Fission Phenomena

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OUTLINE

- Historical milestones
- Cell division
- Rayleigh’s instability
- Liquid drop model
- Macroscopic-microscopic method
- Single particle models, shell and pairing corrections
- Unified approach of cold fission, $\alpha$-decay and heavy ion radioactivities
- Applications: radioactive beams; power reactors, etc
Historical milestones

- 1939 Induced fission. Liquid drop model
- 1940 Spontaneous fission
- 1946 $\alpha$ accompanied (ternary) fission
- 1962 Fissioning shape isomers
- 1967 Macroscopic-microscopic method
- 1980 Prediction of heavy particle radioactivities
- 1981 Cold fission
- 1984 Cluster radioactivity experimentally confirmed
- 1989 Fine structure in $^{14}$C decay
- 1998 $\alpha$ and $^{10}$Be accompanied (ternary) cold fission
Discovery of nuclear fission (1939)

Induced fission: Otto Hahn (Nobel prize 1944) born in Frankfurt, Lise Meitner and Fritz Strassmann — E. Fermi Award 1966. Otto Frisch, L. Meitner’s nephew, borrowed the name FISSION from biology (cell division).

Spontaneous fission (1940): G. N. Flerov and K. A. Petrzhak
Cell division — binary fission (I)

Reproduction of cells. The circular DNA molecule is replicated; the cell splits into two identical cells, each containing an exact copy of the original cell’s DNA. Red blood cells divide at a rate of 2.5 million per second. Nerve cells lose their capability to divide once they reach maturity. Cancer cells undergo rapid divisions (the daughter divides before reaching functional maturity).

Prokaryote (usually 0.5 – 2μm) is a type of cell that lacks a membrane-bound nucleus and has no membrane organelles; a bacterium.

Figs. 1, 2 Rod-Shaped Bacterium (www.DennisKunkel.com)
Nobel Prize in Physiology or Medicine 2001: Leland Hartwell, Tim Hunt and Paul Nurse for their discoveries of “key regulators of the cell cycle”.

Eukaryotic cells have a nucleus. Few cells reproduce by budding (e.g. yeast), where the daughter cell grows out of the parent and gradually increases in size. Yeast are utilized to ferment the sugars of rice, wheat, barley, and corn to produce alcoholic beverages and in the baking industry to expand dough. Yeast is also used as a model for cell division.
Rayleigh’s instability

John William Strutt, Lord Rayleigh (1842–1919) one of the few members of higher nobility who won fame as a scientist. His book, Theory of Sound, was published in two volumes in 1878. Nobel Prize in 1904 (discovery of argon).

The critical value of the ratio length/width is about 4.5 or \( \frac{2l}{r} \approx 9 \).

Rayleigh studied the stability of an infinite jet of fluid. The capillarity instability appears when, for large \( \lambda = 2l \), the decrease of energy due to volume conservation dominates that due to the increase of surface energy.

On receiving the Order of Merit in 1902 he said: “the only merit of which I personally am conscious was that of having pleased myself by my studies, and any results that may be due to my researches were owing to the fact that it has been a pleasure for me to become a physicist.”
N. Bohr developed nuclear liquid drop model inspired by Rayleigh’s works.

Nuclear scission: U. Brosa, S. Grossmann and A. Müller,


Nanophysics: C. Bréchignac et al.,


Fragmentation occurs beyond the 4.5 critical value.

Dendritic (fractal) shape by deposition of silver clusters on graphite.
Collective coordinates: separation distance of the fragments, neck radius, mass and charge asymmetry, deformation of each fragments, etc.

Expansion in terms of spherical harmonic, $Y_{\lambda\mu}$, or Legendre polynomial, $P_m$.

A point on the surface

$$R(\theta, \varphi) = R_0 \left[ 1 + \alpha_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) \right]$$

$R_0 = r_0 A^{1/3}$. $\alpha_{00}$ determined from volume conservation $V = (4/3)\pi R_0^3$.

Radius is real: $(\alpha_{\lambda\mu})^* = (-)^{\mu} \alpha_{\lambda - \mu}$. $\lambda = 2$ quadrupole deformation.

$\lambda = 3$ octupole deformation. $\lambda = 4$ hexadecapole deformation.
Nuclear shape parametrization (II)

For axial symmetry along $z$ all $\alpha_{\lambda \mu} = 0$. $\beta_{\lambda} \equiv \alpha_{\lambda 0}$. Legendre polynomials.

$$R(\theta) = \frac{R_0}{\lambda} \left[ 1 + \sum_{1}^{N} \alpha_n P_n(\cos \theta) \right]$$

$\lambda$ determined from the volume conservation.

Reflection symmetric shapes: deformation parameters $\alpha_n$ are all even.

S. Cohen & W. J. Swiatecki, Ann. Phys. (NY) 22 (1963) 406, have used 9 deformation parameters with $n = 2, 4, 6, \ldots 18$ to describe the saddle-point shapes. This parametrization can not be employed for separate fission fragments. For that purpose one can use two-center like shapes (intersected spheres or ellipsoidal shapes, etc).
Spheroidal deformation

Lengths in units of \( R_0 = 1.2249A^{1/3} \text{ fm.} \)

Vol. conserv. \( \omega_z^2 \omega_z = (\omega_0^0)^3 \hbar \omega_0^0 = 41A^{-1/3} \text{ MeV} \) \( \hbar^2/M \approx 41.5 \text{ MeV} \cdot \text{fm}^2 \)

Quadrupolar deformation: \( \varepsilon = \frac{3(c-a)}{2c+a} \). Harmonic oscill. freq.

\[
\omega_{\perp}(\varepsilon) = \omega_0 \left(1 + \frac{\varepsilon}{3}\right) ; \quad \omega_z(\varepsilon) = \omega_0 \left(1 - \frac{2\varepsilon}{3}\right).
\]

S. G. Nilsson 1955
Two intersected spheres. Volume conservation and \( R_2 = \text{const.} \). One deformation parameter: separation distance \( R \). Surface equation \( \rho = \rho(z) \). Initial \( R_i = R_0 - R_2 \). Touching point \( R_t = R_1 + R_2 \).

Example: \(^{232}\text{U} \rightarrow ^{14}\text{C} + ^{208}\text{Pb}\)

Two center shell model potential (right)

Sequence of shapes
Liquid drop model

Nucleus considered a uniformly charged drop. Two variants: Myers-Swiatecki (LDM) and Yukawa-plus-exponential (Y+EM). LDM (surface + Coulomb) deformation energy

\[ E_{LDM} = E - E^0 = (E_{s} - E_{s}^0) + (E_{C} - E_{C}^0) \]

\[ = E_{s}^0(B_s - 1) + E_{C}^0(B_C - 1) \]

For spherical shapes \( E_{s}^0 = a_{s}(1 - \kappa I^2)A^{2/3} \); \( I = (N - Z)/A \);
\( E_{C}^0 = a_{c}Z^2A^{-1/3} \). Nuclear fissility \( X = E_{C}^0/(2E_{s}^0) \).

Parameters obtained by fit to experimental data on nuclear masses, quadrupole moments and fission barriers: \( a_{s} = 17.9439 \text{ MeV}, \kappa = 1.7826, a_{c} = 3e^2/(5r_0), e^2 = 1.44 \text{ MeV} \cdot \text{fm}, r_0 = 1.2249 \text{ fm.} \)
Shape dependent $B_s$ and $B_C$

$B_s$ is proportional with surface area $B_s = \frac{d^2}{2} \int_{-1}^{+1} \left[ y^2 + \frac{1}{4} \left( \frac{dy^2}{dx} \right)^2 \right]^{1/2} \, dx$

In cylindrical coordinates with -1, +1 intercepts on the symmetry axis $y = y(x)$ or $y_1 = y(x')$ is the surface equation. $d = (z'' - z')/2R_0$ – seminuclear length in units of $R_0$. Assume uniform charge density, $\rho_{0e} = \rho_{1e} = \rho_{2e}$. D.N. Poenaru et al., Comp. Phys. Comm. 16 (1978) 85, 19 (1980) 205. $K$, $K'$ – complete elliptic integrals of the 1st and 2nd kind. $D = (K - K')/k^2$.

$$B_C = \frac{5d^5}{8\pi} \int_{-1}^{+1} dx \int_{-1}^{+1} dx' F(x, x')$$

$$F(x, x') = \{yy_1[(K - 2D)/3] \cdot \left[ 2(y^2 + y_1^2) - (x - x')^2 + \frac{3}{2}(x - x') \left( \frac{dy_1^2}{dx'} - \frac{dy_2^2}{dx} \right) \right] + K \left\{ y^2 y_1^2/3 + \left[ y^2 - \frac{x - x'}{2} \frac{dy^2}{dx} \right] \left[ y_1^2 - \frac{x - x'}{2} \frac{dy_1^2}{dx'} \right] \right\} a_{\rho}^{-1} \}$$
LDM PES and saddle-point shapes

Potential energy surfaces (PES) for \(^{106}\)Te (left) and \(^{232}\)Th (right)

Saddle point shapes for fissility parameter \(X = 0.60, 0.70, 0.82\) \((^{170}\text{Yb}, \, ^{204}\text{Pb}, \, ^{252}\text{Cf} \text{ nuclei})\) obtained by solving an integro-differential equation.

Macroscopic-microscopic method

Also extended to atomic cluster physics. $E_{\text{def}} = E_{LDM} + \delta E$.

V.M. Strutinsky (Nucl. Phys. A 95 (1967) 420) gave a microscopic definition of the shell and pairing corrections, $\delta E = \delta U + \delta P$, and the method of calculation based on the single-neutron and single-proton energy levels of deformed shell models.

The two-humped fission barrier of actinides (top), obtained by adding the shell and pairing corrections, can explain many properties of nuclei which were not understood within LDM, including the mass asymmetry of fission fragments (bottom).

V. M. Strutinsky and S. Polikanov received the APS 1978 Tom W. Bonner Prize “For their significant contributions to the discovery and elucidation of isomeric fission. Their work has vastly expanded our understanding of the role of the single particle states on the total energy of heavy deformed nuclei. Their discoveries have had a crucial impact on the possible stability of very heavy nuclei.”
Harmonic oscillator. Energy levels

\[ V = M \left( \omega_\perp^2 \rho^2 + \omega_z^2 z^2 \right)/2 \]

\[ N = n_\perp + n_z \] main quantum number

\[ n_\perp = 0, 1, 2, \ldots, N \]

\[ E/(\hbar \omega_0) = N + 3/2 + \epsilon (n_\perp - 2N/3) \]

\( (N + 1)(N + 2) \) nucleons in a closed shell (degeneracy).

Spherical magic numbers at \( \epsilon = 0 \) \( \frac{(N + 1)(N + 2)(N + 3)}{3} = 2, 8, 20, 40, 70, 112, 168 \ldots \)

Superdeformed magic numbers at \( \epsilon = 0.6 \) \( 2, 4, 10, 16, 28, 40, 60, 80, 110, 140, 182 \ldots \) \( c/a = 2 \).

Hyperdeformed magic numbers at \( \epsilon = 6/7 \) \( c/a = 3 \).

\[ \epsilon_i = E_i / \hbar \omega_0^0 = \left[ E/(\hbar \omega_0) \right] \left[ 1 - \epsilon^2 \left( 1/3 + 2\epsilon/27 \right) \right]^{-1/3} \]
Harmonic oscillator. Wave functions

\[ \Psi = |n_r m n_z \Sigma \rangle = \psi_{n_r}^m (\rho) \Phi_m (\varphi) \psi_{n_z} (z) \chi (\Sigma) \]

\[ n_r = 0, 1, 2, \ldots, n_\perp \quad n_z = N - n_\perp \quad m = n_\perp - 2n_r \]

\[ \psi_{n_r}^m (\rho) = \frac{\sqrt{2}}{\alpha_\perp} N_{n_r}^m \eta^{|m|/2} e^{-\eta^2/2} L_{n_r}^{|m|} (\eta) = \frac{\sqrt{2}}{\alpha_\perp} \psi_{n_r}^m (\eta) \]

Laguerre and Hermite polynomials. \( \Phi_m (\varphi) = \frac{1}{\sqrt{2\pi}} e^{im \varphi} \)

\[ \psi_{n_z} (z) = \frac{1}{\sqrt{\alpha_z}} N_{n_z} e^{-\xi^2/2} H_{n_z} (\xi) = \frac{1}{\sqrt{\alpha_z}} \psi_{n_z} (\xi) \]

Dimension-less variables \( \eta = \rho^2 / \alpha_\perp^2 \quad \xi = z / \alpha_z \)

\[ \alpha_\perp = \sqrt{\hbar / (M \omega_\perp)} \quad \alpha_z = \sqrt{\hbar / (M \omega_z)} \]

Norm. ct. \( (N_{n_r}^m)^2 = \frac{n_r!}{(n_r + |m|)!} \quad (N_{n_z})^2 = \frac{1}{\sqrt{\pi 2^{n_z} n_z!}} \)
Two center shell model (I)


Applied to binary molecules.

The Hamiltonian, $H$, is a sum of the kinetic energy, $-(\hbar^2/2M)\Delta$, and two potential terms: along the axis perpendicular to the symmetry axis is a harmonic oscillator $V_\rho = (m\omega_\rho^2/2)\rho^2$, and along the symmetry axis has two-centers $-z_1$ and $+z_1$

$$V_z = \frac{m\omega_z^2}{2} \begin{cases} (z - z_1)^2, & z > 0 \\ (z + z_1)^2, & z < 0 \end{cases}$$
Two center shell model (II)

One can separate the variables in the Schrödinger equation $H\Psi = E\Psi$ as

$$\Psi(\rho, \varphi, z) = R(\rho)\Phi(\varphi)Z(z),$$

where $\Phi = e^{im\varphi}/\sqrt{2\pi}$,

$$R = \eta^{m/2}e^{-\eta/2}L_n^m(\eta),$$

with $\eta = \rho^2/\alpha_{\perp}^2$ and the quantum numbers $m = (n_{\perp} - 2i)$ with $i = 0, 1, \ldots$ up to $(n_{\perp} - 1)/2$ for an odd $n_{\perp}$ or to $(n_{\perp} - 2)/2$ for an even $n_{\perp}$. $L_n^m(x)$ is the associated Laguerre polynomial and $\alpha_{\perp} = \sqrt{\hbar/m\omega_\rho}$ has the dimension of a length. The wave function in the dimension-less variable is given in terms of a Hermite function, with $\nu_n$ nonintegers.

$$\langle x|\nu_n \rangle = \begin{cases} 
  c_n e^{-x^2/2} H_{\nu_n} \left( \frac{z-z_1}{\alpha} \right) , & z > 0 \\
  (-1)^n c_n e^{-x^2/2} H_{\nu_n} \left( -\frac{z+z_1}{\alpha} \right) , & z < 0 
\end{cases}$$
The average potential felt by one nucleon can be either of finite depth (generalized Woods-Saxon or folded Yukawa) or of infinite depth (modified harmonic potential or the two-center shell model (TCSM)). We assume to know the doubly degenerate discrete energy levels 
\[ \epsilon_i = E_i / \hbar \omega_0 \] in units of \( \hbar \omega_0 = 41 A^{-1/3} \), arranged in order of increasing energy.

The smoothed-level distribution density is obtained by averaging the actual distribution over a finite energy interval \( \Gamma = \gamma \hbar \omega_0 \), with \( \gamma \simeq 1 \),

\[ \tilde{g}(\epsilon) = \{ \sum_{i=1}^{n_m} [2.1875 + y_i (y_i (1.75 - y_i / 6) - 4.375)] e^{-y_i} \} (1.77245385 \gamma)^{-1} \]

where \( y = x^2 = [(\epsilon - \epsilon_i) / \gamma]^2 \). The summation is performed up to the level \( n_m \) fulfilling the condition \( |x_i| \geq 3 \).
The Fermi energy, \( \tilde{\lambda} \), of this distribution is given by

\[
N_p = 2 \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) d\epsilon
\]

with \( N_p = Z \) for proton levels and \( N_p = A - Z \) for neutron levels, leading to a non-linear equation in \( \tilde{\lambda} \), solved numerically.

The total energy of the uniform level distribution

\[
\tilde{u} = \tilde{U} / \hbar \omega_0^0 = 2 \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) \epsilon d\epsilon
\]

In units of \( \hbar \omega_0^0 \) the shell corrections are calculated for each deformation \( \varepsilon \)

\[
\delta u(n, \varepsilon) = \sum_{i=1}^{n} 2\epsilon_i(\varepsilon) - \tilde{u}(n, \varepsilon)
\]

\( n = N_p / 2 \) particles. Then \( \delta u = \delta u_p + \delta u_n \).
Pairing corrections (I)

Doubly degenerate levels \( \{ \epsilon_i \} \) in units of \( \hbar \omega_0 \). \( Z/2 \) levels are occupied. \( n \) levels below & \( n' \) above Fermi energy contribute to pairing, \( n = n' = \Omega \tilde{g}_s / 2 \). Cutoff energy, \( \Omega \simeq 1 \gg \tilde{\Delta} = 12 / \sqrt{\tilde{A} \hbar \omega_0} \). The gap \( \Delta \) and Fermi energy \( \lambda \) are solutions of the BCS eqs:

\[
0 = \sum_{k_i}^{k_f} \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}} ; \quad \frac{2}{G} = \sum_{k_i}^{k_f} \frac{1}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}
\]

\( k_i = Z/2 - n + 1, \quad k_f = Z/2 + n', \quad \frac{2}{G} \simeq 2\tilde{g}(\tilde{\lambda}) \ln \left( \frac{2\Omega}{\tilde{\Delta}} \right) \).
As a consequence of the pairing correlation, the levels below the Fermi energy are only partially filled, while those above the Fermi energy are partially empty. Occup. probab. by a quasiparticle \( u_k \) or hole \( v_k \)

\[
v_k^2 = \left[ 1 - (\epsilon_k - \lambda)/E_k \right] / 2; \quad u_k^2 = 1 - v_k^2.
\]

Quasip. energy \( E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2} \).

The pairing correction \( \delta p = p - \tilde{p} \), represents the difference between the pairing correlation energies for the discrete level distribution

\[
p = \sum_{k=k_i}^{k_f} 2u_k^2 \epsilon_k - 2 \sum_{k=k_i}^{Z/2} \epsilon_k - \frac{\Delta^2}{G}
\]

and for the continuous level distribution \( \tilde{p} = -(\tilde{g} \tilde{\Delta}^2)/2 = -(\tilde{g}_s \tilde{\Delta}^2)/4 \). Compared to shell correction, the pairing correction is out of phase and smaller. One has again

\[
\delta p = \delta p_p + \delta p_n, \quad \text{and} \quad \delta e = \delta u + \delta p.
\]
Results for $^{240}$Pu and $^{304}$120

Shell corrections for $^{240}$Pu. Deformed harmonic oscillator energy levels. No spin-orbit coupling. Remark the smoothing due to the pairing correction.

PES vs $R$ and $\eta$ for a superheavy nucleus with $Z = 120$ and $A = 304$. The valleys due to the doubly magic fragments $^{208}$Pb and $^{132}$Sn are shown. Such cold valleys were used in the sixtieth by Walter Greiner to motivate the search for SHs, and the development of Heavy Ion Physics worldwide and in Germany, where GSI was built. Itkis et al. exp. confirmed the superasymmetric shoulder of fission fragment mass distributions.
Shell effects explain the mass asymmetry. Nuclear shape obtained as a solution of integro-differential equation.

Britannica: “Heavy-ion radioactivity. In 1980 A. Sandulescu, D.N. Poenaru, and W. Greiner described calculations indicating the possibility of a new type of decay of heavy nuclei intermediate between alpha decay and spontaneous fission. The first observation of heavy-ion radioactivity was that of a 30-MeV carbon-14 emission from radium-223 by H.J. Rose and G.A. Jones in 1984.”
Our models

- Fragmentation and the asymmetric two center shell model
- Alpha-decay like theory
- Numerical superasymmetric fission (NuSAF) model
- Analytical superasymmetric fission (ASAF)

D. N. Poenaru, W. Greiner (Eds):

- *Nuclear Decay Modes*, (IOP, Bristol, 1996).
Basic relationships

Parent → emitted ion + daughter nucleus, $^{A}Z \rightarrow ^{A_{e}}Z_{e} + ^{A_{d}}Z_{d}$

Measurable quantities

- Kinetic energy of the emitted cluster $E_{k} = QA_{1}/A$ or the released energy $Q = M - (M_{e} + M_{d}) > 0$.
- Decay constant $\lambda = \ln 2/T$ or Half-life ($T < 10^{32}$ s) or branching ratio $b_{\alpha} = T_{\alpha}/T$ ($b_{\alpha} > 10^{-17}$)

Model dependent quantities ($\lambda = \nu SP_{s}$)

- $\nu$ frequency of assaults or $E_{v} = h\nu/2$
- $S$ preformation probability
- $P_{s}$ penetrability of external barrier
Fission theory

Shape parameters: fragment separation, $R$, and mass asymmetry

$$\eta = (A_d - A_e)/A.$$ 


Similarly $P = \exp(-K_s)$ for external barrier.

Action integral calculated within Wentzel-Kramers-Brillouin (WKB) quasiclassical approximation

$$K_{ov} = \frac{2}{\hbar} \int_{R_i}^{R_t} \sqrt{2B(R)E(R)} dR$$

$E$ – Potential barrier

$B = \mu$ – Nuclear inertia = reduced mass for $R \geq R_t$
Experimental masses

Cluster emitters

Most probable emitted clusters with different colors.

Fission fragments of $^{252}\text{Cf}$ are neutron-rich nuclei with $Q_\alpha < 0$, hence they are not $\alpha$ emitters.
Unified approach: CF; HPR, and $\alpha$-d

Three valleys: cold-fission (almost symmetrical); $^{16}$O radioactivity, and $\alpha$-decay

$^{234}$U half-lives spectrum (short $T_u$ up)
Experimental confirmations

Rare events in a strong background of $\alpha$ particles

Detectors:

- Semiconductor telescope + electronics
- Magnetic spectrometers (SOLENO, Enge split-pole)
- Solid state nuclear track det. (SSNTD). Cheap and handy. Need to be chemically etched then follows microscope scanning

Experiments performed in Universities and Research Institutes from: Oxford; Moscow; Orsay; Berkeley; Dubna; Argonne; Livermore; Geneva; Milano; Vienna, and Beijing.
Natural radioactive family

Compare $\alpha$ and $\beta^-$ to $^{14}\text{C}$ and $^{24}\text{Ne}$ decays
Systematics ($T_{1/2}$)

Calculated lines within ASAF model and exp. points. Open squares — new candidates.
Systematics \((T_{1/2})\) for few \(\alpha\) emitters

Very strong (short half-life) compared to cluster decay
Candidates for future experiments

$^{220,222,223}$Fr, $^{223,224}$Ac, and $^{225}$Th as $^{14}$C emitters
$^{229}$Th for $^{20}$O radioactivity
$^{229}$Pa for $^{22}$Ne decay mode
$^{230,232}$Pa, $^{231}$U, and $^{233}$Np for $^{24}$Ne radioactivity
$^{234}$Pu for $^{26}$Mg decay mode
$^{234,235}$Np and $^{235,237}$Pu as $^{28}$Mg emitters
$^{238,239}$Am and $^{239-241}$Cm for $^{32}$Si radioactivity
$^{33}$Si decay of $^{241}$Cm

Universal curves (I)

Approximations: $\log S = [(A_e - 1)/3] \log S_\alpha, 
\nu(A_e, Z_e, A_d, Z_d) = \text{constant. From fit to } \alpha \text{ decay: } 
S_\alpha = 0.0160694 \text{ and } \nu = 10^{22.01} \text{ s}^{-1}.$

$$
\log T = - \log P - 22.169 + 0.598(A_e - 1)
$$

$$
- \log P = c_{AZ} \left[ \arccos \sqrt{r} - \sqrt{r(1-r)} \right]
$$

$$
c_{AZ} = 0.22873(\mu_A Z_d Z_e R_b)^{1/2}, \quad r = R_t/R_b, \quad R_t = 1.2249(A_d^{1/3} + A_e^{1/3}), \quad R_b = 1.43998 Z_d Z_e / Q, \text{ and } \mu_A = A_d A_e / A.
$$

Universal curves (II)

Geiger-Nuttal plot \( T_\alpha = f(\text{range of } \alpha \text{ in air}) \)
\[
\log T = f\left(1/Q^{-1/2}\right)
\]
Fission approach of HPR summary

- Up to now the ASAF model predictions have been confirmed
- The strong shell effects of the daughter $^{208}\text{Pb}$ were not fully exploited
- New experimental searches can be performed
- The expected half-lives can be estimated by using the universal curves
Submarine and fission bombs

First application of the enormous energy released, $Q$, in fission (Einstein’s $E = mc^2$) and the chain reaction. Controlled chain reaction in the reactor used in submarine.

Uncontrolled induced fission chain reaction in bombs (supercritical mass). The Little Boy is a gun that fired one mass $^{235}$U at another mass of $^{235}$U, creating a supercritical mass. The pieces should be brought together in a time shorter than the time between spontaneous fissions. Then the initiator introduces a burst of neutrons and the chain reaction begins, continuing until the energy released becomes so great that the bomb blows itself apart.

Fatman is an implosion-type bomb because $^{239}$Pu had an important spontaneous fission yield.
235\text{U} + n \rightarrow \text{fiss. fragm.} + 2 \text{ (or 3) } n + 200 \text{ MeV} \text{ Most probable pair of fission fragments: } 92\text{Kr}, 142\text{Ba} 

Controlled induced fission chain reaction: neutrons released in fission produce an additional fission in at least one further nucleus. This nucleus in turn produces neutrons, and the process repeats.

Nuclear power plants provide about 17% of the world’s electricity (France: 75%). Nuclear reactors are used for supplying heat for electricity generation, domestic and industrial heating, desalination, and naval propulsion, for providing neutron beams for research purposes, and for making radioactive isotopes.
Neutron rich radioactive nuclei are produced through fission of actinide targets bombarded with protons from a primary accelerator. Fission of actinide targets can produce a wide variety neutron rich species with production rates as high as $10^{-4}$ radioactive nuclei per incident proton. There are two basic ways of making radioactive beams: ISOL and IN-FLIGHT separation of the recoils methods.