Transport coefficients of the dense QGP along the chiral PT

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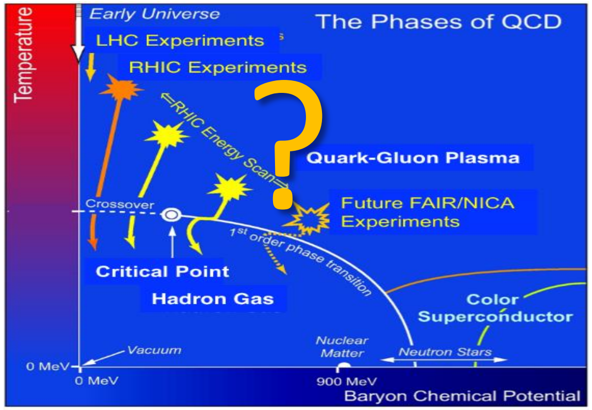
In collaboration with David Fuseau, Joerg Aichelin

Transport meeting
December 10 2020

based on arXiv:2011.03505



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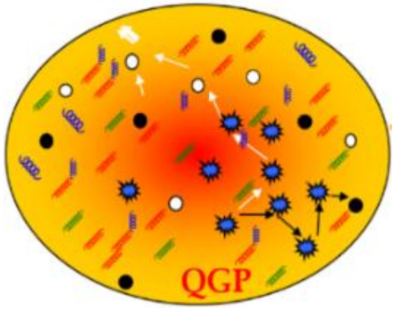
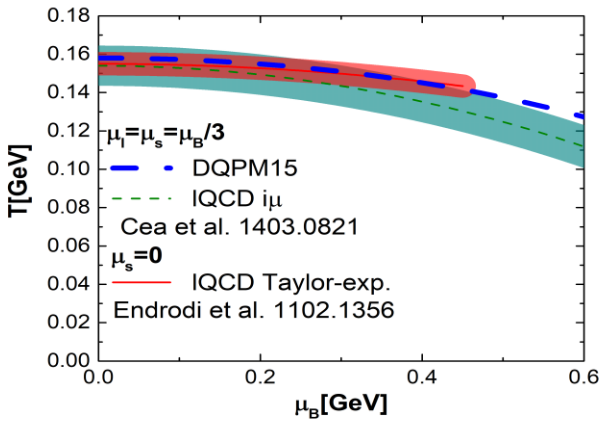


QGP in equilibrium: DQPM and PNJL

Transport coefficients at finite T and μ_B
1.) crossover (DQPM model)
2.) CEP and 1st order phase transition (PNJL model)

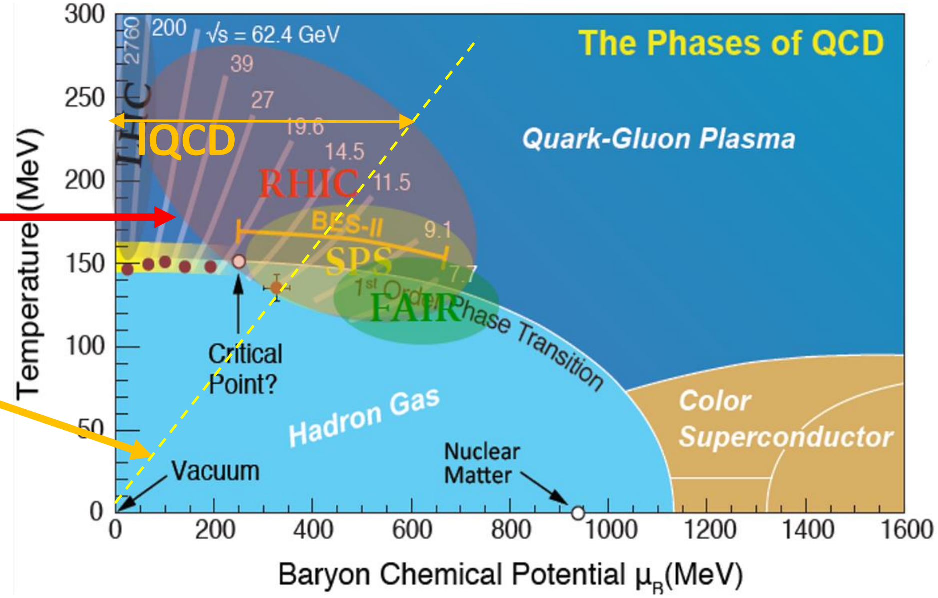


Motivation: QGP at finite baryon density

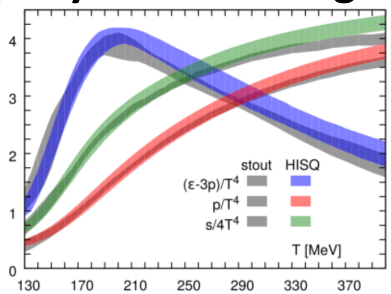
- Explore the **QCD phase diagram** at finite temperature and chemical potential through heavy-ion collisions
- **Available information:**
  - Experimental data at SPS, BES at RHIC
  - Lattice QCD calculations

(only for vanishing μ_B)

Theoretical scketch of phase diagram



EoS



$\sigma(\sqrt{s}, m_q, m_q, T, \mu_B)$

$m(T, \mu_B)$

- How to learn about degrees-of-freedom of QGP ? ➔

HIC simulations – transport models

! Problem: Transport models need an input for the partonic phase: cross-sections, masses, ...

Solution: effective models

QGP in equilibrium: DQPM and PNJL

Properties of QGP: transport coefficients

Hydrodynamics

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \Delta T^{\mu\nu}$$

$$J_B^\mu = n_B u^\mu + \Delta J_B^\mu$$

$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases}$$

input for hydro simulations

$$\Delta T^{\mu\nu} = \eta \left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \zeta \Delta^{\mu\nu}\partial_\rho u^\rho$$

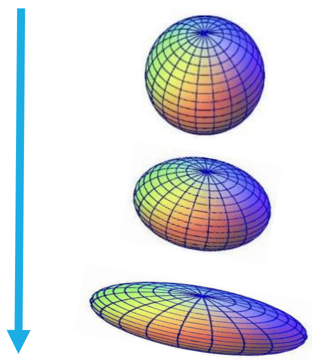
$$\Delta J_B^\mu = \kappa_B D^\mu \left(\frac{\mu_B}{T}\right)$$

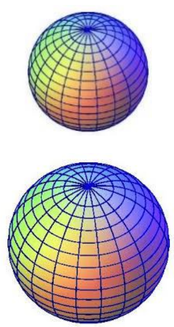
$$D^\mu = \Delta^{\alpha\nu}\partial_\nu \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Shear viscosity
Resistance to deformation
$\eta \, \nabla^{<\mu} u^{\nu>}$

Bulk viscosity
Resistance to expansion
$-\zeta \, \nabla u$

Baryon/electric charge
diffusion coefficients
$\kappa_B \, \nabla^\mu \frac{\mu_B}{T}$





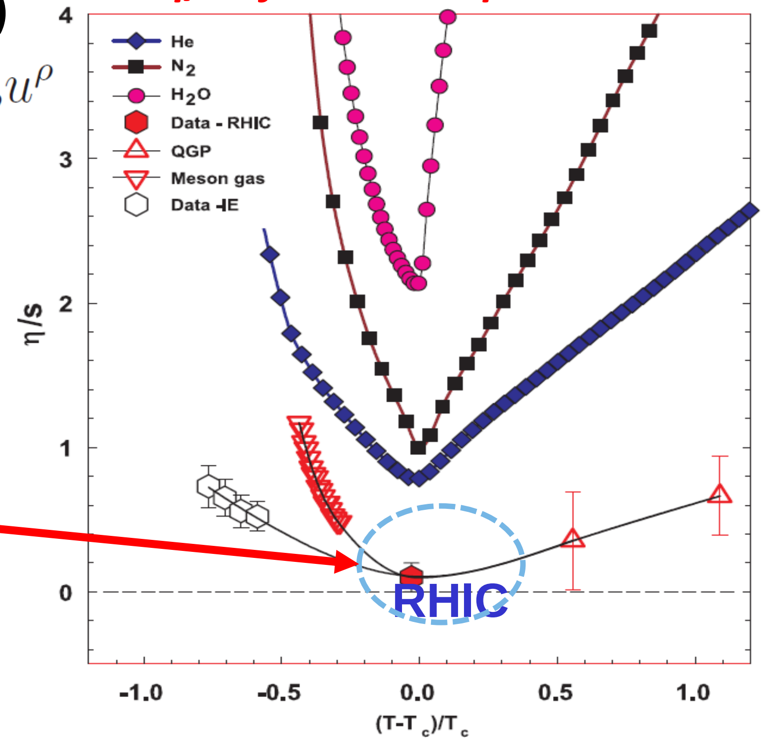
Transport coefficients of QGP

Hydrodynamical model(macroscopic description)

$$\Delta T^{\mu\nu} = \eta\left(D^{\mu}u^{\nu} + D^{\nu}u^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\rho}u^{\rho}\right) - \zeta\Delta^{\mu\nu}\partial_{\rho}u^{\rho}$$

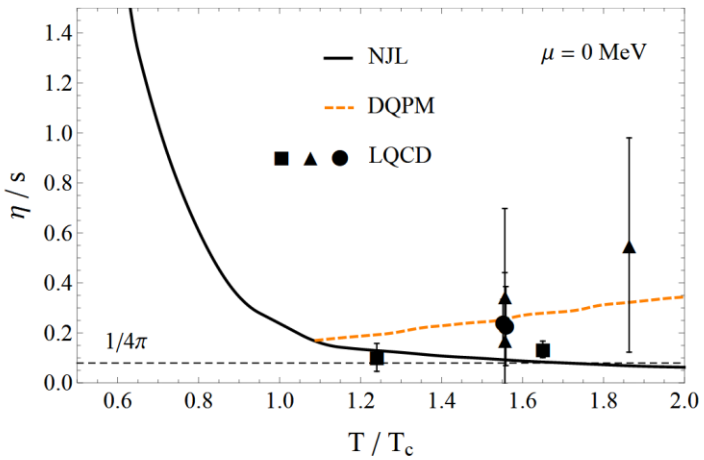
input for hydro simulations

η/s of various liquids



Shear viscosity to entropy density ratio is extremely small
QGP is the most ideal liquid!

Model predictions:



! Different models using the same EoS can have completely different transport coefficients!

Transport coefficients: approaches

- **Kubo formalism: transport coefficients are expressed through correlation functions of stress-energy tensor**

used in lattice QCD, transport approaches(hadrons), effective models

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[\mathcal{S}^{ij}(t,\mathbf{x}),\,\mathcal{S}^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t) \qquad \mathcal{S}^{ij} = T^{ij} - \delta^{ij}\mathcal{P}$$

$$\zeta = \frac{1}{2}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\langle[\mathcal{P}(t,\mathbf{x}),\,\mathcal{P}(0,\mathbf{0})]\rangle\theta(t) \qquad \mathcal{P} = -\tfrac{1}{3}T^i{}_i$$

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)
A. Harutyunyan et al, PRD 95, 114021, (2017)

Kinetic theory:

- **Relaxation time approximation(RTA) : consider relaxation time** $\frac{df_a^{eq}}{dt} = C_a = -\frac{f_a^{eq}\phi_a}{\tau_a}$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011)

$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

- **Chapman-Enskog : expand the distribution in terms of the Knudsen number**

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

And more!

Holographic models: AdS/CFT correspondence

D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006)
M. Attems et al , JHEP 10 (2016), 155.

Relaxation Time Approximation

- **Boltzmann equation** $f_a = f_a^{\text{eq}}(1+\phi_a)$

$$\frac{df_a^{\text{eq}}}{dt} = \mathcal{C}_a = -\frac{f_a^{\text{eq}}\phi_a}{\tau_a}$$

RTA: system equilibrates within the relax time τ, Express collisional Integral via τ and fa

- **Relaxation times:**

$$\frac{1+d_a f_a^{\text{eq}}}{\tau_a(E_a^*)} = \sum_{bcd} \frac{1}{1+\delta_{ab}} \int d\Gamma_b^* \, d\Gamma_c^* \, d\Gamma_d^* \, W(a,b|c,d) f_b^{\text{eq}} (1+d_c f_c^{\text{eq}})(1+d_d f_d^{\text{eq}})$$ + (cd), (bc)

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \underline{\Delta T^{\mu\nu}} \qquad J_B^\mu = n_B u^\mu + \underline{\Delta J_B^\mu}$$

$$\Delta T^{\mu\nu} = \eta\left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \zeta \Delta^{\mu\nu}\partial_\rho u^\rho$$

$$\Delta J_B^\mu = \lambda \left(\frac{n_B T}{w}\right)^2 D^\mu \left(\frac{\mu_B}{T}\right)$$

hydrodynamics

Energy-momentum tensor and baryon diffusion current can be expressed using fa :

$$T^{\mu\nu}(f_a, m_{q,g}), J_B^\mu(f_a, m_{q,g})$$

Obtain the transport coefficients using conservation laws, and fa:

$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases} \Rightarrow \quad \eta^{\text{RTA}}(T,\mu_B) = \frac{1}{15T} \sum_{i=q,\bar{q},g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p},T,\mu_B) \, d_i(1 \pm f_i) f_i$$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011).

Relaxation time and scattering rate

$$\delta f_i = f_i^{(0)}\phi_i (\text{while } \phi_j = \phi_c = \phi_d = 0\ ).$$

$$\frac{\partial f_i}{\partial t} + v_i \cdot \nabla f_i = \sum_{jcd} \frac{1}{1+\delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i,j|c,d)(f_c f_d - f_i f_j)$$

$$= \sum_{jcd} \frac{1}{1+\delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i,j|c,d)(-\delta f_i f_j^{(0)}) = -\Gamma_i(p,T,\mu)\delta f_i,$$

$$1)\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

$$2)\tau_i(T,\mu_B) = \frac{1}{2\gamma_i(T,\mu_B)},$$

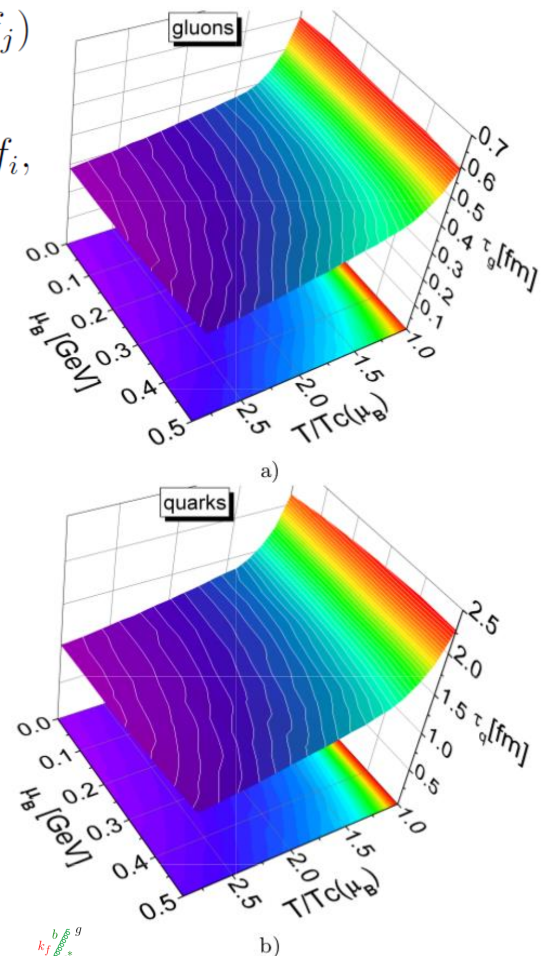
- **on-shell scattering (interaction) rates**

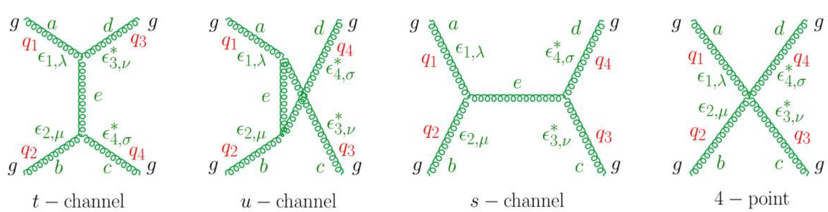
$$\Gamma_i^{\text{on}}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\ f_j(E_j,T,\mu_q)$$

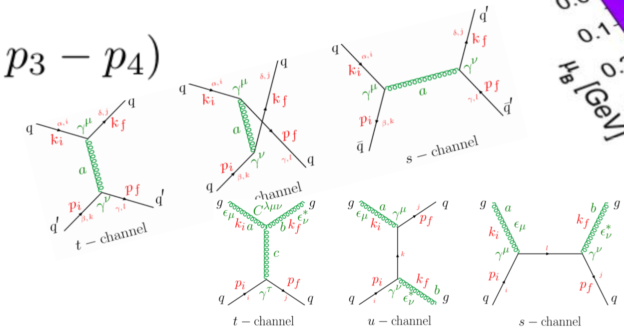
$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1\pm f_3)(1\pm f_4)$$

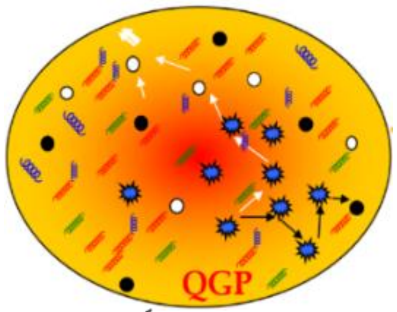
$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\,(2\pi)^4\delta^{(4)}\,(p_i+p_j-p_3-p_4)$$

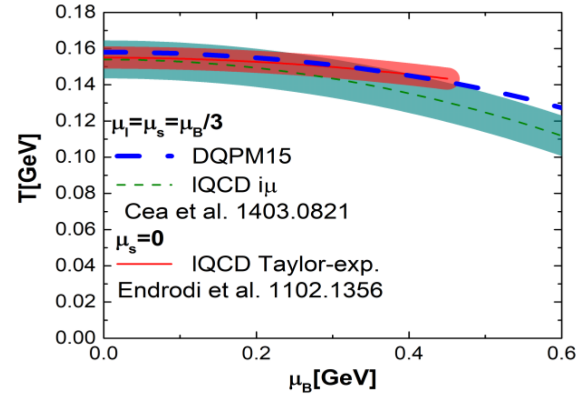
Relaxation times(DQPM)











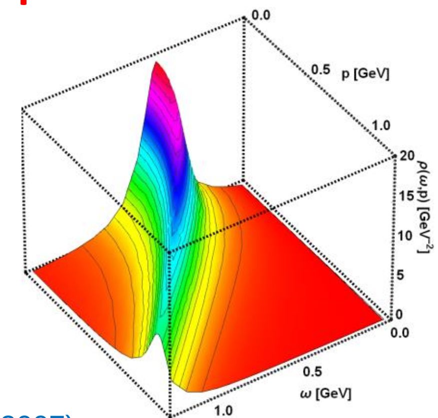
QGP in equilibrium:

Dynamical QuasiParticle Model (DQPM)

DQPM: consider the effects of the nonperturbative nature of the strongly interacting quark-gluon plasma (sQGP) constituents (vs. pQCD models)

- **The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:**

$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$
$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$



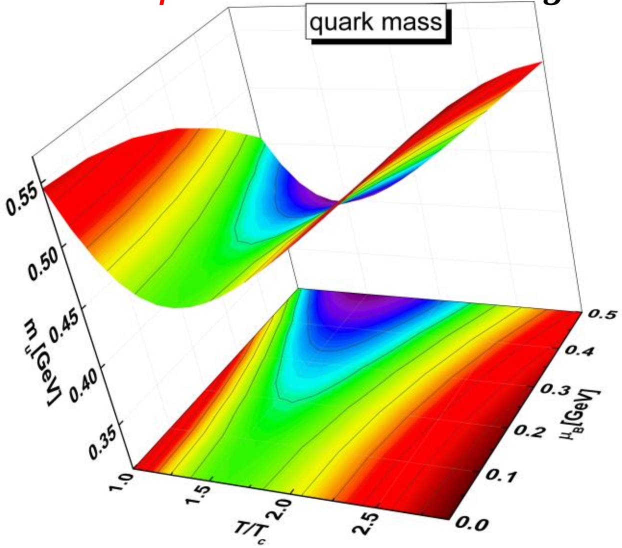
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

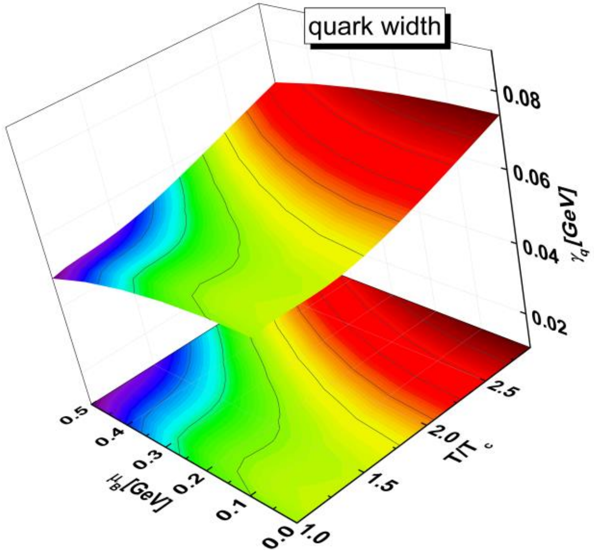
Dynamical QuasiParticle Model (DQPM)

- **Resummed properties of the quasiparticles are specified by scalar complex self-energies:**

gluon propagator: $\Delta^{-1} = P^2 - \Pi$ & quark propagator $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$ & quark self-energy: $\Sigma_q = M_q^2 - i2\,\gamma_q\omega$

- **Re Π, Σ_q : thermal mass (M_g, M_q) Im Π, Σ_q : interaction width (γ_g, γ_q)**





$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B)\left(T^2 + \frac{\mu_q^2}{\pi^2}\right) \qquad \gamma_{q,g}(T,\mu_B) = \frac{c_{A,F}}{3}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

- **Modeling of the quark/gluon masses and widths (inspired by HTL calculations)**

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM g^2: fixed within s(lQCD) at μ_B=0

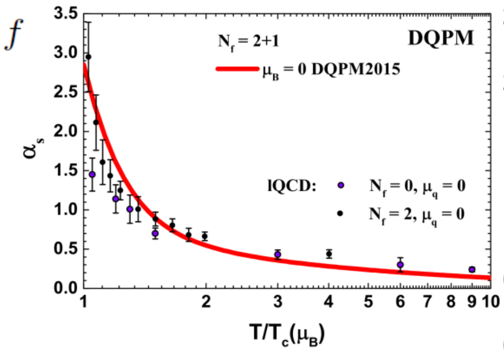
- Input: entropy density as a f(T , $\mu_B = 0$)

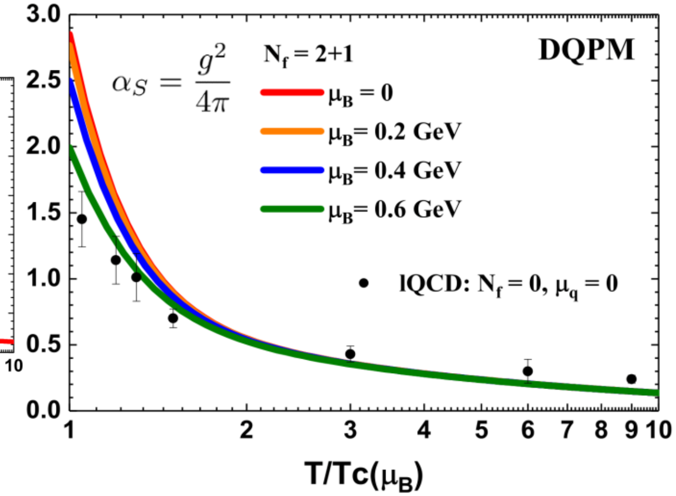
$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$
$$s_{SB}^{QCD} = 19/9\pi^2T^3$$
$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

fit S from QP to S from lQCD

↓

fix the model parameters

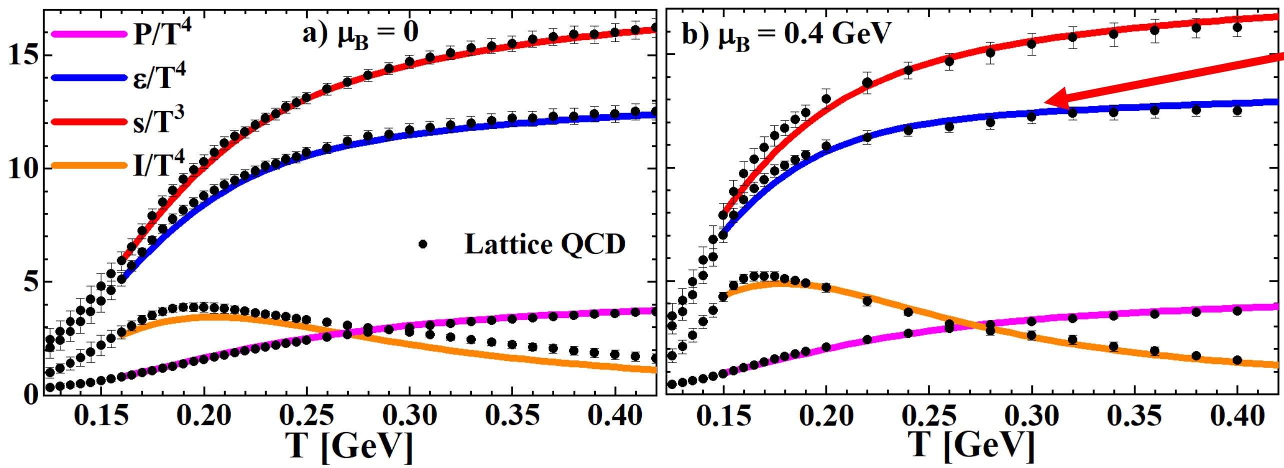




- Scaling hypothesis at finite $\mu_B \approx 3\mu_q$

$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right)$$ with the effective temperature $$T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$

Input:
lattice EoS
$\mu_B = 0$ (dots)

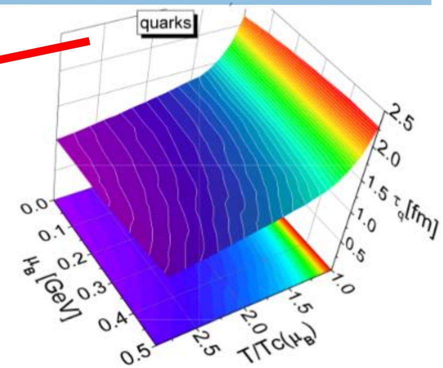


Output:
(lines)
DQPM EoS
$\mu_B > 0$

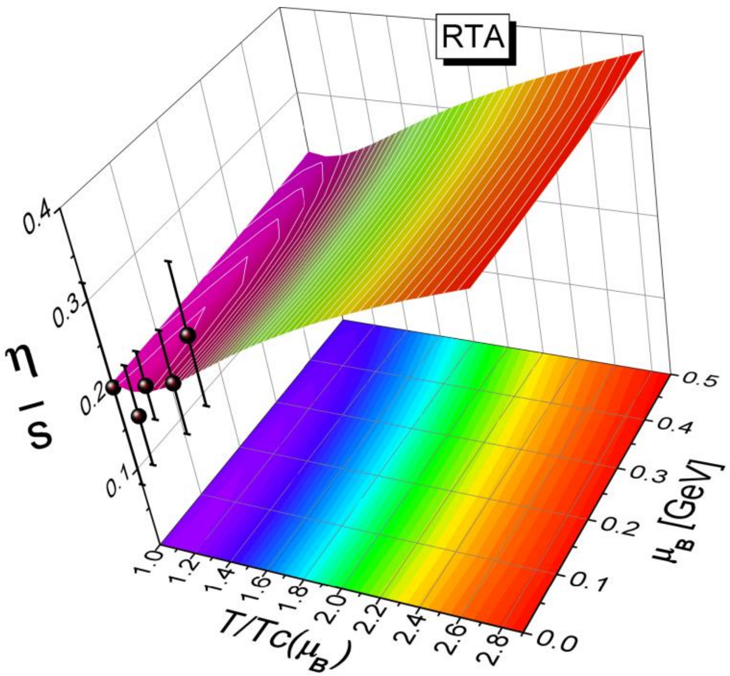
Transport coefficients: specific shear viscosity

Relaxation times

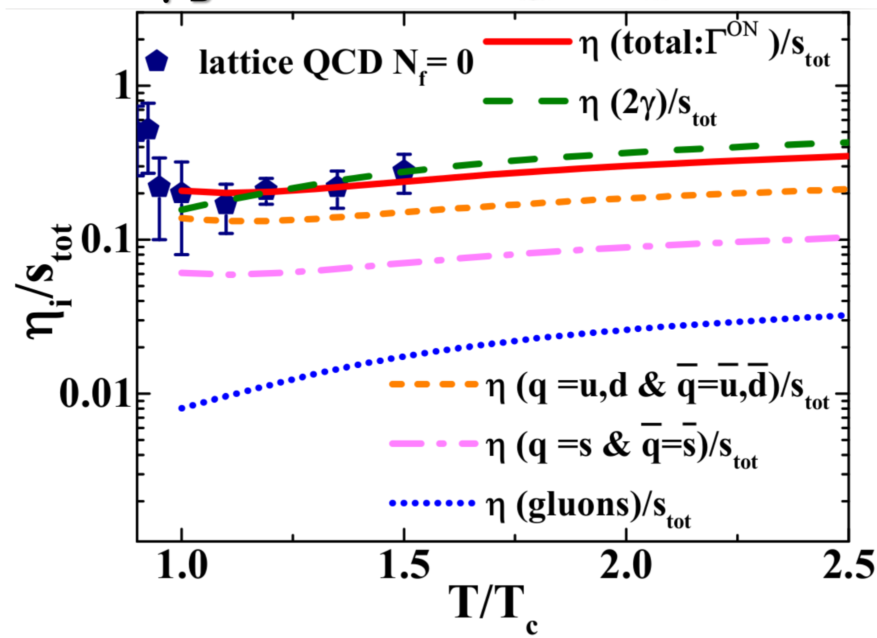
$$\eta^{\rm RTA}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B)\, d_i(1\pm f_i)f_i$$



μ_B =0



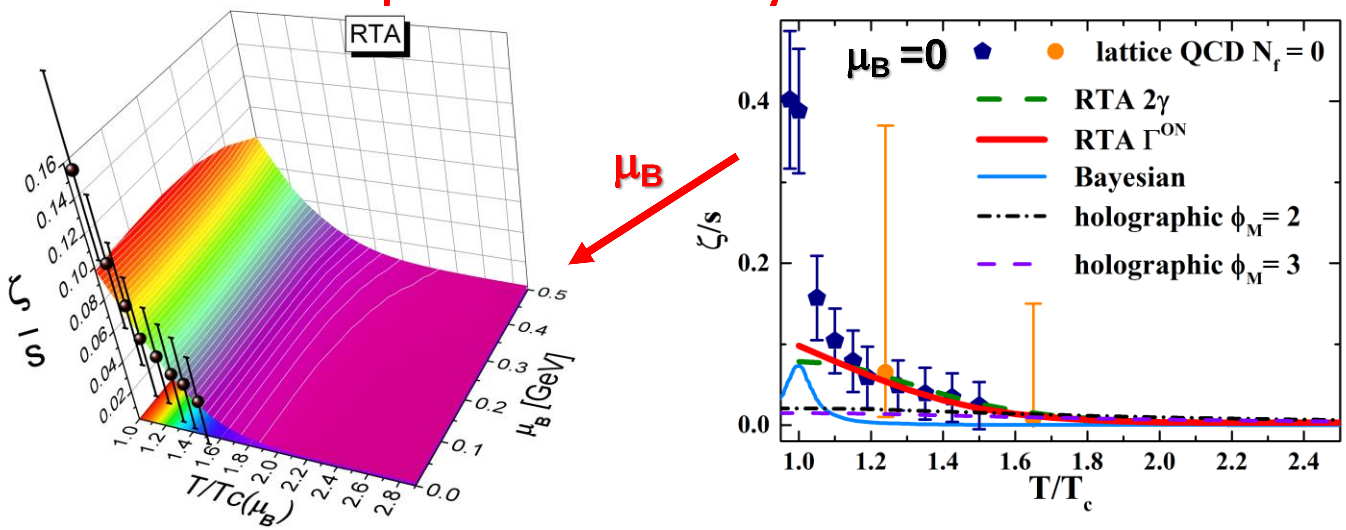
μ_B



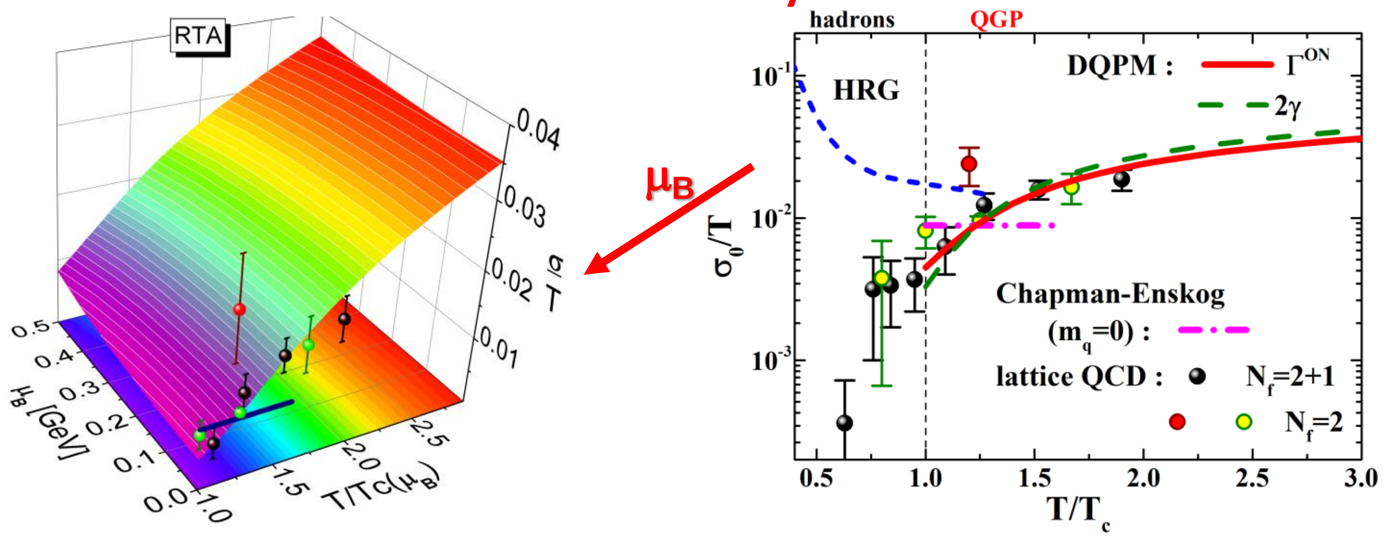
- Main contribution comes from light quarks and anti-quarks

Transport coefficients: increasing with μ_B

Specific bulk viscosity



Electric conductivity



Polyakov Nambu Jona-Lasinio (PNJL) model

- **Effective lagrangian with the same symmetries for the quark dof as QCD**

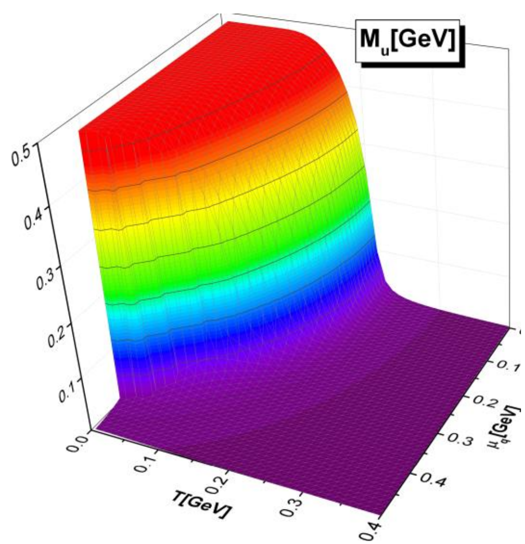
$$\mathscr{L}_{PNJL} = \sum_i \bar{\psi}_i (i\not{D} - m_{0i} + \mu_i \gamma_0)\psi_i$$
$$+ G \sum_a \sum_{ijkl} \left[(\bar{\psi}_i \; i\gamma_5 \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \; i\gamma_5 \tau^a_{kl} \psi_l) + (\bar{\psi}_i \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \tau^a_{kl} \psi_l) \right]$$
$$- K \det_{ij} \left[\bar{\psi}_i \; (-\gamma_5) \psi_j \right] - K \det_{ij} \left[\bar{\psi}_i \; (+\gamma_5) \psi_j \right]$$
$$- \mathcal{U}(T; \Phi, \bar{\Phi}) \; .$$

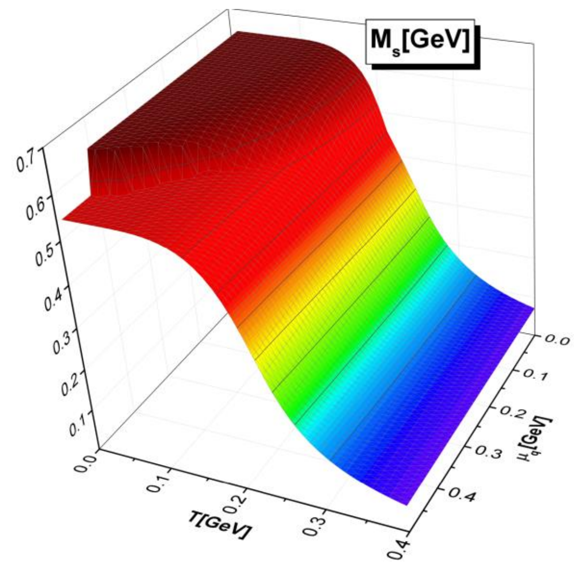
5 parameters fixed by vacuum values K,π masses, η-η'mass splitting , π decay constant, Chiral condensate

Polyakov potential fitted to the YM

J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205
D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203

- **1st order PT at high μ_B (sudden change of q and meson masses)**



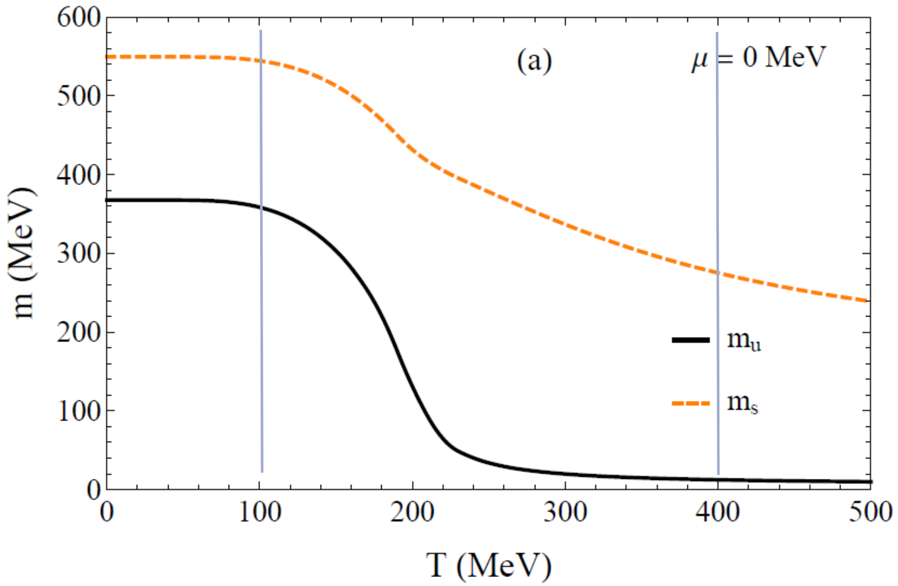


Quark masses NJL and PNJL

- Gap equation + minimization of the grand potential → Chiral masses (M_l, M_s)

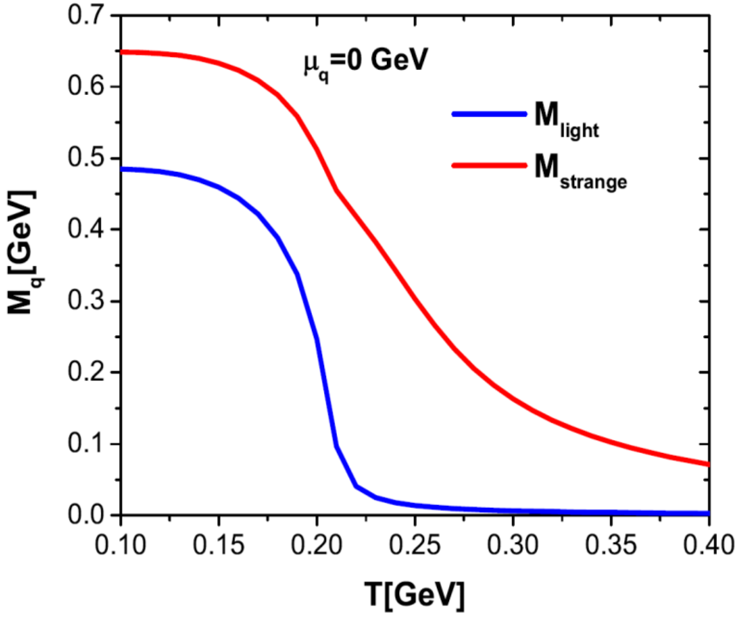
$$m_i = m_{0i} - 4G\langle\langle\bar{\psi}_i\psi_i\rangle\rangle + 2K\langle\langle\bar{\psi}_j\psi_j\rangle\rangle\langle\langle\bar{\psi}_k\psi_k\rangle\rangle$$

Chiral masses(NJL)



R. Marty et al. PRC 88 (2013) 4 045204

Chiral masses(PNJL)



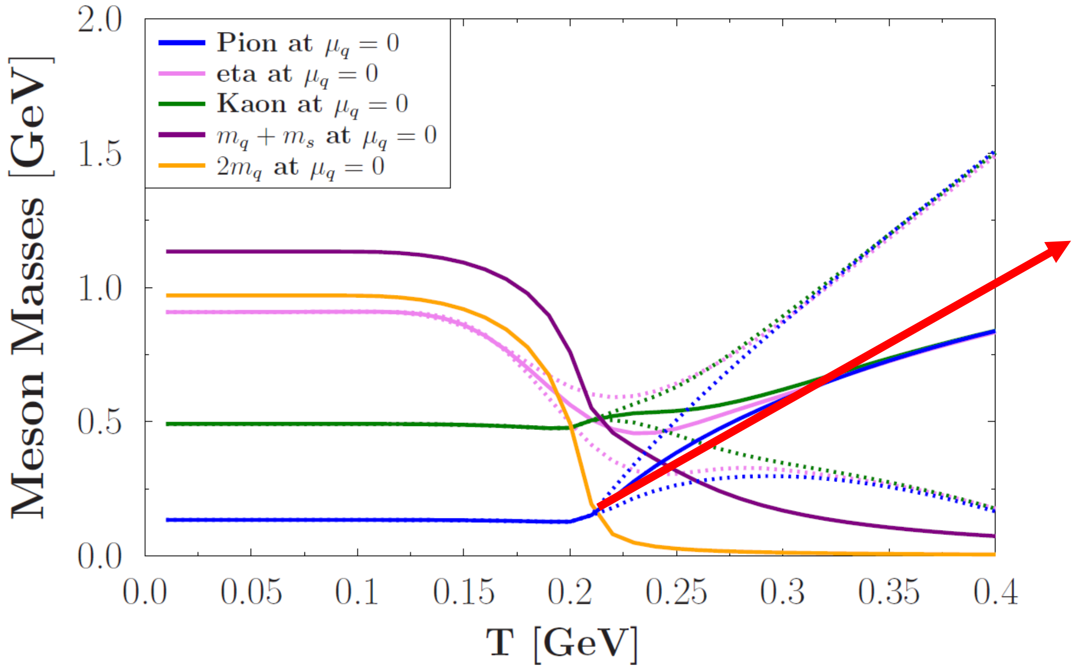
- in PNJL transition is steeper than in NJL

Mesons in PNJL

- **The meson pole mass and the width can be obtained by**

$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$

PNJL

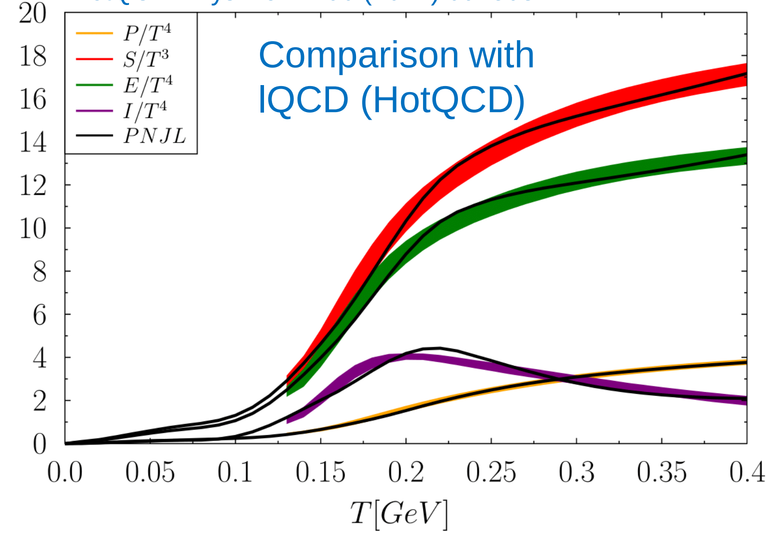


At T=0 good agreement with the physical masses
After T>T_{Mott} mesons become unstable

Polyakov Nambu Jona-Lasinio (PNJL)model: EOS

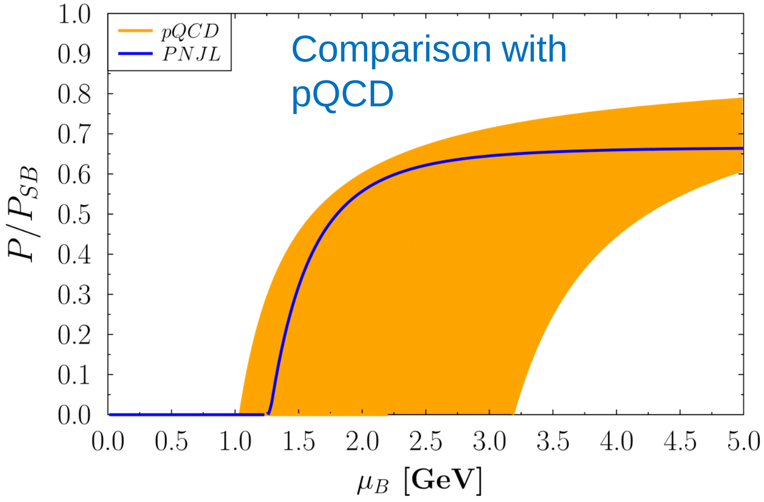
- **PNJL allow for predictions for finite T and μ_B:** D. Fuseau, T. Steinernert, J.Aichelin PRC 101 (2020) 6 065203
- **Parameters fixed, EoS at $\mu_B = 0$:**

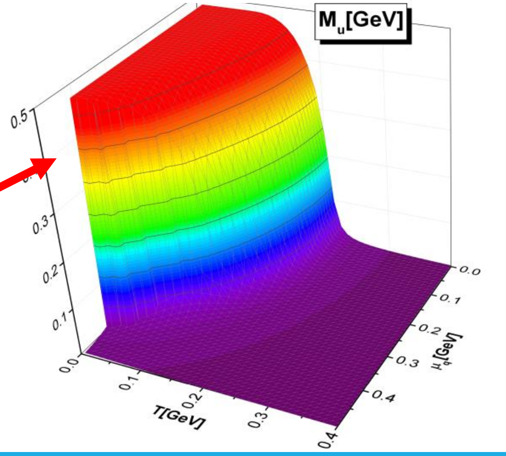
HotQCD Phys.Rev. D90 (2014) 094503



- **EoS at high μ_B:**

pQCD: A.Kurkela, A.Vuorinen, PRL 117 (2016)4 042501





- **CEP: (T , μ_B) = (110,960) MeV , μ_B/T = 8.73**
- **1st order PT at high μ_B (sudden change of q and**
- **meson masses)**

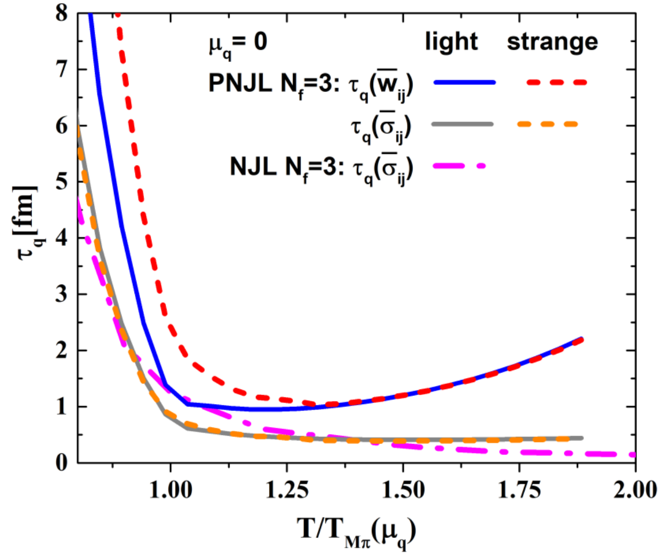
Relaxation time: PNJL

$$1)\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

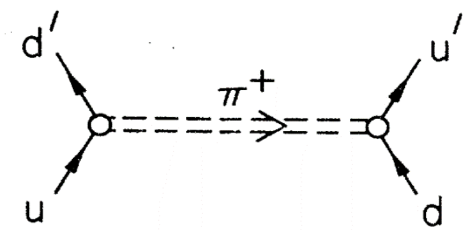
- **on-shell scattering (interaction) rates**

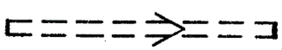
$$\Gamma_i^{\rm on}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i}\sum_{j=q,\bar{q},g}\int\frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\; f_j(E_j,T,\mu_q)$$
$$\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(1\pm f_3)(1\pm f_4)$$
$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\;(2\pi)^4\delta^{(4)}\left(p_i+p_j-p_3-p_4\right)$$

Relaxation times(PNJL)



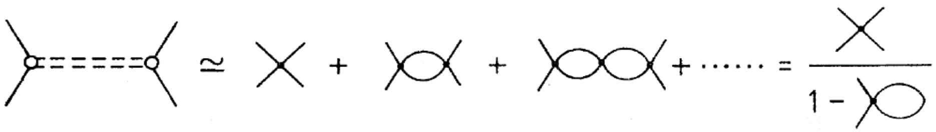
4 point interaction -> meson exchange



 $= (i\gamma_5)\tau^{(-)}\dfrac{-ig^2_{\pi qq}}{k^2 - m^2_\pi}(i\gamma_5)\tau^{(+)}$

meson propagator $\quad \mathscr{D} = \dfrac{2ig_m}{1 - 2g_m\Pi^{\pm}_{ff'}(k_0,\vec{k})}$

Effective interaction in RPA



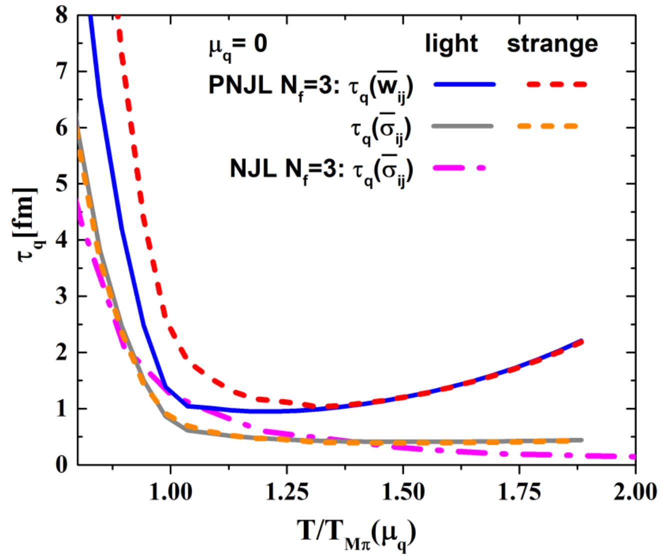
Relaxation time: PNJL

$$1)\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

- **on-shell scattering (interaction) rates**

$$\Gamma_i^{\rm on}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i}\sum_{j=q,\bar{q},g}\int\frac{d^3p_j}{(2\pi)^3 2E_j}\; d_j\, f_j(E_j,T,\mu_q)$$
$$\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(1\pm f_3)(1\pm f_4)$$
$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\;(2\pi)^4\delta^{(4)}(p_i+p_j-p_3-p_4)$$

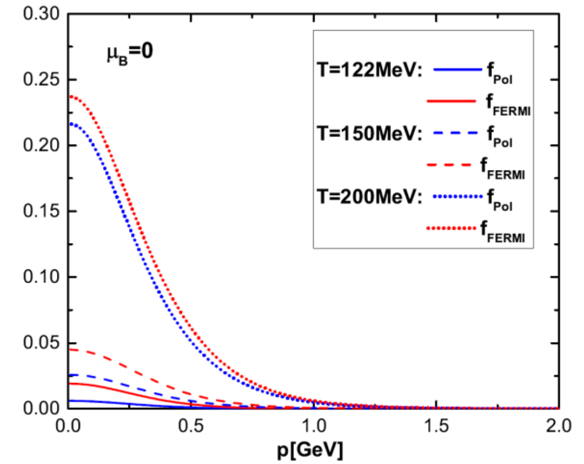
Relaxation times(PNJL)



Modified distribution function: Polyakov loop contributions

$$f_q \to f_q^{\Phi}(\mathbf{p},T,\mu) = \frac{(\bar{\Phi}+2\Phi e^{-(E_\mathbf{p}-\mu)/T})e^{-(E_\mathbf{p}-\mu)/T}+e^{-3(E_\mathbf{p}-\mu)/T}}{1+3(\bar{\Phi}+\Phi e^{-(E_\mathbf{p}-\mu)/T})e^{-(E_\mathbf{p}-\mu)/T}+e^{-3(E_\mathbf{p}-\mu)/T}},$$

$$f_{\bar{q}} \to f_{\bar{q}}^{\Phi}(\mathbf{p},T,\mu) = \frac{(\Phi+2\bar{\Phi} e^{-(E_\mathbf{p}+\mu)/T})e^{-(E_\mathbf{p}+\mu)/T}+e^{-3(E_\mathbf{p}+\mu)/T}}{1+3(\Phi+\bar{\Phi} e^{-(E_\mathbf{p}+\mu)/T})e^{-(E_\mathbf{p}+\mu)/T}+e^{-3(E_\mathbf{p}+\mu)/T}}.$$



Relaxation time: increases with μ_B

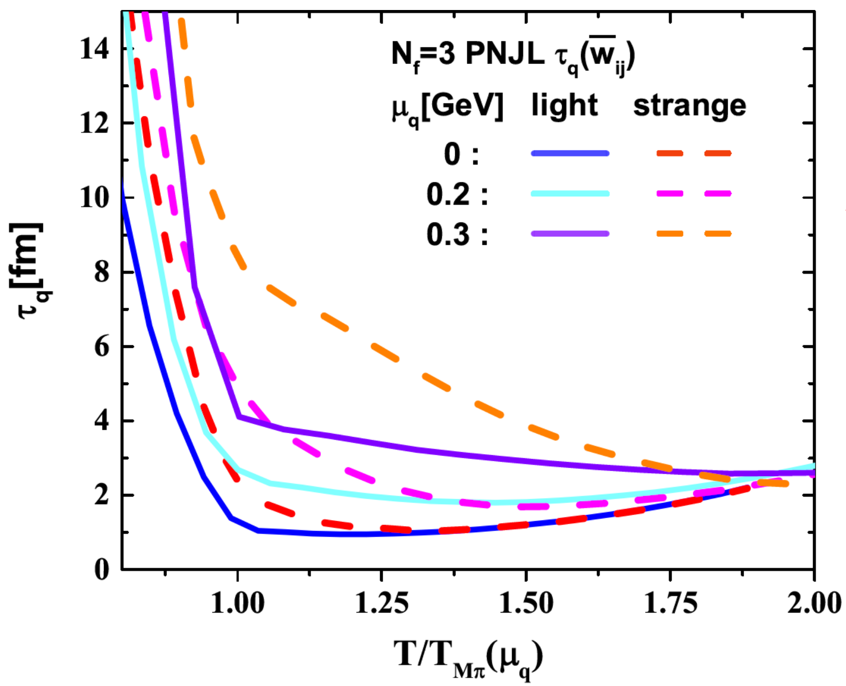
$$1)\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

$$\tau_i^{-1}(T,\mu_q) = \frac{1}{n_i(T,\mu_q)} \int \frac{\mathrm{d}^3 p_i}{(2\pi)^3} d_q f_i^{(0)} \tau_i^{-1}(p_i,T,\mu_q)$$

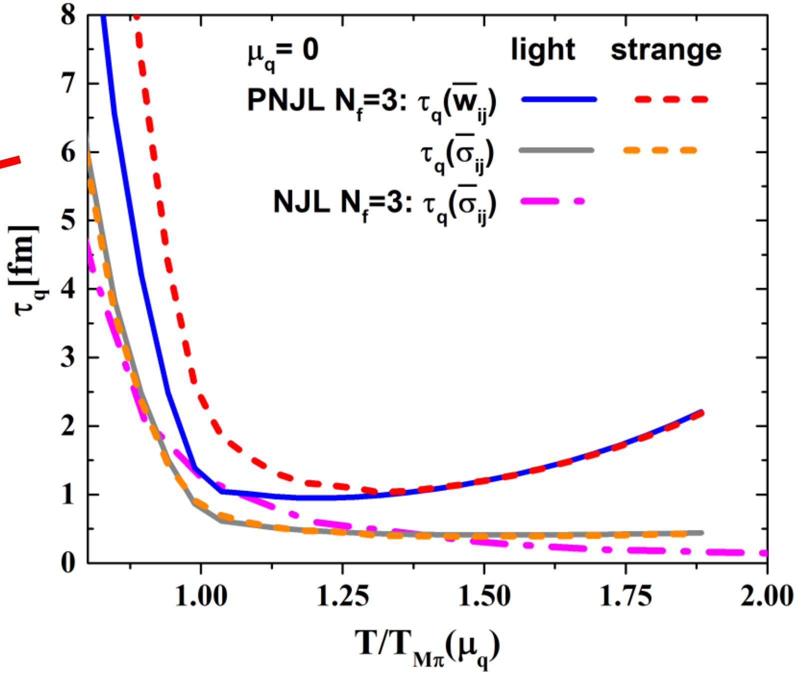
- **on-shell scattering (interaction) rates**

$$\Gamma_i^{\rm on}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\ f_j(E_j,T,\mu_q)$$

$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1\pm f_3)(1\pm f_4)\,|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\ (2\pi)^4 \delta^{(4)}\left(p_i+p_j-p_3-p_4\right)$$



$\mu_B = 3\mu_q$

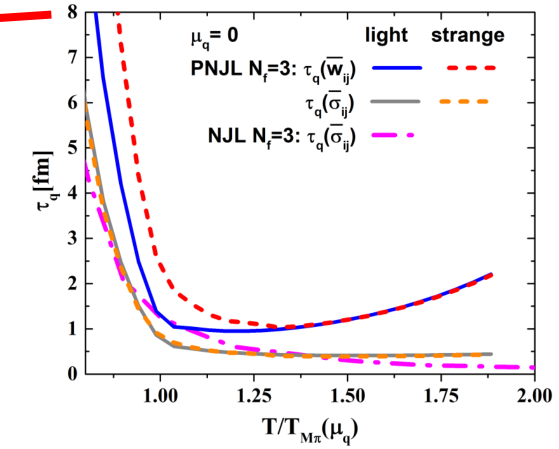


Specific shear viscosity at high μ_B

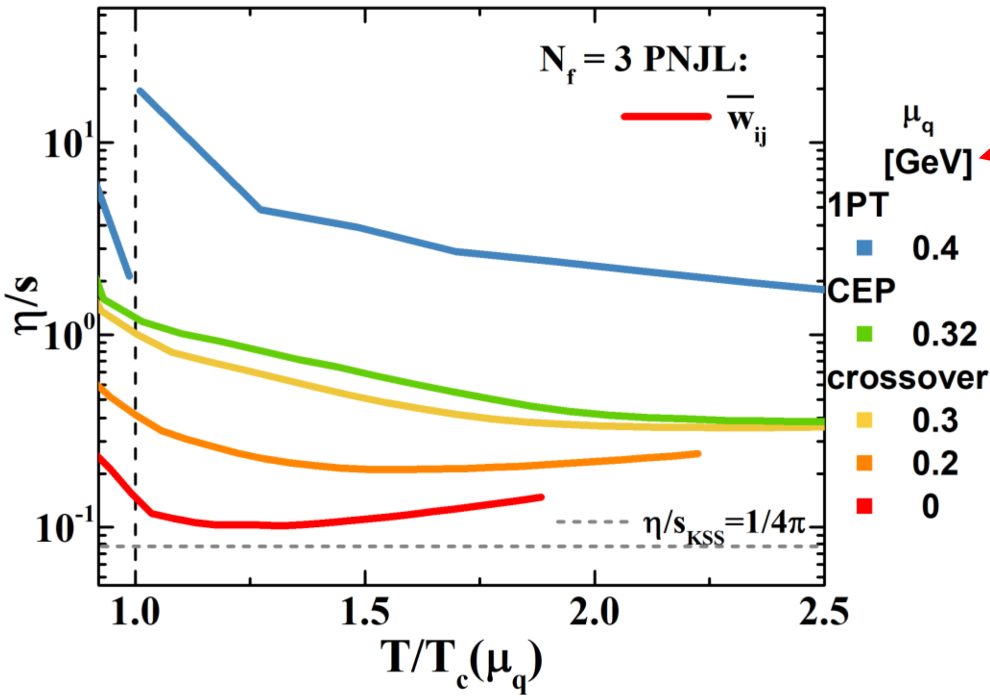
$$\eta^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B)\, d_q f_i^{\phi}$$

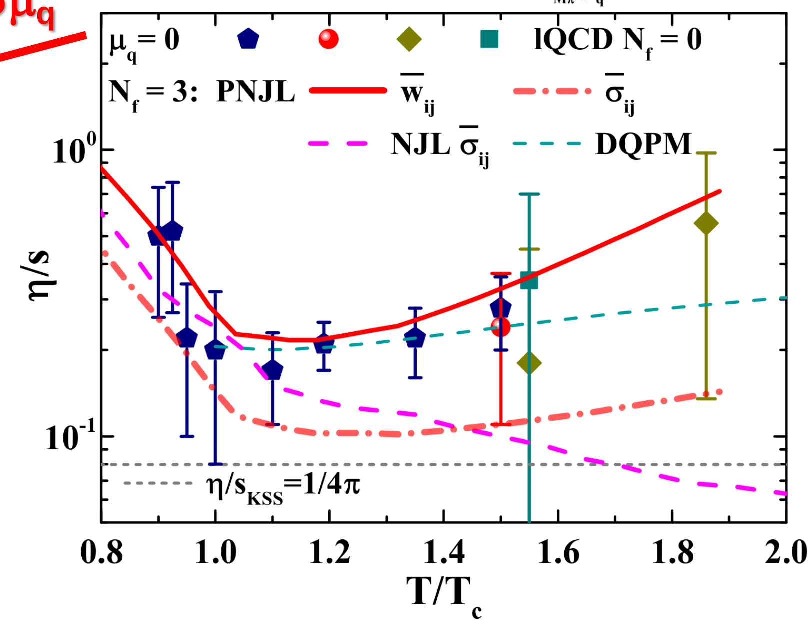
$$f_i^{\phi} = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1 + 3\phi e^{-(E_i\mp\mu)/T} + 3\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

with Polyakov loops



$\mu_B = 3\mu_q$





In agreement w Nf=2 NJL results C. Sasaki et al, NPA 832 (2010)

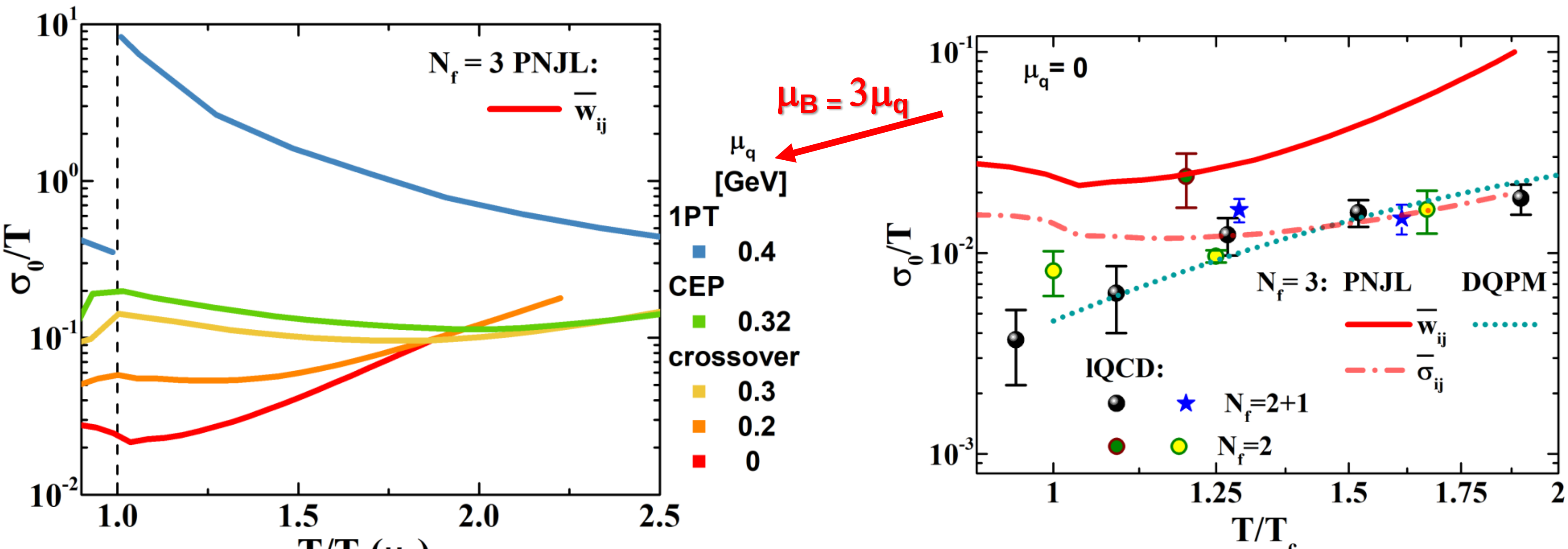
PNJL results:arXiv:2011.03505

Electric conductivity at high μ_B

$$\sigma_0^{\rm RTA}(T,\mu_B) = \frac{e^2}{3T}\sum_{i=q,\bar q} q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\,\tau_i(\mathbf{p},T,\mu_B)\, d_q f_i^\phi$$

$$f_i^\phi = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar\phi e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1+3\phi e^{-(E_i\mp\mu)/T} + 3\bar\phi e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

with Polyakov loops



PNJL results:arXiv:2011.03505

Summary / Outlook

- Transport coefficients at finite T and μ_B have been found using the (T, μ_B)-dependent cross sections in the DQPM and PNJL models
- At $\mu_B = 0$ good agreement with the Bayesian analysis estimations and lQCD estimations of QGP transport coefficients
- At large values of μ_B(1.2 GeV in this work) presence of the 1st order phase transition changes T dependence of transport coefficients drastically and a discontinuity can be seen approaching the Tc

- **Outlook:**
  - More precise EoS large μ_B
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Thank you for your attention!

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