The mass of a quark-squark bound state from QCD sum rules

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We give an estimate of the expected mass range for a possible 2-quark 2-squark bound state, using QCD sum rules. The sum rules are modified, taking mass effects into account, so that a treatment of heavy squarks becomes justified. The influence of the higher-order mass corrections and the possibility of a bound state mass below the squark mass are discussed.

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1 Introduction

The introduction of supersymmetry by Wess and Zumino [1] was followed by an extended search for supersymmetric particles. The squark masses are theoretically [2] and experimentally [3] predicted to be at least about 20 GeV. The search for squarks or bound states of supersymmetric particles was unsuccessful so far. One possibility to find experimental evidence for supersymmetry is to look for supersymmetric bound states in $e^+e^-$ collisions [4]. Such experiments give an upper bound for the leptonic width of a squark-antisquark bound state. Unfortunately the leptonic width of such a state is predicted to be much lower than these experimental bounds and the situation becomes worse the bigger the bound state mass is [5]. A way out of this dilemma would be the existence of a supersymmetric bound state with lower mass than the sum of the masses of their constituents.

An appropriate method for studying the lowest lying bound states are QCD sum rules [6]. In this context the first idea coming in mind is to look at the mass of a two squark bound state, the $\rho_S$-Meson with $J^{PC} = 1^{--}$. A natural extension of QCD sum rules predict a $\rho_S$-Meson mass of about 70 GeV, where the squark mass was choosen much smaller [7], so that the $\rho_S$-Meson seems not to be a promising candidate. We want to analyse the more complicated bound state composed of 2 quarks and 2 squarks using again QCD sum rules.
In the following section the problem is formulated in detail. Especially the relevance of higher-order mass corrections for the sum rule is discussed. The third section deals with the calculation of the Wilson coefficients with special attention to some new IR-problems appearing due to the generalization of the concept of QCD sum rules to particles with finite mass. In the fourth section the sum rules are calculated and interpreted including all relevant higher-order contributions of the squark mass. The article closes with some phenomenological considerations.

2 A four particle bound state

To consider a bound state composed of supersymmetric particles the formalism of QCD sum rules has to be extended. While the quarks can be treated as massless particles, the squark mass is large and must be treated consistently. We also allow for new condensates — unknown a priori — which lead to additional operators in the operator product expansion (OPE) [8].

The sum rule is calculated starting from the polarization function, which is defined by a two-point-function

$$
\Pi(k^2) = -\frac{ie^2}{3} \int d^4(x-y) \ e^{ik(x-y)} g_{\mu\nu} <0| T\{J^\mu(x), J^\nu(y)\}|0 >
$$

(1)

where $T$ denotes the time ordered product. The current $J^\mu(x)$ contains an incoming and outgoing quark $\psi$ and squark $\phi$ respectively:

$$
J^\mu(x) = g \ \bar{\psi}(x) \gamma^\mu \psi(x) \ \bar{\phi}(x) \phi(x)
$$

(2)

The present sum rule is calculated in lowest order of QCD perturbation theory. Thus the exchange of gluons and the coupling to the gluon condensate are neglected.

Usually QCD sum rule calculations are carried out in coordinate space. In the case of massless particles this choice simplifies the calculations: the propagators are as simple as in momentum space, but the number of integrations is reduced to one, while in momentum space the number of integrations is equal to the number of loops in the corresponding Feynman-diagram. However, in the case of particles with non-vanishing masses the propagators become very cumbersome, so that in most cases the momentum space will be more convenient.

To find the explicit form of the OPE of the polarization function Eq.(1), the time-ordered product has to be expanded into normal-ordered products with attention to the nonvanishing vacuum condensates. In this way one gets sixteen terms,
each of them corresponding to one of the following diagrams: The perturbative diagram (see Figure 1), which corresponds to the fully contracted term without any

![Diagram](image)

**Figure 1:** Perturbative diagram with two scalar (thin lines) and two fermion propagators (fat lines)

condensates

\[
- i < 0 \mid \text{Tr}\{S(x - y)\gamma^\mu S(y - x)\gamma^\nu\} \text{ Tr}\{\Delta(y - x)\Delta(x - y)\} \mid 0 >
\]

(3)

where \(S(x - y)\) and \(\Delta(x - y)\) are the quark- and the quark-propagators respectively. The traces are taken in color- and Lorentz-space in the case of fermionic propagators and in color-space only in the case of scalar propagators.

There are eight terms containing one non-contracted pair of quark field operators of the form

\[
+ i < 0 \mid \gamma^\nu S(y - x)\gamma^\mu : \overline{\psi}(y)\psi(x) : \text{Tr}\{\Delta(y - x)\Delta(x - y)\} \mid 0 >
\]

(4)

which all vanish independently of the scalar part, for the condensates of massless fermions are proportional to \(\delta_{ij}\), such that a trace over an odd number of \(\gamma\)-matrices appears in the fermionic part of the expression.

The term without any contraction is vanishing as well, because it contains no propagator, so that there can’t exist any momentum flow through the diagram.

Six diagrams are left with one or two non-contracted pairs of squark field operators:

- \( i < 0 \mid \overline{\psi}(x)\gamma^\mu \psi(x) \overline{\psi}(y)\gamma^\nu \psi(y) : \Delta(x - y) : \overline{\phi}(x)\phi(y) : \mid 0 > \)
- \( i < 0 \mid \overline{\psi}(x)\gamma^\mu \psi(x) \overline{\psi}(y)\gamma^\nu \psi(y) : \Delta(y - x) : \phi(x)\overline{\phi}(y) : \mid 0 > \)
- \( - i < 0 \mid \overline{\psi}(x)\gamma^\mu \psi(x) \overline{\psi}(y)\gamma^\nu \psi(y) : \text{Tr}\{\Delta(y - x)\Delta(x - y)\} \mid 0 > \)
- \( + - i < 0 \mid \text{Tr}\{S(x - y)\gamma^\mu S(y - x)\gamma^\nu\} : \overline{\phi}(x)\phi(x) \overline{\phi}(y)\phi(y) : \mid 0 > \)
- \( - i < 0 \mid \text{Tr}\{S(x - y)\gamma^\mu S(y - x)\gamma^\nu\} \Delta(x - y) : \overline{\phi}(x)\phi(y) : \mid 0 > \)
- \( - i < 0 \mid \text{Tr}\{S(x - y)\gamma^\mu S(y - x)\gamma^\nu\} \Delta(y - x) : \phi(x)\overline{\phi}(y) : \mid 0 > \)

(5)
each of the terms corresponding to one of the diagrams in the Figures 2–5.

![Diagram 2](image2.png)  
Figure 2: Diagram for the operator with two quark and one squark condensate. There are two diagrams of this type.

![Diagram 3](image3.png)  
Figure 3: Diagram for the operator with one squark condensate. Again there are two diagrams of this type.

![Diagram 4](image4.png)  
Figure 4: Diagram for the operator with two quark condensates.

![Diagram 5](image5.png)  
Figure 5: Diagram for the operator with two squark condensates.

New condensates are to be introduced: a two squark condensate and a four squark condensate. The latter will be approximated by the square of the two squark condensate, so that no additional parameter appears. The six particle condensate factorizes into a known quark condensate and again a four squark condensate. So in the present approximation — being completely analogous to the treatment of high dimension quark condensates — there is only the two squark condensate appearing as a new parameter. In view of the search for a bound state with low mass it will be convenient to vary the value of the squark condensate in a reasonable range to minimize the resulting mass of the bound state.

The matrix elements (5) contain expressions of the type \( <0 | \psi(x) \overline{\psi}(y) : 0 > \), which are usually taken to be proportional to the quark condensates \( < \overline{\psi} \psi > \). We want to stress that the coupling to the squark condensate is analogous. There is, however, an important difference: We showed in a previous paper [9] that for massive particles the coupling to the corresponding condensates is much more complex than for massless ones. So the relation between \( <0 | : \phi(x) \overline{\phi}(y) : 0 > \) and the squark condensate \( < \overline{\phi} \phi > \) is more involved than the corresponding relation for the quarks. The importance of the mass corrections raises with increasing ratio \( m_s/m \),

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where $m_s$ is the squark mass and $m$ the mass of the bound state. In the problem under consideration we are just looking for a bound state with lower mass than its constituents, the squarks. It seems unavoidable to treat all mass corrections correctly, so that the squark condensate has to be identified with (using $\xi^2 = (x - y)^2$):

$$< 0 | : \bar{\phi}_\alpha(x) \phi_\beta(y) : | 0 > = \frac{1}{N_c} \delta_{\alpha \beta} < \bar{\phi} \phi > \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!(n + 1)!} \left( \frac{m_s^2 \xi^2}{4} \right)^n$$

Having in mind the relation of this expression to the Bessel function, it may be interpreted as a special kind of nonperturbative squark field propagator between $x$ and $y$.

One can recover the discussed identification of vacuum expectation value of a normal-ordered nonlocal product of fields with the corresponding vacuum condensate — remaining correct for massless particles — as the first term of the series, that is in lowest order in the squark mass.

## 3 Calculation of the Wilson coefficients

In the previous section we have clarified which operators appear in the OPE and how the corresponding Wilson coefficients look like (see Figures 1–5). Furthermore we showed that mass corrections to all terms containing a squark condensate possibly may become important. These corrections are not expected to be large in the case of quark condensates, for the mass of the fermions is considered to be small. In order to treat the squark mass consistently, the squark propagator will be expanded in the mass:

$$\Delta(p) = - \frac{1}{p^2 - m_s^2 + i \epsilon} = - \sum_{n=0}^{\infty} \frac{m_s^{2n}}{(p^2 + i \epsilon)^{n+1}}$$

Except for the perturbative graph, which is suppressed by the factor $(2\pi)^{-12}$ all graphs will be treated to all orders in the squark mass.

For the calculation of the Wilson coefficient, dimensional regularization will be used [10]. Having in mind the explicit form of the sum rule we are using

$$m^2 \approx \int_0^\infty ds \frac{\text{Im}\Pi(s)}{\text{Im}\Pi(s)} e^{-st} \int_0^\infty ds \frac{\text{Im}\Pi(s)}{\text{Im}\Pi(s)} e^{-st}$$

$(t = 1/M^2$ is the Borel-parameter and $m$ is the bound state mass), we will have to regularize the imaginary part of the above diagrams. In the following some comments concerning the regularization of the different diagrams and the corresponding
results are given ($D_n^{(m)}$ denotes respectively the $m$-th order in the squark mass $m_s^2$ of the diagram in Figure $n$. We use $s = k^2$).

The perturbative graph in fig. 1 gives the Wilson coefficient of the unit operator. It is a three-loop integral in momentum space, which has to be regularized successively loop by loop [11]. After two trivial one-loop regularizations there remains a convolution product of two loops. The lowest order term in the squark mass of the expanded squark propagators reads

$$D_1^{(0)}(k^2) = -i g^2 \frac{4N_c^2 \pi^4}{3(2\pi)^8} \int \frac{d^4q}{(2\pi)^4} (k - q)^2 \left\{ 2 - \gamma + \ln \left( -\frac{4\pi \mu^2}{q^2} \right) \right\} \times \left\{ \frac{5}{3} - \gamma + \ln \left( -\frac{4\pi \mu^2}{(k - q)^2} \right) \right\} , \quad (9)$$

where $\mu^2$ is the renormalization scale. The first bracket originates from the squark loop, while the second one originates from the quark loop. Expanding this product one gets four types of integrals, two of which vanish, while the remaining integral is purely UV-divergent and leads to the imaginary part:

$$\text{Im}D_1^{(0)}(s) = g^2 \frac{N_c^2 \pi^7}{54(2\pi)^{12}} s^3 \Theta(s) \quad (10)$$

For the first order squark mass contribution, the corresponding expression is

$$\text{Im}D_1^{(1)}(k^2) = -i g^2 \frac{4N_c^2 \pi^4}{3(2\pi)^8} \int \frac{d^4q}{(2\pi)^4} \frac{2m_s^2}{q^2} (k - q)^2 \left\{ \gamma - \ln \left( -\frac{4\pi \mu^2}{q^2} \right) \right\} \times \left\{ \frac{5}{3} - \gamma \ln \left( -\frac{4\pi \mu^2}{q^2} \right) \right\} \quad (11)$$

leading to the imaginary part:

$$\text{Im}D_1^{(1)}(s) = g^2 \frac{4N_c^2 \pi^7}{9(2\pi)^{12}} m_s^2 s^2 \left( \frac{10}{3} - \gamma + \ln \left( -\frac{4\pi \mu^2}{s} \right) \right) \Theta(s) \quad (12)$$

All higher order terms are neglected because of the already mentioned supression by the factor $(2\pi)^{-12}$.

The graph in Figure 3 and the corresponding one where the other squark line is cut, are the Wilson coefficients to the operator with one squark condensate. Because of the condensate one has to solve a two-loop integral only. Two different expansions in the squark mass enter in this expression, one from the squark propagator (7) and one from the nonperturbative propagator associated to the condensate (6). Here
the influence of the mass corrections becomes important. The lowest order term ($\sim m_s^0$) is

$$D_s^{(0)}(k^2) = \frac{8N_c \pi^2 g^2}{3(2\pi)^4} < \phi \phi >$$

Again the same integrand as in Eq. (9) for the diagram without condensates enters here. The integral is purely UV-divergent and leads to the imaginary part:

$$\text{Im} D_s^{(0)}(s) = \frac{4N_c \pi^5 g^2}{9(2\pi)^8} < \phi \phi > s^2 \Theta(s)$$

However, the first order term

$$D_s^{(1)}(k^2) = \frac{8N_c \pi^2 g^2}{3(2\pi)^4} m_s^2 < \phi \phi >$$

contains IR-divergencies, which may be verified by power counting. An UV- and IR-divergent integral leads to a divergent imaginary part, which is unphysical. One can get rid of this problem by introducing an IR-cutoff before regulating the UV-divergency. In the resulting expression the limit of vanishing cutoff is well defined and no additional divergencies appear (see [9] for a more detailed discussion). The final result reads:

$$\text{Im} D_s^{(1)}(s) = \frac{8N_c \pi^5 g^2 m_s^2 s}{3(2\pi)^8} < \phi \phi > \left\{ \frac{3}{2} + \ln \left( \frac{16 \pi^2}{s} \right) \right\} \Theta(s)$$

It should be mentioned, that the same result may be obtained by an expansion of the squark propagator without any additional term coming from the condensate. This may be realized by analyzing the diagrams with one squark condensate before moment-integration:

$$D_s^{(0+1+2)}(k^2) = \frac{8iN_c \pi^2 g^2}{3} < \phi \phi > \int \frac{d^4 q}{(2\pi)^4} (k-q)^2 \left\{ \frac{5}{3} - \gamma + \ln \left( -\frac{4 \pi^2}{(k-q)^2} \right) \right\}$$

Fermion-Loop

$$\int \frac{d^4 p}{(2\pi)^4} \delta^4(p) \left\{ 1 + \frac{m_s^2}{8} + \frac{m_s^4}{192} \right\}$$

squark condensate

$$\times \left\{ \frac{1}{(p-q)^2 + i \epsilon} + \frac{m_s^2}{((p-q)^2 + i \epsilon)^2} + \frac{m_s^4}{((p-q)^2 + i \epsilon)^3} \right\}$$

squark propagator
Carrying out the differentiations and restricting the expression to the first order in \( m_s^2 \) one gets an expression which suggests that one has neglected all higher contributions of the condensate:

\[
D_3^{[0+1+2]}(k^2) = \frac{8i\pi^2 g^2}{3} \left< \bar{\phi} \phi \right> \left\{ \frac{5}{3} - \gamma + \ln \left( \frac{4\pi \mu^2}{(k-q)^2} \right) \right\} \\
\int \frac{d^4p}{(2\pi)^4} \delta^4(p) \left\{ \frac{1}{(p-q)^2 + i\epsilon} + \frac{m_s^2}{((p-q)^2 + i\epsilon)^2} + \frac{2m_s^4}{((p-q)^2 + i\epsilon)^3} \right\}
\]

This means, that the first order mass correction due to the squark condensate, are already included in an exact treatment of the mass in the squark propagator. An effect of mass corrections due to the condensate emerge in second order of \( m_s^2 \) only and leads to a relative factor of 2. This contribution reads before regularization:

\[
D_3^{(4)}(k^2) = \frac{16\pi^2 g^2 m_s^4}{3(2\pi)^4} \left< \bar{\phi} \phi \right> i \int \frac{d^4q}{(2\pi)^4} \frac{k^2 - 2k \cdot q + q^2}{(q^2 + i\epsilon)^3} \ln \left( \frac{4\pi \mu^2}{(k-q)^2} \right)
\]

Here the only physical term is the one proportional to \( q^2 \) which is UV- and IR-divergent, while the other two terms (\( \sim k^2 \) and \( \sim 2k \cdot q \)) have unphysical IR-divergencies only. The remaining double divergent integral is of a similar type as Eq. (15) and therefore leads to an imaginary part of the same structure than Eq. (16)

\[
\text{Im} D_3^{(4)}(s) = \frac{N_c g^2 \pi m_s^4}{3(2\pi)^4} \left< \bar{\phi} \phi \right> \ln \left| \frac{s}{16\pi \mu^2} \right| \Theta(s)
\]

Comparing the expressions (13), (15) and (19) one realizes that the higher the order in the squark mass the lower the order in the momentum \( q^2 \) in the moment-integral, so that the UV-divergencies disappear for higher orders in the squark mass. This is the case for the third order term of the one squark condensate graph, which gets an additional \( 1/q^2 \) in Eq.(19), so that no UV-divergency exists any more. Consequently the third and all higher order terms have no influence on the physical imaginary part which enters into the sum rule and the Wilson-coefficient is treated exactly with respect to the squark mass although the calculation is stopped at the third order level \( \sim m_s^6 \). The result, that only a finite number of mass corrections are relevant for sum rule calculations, holds in general [9].

The diagram with two quark condensates (see Figure 4) can be regularized without any problems to all orders in the squark mass, for there is no squark condensate, which would lead to mass corrections. The corresponding integral

\[
D_4(k^2) = \frac{ig^2}{3N_c} \left< \bar{\psi} \psi \right>^2 \int \frac{d^4p}{(2\pi)^4} Tr \left\{ \Delta(p)\Delta(p-k) \right\}
\]

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is just the one-squark-loop integration which can be regularized using the exact squark propagators.

\[
\text{Im} D_4(s) = -\frac{\pi^3 g^2}{3(2\pi)^4} <\bar{\psi}\psi>^2 \sqrt{1 - \frac{4m^2}{s}} \Theta(s - 4m^2) \quad (22)
\]

Also the diagram with two squark condensates (see Figure 5) is calculated to all orders in the squark mass although there are infinitely many terms due to condensate mass corrections. This graph consists mainly of a massless fermion (quark) loop, which is already known from Eq (9). In addition there is an integral over the product of two series (6) — each corresponding to one squark condensate — and a \( \delta \)-distribution from energy-momentum conservation.

\[
D_5(k^2) = \frac{4\pi^2 g^2}{3} <\bar{\phi}\phi>^2 \sum_{n,m=0}^{\infty} \frac{1}{n!(n+1)!m!(m+1)!} \\
\int \frac{d^4q}{(2\pi)^4}(k-q)^2 \left\{ \frac{5}{3} - \gamma + \ln \left( -\frac{4\pi^2}{(k-q)^2} \right) \right\} \left( \frac{m^2 \Box_q}{4} \right)^n \delta^4(q) \\
\]

Following the general argument stated above only a finite number of these corrections contribute to the imaginary part of the polarization function, for only a finite number of terms is UV-divergent. It turns out in the case of diagram 5 that UV-divergencies appear for the lowest and the first order squark mass corrections only. The final answer is

\[
\text{Im} D_5(s) = \frac{4\pi^3 g^2}{3(2\pi)^4} <\bar{\phi}\phi>^2 \left( s + 2m^2 \right) \Theta(s) \quad . \quad (24)
\]

The second order \( \sim m^4 \) and all higher order terms do not contribute to the physical imaginary part of the Wilson coefficient corresponding to the 2 squark condensate operator.

The same argument remains correct for the diagrams in Figure 2 with the difference that only the term in lowest order in the mass contributes to the imaginary part. The graph contains one momentum integral over one squark propagator, the series (6) and a \( \delta \)-distribution.

\[
D_2(k^2) = \frac{2\pi^2 g^2}{3N_c} <\bar{\phi}\phi> <\bar{\psi}\psi>^2 \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \\
\int \frac{d^4q}{k^2 - 2k\cdot q + i\epsilon} \left( \frac{m^2 \Box_q}{4} \right)^n \delta^4(q) , \quad (25)
\]

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where we have made use of the general relation

\[ \int d^4q \ f(q^2, q, \ldots) \Delta_{NP}(q) = \int d^4q \ f(m^2, q, \ldots) \Delta_{NP}(q) \]  

(26)

\[ \Delta_{NP} \] denotes the nonperturbative propagator defined by Eq. (6). Eq. (26) is valid only in lowest order of \( \alpha_s \). This restriction does not affect the present sum rule calculation, as perturbative QCD-corrections are not included from the beginning. Perturbative corrections to the sum rule should be small in the squark mass range considered here. The imaginary part of Eq. (25) to all orders in the squark mass becomes:

\[ \text{Im} D_2(s) = -\frac{2\pi g^2}{3N_c} <\bar{\phi}\phi><\bar{\psi}\psi>^2 \delta(s) \]  

(27)

4 Evaluation of the sum rule

In the previous section the Wilson coefficients to all relevant operators were calculated in lowest order perturbation theory and to all orders in the squark mass \( m_\tilde{q}^2 \). The whole imaginary part of the polarization function entering in expression (8) is the sum of all these contributions:

\[ \text{Im} \Pi(s) = \sum_{i=1}^{5} \sum_{j=0}^{n_i} D_i^{(j)} , \]

where \( n_i \) denotes the number of relevant orders of the \( i \)-th graph. With some stamina the Borel integrations in (8) can be performed, giving:

\[
f_0^{s_0} ds \ \text{Im} \Pi(s) e^{-s/M^2} \
= g^2 \frac{N_c^2 \pi^7}{54(2\pi)^{12}} \left\{ 6M^8 - e^{-s_0/M^2} \left( 6M^8 + 6s_0 M^6 + 3s_0^2 M^4 + s_0^3 M^2 \right) \right\} \\
+ g^2 \frac{4N_c^2 \pi^7}{9(2\pi)^{12}} m_\tilde{q}^2 \left( \frac{10}{3} - \gamma \right) \left\{ 2M^6 - e^{-s_0/M^2} \left( 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right) \right\} \\
- g^2 \frac{4N_c^2 \pi^7}{9(2\pi)^{12}} m_\tilde{q}^2 \left\{ 2M^6 \left( \frac{3}{2} - \gamma - \ln \left( \frac{4\pi \mu^2}{M^2} \right) - E_1(s_0) \right) \\
- e^{-s_0/M^2} \left( 3M^6 + s_0 M^4 + \ln \left( \frac{s_0}{4\pi \mu^2} \right) \left( 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right) \right) \right\} \\
+ g^2 \frac{4N_c \pi^5}{9(2\pi)^8} <\bar{\phi}\phi> \left\{ 2M^6 - e^{-s_0/M^2} \left( 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right) \right\} \\
+ g^2 \frac{4N_c \pi^5}{(2\pi)^8} m_\tilde{q}^2 <\bar{\phi}\phi> \left\{ M^4 - e^{-s_0/M^2} \left( M^4 + s_0 M^2 \right) \right\} \\
- g^2 \frac{8N_c \pi^5}{3(2\pi)^8} m_\tilde{q}^2 <\bar{\phi}\phi> \left\{ M^4 \left( 1 - \gamma - \ln \left( \frac{16\pi \mu^2}{M^2} \right) - E_1 \left( \frac{s_0}{M^2} \right) \right) \right\} 
\]
\[-e^{-s_0/M^2} \left( M^4 + \ln \left( \frac{s_0}{16\pi\mu^2} \right) M^4 + s_0 M^2 \right) \} \\
+ g^2 \frac{16N_c \pi^5}{3(2\pi)^8} m_s^4 < \phi \phi > M^2 \left\{ -\gamma - \ln \left( \frac{16\pi\mu^2}{M^2} \right) - E_1 \left( \frac{s_0}{M^2} \right) \right. \\
\left. \left. - \ln \left( \frac{s_0}{16\pi\mu^2} \right) e^{-s_0/M^2} \right\} \right)
- g^2 \frac{\pi^3}{3(2\pi)^4} < \phi \phi > 2 \int_{4m_s^2}^{s_0} ds \sqrt{1 - \frac{4m_s^2}{s}} e^{-s/M^2} \\
+ g^2 \frac{4\pi^3}{3(2\pi)^4} < \phi \phi > 2 \left\{ M^4 - e^{-s_0/M^2} \left( M^4 + s_0 M^2 \right) \right\} \\
+ g^2 \frac{8\pi^3}{3(2\pi)^4} m_s^2 < \phi \phi > 2 M^2 \left( 1 - e^{-s_0/M^2} \right) \\
- g^2 \frac{2\pi}{3N_c} < \phi \phi > < \bar{\psi} \psi > 2 \right) \right)
}

and

\[
f_{0}^{s_0} ds \ s \ \text{Im} \Pi(s) \ e^{-s/M^2} \\
= g^2 \frac{N_c^2 \pi^7}{54(2\pi)^{12}} \left\{ 24M^{10} - e^{-s_0/M^2} \left( 24M^{10} + 24s_0 M^8 \\
+ 12s_0^2 M^6 + 4s_0^3 M^4 + s_0^4 M^2 \right) \right\} \\
+ g^2 \frac{4N_c^2 \pi^7}{9(2\pi)^{12}} m_s^2 \left( \frac{10}{3} - \gamma \right) \left\{ 6M^8 - e^{-s_0/M^2} \left( 6M^8 + 6s_0 M^6 + 3s_0^2 M^4 + s_0^3 M^2 \right) \right\} \\
- g^2 \frac{4N_c^2 \pi^7}{9(2\pi)^{12}} m_s^2 \left\{ 6M^8 \left( \frac{11}{6} - \gamma - \ln \left( \frac{4\pi\mu^2}{M^2} \right) - E_1 \left( \frac{s_0}{M^2} \right) \right) \\
- e^{-s_0/M^2} \left( \ln \left( \frac{s_0}{4\pi\mu^2} \right) \left( 6M^8 + 6s_0 M^6 + 3s_0^2 M^4 + s_0^3 M^2 \right) \right) \\
+ 11M^8 + 5s_0 M^6 + s_0^2 M^4 \right\} \right) \\
+ g^2 \frac{4N_c \pi^5}{9(2\pi)^8} < \phi \phi > \left\{ 6M^8 - e^{-s_0/M^2} \left( 6M^8 + 6s_0 M^6 + 3s_0^2 M^4 + s_0^3 M^2 \right) \right\} \\
+ g^2 \frac{4N_c \pi^5}{(2\pi)^8} m_s^2 < \phi \phi > \left\{ 2M^6 - e^{-s_0/M^2} \left( 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right) \right\} \\
- g^2 \frac{8N_c \pi^5}{3(2\pi)^8} m_s^2 < \phi \phi > \left\{ 2M^6 \left( \frac{3}{2} - \gamma - \ln \left( \frac{16\pi\mu^2}{M^2} \right) - E_1 \left( \frac{s_0}{M^2} \right) \right) \\
- e^{-s_0/M^2} \left( 3M^6 + s_0 M^4 + \ln \left( \frac{s_0}{16\pi\mu^2} \right) \left( 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right) \right) \right\} \\
- g^2 \frac{16N_c \pi^5}{3(2\pi)^8} m_s^4 < \phi \phi > \left\{ M^4 \left( 1 - \gamma - \ln \left( \frac{16\pi\mu^2}{M^2} \right) - E_1 \left( \frac{s_0}{M^2} \right) \right) \right\} \\
\right)
\[- e^{-s_0/M^2} \left( M^4 + \ln \left( \frac{s_0}{16\pi\mu^2} \right) \left( M^4 + s_0 M^2 \right) \right) \}
\]

\[- g^2 \frac{\pi^3}{3(2\pi)^4} \left\{ \phi \psi \right\}^2 \int_{4m_s^2}^{\infty} ds \, s \left( 1 - \frac{4m_s^2}{s} \right) e^{-s/M^2} \]

\[+ \quad g^2 \frac{4\pi^3}{3(2\pi)^4} \left\{ \phi \phi \right\}^2 \left\{ 2M^6 - e^{-s_0/M^2} \left[ 2M^6 + 2s_0 M^4 + s_0^2 M^2 \right] \right\} \]

\[+ \quad g^2 \frac{8\pi^3}{3(2\pi)^4} m_s^2 \left\{ \phi \phi \right\}^2 \left\{ M^4 - e^{-s_0/M^2} \left( M^4 + s_0 M^2 \right) \right\} \]

(30)

where \( E_1 \) is the exponential integral defined by

\[
\int_1^\infty dx \ln(x) e^{-ax} = \frac{M^2}{a} E_1 \left( \frac{a}{M^2} \right) \]

(31)

We have left the contribution of the graph with two quark condensates in integral form, as we do not know its analytical solution, and it will be performed numerically. The contributions of each graph in a given order of the squark mass can be identified by looking for the corresponding combination of condensates and powers of \( m_s^2 \). The mass of the four-particle bound state we are looking for is determined by the ratio of the two integrals (see Eq.(8)).

The sum rule given above depends on several parameters, which are to be discussed now. The coupling constant \( g^2 \) factors in both integrals, so that the ratio reduces to a \( g^2 \) independent expression. The number of colours \( N_c \) is fixed to 3 in the following and for the renormalization point \( \mu \) we choose the squark mass \( m_s \).

The bound state mass resulting from the sum rule must not depend strongly on this parameter to get a reliable result. This turns out to be guaranteed to a good degree of accuracy. The sum rule is evaluated as a function of the Borel transformation parameter \( M \). We will have to look for a region of the Borel parameter, where the function \( m(M) \) is flat to get a meaningful result for the bound state mass \( m \).

The threshold of the continuum \( \sqrt{s_0} \) is fixed to three times the squark mass. This allows to have some bound states with a mass below the threshold. The bound state mass has to be independent of \( s_0 \), which is approximately correct. Multiplying the threshold with 2 leads to a change of the bound state mass smaller than 3%.

The value of the squark condensate is unknown a priori, but it may be correlated to the squark mass. The higher the squark mass the lower we expect the corresponding condensate to be, in the same way as the charm quark condensate is much smaller than the up quark condensate. We will choose the following parametrisation

\[ \left\{ \phi \phi \right\}^{1/2} = \left( M/m_s \right)^p \]

(32)
where $I$ is interpreted as the value of the squark condensate at a squark mass of 1 GeV and $p > 0$ controls the decrease of the condensate with increasing squark mass.

The quark condensate is known to be $< \bar{\psi} \psi >^{1/3} \approx 0.2$ GeV. For the squark mass we choose different values ranging between several hundred MeV and several GeV corresponding to some unknown light scalar particles and squarks respectively. The quark and the squark condensates are treated as parameters and are varied in a reasonable range with the objective to minimize the resulting bound state mass $m$.

Typical results are shown in Figs. 6 to 8) where one observes a stable region of the sum rule over a sufficient large range of the Borel parameter $(2m_s < M < 4m_s)$, so that a prediction for the bound state mass is possible. Its value is just a bit higher than twice the squark masses, but always lower than the continuum threshold.

It turns out that it is not possible to get a considerable change of the bound state mass varying the value of the quark condensate (see fig. 9 for $m_s = 8$ GeV), the value of the squark condensate at a squark mass of 1 GeV defined in Eq. 32 (see fig. 10) or the power of the relation between the squark condensate and the squark mass (see Eq. 32 and fig. 11).

The conclusion is: sum rules do not predict any possibility to construct a bound state of two quarks and two squarks, which is considerably lighter than twice the squark mass. This remains correct over the whole range of scalar quark masses we have analyzed.

In addition we may use the results to check the importance of the mass corrections due to the nonlocal squark condensate and to compare this to our general expectations. Their importance is correlated with the ratio of the bound state mass $m$ and the constituent squark mass $m_s$, which turned out to be of the order 0.5 in this special calculation. Analyzing the importance of the first-order mass correction in (6) we expect a 5% correction to the sum rule. Further more we expect that this correction will vanish for very small squark masses. On the other hand the corrections will vanish for very big squark masses, for in this domain the squark condensate is suppressed by the relation (32) and kills the mass corrections. So the expected 5% correction will appear for intermediate squark masses only. Indeed, this is confirmed by the comparison of the sum rule with and without these higher mass corrections (see fig. 12). The effect is disappearing for squark masses.
bigger than 8 GeV. The squark mass domain, in which the correction appears, can be shifted varying the parameters $I$ and $p$ determining the value of the squark condensate, like it should be.

The effect of the higher mass corrections due to the nonlocal condensate (6) turned out to be small in this special calculation. But they are nevertheless an unavoidable component of a consistent sum rule calculation.

5 Conclusions

We have calculated the bound state mass of a two squark-two quark system with the QCD sum rule method. This technique was extended to treat finite mass particles consistently. We found that the vacuum expectation value of nonlocal field products can be identified with the corresponding condensate in lowest order only, and that higher order mass corrections may become important — especially if one is interested in relative small bound state masses.

For calculations of Wilson coefficients the number of mass correction terms turns out to be finite, being a general result due to the structure of the integrals. The mass of the two squark two quark bound state is predicted to be more than twice the squark mass, so that there is no indication for a light supersymmetric state constructed of two fermions and two bosonic supersymmetric partners from QCD sum rules.

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References


\[ s_0^{1/2} = 3m_s, \quad <qq>^{1/3} = 0.2 \text{GeV}, \quad <ss>^{1/2} = 0.1/m_s \text{GeV}^2 \]

Figure 6: Small scalar particle masses \( m_s \). The resulting bound state mass is bigger than the sum of the constituent masses (2\( m_s \)), independently of \( m_s \).
\[ s_0^{1/2} = 3m_s, \quad \langle qq \rangle^{1/3} = 0.2 \text{GeV}, \quad \langle ss \rangle^{1/2} = 0.1/m_s \text{GeV}^2 \]

Figure 7: The same as Fig. 6 for intermediate squark masses \( m_s \).
$s_0^{1/2} = 3m_s$, $<qq>^{1/3} = 0.2\text{GeV}$, $<ss>^{1/2} = 0.1/m_s\text{GeV}^2$

Figure 8: The same as Fig. 6 for large squark masses $m_s$. 
Figure 9: The bound state mass in dependence of the quark condensate in the case of a squark mass of $m_s = 8\,\text{GeV}$. Within a reasonable range of values there is no possibility of lowering the bound state mass except a small effect for a quark condensate of $5\,\text{GeV}$.
Figure 10: The bound state mass as a function of the squark condensate in the case of a squark mass of $m_s = 8\,\text{GeV}$. Within a reasonable range of values there is no possibility of lowering the bound state mass.
$s_0^{1/2} = 3m_s$, $<qq>^{1/3} = 0.2\text{GeV}$, $<ss>^{1/2} = (0.1\text{GeV}/m_s)^p\text{GeV}$, $m_s = 8\text{GeV}$

Figure 11: The bound state mass as a function of the power $p$ determining the ratio between the squark condensate and the squark mass (see Eq. (32)) in the case of a squark mass of $m_s = 8\text{GeV}$. There is no possibility of lowering the bound state mass.
$s_0^{1/2} = 3m_s$, $<qq>^{1/3} = 0.2\text{GeV}$, $<ss>^{1/2} = 2/m_s\text{GeV}^2$

Figure 12: The bound state mass including ($m$) and not including ($m_0$) the higher mass corrections due to the nonlocal squark condensate, which is chosen to be $(2/m_s \text{GeV}^2)$. The comparison of the two sum rules leads to a 5% correction for squark mass values in the range of some GeV.