MEASUREMENT OF THE VACUUM POLARIZATION IN MUONIC ATOMS

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The vacuum polarization corrections of the muonic 4f → 3d and 5g → 4f transitions in Bi and Pb have been measured. The overall accuracy obtained for the vacuum polarization correction is 1%. Experiment and theory agree if higher order corrections are taken into account.

Radiative corrections are important for the level energies in atoms. The Lamb-shift of the 2s level in hydrogen amounts to 1057.77 ± 0.10 MHz [1], corresponding to about 4.37 × 10⁻⁶ eV. Only 2.6% of this shift is due to the vacuum polarization [2], while the self-energy is dominating. The opposite is true muonic atoms. Owing to the high mass of the muon, the self-energy is small, and the vacuum polarization is large because of the smaller muonic Bohr radii. Muonic atoms are therefore suitable for tests of the vacuum polarization. Earlier evidence for the vacuum polarization effect in muonic atoms was obtained by Koslov et al. [3], and the effect was deduced [4] from precision measurements on muonic phosphorus [5, 6] with an accuracy of 4%. Recently, Anderson et al. [7] checked the effect on various muonic X-ray transitions in lead with similar accuracy. A continuing effort seemed to be justified, in particular since higher order corrections are just at the limit of the accuracy achieved so far.

For the present experiment [8], muonic X-ray transitions were selected where the vacuum polarization correction is large compared to other corrections such as the finite size effect, the nuclear polarization, and the screening of the muon by the K-electrons. We chose the transitions 5g → 4f and 4f → 3d in Bi and natural Pb.

The experiments were carried out at the CERN muon channel. The set-up was described earlier [9]. A usual telescope technique in connection with two Ge detectors was used, and only those Ge detector signals were measured which coincided with a telescope signal, i.e. with a stopped-particle event. Typical resolution values were 3.3 keV at 938 keV, and 2.3 keV at 437 keV. In order to measure calibration and X-ray lines under exactly the same conditions, these were measured simultaneously. This was achieved by placing the calibration sources close to the target and enlarging the coincidence length of the telescope signal. Thus muonic X-rays are measured in true coincidence with a stopped-particle signal, and γ-rays from the calibration source are measured in accidental coincidence. As calibration sources we used 54Mn, 88Y, 192Ir, and 207Bi [10]. In addition, the annihilation line was used as a calibration standard.

The evaluation of the spectra was carried out using the usual techniques [9]. The slight asymmetry of the lines, especially in the higher energy region, was taken into account by different fitting procedures.

First the asymmetry of the line-shape was approximated by adding to the Gaussian shaped line one or more artificial symmetric satellite lines. Furthermore, fits were made where the asymmetric part of the line was excluded. Finally only the upper parts of the lines were fitted. The comparison of the three methods gave an estimate of possible errors introduced by the evaluation.

In order to have an estimate of a shift introduced by muonic satellite lines, the energies and
relative intensities of these lines were calculated. Most of them are separated from the principal lines, except for several \(5 \rightarrow 4\) transitions, whose intensities amounted to only 0.4% of the total \(5g \rightarrow 4f\) transition intensity. The influence of the unresolved satellite lines on the line position can be neglected, yet they were included in the fit.

The HFS splitting of the muonic Bi-lines was calculated and also included in the fit. Finally, the isotopic shifts of the three Pb isotopes \(^{206}\text{Pb}, {^{207}\text{Pb}}, \text{and}^{208}\text{Pb}\) were calculated with well-known parameters of the Fermi-type charge distribution [7], and were taken into account in the fit of the \(4f \rightarrow 3d\) transitions in natural lead.

The experimental results obtained are listed in table 1. Column 1 shows the elements and the charge distribution parameters for \(^{209}\text{Bi}\) [11] and \(^{208}\text{Pb}\) [7]. The experimental energies of the transitions indicated in col. 2 are listed in col. 3.

These data were obtained from various measurements (three runs for Pb and two runs for Bi) utilizing different Ge detectors and multichannel analyzers. The errors consist of statistical errors, calibration errors and evaluation errors, the latter being the main contribution. The individual errors are consistent with the final experimental error given in col. 3.

In order to be able to compare these data with theoretical predictions of the vacuum polarization terms, theoretical values for the transition energies are listed in col. 4. They were obtained by numerical integration of the Dirac equation using a Fermi-type charge distribution with the parameters of col. 1. The errors of the Fermi parameters cause an uncertainty of the transition energies, which is indicated as an error of the theoretical energies in col. 4. Additionally, the following corrections were applied and included in the values of col. 4:

- a) The potential of the vacuum polarization of the order \(\alpha\) (fig. 1a) due to virtual \(e^+e^-\) pairs was calculated exactly. The corresponding vacuum polarization correction was obtained by perturbation calculation, using the wave functions from the numerical integration of the Dirac equation. The values are listed in col. 5 of table 1.
- b) The potentials of the higher order vacuum polarization terms \(\alpha^2\) and \(\alpha^3\) were calculated according to the paper by Fricke [12]. The corrections of the order \(\alpha^2\) originate from the reducible graph (fig. 1b) and from the three irreducible graphs (fig. 1c, 1d, 1e). The reducible graph yields the values \(E_{\text{red}}\) listed in col. 6 of table 1, whereas the values \(E_{\text{irr}}\) listed in col. 7.
result from the irreducible graphs. The corrections of the order $\alpha^3$ are listed in col. 8.

c) The self-energy correction, the energy shifts due to the anomalous magnetic moment of the muon, and the $\mu^+\mu^-$ pair contribution to the first order vacuum polarization were obtained from Barrett [13] for Bi and from Anderson et al. [7] for Pb. These corrections are listed in col. 9. Contributions from $\pi^+\pi^-$ pairs as well as from other hadron pairs were neglected.

d) The energy shift caused by the screening of the muon by the atomic electrons depends on the number of electrons present when the transitions in question occur. From calculations on the muonic cascade which takes place via Auger and radiative transitions, we deduce that less than 10% of the K-electrons have been ejected. It is more difficult to estimate the number of L-electrons ejected. However, since the K-electrons contribute between 85-90% to the total screening shift, the error introduced by this uncertainty is small. The shift due to the two K-electrons was calculated by using point nucleus Dirac wave functions to deduce the potential caused by the K-electrons and point nucleus Schrödinger wave functions for the muon. The result was $-76$ and $-79$ eV for the $5g \rightarrow 4f$ transitions and $-48$ and $-50$ eV for the $4f \rightarrow 3d$ transitions in Pb and Bi, respectively. Recently, more elaborate calculations have been performed by Fricke [14], who used a total self-consistent approach with a Hartree-Fock-Slater program in which the electrons as well as the muon are included. The values obtained this way for the case where only two K-electrons are present have been used and are listed in col. 10. The unlikely assumption that all electrons are present would increase the shifts by 10-15%.

e) The values of the shifts due to nuclear polarization are listed in col. 11. The nuclear polarization was taken from the paper by Cole [15].

The experimental values for the vacuum polarization corrections are listed in col. 13. They are obtained by subtracting the theoretical values of col. 4 reduced by the calculated vacuum polarization correction of col. 12 from the experimental transition energies of col. 3. The quoted errors consist of the experimental errors of col. 3 and the uncertainties in the theoretical energies, arising from the errors of the Fermi parameters, col. 1. These values have to be compared with the calculated values of col. 12. The agreement is particularly good in the case of the $5g \rightarrow 4f$ transitions.

Since the corrections which are not well-known in muonic atoms, such as the self-energy, the nuclear polarization, and the finite size effect, are much smaller in $5g \rightarrow 4f$ than in $4f \rightarrow 3d$ transitions, the $5g \rightarrow 4f$ transitions are much more suitable for vacuum polarization measurements than are the $4f \rightarrow 3d$ transitions. Furthermore, the experimental conditions in the 500 keV region are more favourable than in the 1 MeV region. Therefore, the essential information about the vacuum polarization correction is obtained from $5g \rightarrow 4f$ transitions.

The accuracy of the measured vacuum polarization values for Bi and $^{208}$Pb is 4.0% and 1.9% for the $4f \rightarrow 3d$ transitions, respectively, and 2.2% and 1.8% for the $5g \rightarrow 4f$ transitions, respectively. The weighted mean of the relative deviations $(E_{\text{VP}} - E_{\text{ex}})/E_{\text{VP}}$ of the experimental vacuum polarization values (col. 13) from the calculated values (col. 12) amounts to $(-0.6 \pm 0.8)\%$. The weighted mean of the relative deviation of the experimental vacuum polarization values (col. 13) from the first order vacuum polarization values (col. 5) amounts to $(-2.3 \pm 0.8)\%$. Hence, the agreement between theory and experiment is improved significantly by including these higher order terms. Screening by the total electron cloud as described under point (d) would cause a decrease of both of the above values by 0.3%, and will, therefore, not change the conclusions derived from the data presented.

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References


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