Color

Outline:
- visible light, photoreceptor space
- psychophysics of color matching
- color spaces
- color constancy
- color in object recognition
- some lightness & color illusions
Credits: major sources of material, including figures and slides were:

- Mallot, H.-P., Computational Vision
- and various resources on the WWW
Vision is really, really hard!

- forward problem (graphics): from world to images has unique solution
- inverse problem (vision): from images to description of the world does not have unique solution: infinitely many scenes can give rise to the same image
Visible Spectrum

spectral radiance: radiance emitted in a range of wavelength \([\lambda, \lambda + \delta\lambda]\)

\[L^\lambda (x, \theta, \varphi) \quad [\text{Wm}^{-3}\text{sr}^{-1}]\]
Black body radiators

- Construct a hot body with near-zero albedo (black body)
  - Easiest way to do this is to build a hollow metal object with a tiny hole in it, and look at the hole.
- The spectral power distribution of light leaving this object is a simple function of temperature

\[
E(\lambda) \propto \left( \frac{1}{\lambda^5} \right) \left( \frac{1}{\exp(hc/k\lambda T) - 1} \right)
\]

- This leads to the notion of color temperature --- the temperature of a black body that would look the same
Measurements of relative spectral power of sunlight, made by J. Parkkinen and P. Silfsten. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”. Mnemonic is “Richard of York got blisters in Venice”.

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Violet Indigo Blue Green Yellow Orange Red
Relative spectral power of two standard illuminant models --- D65 models sunlight, and illuminant A models incandescent lamps. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”.

Violet Indigo Blue Green Yellow Orange Red
Measurements of relative spectral power of four different artificial illuminants, made by H. Sugiura. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm.
Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E. Koivisto.
Helmholtz: three receptors

Vision as unconscious inference

Helmholtz
Figure 2  The relative sensitivities of the Short- (S), Middle- (M), and Long- (L) wavelength sensitive cones as a function of wavelength. Each curve was normalized to its maximum. The curves show the cones sensitivity profiles derived by Stockman & Sharpe (2000). The M- and L-cones’ sensitivities overlap to a large extent and include almost the entire visible spectrum.
3-dim. Receptor Space

- cone excitation/photon catch, assuming linearity

\[ e_s = \int \phi_s(\lambda)I(\lambda) d\lambda \]
\[ e_m = \int \phi_m(\lambda)I(\lambda) d\lambda \]
\[ e_l = \int \phi_l(\lambda)I(\lambda) d\lambda \]

- metamerism: different spectral distributions that lead to the same receptor activation appear identical
Figure 3  Excitations produced in L- and M-cones by various fruit objects. Measurements were taken in a small area of each fruit with a Photo Research PR 650 spectroradiometer. The spectra were then multiplied with the cone absorption functions and normalized to the maximum excitations in each cone type, which was caused by a clove of garlic (appearing white under daylight conditions).
Hering: opponent colors

[Diagram showing opponent colors: Red/Green, Blue/Yellow, Black/White receptors]
Figure 4  Isoluminant plane of the DKL color space proposed by Derrington et al. (1984). At the center is a neutral white. Along the L-M axis, the excitation of the L- and M-cones covary to keep their sum constant. Along the S axis, only the excitation of the S-cones varies. The thin lines show the location of various basic colors in this space, as determined by a color naming experiment. The DKL color space also includes a luminance (S+M+L, not shown in this figure) axis, going through the origin and perpendicular to the other two. Along the luminance axis, the excitation of the three cones varies in proportion. A light in DKL space is specified by its elevation, the angle between a vector joining it to the origin, and its projection onto the isoluminant plane, and by its azimuth, the angle between its projection on the isoluminant plane and the L-M axis.
Opponent channels and PCA

- Buchsbaum & Gottschalk (1983):
  - PCA on cone excitations yields the following first three PCs: bright/dark, red/green, blue/yellow
  - the opponent colors are a “natural” basis for representing the signal
Cortical Representation of Color

Figure 5 | Segregation and integration in V2. The graph shows the proportions of cells selective for colour, orientation, direction of motion and size in different cytochrome oxidase (CO) compartments (thick stripes, thin stripes and interstripes) of macaque monkey area V2. The data are from six studies. The heavy black lines represent the means across all six studies. Despite the different methods used in these studies, the results show remarkable agreement.
Psychophysical Color Space

- color matching experiments: “adjust intensities of the three primary sources to match the colors”
- spectrum of the primaries: $b_j(\lambda), j = 1, 2, 3$

Note: primaries must be chosen such that mixing of two of them can’t produce the third

3 stimulus values
Subtractive matching

- Some colors can’t be matched like this: instead, must write
  \[ M + a \, A = b \, B + c \, C \]
- This is **subtractive** matching.
- Interpret this as \((-a, b, c)\)
- Problem for building monitors: Choose R, G, B such that positive linear combinations match a large set of colors
Grassman's Laws

- if we mix two testlights, then mixing the matches will match the result

\[ a_{1\alpha}b_1 + a_{2\alpha}b_2 + a_{3\alpha}b_3 \leftrightarrow T^\alpha \text{ and } a_{1\beta}b_1 + a_{2\beta}b_2 + a_{3\beta}b_3 \leftrightarrow T^\beta \]

\[ \Rightarrow (a_{1\alpha} + a_{1\beta})b_1 + (a_{2\alpha} + a_{2\beta})b_2 + (a_{3\alpha} + a_{3\beta})b_3 \leftrightarrow T^\alpha + T^\beta \]

- lights that are matched by the same set of weights match each other

\[ a_{1}b_1 + a_{2}b_2 + a_{3}b_3 \leftrightarrow T^\alpha \text{ and } a_{1}b_1 + a_{2}b_2 + a_{3}b_3 \leftrightarrow T^\beta \Rightarrow T^\alpha \leftrightarrow T^\beta \]

- matching is linear: (k>0)

\[ a_{1}b_1 + a_{2}b_2 + a_{3}b_3 \leftrightarrow T^\alpha \Rightarrow (ka_{1})b_1 + (ka_{2})b_2 + (ka_{3})b_3 \leftrightarrow kT^\alpha \]
spectral tri-stimulus values / color-matching function

• Question: how do you have to mix the three primaries to obtain the color perception produced by monochromatic light of wavelength lambda?

• If you know how to mix the primaries to obtain the color of light of lambda, under linear assumptions you can mix the primaries to obtain the color of any spectrum by decomposing it into its constituent wavelength:

\[ a_i = \int \bar{b}_i(\lambda)I(\lambda)d\lambda \]
Linear color spaces

- A choice of primaries yields a linear color space --- the coordinates of a color are given by the weights of the primaries used to match it.
- Choice of primaries is equivalent to choice of color space.

- **RGB**: primaries are monochromatic energies are 645.2nm, 526.3nm, 444.4nm.
- **CIE XYZ**: Primaries are imaginary, but have other convenient properties. Color coordinates are \((X,Y,Z)\), where \(X\) is the amount of the \(X\) primary, etc.
  - Usually draw \(x, y\), where
    \[
    x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z}
    \]
RGB: primaries are monochromatic, energies are 645.2nm, 526.3nm, 444.4nm. Color matching functions have negative parts $\rightarrow$ some colors can be matched only subtractively.
CIE XYZ: Color matching functions are positive everywhere, but primaries are imaginary. Usually draw $x$, $y$, where $x = X/(X+Y+Z)$ and $y = Y/(X+Y+Z)$. 
A qualitative rendering of the CIE (x,y) space. The blobby region represents visible colors. There are sets of (x, y) coordinates that don’t represent real colors, because the primaries are not real lights (so that the color matching functions could be positive everywhere).
A plot of the CIE (x,y) space. We show the spectral locus (the colors of monochromatic lights) and the black-body locus (the colors of heated black-bodies). I have also plotted the range of typical incandescent lighting.
HSV Color Space
## Human Color Discrimination

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Hue</th>
<th>Saturation</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perceivable steps</td>
<td>ca. 200</td>
<td>ca. 20</td>
<td>ca. 500</td>
</tr>
</tbody>
</table>
Land's Famous Experiment

Photometer reading (1, .3, .3)

Audience name "Red"

White light

Coloured light

Photometer reading (1, .3, .3)

Audience name "Blue"
Lightness Constancy

- Lightness constancy
  - how light is the surface, independent of the brightness of the illuminant
- issues
  - spatial variation in illumination
  - absolute standard
- Human lightness constancy is very good
- Assume
  - frontal 1D “Surface”
  - slowly varying illumination
  - quickly varying surface reflectance
Horn's version of “Retinex”
Thresholded $\frac{d\log p}{dx}$

Integrate This to get
Lightness Constancy in 2D

- Differentiation, thresholding are easy
  - integration isn’t
  - problem - gradient field may no longer be a gradient field
- One solution
  - Choose the function whose gradient is “most like” thresholded gradient
- This yields a minimization problem
- How do we choose the constant of integration?
  - average lightness is grey
  - lightest object is white
  - ?
Figure 3 | Colour constancy and induction. Eight squares were cut from coloured papers and were illuminated with a greenish light to the left and a reddish light to the right. The mechanisms of colour constancy make the eight squares on the left seem identical to the eight squares on the right. In fact, the four squares in column 1 were printed with the same ink as the four squares in column 4. They appear different because they are printed on a slightly different background. The change in background was due to the illumination change from green to red light. The four squares in the first column appear similar to those in the third column because the local colour difference to the background is identical for these two columns (1 and 3), as it is for columns 2 and 4.
- The received light depends on both the spectral albedo of the surface and the spectral radiance of the illuminant.

\[ \int_{\lambda} \sigma(\lambda) \rho(\lambda) E(\lambda) d\lambda \]

**Incoming spectral radiance**

\[ E(\lambda) \]

**Spectral albedo**

\[ \rho(\lambda) \]

**Outgoing spectral radiance**

\[ E(\lambda) \rho(\lambda) \]

**Receptor response of k\(^{th}\) receptor class**
Notice how the color of light at the camera varies with the illuminant color; here we have a uniform reflectance illuminated by five different lights, and the result plotted on CIE x,y.
Notice how the color of light at the camera varies with the illuminant color; here we have the blue flower illuminated by five different lights, and the result plotted on CIE x,y. Notice how it looks significantly more saturated under some lights.
Notice how the color of light at the camera varies with the illuminant color; here we have a green leaf illuminated by five different lights, and the result plotted on CIE x,y.
Problem Statement

- You want to know the reflectances of the objects, but all you observe is the light reflected, which depends on the reflectances and the spectrum of the light source(s).
- The simplest case: flat scene (no shading), uniform illumination, lambertian objects.
- Even in this case the problem is underconstrained: more unknowns than equations to estimate them.
- Example: consider white surface under red light vs. red surface under white light.
- Need to introduce additional assumptions.
- Different assumptions give rise to different approaches/algorithms.
Some Assumptions

• there are surfaces of known reflectance in the environment (carry a color sample with you)
• the brightest patch in the image is white
• find specularities, use them to estimate spectrum of light source
• the average color of the scene is known
• it is known which lighting can give rise to what colors
Grey World Assumption I

- Consider making a scatter plot of the colors of all the pixels in the current scene
- The grey world assumptions says that the bulk of the distribution should be centered around the black/white axis for a normal (white) illumination

- **Idea**: if it doesn’t fall there, find a transform that brings it there; hope for the best

- For example: extract first PC, shift and rotate cloud
Grey World Assumption II

• **Problem:** the assumption is often violated, consider looking at a green meadow: you don’t want all the nice green to turn to grey!

• **One approach:** don’t consider full distribution but only which colors are present at all, i.e. a small patch of red somewhere in the scene gets the same “weight” as many pixels of a particular shade of green.

• **But still:** you need a good sampling of maybe 10 or so different surface colors to get good results

• **Idea:** accumulate these across time, make adaptation slow (time scale of minutes)
Normalizing the Gamut

- Gamut: convex hull of pixel colors in an image
- Assume you know the possible gamuts of scenes under normal (white) illumination
- Given a scene, look at its gamut. Under varying illumination it will not lie within the standard gamut
- Find a transform that brings the convex hull of the transformed colors back in the standard
A unifying correlational approach

Finlayson, Hordley, Hubel (2001):

- Basic approach:
  - discretize chromaticity space into N x N bins (ignore brightness)
  - make probabilistic model of which colors can show up under which of a finite set of illuminations, this gives a "correlation matrix" $W$
  - for new scene: collect all the colors showing up
  - find light source that is most likely to have generated this combination of colors
Building the correlation matrix $W$

NxN chromaticity space (2d)

illuminant

Fig. 1. Three steps in building a correlation matrix. (a) We first characterize which image colors (chromaticities) are possible under each of our reference illuminants. (b) We use this information to build a probability distribution for each light. (c) Finally, we encode these distributions in the columns of our matrix.
To estimate illuminant in a scene:

- first, calculate histogram of colors present in image

\[ N^2 \times 1 \text{ vector } h: \]

\[ h(x_i * y_i) = \frac{\text{count}_i}{N_{\text{pix}}} \]

\[ \text{count}_i = \sum_{j=1}^{N_{\text{pix}}} c_j, \quad c_j = \begin{cases} 
1 & \text{if } C_{im}(j) = (x_i, y_i) \\
0 & \text{otherwise.} \end{cases} \]

- thresholding: which color is present at all

\[ \text{thresh}(x) = \begin{cases} 
1, & \text{if } x > 0 \\
0, & \text{otherwise.} \end{cases} \]

\[ \text{thresh}([h_1, h_2, \ldots, h_N]^t) = [\text{thresh}(h_1), \text{thresh}(h_2), \ldots, \text{thresh}(h_N)]^t. \]

With these definitions, \( v \) can be expressed:

\[ v = \text{thresh}(\text{chist}(C_{im})). \]
• calculate “likelihood” of each light source w/ dot product

\[ l = v^t M = \text{thresh}(\text{chist}(C_{im}))^t M \]

• estimated light source is the one maximizing likelihood

\[ \hat{c}_E = \text{thresh}2(\text{thresh}(\text{chist}(C_{im}))^t M)C_{ill}, \]

\[ N_{ill \times 2} \text{ matrix of illuminant chromaticities} \]

\[ \text{thresh}2(h) = h' \quad h'_i = \begin{cases} 1, & \text{if } h_i = \max(h) \\ 0, & \text{otherwise.} \end{cases} \]
Fig. 2. Solving for color constancy in three stages. (a) Histogram the chromaticities in the image. (b) Correlate this image vector $\mathbf{v}$ with each column of the correlation matrix. (c) This information is used to find an estimate of the unknown illuminant, for example, the illuminant which is most correlated with the image data.

$$l = \mathbf{v}^t M = \text{thresh}(\text{chist}(C_{im}))^t M$$
• When is this justified?
• Assume you know $Pr(c|E)$.

• Then use Bayes rule:

$$Pr(E|c) = \frac{Pr(c|E)Pr(E)}{Pr(c)}.$$

$$Pr(E|C_{im}) = \frac{Pr(C_{im}|E)Pr(E)}{Pr(C_{im})}.$$

• Assume chromaticities independent, ignore denominator

$$Pr(E|C_{im}) = \left[ \prod_{c \in C_{im}} Pr(c|E) \right] Pr(E).$$

• If we assume illuminants are equally likely:

$$Pr(E|C_{im}) = k \prod_{c \in C_{im}} Pr(c|E),$$
• Now define log likelihood:

\[ l(E|C_{im}) = \sum_{c \in C_{im}} \log(Pr(c|E)) \]

• Finally, define:

\[ M_{Bayes} \text{ whose } ij \text{th entry is:} \]

\[ \log(Pr(\text{image chromaticity } i|\text{illuminant } j)). \]

It follows that the correlation vector \( \mathbf{l} \) defined in (7) becomes the log-likelihood \( l(E|C_{im}) \):

\[ l(E|C_{im}) = \text{thresh}(\text{chist}(C_{im}))^t M_{Bayes} \]  \hspace{1cm} (15)

and our estimate of the scene illuminant can be written:

\[ \hat{c}^E = \text{thresh2}(\text{thresh}(\text{chist}(C_{im}))^t M_{Bayes})C_{ill}. \]  \hspace{1cm} (16)

• Note: we have likelihood of each illuminant, not just max.
Notes:

• this method is very simple…
• … but it generalizes a number of other methods
• e.g., for $M = I$, we obtain a grey world method:

\[ p^E = \text{mean}(RGB_{im}). \]

• can be written as:

\[ p^E = \text{hist}(RGB_{im})^t \mathbb{I} RGB_{ill}, \]

• every pixel votes for its color being the ill. color
• thresholded version (large surfaces don’t dominate):

\[ p^E = \text{thresh(hist}(RGB_{im})^t \mathbb{I} RGB_{ill}. \]
Notes, cont’d:

- e.g., for $M_{For}(ij)$ in (0,1) such that $M_{For}(ij) = 1$ iff color $i$ can be observed under illumination $j$, we get a gamut mapping technique

  - likelihood now is:
    $$l = \text{thresh}(\text{hist}(RGB_{im})^t)M_{For},$$

  - many surfaces will have same (maximal, $N_{surf}$) correlation, just take the mean of all of those by calculating
    $$\hat{p}^E = \text{thresh2}(\text{thresh}(\text{hist}(RGB_{im})^t M_{For})RGB_{ill},$$

- can also do a 2d version of Gamut mapping
- also relations to other methods
Performance and Examples

![Graph showing performance and examples]
Fig. 4. Left to right: raw camera image, correction based on measured illuminant, Gray-World, and Color by Correlation. Images were taken under daylight D50 (top) and simulated D65 (bottom).
Fig. 5. Left to right: raw camera image, correction based on measured illuminant, 2D Gamut Mapping, and Color by Correlation. Images were taken under Illuminant A (top) and cool white fluorescent (bottom).
Color in Object Recognition

Human Vision:
- color improves object recognition and scene memory
- object recognition influences color perception: banana-shaped objects look slightly more yellow (?)
- synesthesia, e.g. color-grapheme

Computer Vision:
- for constant illumination: color very helpful, in fact some researchers try to recognize objects only with color!
- for varying illumination: even without color constancy mechanisms color can be a helpful additional cue
Derain
Example: Cluttered Scenes

Triesch & Eckes (1998)

- 11 objects, 70 scenes with 190 placed objects,
- 50% with complex backgrounds,
- object models have Gabor and color features which
- matching is performed in stereo image pairs which gives disparity of object

Jochen Triesch, UC San Diego, http://cogsci.ucsd.edu/~triesch
Algorithm for Scene Analysis:

1. Initialization: mark all image areas as \textit{free}, then match all object models.

2. Find \textit{candidate} objects. IF set of candidates is empty THEN END.

3. Determine closest candidate object.

4. Accept or reject closest candidate object.

5. If accepted, mark corresponding image region as \textit{occupied} and update node visibilities of all remaining object models and re-compute their match similarities.

6. Go to step 2.
Example results:

Extension with color features and matching in stereo image pairs drastically improves performance
Example: Hand Postures

Triesch & von der Malsburg (2001)

Why hand postures?
- sign language
- gestures (HCI)
- action recognition

very challenging: complex backgrounds, strong deformations
Graphs use texture, color, and color-texture features types.

* train on 72 images, test on 942 images
* Result: error rates halved by integrating color:
  * simple background: 93% vs. 83%
  * complex background: 86% vs. 70%
A Robot Vision System
Lightness & Color Illusions

Hermann grid:

foveal representation

extra-foveal representation
“double” illusion: illusory circle composed of illusory bright patches at the intersections
Mach bands:
Discounting slowly varying change: Craik-O’Brien-Cornsweet illusion
Discounting slowly varying change: Craik-O’Brian-Cornsweet illusion
Context effects:
Boynton illusion:
Bezold illusion & color assimilation: