. $R$ to be “small” $\Rightarrow$ gradient expansion $\Rightarrow$ “coarse graining”

\[
2p \cdot \frac{\partial}{\partial R} iD^{12}(R, p) = \text{Coll}^{12}(R, p)
\]
Gradient expansion of collision term

\[ 2p \cdot \frac{\partial}{\partial R} iD^{12}(R,p) = \text{Coll}^{12}(R,p) \]

- if \( \Phi \) beyond pure two-point level \( \Rightarrow \) memory + spatial correlations
- simplify further by introducing \( \text{Coll}^{12}_{\text{loc}} \):
  - diagrams evaluated at reference point \( R \)
- usual momentum Feynman rules with \( D_{12}(R,p) \)
- to have exact conservation laws add 1\(^{\text{st}}\)-order \( \partial_R \) correction

\[ D\left(\frac{x_i + x_j}{2},p\right) \simeq D(R,p) + \frac{1}{2} [(x_i + x_j) - R] \cdot \frac{\partial}{\partial R} D(R,p) \]

- for local Green’s functions and self-energies

\[ iD^{12}(R,p) = f(R,p)A(R,p), \quad A(R,p) = -2 \text{Im} D_{\text{ret}}(R,p) \]

- as in equilibrium with off-equilibrium phase-space distrib. \( f(R,p) \)
- usual local Dyson equation for retarded Green’s function

\[ D_{\text{ret}}(R,p) = \frac{1}{p^2 - m^2 - \Sigma_{\text{ret}}(R,p)} \]
Diagrammar for gradient expansion

\[ \frac{1}{2} (\partial_i + \partial_j) G(i, j) \rightarrow \partial_x G(X, p), \]

\[ -i(x_i - x_j) \rightarrow -(2\pi)^4 \frac{\partial}{\partial p} \delta(p) \]

- arbitrary two-point function \( M(x_1, x_2) \) with internal points \( x_3, \ldots \)

\[ \Diamond \{ M(1, 2) \} = \Diamond \]

\[ M'(x_1, x_2; x_3, x_4) = \frac{\delta M(x_1, x_2)}{\delta i G(x_4, x_3)} \]

- collision term \( \Rightarrow \) convolution integral

\[ \Diamond \{ C(X, p) \} = \Diamond \]

\[ = \{ A(X, p), B(X, p) \} + A(X, p) \Diamond \{ B(X, p) \} + \Diamond \{ A(X, p) \} B(X, p). \]
transport equation in Kadanoff-Baym form
\[
p \cdot \frac{\partial}{\partial R} iD^{12}(R, p) + \left\{ \text{Re } \Sigma_{\text{ret}}, iD^{12} \right\}_{pb} + \left\{ i\Sigma^{12}, \text{Re } D_{\text{ret}} \right\}_{pb} = C^{12}_{\text{loc}} + C^{12}_{\text{mem}}
\]
then Noether currents exactly conserved also after gradient expansion

problem: 2\textsuperscript{nd} Poisson bracket ("back-flow term") cannot be represented in test-particle Monte Carlo

Botermans-Malfliet ansatz
\[
i\Sigma^{12}(R, p) = -f(R, p)\Gamma(R, p), \quad \Gamma(R, p) = -2 \text{Im } \Sigma_{\text{ret}}
\]
valid up to 1\textsuperscript{st}-order gradients

Caveat: in conservation laws from BM ansatz
\[
A(R, p) \rightarrow B(R, p) := \frac{\partial}{\partial p_0} \left[ 2 \text{Im } \ln(D_{\text{ret}}^{-1}) - \text{Re } G_{\text{ret}}\Gamma \right]
\]
for narrow resonances (BW approximation) \( B \simeq \frac{1}{2}A^2\Gamma \)
for test-particle off-shell method ⇒ see W. Cassing’s talk
Caveat: possible trouble with tachyons
  - transition to semi-class. particle picture ↔ WKB/eikonal approximation
  - particle velocity ⇒ group velocity superluminal around resonance
  - no trouble in wave picture (see Sommerfeld+Brillouin 1913!)
in transport codes: resonances propagated as particles
subject to decay with probability $\exp(-\Gamma \Delta t)$
but $\Gamma = \Gamma(M)$ (vacuum) or even $\Gamma = \Gamma(M, \vec{p})$ (in med)
in virial expansion (formally expansion of $D$ around $D_{\text{vac}}$ ⇒ “thermodynamics” in terms of $S$ matrix [Dashen, Ma, Bernstein 1969]
correct lifetime from KB equations [Leupold, NPA 695, 377 (2001)]
$$\tau = 2p_0 B_{\text{vac}} = \frac{\partial \delta}{\partial p_0}$$
also from resonant wave propagation [Danielewicz, Pratt, PRC 53, 249 (1996)] ⇒ “delay time”: $\partial \delta / \partial E$
Application: Lifetime of an “off-shell resonance”

example: \( \Delta(1232) \) (from [Leupold, NPA 695, 377 (2001)])
Summary

- propagation of instable resonances great challenge for transport
- start from self-consistent $\Phi$ derivable approximations
- approximate Kadanoff-Baym equations for Wigner transformed single-particle GF
- gradient expansion $\Rightarrow$ coarse-grained dynamics $\Rightarrow$ semi-classical transport equations $\Rightarrow$ positive phase-space distributions
- Kadanoff-Baym form: exact conservation laws for Noether currents for complete 1$^{st}$-order gradient expansion
- Botermans-Malfliet form: feasibility as test-particle MC
- finite width $\Rightarrow$ “off-shell potential”
- Caveat: danger of superluminal particles; pragmatically solved in GiBUU, pHSD (...?)
- has intuitive physical interpretation (at least in simplifying limits)