COGSCI 275: “Visual Modeling”
Homework 1
due: Friday 2005-01-21, at the beginning of class

Please check the course web page for information and announcements:
http://cloudbreak.ucsd.edu/~triesch/courses/275vision/.
Exercises marked with a star are voluntary. You can work in teams of two, i.e. there can be two names on
a single solution. Where appropriate, hand in all plots as well as a listing of your code, or send them via
email.

1. Dense Packing of Spheres

Complex eyes of insects typically have a hexagonal packing of their ommatidia. Consider the problem of
packing circles (ommatidia) into an infinite plane (the complex eye) in the densest possible fashion. More
specifically, calculate what fraction of the plane will be covered by the spheres if they are arranged (a) in a
square lattice fashion and (b) in a hexagonal lattice fashion. Compare the results.

Note: it turns out that for an infinite plane, the hexagonal packing is indeed the optimal pattern. This is
known as Thue’s theorem, after the Norwegian mathematician Axel Thue. The problem of packing spheres
into finite size areas is much harder.

2. Lambert Cosine Law

Consider a lambertian ellipsoid given by

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

where \( a \neq b \neq c \) (i.e., the ellipsoid is scalene) under parallel illumination. Derive what the ellipsoid would
look like from a particular direction, say the z-axis, under orthographic projection, under the assumption of
parallel light coming from a particular direction \( \hat{l} \). Use Matlab to plot an image of the ellipsoid.

Now change the direction of \( \hat{l} \), thereby generating many such images, such that the light source rotates around
the ellipsoid. Generate a movie with these images.

3. Autocorrelation, Redundancy in Natural Images

Natural images exhibit a high degree of redundancy. One way to see this is to consider the autocorrelation.
Consider the intensity of a pixel at location \((x, y)^T\) and that at neighboring location \((x + \Delta x, y)^T\). We can
view their intensities as random variables and calculate their correlation coefficient. Do this for the natural
image on the course web page by sampling 1000 such pixel pairs for a particular offset \( \Delta x \). Now repeat for
different values of \( \Delta x \). Plot a graph of the correlation coefficient as a function of \( \Delta x \). This is (essentially)
the autocorrelation function. Interpret the result. Are the brightness values of nearby pixels independent?
4. DoG Processing in the Retina: Redundancy Reduction

A simple linear filter model for retinal ganglion cells describes their output as the convolution of the image with a radially symmetric difference of gaussian (DoG) kernel:

\[ K(r) = \frac{1}{2\pi \sigma_c^2} e^{-r^2/2\sigma_c^2} - \frac{1}{2\pi \sigma_s^2} e^{-r^2/2\sigma_s^2} , \]

where \( r^2 = x^2 + y^2 \), \( \sigma_c \) and \( \sigma_s \) are the width of the center and surround parts, respectively (\( \sigma_c < \sigma_s \)). Apply such a filter to the natural image from the web page and display the result.

Now repeat the autocorrelation analysis from above for the processed image. Compare the result to the one above. Interpret the result.

General Hints:

Matlab can be quite efficient if used right. The most important thing to remember is to avoid programming with loops and use Matlab’s matrix operations wherever possible. Check the tutorials on the course web page for more details.

Have fun!