Diagnosing a new state of matter in collisions of atomic nuclei at very high energy

recent reviews:


pbm and J. Wambach, Rev. Mod. Phys. (2009) in print
arXiv:0801.4256

see also: Heavy Ion Collisions at the LHC – Last Call for Predictions

FIAS, June 18, 2009
Short reminder: the building blocks of matter

**FERMIIONS** matter constituents spin = 1/2, 3/2, 5/2, ...

### Leptons spin = 1/2

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_L$</td>
<td>$(0-0.13)\times10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>electron</td>
<td>0.000511</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_M$</td>
<td>$(0.009-0.13)\times10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>muon</td>
<td>0.106</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_H$</td>
<td>$(0.04-0.14)\times10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>tau</td>
<td>1.777</td>
<td>-1</td>
</tr>
</tbody>
</table>

### Quarks spin = 1/2

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Approx. Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.002</td>
<td>2/3</td>
</tr>
<tr>
<td>$d$</td>
<td>0.005</td>
<td>-1/3</td>
</tr>
<tr>
<td>$c$</td>
<td>1.3</td>
<td>2/3</td>
</tr>
<tr>
<td>$s$</td>
<td>0.1</td>
<td>-1/3</td>
</tr>
<tr>
<td>$t$</td>
<td>173</td>
<td>2/3</td>
</tr>
<tr>
<td>$b$</td>
<td>4.2</td>
<td>-1/3</td>
</tr>
</tbody>
</table>
**Short reminder: the building blocks of matter**

### BOSONS

**Unified Electroweak** spin = 1

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon $\gamma$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$W^-$</td>
<td>80.39</td>
<td>-1</td>
</tr>
<tr>
<td>$W^+$</td>
<td>80.39</td>
<td>+1</td>
</tr>
<tr>
<td>Z boson</td>
<td>91.188</td>
<td>0</td>
</tr>
</tbody>
</table>

### Strong (color) spin = 1

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluon $g$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Short reminder: the building blocks of matter

Mesons $q\bar{q}$
Mesons are bosonic hadrons. These are a few of the many types of mesons.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Quark content</th>
<th>Electric charge</th>
<th>Mass GeV/c²</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>pion</td>
<td>$u\bar{d}$</td>
<td>+1</td>
<td>0.140</td>
<td>0</td>
</tr>
<tr>
<td>$K^-$</td>
<td>kaon</td>
<td>$s\bar{u}$</td>
<td>−1</td>
<td>0.494</td>
<td>0</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>rho</td>
<td>$u\bar{d}$</td>
<td>+1</td>
<td>0.776</td>
<td>1</td>
</tr>
<tr>
<td>$B^0$</td>
<td>B-zero</td>
<td>$d\bar{b}$</td>
<td>0</td>
<td>5.279</td>
<td>0</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>eta-c</td>
<td>$c\bar{c}$</td>
<td>0</td>
<td>2.980</td>
<td>0</td>
</tr>
</tbody>
</table>

Baryons $qqq$ and Antibaryons $q\bar{q}\bar{q}$
Baryons are fermionic hadrons. These are a few of the many types of baryons.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Quark content</th>
<th>Electric charge</th>
<th>Mass GeV/c²</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>proton</td>
<td>$uud$</td>
<td>1</td>
<td>0.938</td>
<td>1/2</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>antiproton</td>
<td>$\bar{u}\bar{u}\bar{d}$</td>
<td>−1</td>
<td>0.938</td>
<td>1/2</td>
</tr>
<tr>
<td>$n$</td>
<td>neutron</td>
<td>$udd$</td>
<td>0</td>
<td>0.940</td>
<td>1/2</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>lambda</td>
<td>$uds$</td>
<td>0</td>
<td>1.116</td>
<td>1/2</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>omega</td>
<td>$sss$</td>
<td>−1</td>
<td>1.672</td>
<td>3/2</td>
</tr>
</tbody>
</table>
Nucleon

**confinement**

Atomic nucleus

**de-confinement**

Quark-Gluon Matter
Before the collision

normal nuclear matter
\[ \rho_0 = 0.17 \text{ /fm}^3 \]
\[ \varepsilon_0 = 0.16 \text{ GeV/fm}^3 \]

compression and heating

Quark-Gluon Matter
Quark-Gluon Plasma

Phase transition expected for
\[ \rho = 1.2 \text{ /fm}^3 \]
\[ \varepsilon = 3 \text{ GeV/fm}^3 \]

fireball

expansion and freeze-out

non-interacting particles

time
Diagnosing the fireball by measurement of the emitted particles.
Quark-Gluon Plasma

confined matter

density (kg/m$^3$)

temperature (K)

$1 \times 10^{12}$

$2 \times 10^{12}$

$3 \times 10^{12}$

$4 \times 10^{12}$
Critical energy density and critical temperature

\[ T_c = 173 \pm 12 \text{ MeV} \]
\[ \varepsilon_c = 700 \pm 200 \text{ MeV/fm}^3 \]

for the (2 + 1) flavor case: the phase transition to the QGP and its parameters are quantitative predictions of QCD.

The order of the transition is not yet definitively determined, see also:

Lattice QCD calculations for \( \mu_B = 0 \)
Karsch et al, hep-lat/0305025

current status of lattice QCD calculations -- critical temperature

F. Karsch, Erice Workshop, Sept. 2008
Evolution of the Early Universe

Homogeneous Universe in Equilibrium, this matter can only be investigated in nuclear collisions:

- Charge neutrality
- Net lepton number = net baryon number
- Constant entropy/baryon

neutrinos decouple and light nuclei begin to be formed

QCD Phase Boundary
characterizing QGP matter at LHC

equation of state
number of degrees of freedom
transport coefficients (viscosity etc)
velocity of sound
parton energy loss and opacity
susceptibilities
deconfinement

but also, look for the unexpected
Accelerators where ultra-relativistic nuclei collide

<table>
<thead>
<tr>
<th>fixed target</th>
<th>collider</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGS</td>
<td>RHIC</td>
</tr>
<tr>
<td>SPS</td>
<td>LHC</td>
</tr>
<tr>
<td>1987-2000</td>
<td>since 2000</td>
</tr>
<tr>
<td></td>
<td>from 2009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>beam momentum</th>
<th>29·Z GeV/c</th>
<th>450·Z GeV/c</th>
<th>ea250·Z GeV/c</th>
<th>ea7000·Z GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>projectile</td>
<td>p...Au</td>
<td>p...Pb</td>
<td>p...Au</td>
<td>p...Pb</td>
</tr>
<tr>
<td>energy available in c.m. system</td>
<td>Au+Au</td>
<td>Pb+Pb</td>
<td>Au+Au</td>
<td>Pb+Pb</td>
</tr>
<tr>
<td></td>
<td>600 GeV</td>
<td>3200 GeV</td>
<td>40 TeV</td>
<td>1150 TeV</td>
</tr>
<tr>
<td>hadrons produced per collision</td>
<td>900</td>
<td>2400</td>
<td>7500</td>
<td>40000?</td>
</tr>
</tbody>
</table>

compilation: J. Stachel
RHIC experiments: 2 large and 2 small

**PHENIX**: central 2 arm spectrometer plus forward/backward muon arms

**STAR**: large TPC at central rapidity

as well as **PHOBOS** and **BRAHMS** (both completed)
STAR event display

in central AuAu collisions
at RHIC $\sqrt{s} = 200$ GeV
about 7500 hadrons produced (BRAHMS)

about three times as much as at CERN SPS
The Space-Time Evolution of a Relativistic Nuclear Collision

CGC  Initial Singularity  Glasma  sQGP  Hadron Gas

Quark Gluon Plasma $\rightarrow$ Hadronization $\tau \approx 1 - 10 \text{ fm/c}$

Glasma $\rightarrow$ Density Fluctuations, Thermalization $\tau \approx 0.1 - 1 \text{ fm/c}$

Initial Singularity $\rightarrow$ Quantum Fluctuations $\tau \approx 0 - 0.1 \text{ fm/c}$

Initial Nuclei as CGC $\rightarrow$ Coherent, High-density Gluons

one possible view (courtesy Larry McLerran)

Peter Braun-Munzinger
fireball measurements – charged particle multiplicity

thousands of particles (mesons, baryons) are produced in one collision!

Extrapolation to LHC energy??
differing predictions for multiplicity density

Charged multiplicity for $\eta=0$ in central Pb+Pb at $\sqrt{s_{\text{NN}}}=5.5$ TeV

compilation from: arXiv:0711.0974
The fireball emits hadrons from an equilibrium state

- From AGS energy on, all hadron yields in central PbPb collisions reflect grand-canonical equilibration
- Strangeness suppression observed in elementary collisions is lifted

For a recent review see:

pbm, Stachel, Redlich, QGP3, R. Hwa, editor, Singapore 2004, nucl-th/0304013
Hadro-chemistry at RHIC

All data in excellent agreement with thermal model predictions

chemical freeze-out at: $T = 165 \pm 8$ MeV

fit uses vacuum masses

most recent analysis:
A. Andronic, pbm, J. Stachel,
nucl-th/0511071

see also Xu et al., Nucl. Phys. A698(2002) 306;
Broniowski et al., nucl-th/0212052.

Peter Braun-Munzinger
Parameterization of all freeze-out points

note: discovery of the limiting temperature

$T_{\text{lim}} = 160$ MeV

provides connection to QCD phase boundary

Boiling point of hadronic matter – implies phase transition to QGP
Horn structure well described

rapid saturation of contributions from higher resonances in conjunction with additional pions from the sigma describes horn structure well

crucial input is saturation of $T$ due to the phase boundary

solid prediction for LHC energy

Andronic, pbm, Stachel
ArXiv:0812.1186 [nucl-th]
Summary of statistical model interpretation

- hadron yields quantitatively described at all energies by 3 parameters: $T$, $\mu_b$, $V$
- limiting temperature established
- connection to QCD phase boundary

- first data from LHC will provide a crucial test of this picture: does limiting temperature picture survive a 20 fold increase in cm energy?

- anything else would be a major surprize
- already day 1 data from LHC will be decisive
The fireball expands collectively like an ideal fluid.

Hydrodynamic flow characterized by azimuthal anisotropy coefficient $v_2$

$$dN/d\phi = 1 + 2V_2 \cos 2(\phi - \psi) + \ldots$$
Elliptic Flow Results from RHIC

- Initial spatial anisotropy
- Strong pressure gradients
- Azimuthal anisotropy $v_2$
- Sensitivity to early expansion

- Shear viscosity/entropy close to theoretical (AdS/CFT) limit
- $\eta/s = 1/(4\pi)$

Ideal hydro calculations reproduce also the observed mass ordering but fail in detail.

Note the peculiar quark scaling!
QGP and Ultra-cold Quantum Gases

**ultra-cold $^6$Li**
10 nK
$\sim 10^{-12}$ eV

**QGP**
2.1 TK
$\sim 10^8$ eV

collective expansion at strong coupling

Fourier analysis of shape
exploring the importance of viscous effects

calculations by Paul Romatschke

studies imply that \( \eta/s > (2 \cdot \text{AdS/CFT limit}) \)
not compatible with data
summary of all existing data

note: the initial eccentricity must be computed in a model

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$
extrapolations to LHC
$v_2$ scales with eccentricity $\varepsilon$, Knudsen number $K$ describes deviation from ideal hydro, where $K=0$

J.Y. Ollitrault et al, arXiv:0711.0974

$$v_2 = \frac{h\varepsilon}{1 + K/0.7}$$

$$\frac{1}{K} = \frac{\sigma}{S dy} \frac{dN}{\sqrt{3}}$$

$K$(RHIC) = 0.55

expect:

$K$(LHC) < 0.3

important test of ideal fluid scenario and temperature dependent shear viscosity
Summary of RHIC Hydro Results

- spectra and flow well explained by ideal hydrodynamics calculations
- viscosity/entropy density close to AdS/CFT limit
- is hydro limit reached at RHIC, will it be „exceeded at LHC“?
- is viscosity only low near phase boundary?
- is quark scaling universal?

day 1 results from LHC will be decisive
The fireball is opaque for high momentum partons

- suppression of high $p_t$ particles in AA relative to pp collisions
- disappearance of jet-like correlations
- connected to large gluon density in hot (QGP) fireball

schematic picture of energy loss of a fast parton
Jet quenching

- Hard parton scattering observed via leading particles
- Expect strong $\Delta \phi = \pi$ azimuthal correlations

However, the scattered partons may lose energy (several GeV/fm) in the colored medium

$\rightarrow$ momentum reduction (fewer high $p_T$ particles in jet)
$\rightarrow$ no jet partner on other side
Definition of $R_{AA}$

$$R_{AA} = \text{medium/vacuum}$$

$$R_{AA}(p_T) = \frac{d^2 N^{AA} / dp_T d\eta}{T_{AA} d^2 \sigma^{NN} / dp_T d\eta}$$

$\langle N_{\text{binary}} \rangle / \sigma_{\text{inel}}^{p+p}$

no medium effects:

- $R_{AA} < 1$ in regime of soft physics
- $R_{AA} = 1$ at high-$p_T$ where hard scattering dominates

Suppression:

- $R_{AA} \ll 1$ at high-$p_T$
• Direct photons are not suppressed, follow pQCD predictions.
• Common suppression for $\pi^0$ and $\eta$.
• $\varepsilon > 15$ GeV/fm$^3$; $dN_g/dy > 1100$
but what about photons at very large $p_t$?

The elucidation is LHC territory.

$$R_{AA}(p_T) = \frac{\frac{d}{d^2} N^{AA}/dp_T d\eta}{T_{AA} \frac{d}{d^2} \sigma^{NN}/dp_T d\eta}$$
**jet quenching indicative of gluon rapidity density**

<table>
<thead>
<tr>
<th></th>
<th>( \tau_0 [\text{fm}] )</th>
<th>( T [\text{MeV}] )</th>
<th>( \epsilon [\text{GeV}/\text{fm}^3] )</th>
<th>( \tau_{\text{tot}} [\text{fm}] )</th>
<th>( dN^g / dy )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPS</td>
<td>0.8</td>
<td>210-240</td>
<td>1.5-2.5</td>
<td>1.4-2</td>
<td>200-350</td>
</tr>
<tr>
<td>RHIC</td>
<td>0.6</td>
<td>380-400</td>
<td>14-20</td>
<td>6-7</td>
<td>800-1200</td>
</tr>
<tr>
<td>LHC</td>
<td>0.2</td>
<td>710-850</td>
<td>190-400</td>
<td>18-23</td>
<td>2000-3500</td>
</tr>
</tbody>
</table>

I. Vitev, JPG 30 (2004) S791

- Estimates consistent with hydrodynamic analysis

![Graph showing \( \epsilon/\tau^4 \) vs. \( T/T_C \) for SPS, RHIC, and LHC with different flavour scenarios]
further big surprize at RHIC: strong energy loss of heavy quarks

electrons from heavy flavor mesons
strong energy loss
hydrodynamic flow

these data are not well explained, measure heavy quarks „directly“ at LHC
the ultimate hard probes machine

> $10^4$ jets with $E_T > 150$ GeV in one month of PbPb collisions at LHC
Predictions for jet quenching at LHC

S. Wicks and M. Gyulassy
more predictions...

Renk and Eskola

connection to AdS/CFT considerations?

important: perturbative QCD regime may never be reached!
Synopsis of jet quenching predictions for LHC energy

$$R_{PbPb}(p_T=20,50 \text{ GeV}, \eta=0) \text{ in central Pb+Pb at } \sqrt{s_{NN}}=5.5 \text{ TeV}$$

- Wang et al., $\pi^0$, 5% ($\approx 3.3 \sigma_{RHIC}$), WW e-loss + 1d exp., shadowing
- Vitev, $\pi^0$, 10%, GLV+g-feebd.+cold eloss, $dN/d\eta\sim 1.7\sim 3.3(dN/d\eta)_{RHIC}^{R_{RHIC}}$
- Pantuev, charged, $N_{\text{pert}}=350$, $\tau_{QGP}^{\text{form}}=1.2 \text{ fm, } 0.5(\tau_{QGP}^{\text{form}})^{R_{RHIC}}$
- Lokhtin et al., charged, 10% ($dN^{ch}/d\eta\sim 2700$), rad.+coll. eloss in MC
- Kopeliovich et al., $\pi^0$, 10%, early hadronization
- Liu et al., $\pi^+$, $p_T^{\text{highest}}=40$, 10%, 2<=2 w. conv., transv. exp.
- Jeon et al., $\pi^0$, $p_T^{\text{highest}}=40$, 10% ($\lambda=1 \text{ fm}$), BH eloss+QW, $\frac{\Delta E}{E}^{R_{RHIC}}$
- Wicks et al., $\pi^0$, 10%, rad.+coll. eloss, $dN^{ch}/d\eta\sim 1.75\sim 2.9(dN^{ch}/d\eta)_{RHIC}^{R_{RHIC}}$
- Qin et al., charged, 10% ($dN^{ch}/d\eta\sim 2500$), AMY+hydro, $\alpha_s=0.25\sim 0.33$
- Renk et al., $\pi^0$, 10% ($dN^{ch}/d\eta\sim 2500$), BDMPS QW with hydro evol.
- Dainese et al., $\pi^0$, 10%, BDMPS QW with WS, $\hat{q}\sim 2\sim 7q_{RHIC}$
- Cunqueiro et al., $\pi^0$, 10% ($dN^{ch}/d\eta\sim 1500$), percolation
- Capella et al., $\pi^0$, 10% ($dN^{ch}/d\eta\sim 1800$), comovers, kinematics
Summary of jet quenching results

- for $p_t > 3$ GeV hadron production is strongly suppressed compared to pQCD expectations
- only viable explanation: large energy loss of fast partons in fireball
- even heavy quarks lose large amount of energy
- both gluon radiation and collisional energy loss seem important but no unique theoretical description of RHIC data

first month of LHC data with $p_t$ reach up to 50 GeV will bring decisive new insights
The fireballs modifies charmonium production
Charmonium as a probe for the properties of the QGP

the main idea: implant charmonia into the QGP and observe their modification, in terms of suppressed (or enhanced) production in nucleus-nucleus collisions with or without plasma formation


assumptions:
• all charmonia are produced before QGP formation
• suppression takes place in QGP via Debye screening
• some charmonia might survive beyond $T_c$
  $\rightarrow$ sequential suppression pattern due to feeding
Charmonium

- a bound state of charm and anti-charm quarks
- mass of lowest 1-state \((J/\psi) = 3.1\text{GeV}\)
  but its width is only 93 keV
- mass of charm quark 1.2 – 1.6 GeV
- binding energy about 600 MeV
- charm quarks move non-relativistically
Discovery of the J/ψ

p+Be data at 24 GeV from BNL

e+e- annihilation date from
SPEAR at SLAC
PHENIX measurements in Au-Au collisions at RHIC
Debye screening

\[ V(r, T \text{ large}) \quad \text{no bound state} \]

\[ V(r, T \text{ small}) \quad \text{bound state} \]

\[ \sigma = \text{string tension} = 1 \text{ GeV/fm} \]
\[ = 0.2 \text{ GeV}^2 \]
Free energy of a heavy quark-antiquark pair

Color singlet free energy
\[ F_1(T) = U(T) - T S(T) \]

Note: J/ψ is bound by 640 MeV

J/ψ disappears for \( T > 1.1 T_c \)

O. Kaczmarek, F. Zantow, PRD 71(2005)114510

Peter Braun-Munzinger
Upper bounds on dissociation temperatures

condition: thermal width > 2x binding energy

\[ \Gamma(T) \geq 2E_{bin}(T) \]

<table>
<thead>
<tr>
<th>state</th>
<th>( \chi_c )</th>
<th>( \psi' )</th>
<th>( J/\psi )</th>
<th>( \Upsilon' )</th>
<th>( \chi_b )</th>
<th>( \Upsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{dis} )</td>
<td>( T_c )</td>
<td>( 1.2T_c )</td>
<td>( 1.2T_c )</td>
<td>( 1.3T_c )</td>
<td>( 2T_c )</td>
<td></td>
</tr>
</tbody>
</table>
Too much suppression at RHIC in Standard QGP Scenario

standard scenario: all charmonia melt near $T_c$

models tuned for SPS data fail at RHIC

**$J/\psi$ nuclear modification factor $R_{AA}$**

Capella et al. hep-ph/0505032 suppression from co-mover

Grandchamp et al. hep-ph/0306077 suppression (no regen)

Kostyuk et al. hep-ph/0305277 suppression in QGP

PHENIX preliminary

Peter Braun-Munzinger
Quarkonium as a probe for deconfinement at the LHC

charmonium enhancement as fingerprint of deconfinement at LHC energy

Comparison of model predictions to RHIC data: rapidity dependence

Suppression is smallest at mid-rapidity (90 deg. emission) where energy density is largest—a clear indication for charmonium generation at the phase boundary.

A. Andronic, pbm, K. Redlich, J. Stachel
Summary of RHIC data on charmonium production

- major surprize: suppression equal to that observed at SPS
- major surprize: suppression is minimal at midrapidity
- LHC: expect qualitatively new features due to very large charm quark density

**first month of LHC data with a few thousand charmonia will bring decisive new insights**
final comment

the discoveries at RHIC, principally on

thermalization and flow  -->  ideal fluid scenario

jet quenching  -->  parton energy loss in dense fireball

have led to major progress in our understanding of the QGP

These discoveries raise many new questions. The next frontier is at the LHC! The experiments are ready.
A spectacular event with cosmic rays

ALICE has entered the Terascale

a muon bundle:

more than 30 muons, essentially parallel, total momentum $>1\text{TeV}$
additional slides
Viscosity of QCD matter

To the rescue: String theory and lattice QCD.

- General argument [Kovtun, Son & Starinets] based on duality between thermal $N=4$ SYM theory and string theory on a curved background with a black-brane AdS metric:
  \[
  \eta/s = \frac{1}{4\pi} \quad \text{in the strong coupling limit.}
  \]

- Quenched lattice QCD (H. Meyer):
  \[
  \eta/s = 0.134 (0.102) \quad \text{at } T/T_c = 1.65 (1.24).
  \]
Thermal model description of hadron yields

Grand Canonical Ensemble

\[ \ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T)) \]
\[ n_i = \frac{N}{V} = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp((E_i - \mu_i)/T) \pm 1} \]
\[ \mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i^3 \]

for every conserved quantum number there is a chemical potential \( \mu \) but can use conservation laws to constrain:

- Baryon number: \( V \sum_i n_i B_i = Z + N \) \( \rightarrow V \)
- Strangeness: \( V \sum_i n_i S_i = 0 \) \( \rightarrow \mu_S \)
- Charge: \( V \sum_i n_i I_i^3 = \frac{Z - N}{2} \) \( \rightarrow \mu_I \)

This leaves only \( \mu_b \) and \( T \) as free parameter when \( 4\pi \) considered for rapidity slice fix volume e.g. by \( dN_{ch}/dy \)
The QCD phase diagram and chemical freeze-out

Main result: chemical freeze-out points seem to delineate the QCD phase boundary at small baryo-chemical potential ($\mu < 400$ MeV)