COGSCI 108c: “Natural Computation”
Homework Assignment 6,
due: Monday 2002-05-20 (NOTE 2 weeks!)

Please show your work. Partial credit will be given for reasonable attempts — put down what you can. Start early and if you get stuck, please go to sections or office hours for hints/help.

Please check the course web page for information and announcements:
You can work in teams of up to three but everybody needs to hand in his own solution. The exercise parts marked with an asterisk are optional (for those of you who can handle an extra challenge).

Late assignments will lose 10% for the first week late and 50% after that.

This week’s reading: Chapter 7.

(0) (1/2 point) Using the correct formula for projection, do Chapter 4, Problem 1c) in the book (even if you did it correctly the first time).

(A) (2 points) Use the Method of Lagrange Multipliers to solve the following problem: Find the radius, \( r \), and height \( h \) of a cylinder that maximizes the Volume of a cylinder \( \pi * r^2 * h \), whose surface area \( (2 * \pi * r^2 + 2 * \pi * r * h) \) is \( 10\pi \) \( \text{cm}^2 \). Be sure to check that you have found a maximum!

(B) (2 points) Find the point on the line \( x + y = 1 \) that minimizes the function \( f(x, y) = 2x^2 + 3y^2 \) using the method of Lagrange multipliers. Be sure to check that you have found a minimum!

(C) (1 point) Consider the function

\[ \text{logsig}(x) = \frac{1}{1 + \exp(-x)} \]

Show that

\[ \text{logsig}'(x) = \text{logsig}(x)(1 - \text{logsig}(x)) \]

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(D) (continuation of C) Consider a simple (2 Input Units, 1 bias unit, 1 output unit) Neural Network as shown below that you will train in Matlab to learn the AND function.

\[ z = \text{logsig}(x_1 w_1 + x_2 w_2 + 1 w_0) \]

The training patterns are:

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<tr>
<th>i</th>
<th>x0</th>
<th>x1</th>
<th>x2</th>
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Start with \((w_0, w_1, w_2) = (.1, -.1, 2)\). Using the error function

\[ E(w_0, w_1, w_2) = 1/2 * 1/4 \sum_{i=1}^{4} (z_i - t_i)^2 \]

Run all 4 patterns through the network, compute the error and update the weights using the gradient descent rule (repeat 100 times). Use the matlab logsig function.

i) (1 points) Write down the gradient descent rule (use your answer to C)).

ii) (1.5 points) Print out your matlab code

iii) (.33 point) Plot the error function for the first 100 training cycles (1 cycle presents all 4 training patterns) for the case \( \eta = 20 \)

iv) (.33 point) Plot the error function for the first 5 training cycles (1 cycle presents all 4 training patterns) for the case \( \eta = 60 \)

v) (1 point) Explain the difference between the plots

(E*) How would you modify the routine to be less sensitive to \( \eta \)?