# QCD phase diagram with functional methods

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HIC for FAIR
Helmholtz International Center

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HELMHOLTZ
GEMEINSCHAFT

work together with

Jan Lücker, Axel Maas and Jens Müller

# Overview

1. Introduction
   - General
   - Confinement
   - Dynamical chiral symmetry breaking
   - QCD phase diagram

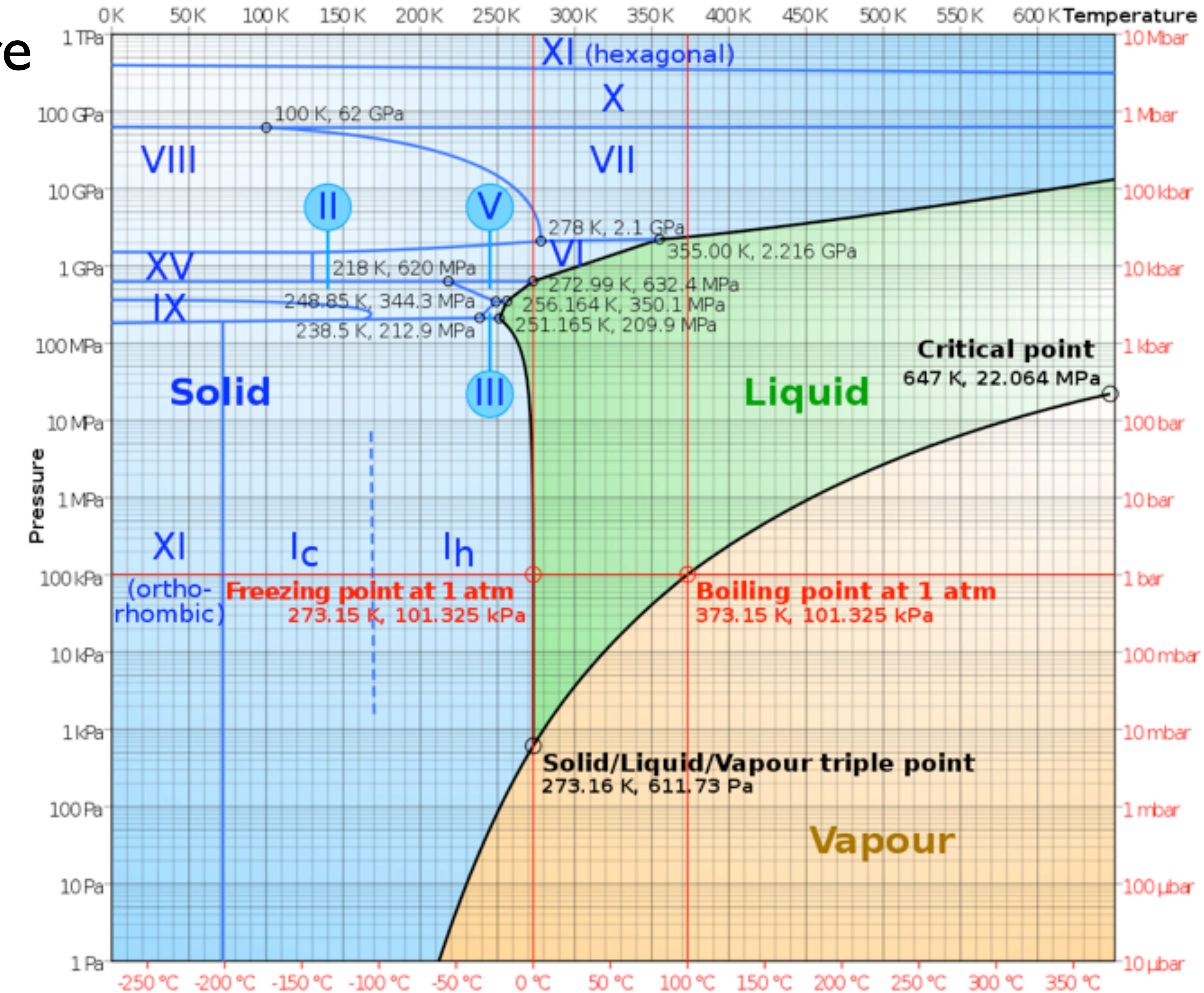
2. QCD with functional methods: Dyson-Schwinger equations
   - Derivation
   - Simple example: pattern of chiral symmetry breaking
   - The gluon propagator
   - Gluons at finite temperature

3. QCD phase diagram
   - Dressed Polyakov-Loops
   - Phase diagram: quenched QCD
   - Transitions of $N_f$=2-QCD, chiral limit
   - Phase diagram: $N_f$=2 vs. $N_f$=2+1

# Phase diagram of water

Source: Wikipedia

Pressure

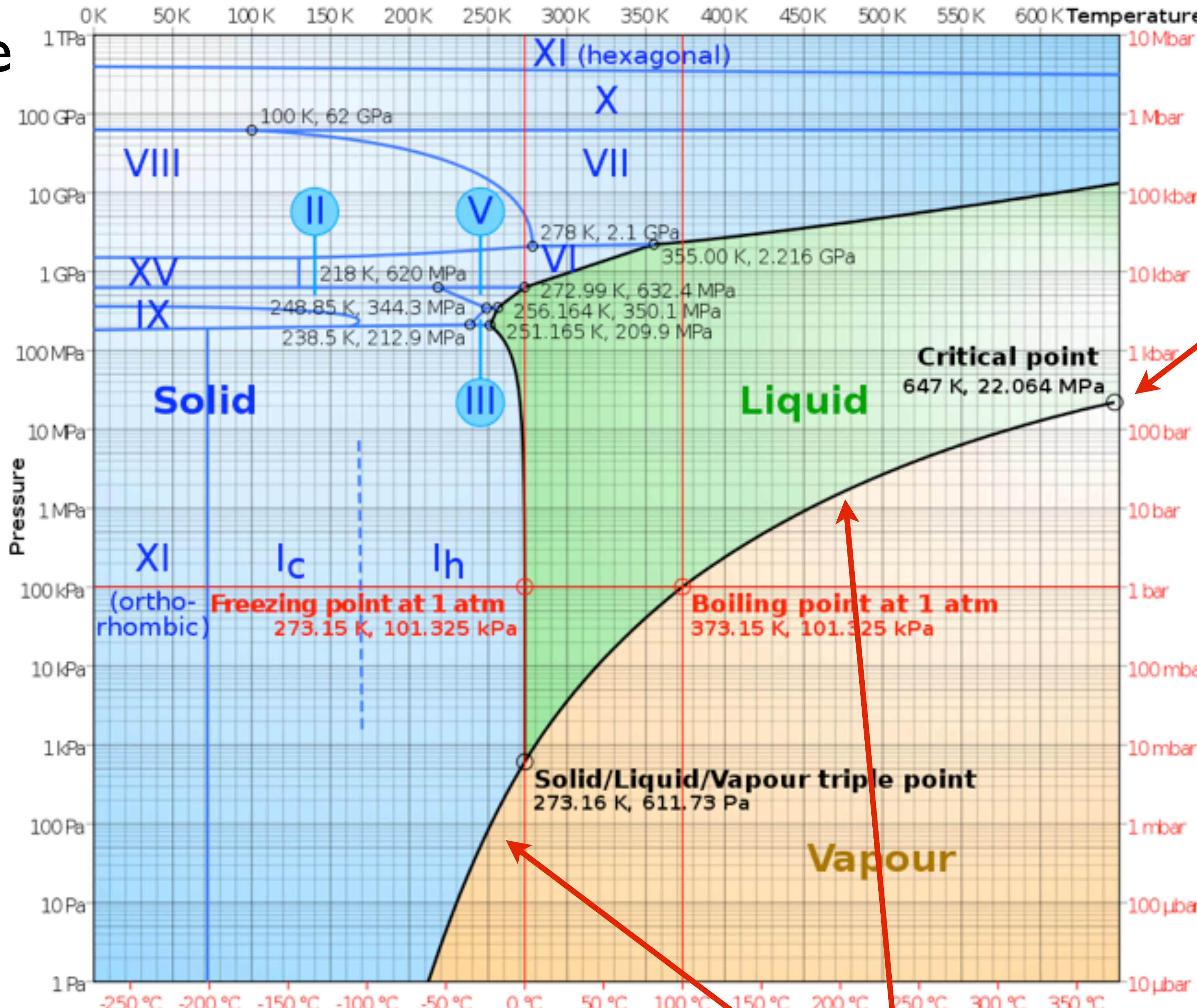



Temperature

# Phase diagram of water

Source: Wikipedia

Pressure




second order

first order

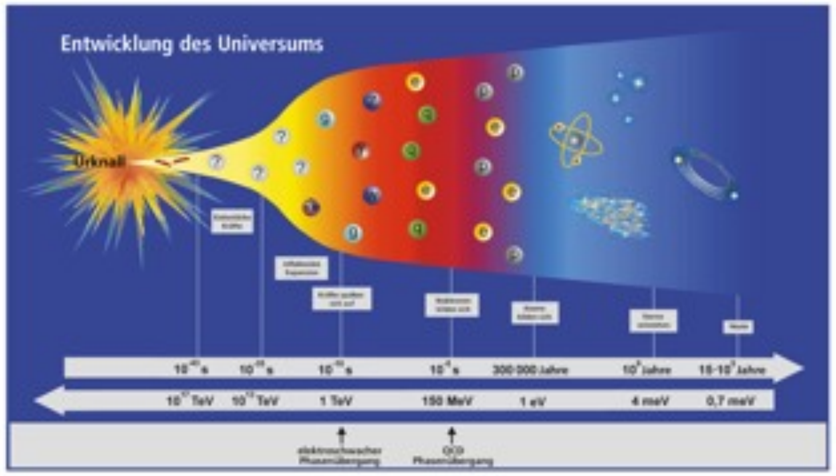
Temperature

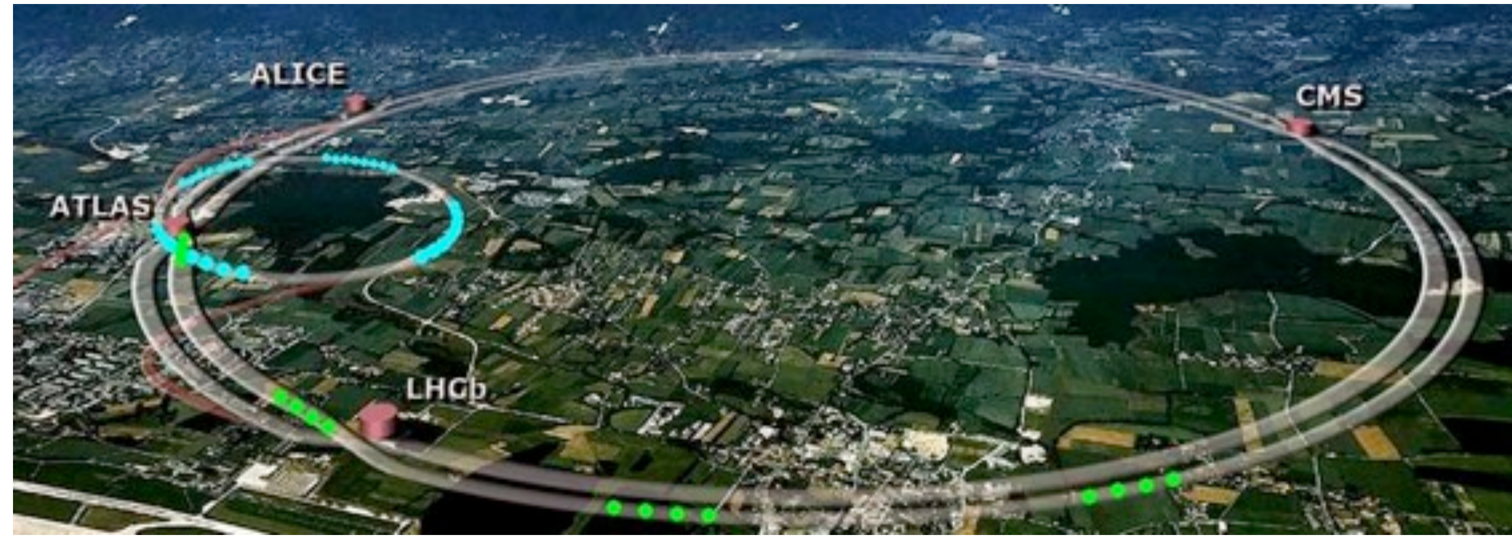


Monday, June 25, 2012

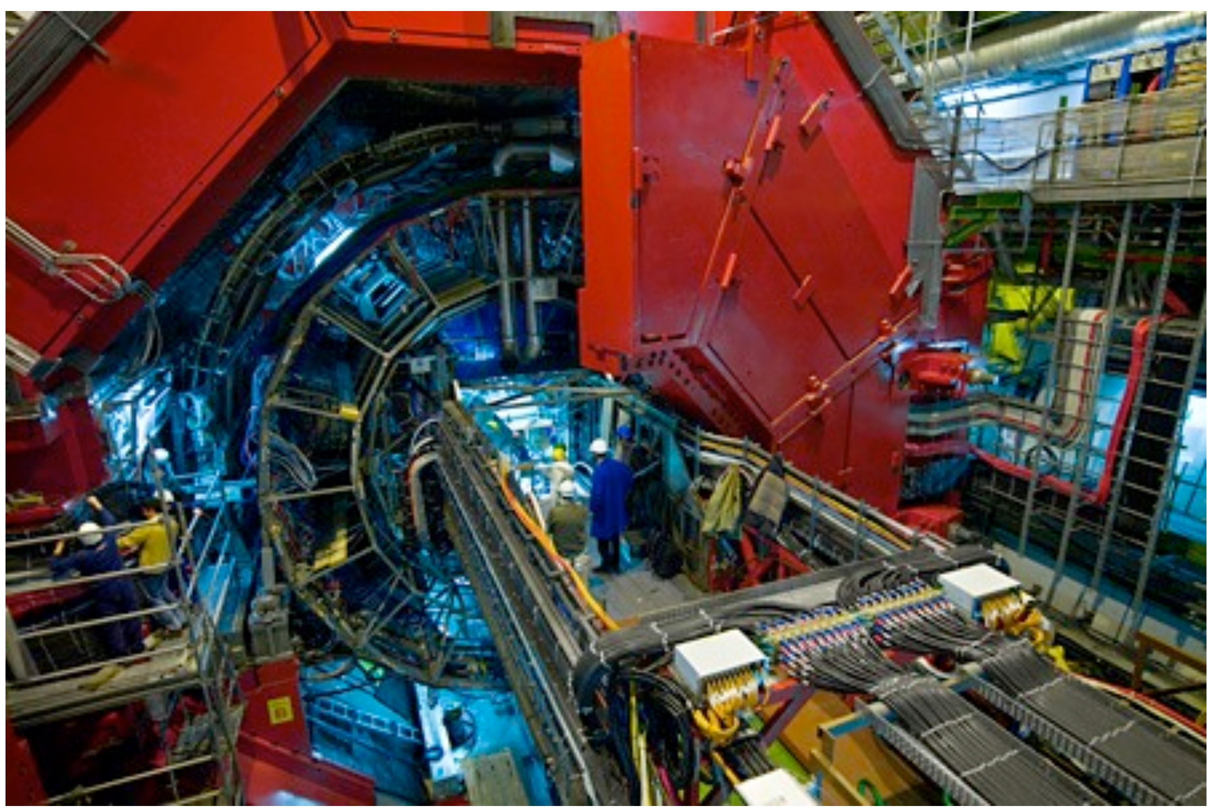
# Connecting small and large scales

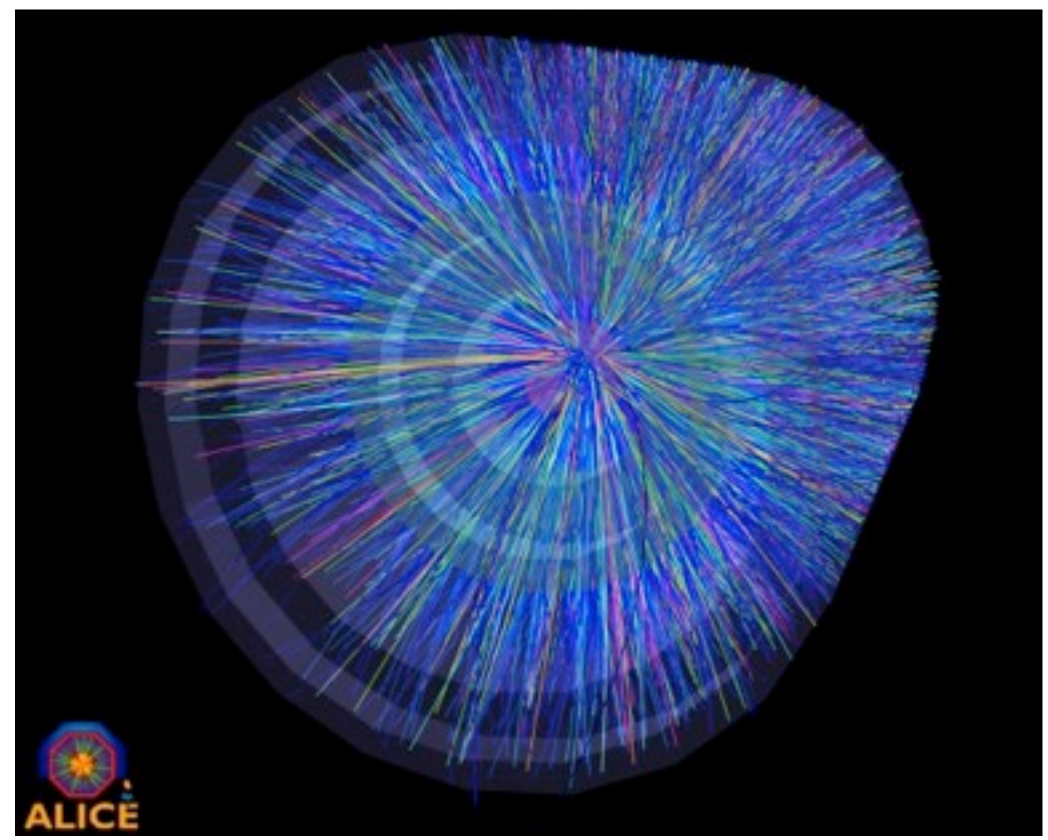
History of our universe...



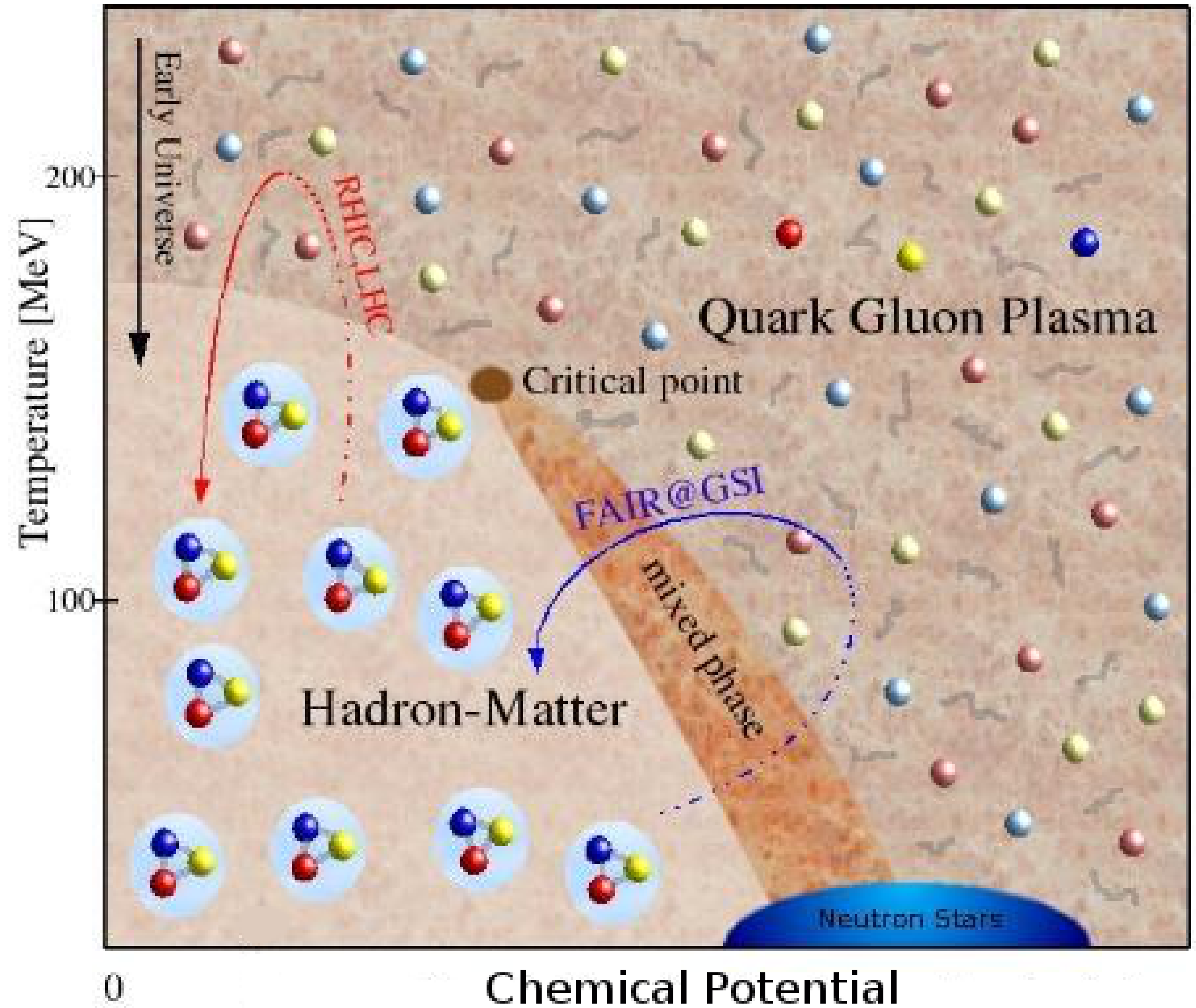

...studied in laboratory








RHIC, ALICE, CBM



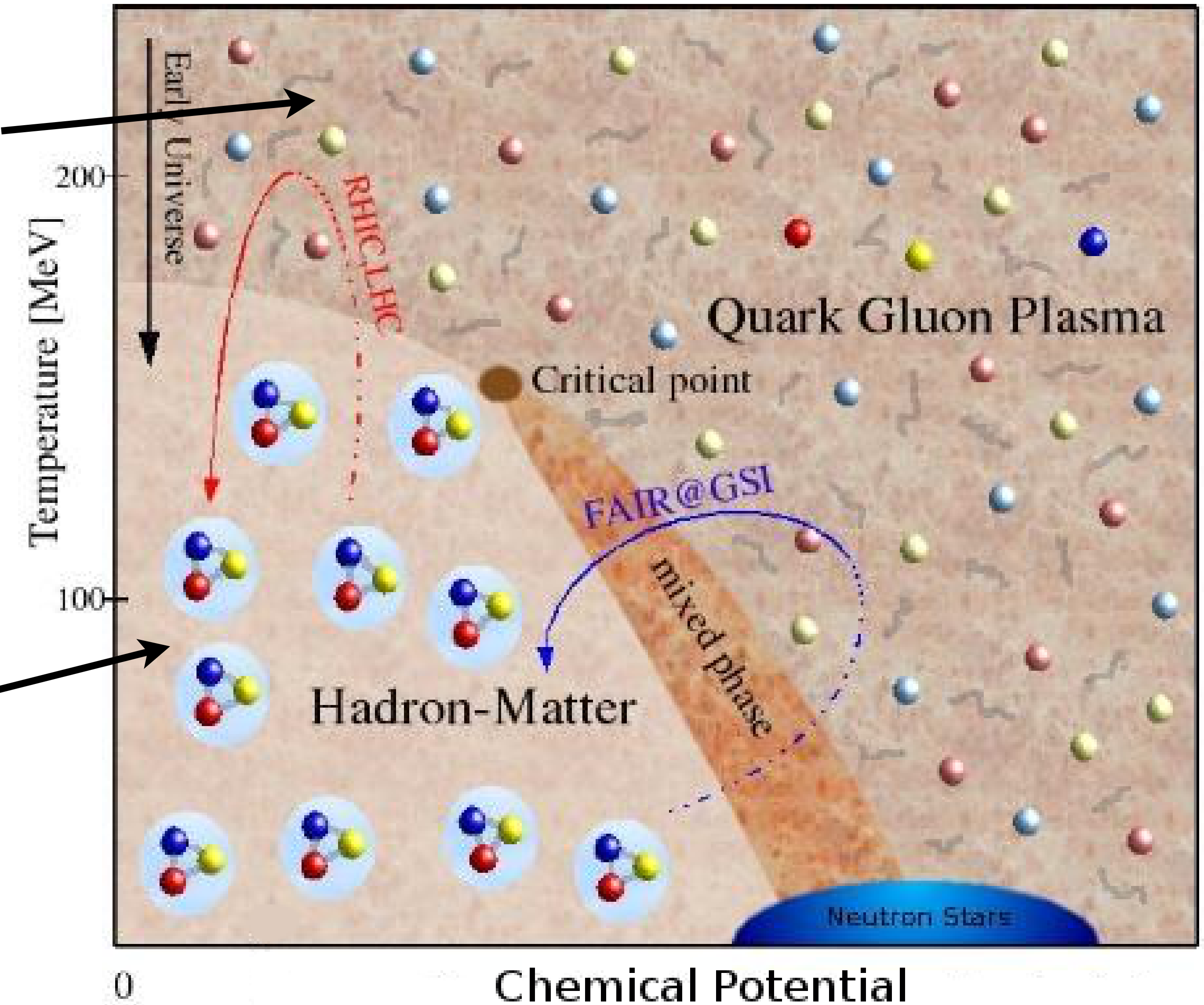

Interesting open questions:

- Existence and location of critical point
- Details of phase transitions
- Properties of Quarks and Gluons in QGP

# QCD phase diagram

Quarks de-confined and (almost) massless

Quarks confined and massive

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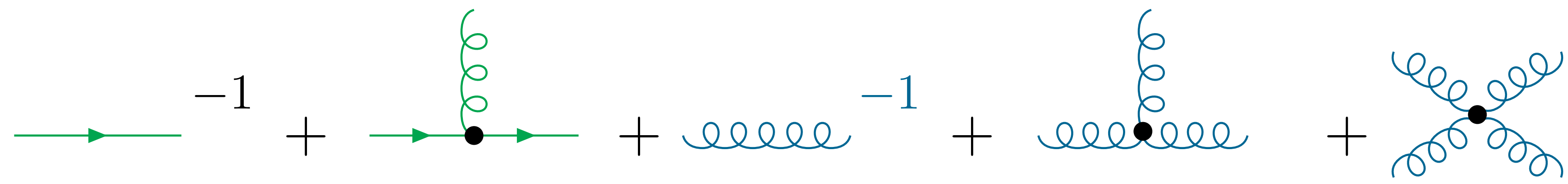
# The QCD generating functional

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \;\exp\left\{ - \int d^4x \Big( \overline{\Psi}\,(i \not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \Big) \right\}$$

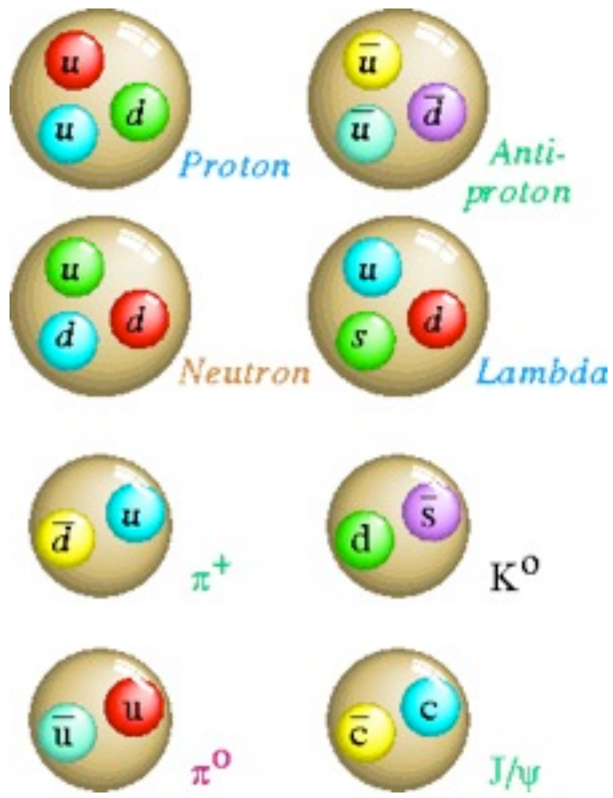
$S_{QCD} = \int d^4x \;\Big($




$\Big)$

- Euclidean space
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D_\mu = \partial_\mu + i g t^a A^a_\mu$
- Landau gauge: $\partial_\mu A^a_\mu = 0$

Color confinement:


We are not detecting quarks,
but baryons, mesons, (tetraquarks, glueballs...).

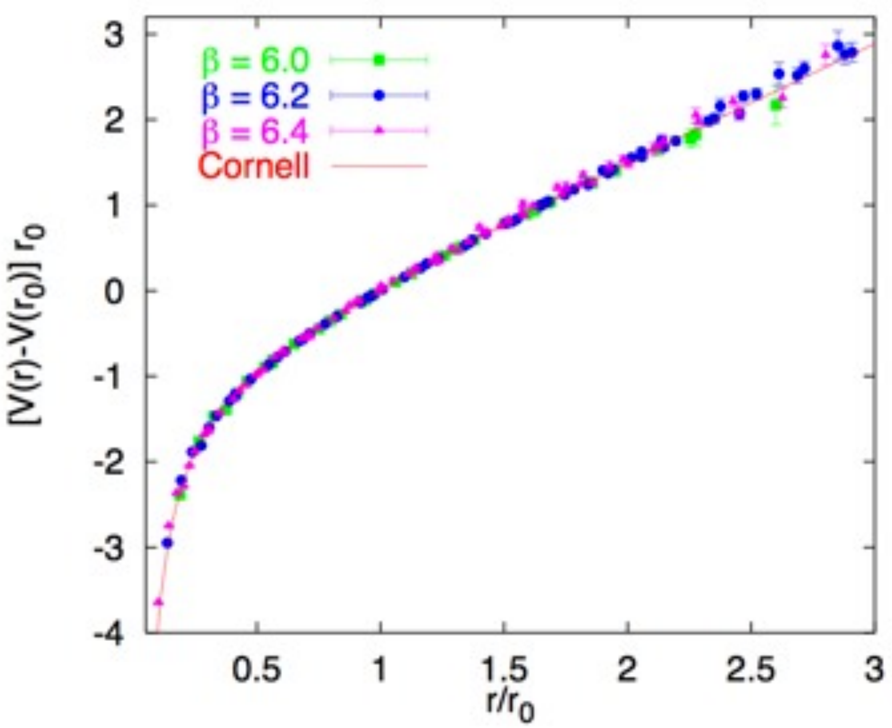
Confinement:

Property of pure Yang-Mills theory: Center symmetry

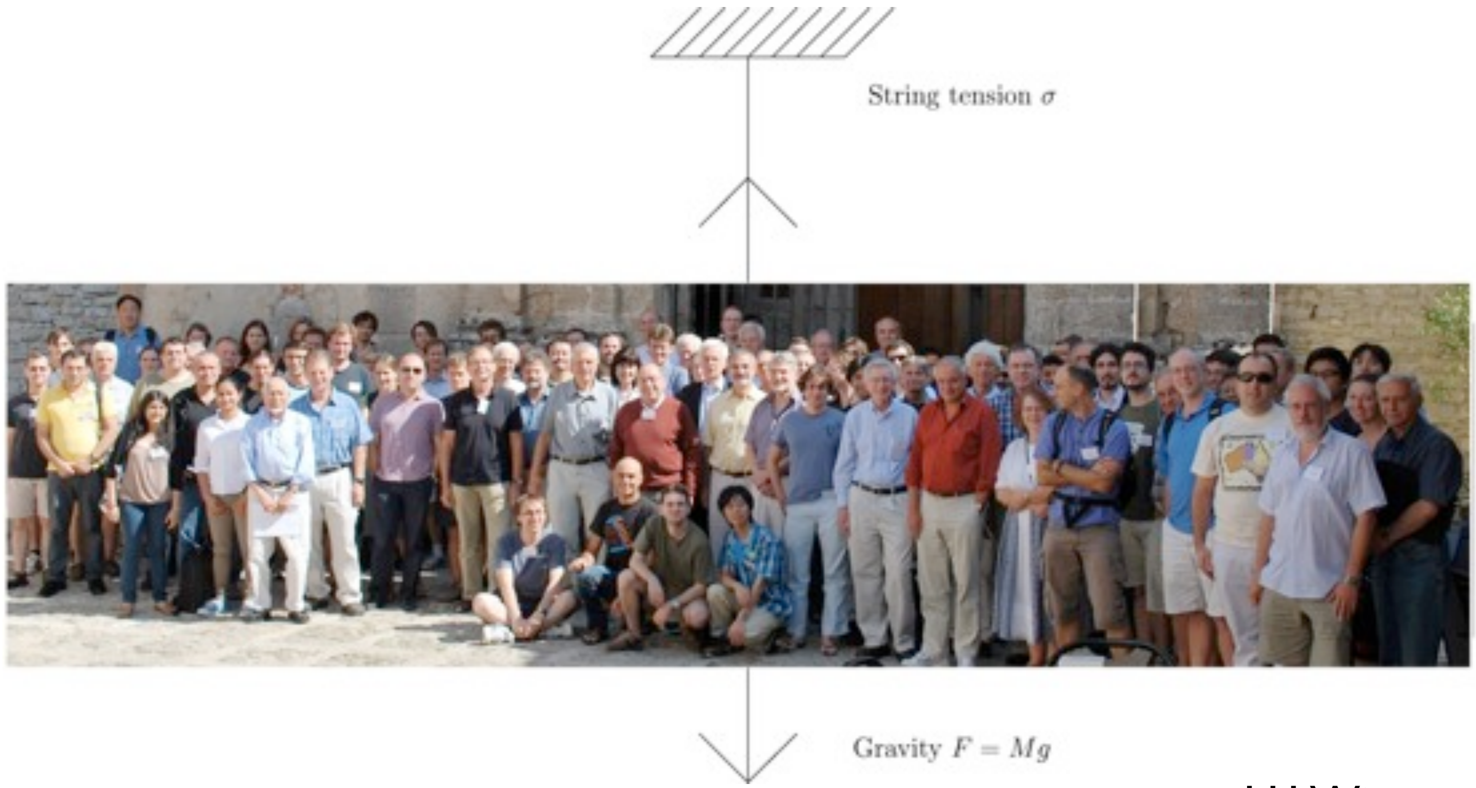
Jeff Greensite, Lecture Notes in Physics 821 (2011) 1.

# Confinement: string tension

Yang-Mills theory with infinitely heavy test quarks:




Bali, Phys. Rept 343 (2001)




U.J.Wiese

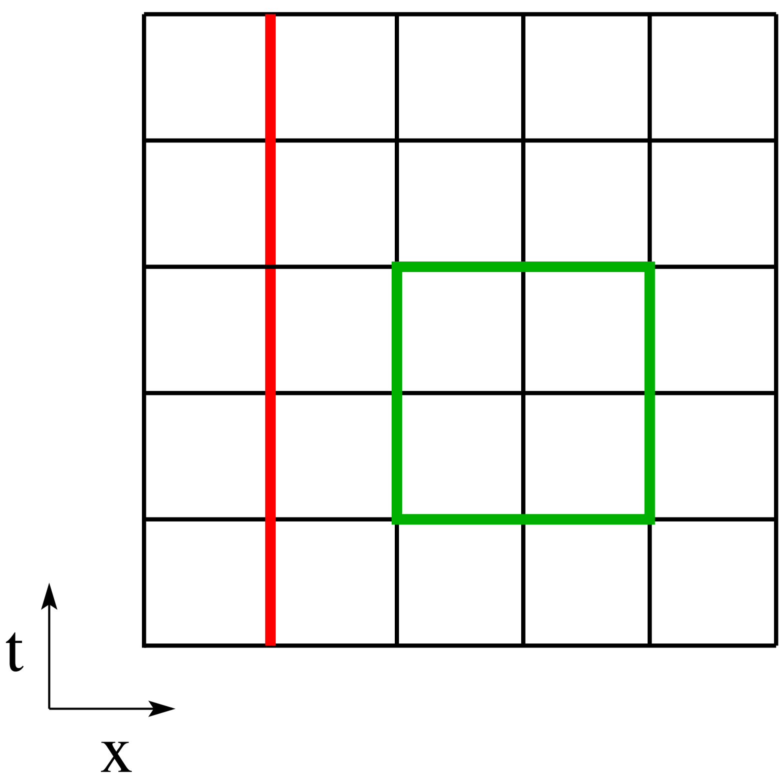
$$E = L \int d^2 x_\perp \frac{1}{2} E_k^a(x) E_k^a(x) = L\sigma$$

- String tension σ
- σ≈1cm thick steel cable
- isolate quark has infinite energy

# Polyakov-Loop and center symmetry

**Wilson-Loop:** $$U(C) = \hat{P} \exp\left[ig \oint_C dx^\mu A_\mu(x)\right]$$

**Polyakov-Loop:** $$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau A_4(\tau, \vec{x})\right]$$




**Center of gauge group SU($N_c$):**

$$z_n = \exp[2\pi i\, n/N_c]\mathbb{1}, \quad n = 0..N_c - 1$$

# Polyakov-Loop and center symmetry

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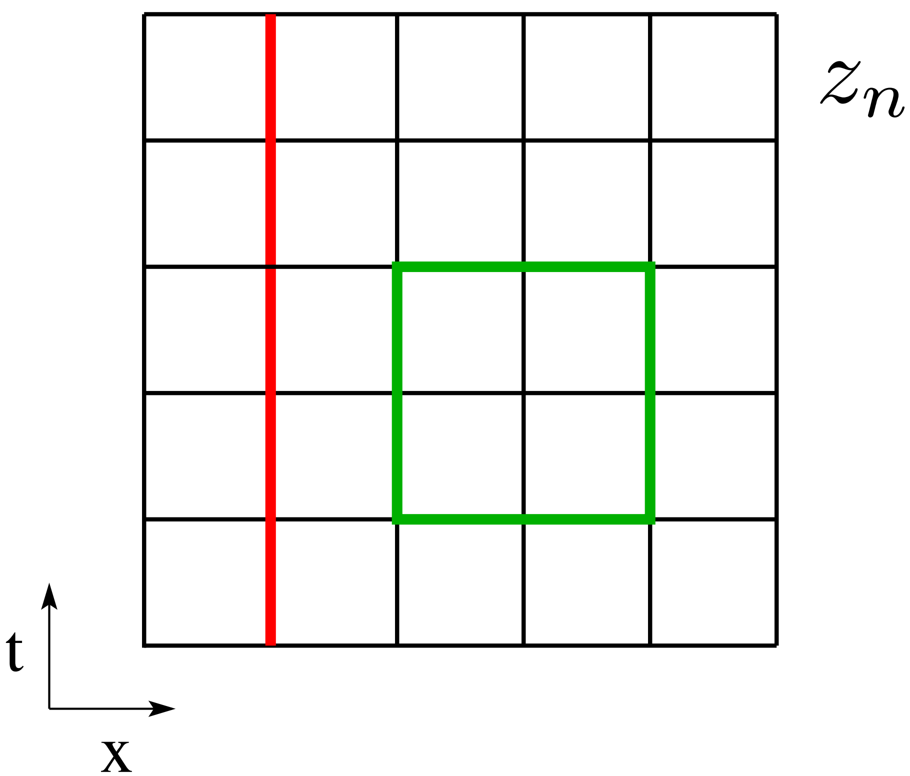


## Center of gauge group SU($N_c$):

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## Center transformation:

$$S_{QCD} \rightarrow S_{QCD}$$
$$\Phi \rightarrow z_n \Phi$$

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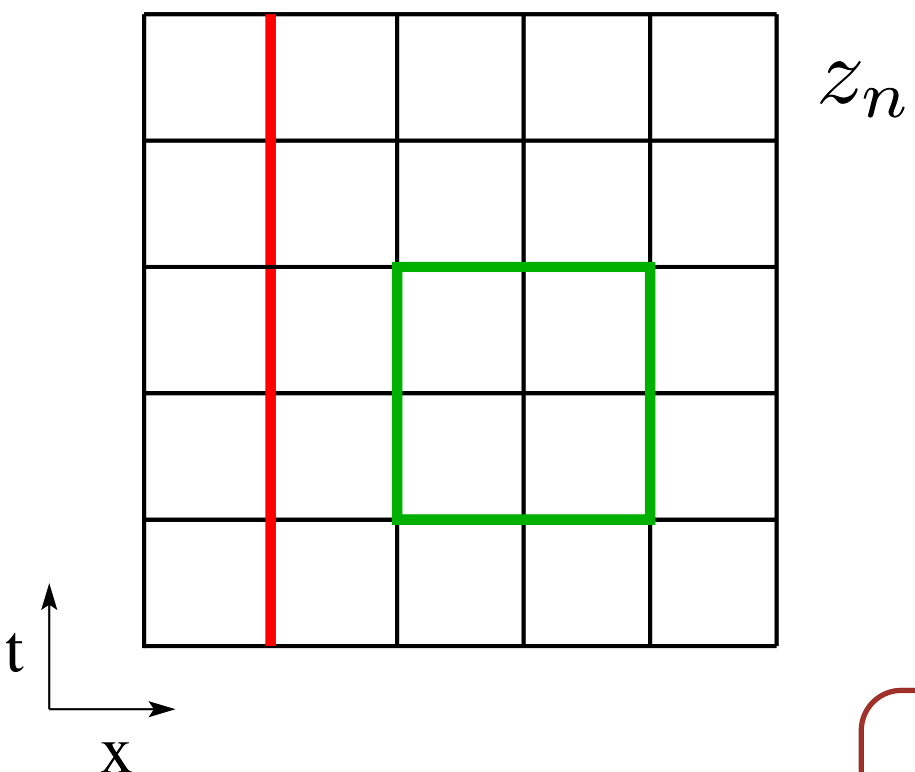


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Center transformation:

$$S_{QCD} \to S_{QCD}$$
$$\Phi \to z_n \Phi$$

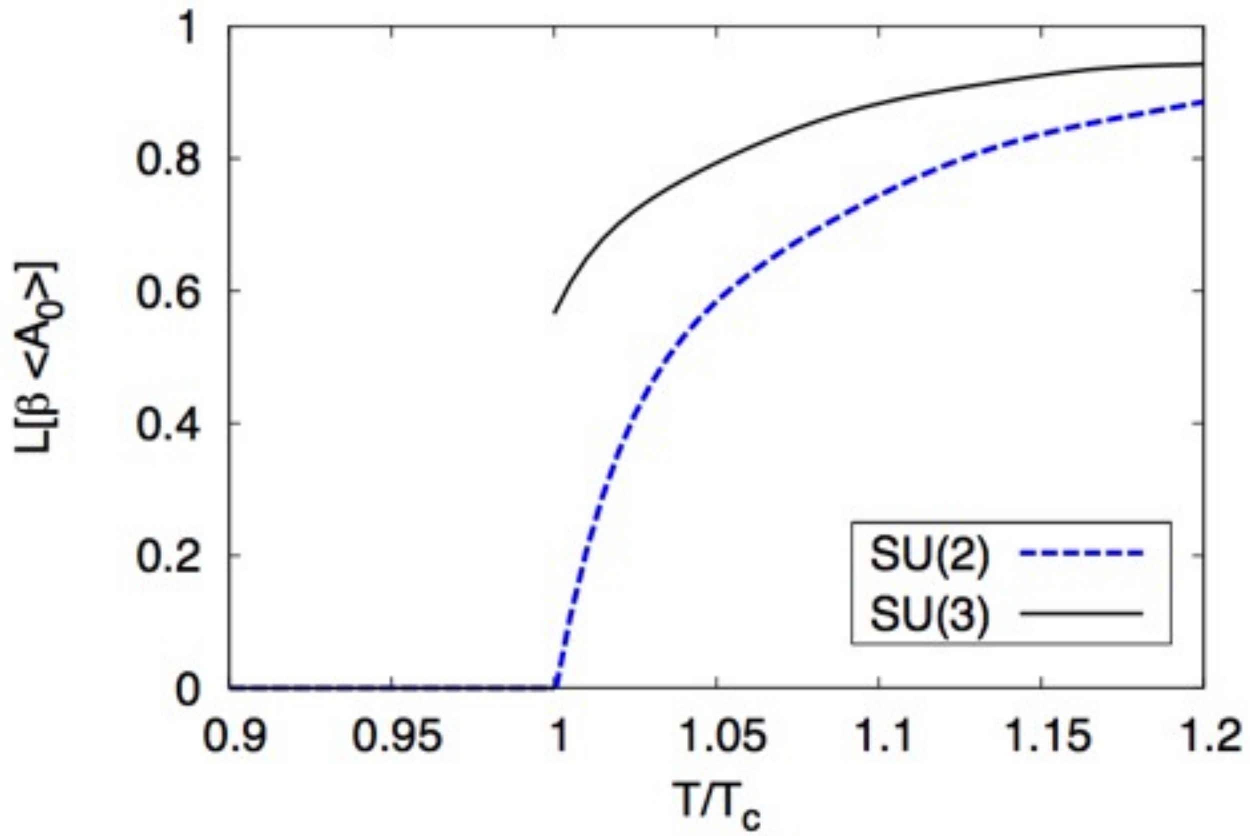
$$\langle Tr\, \Phi \rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

# Energy of an isolated quark

$$\langle Tr\,\Phi \rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

$$\langle Tr\,\Phi \rangle \sim e^{-F_q/T} \qquad F_q = \begin{cases} \infty & \text{unbroken } z_n \text{ symmetry} \\ \text{finite} & \text{broken } z_n \text{ symmetry} \end{cases}$$

$F_q$: free energy of heavy quark



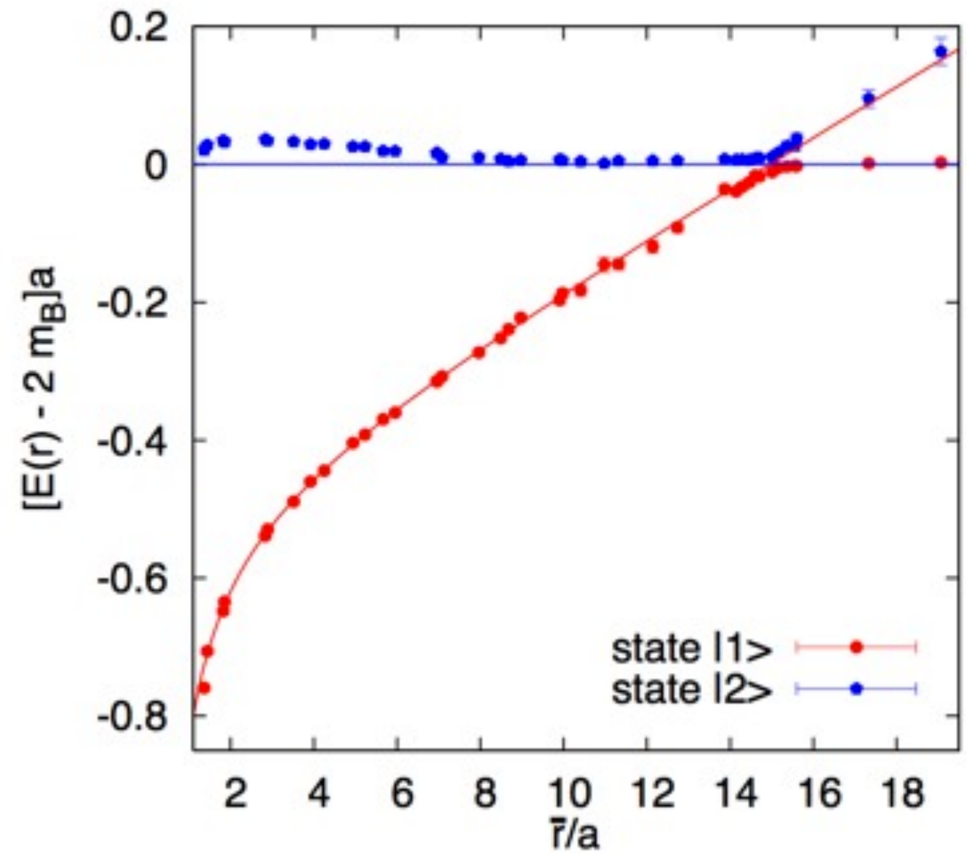

Braun, Gies, Pawlowski, PLB684 (2010)

Order parameter!

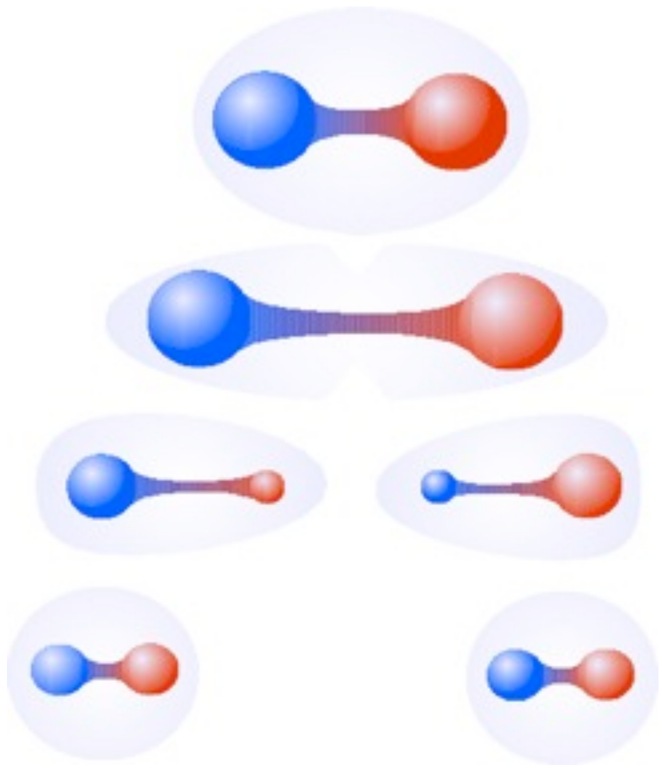
- SU(2): second order
- SU(3): first order

# Confinement: string breaking

QCD:




Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513



$$\langle Tr\,\Phi \rangle \sim e^{-F_q/T} \neq 0$$



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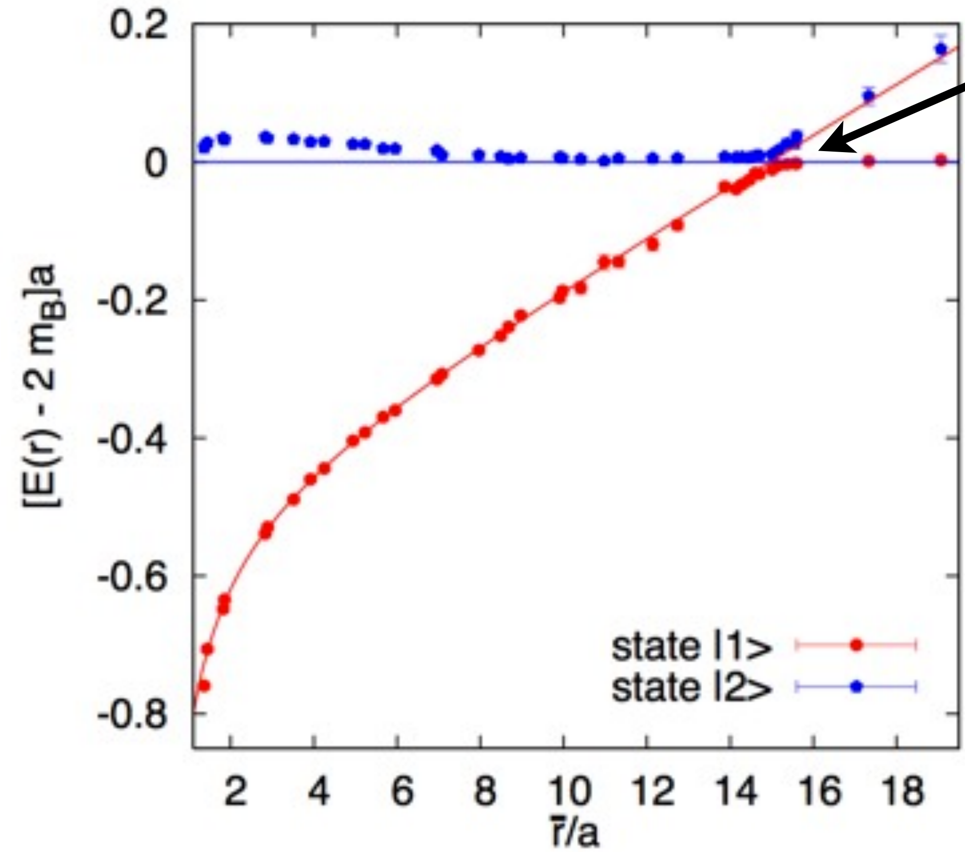
# Confinement: string breaking

QCD:

string breaking

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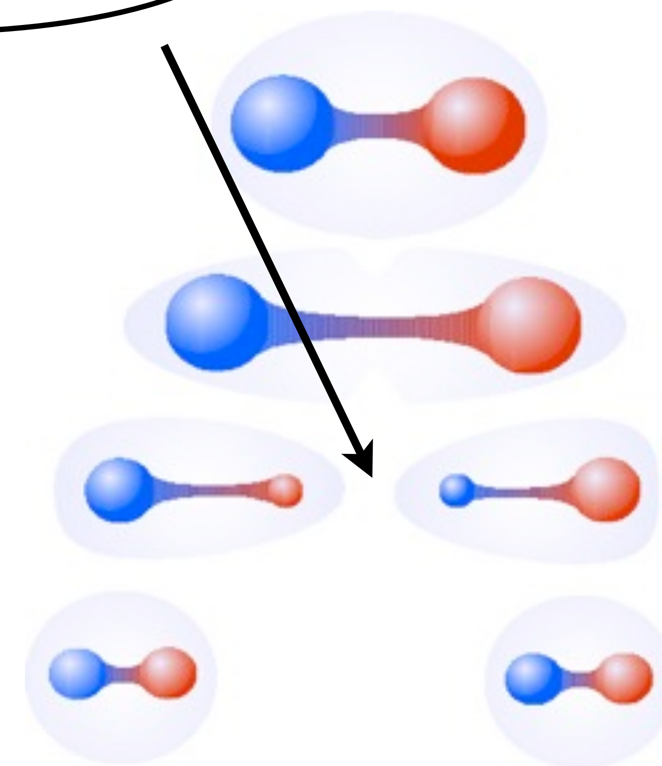

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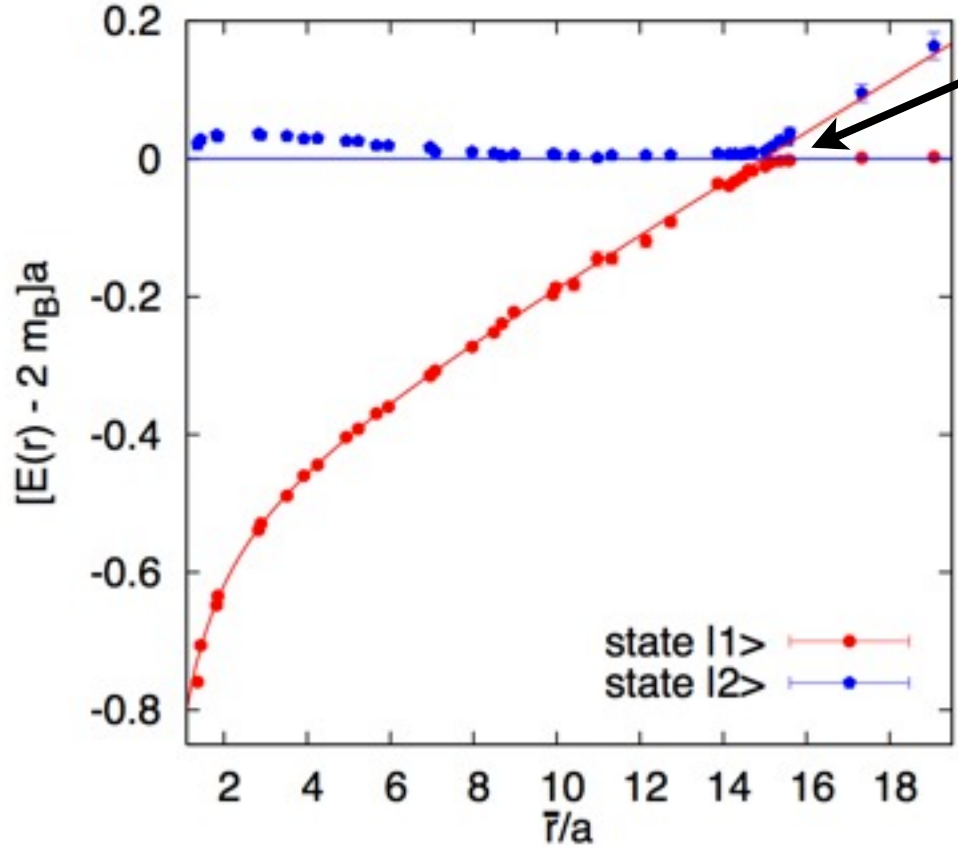
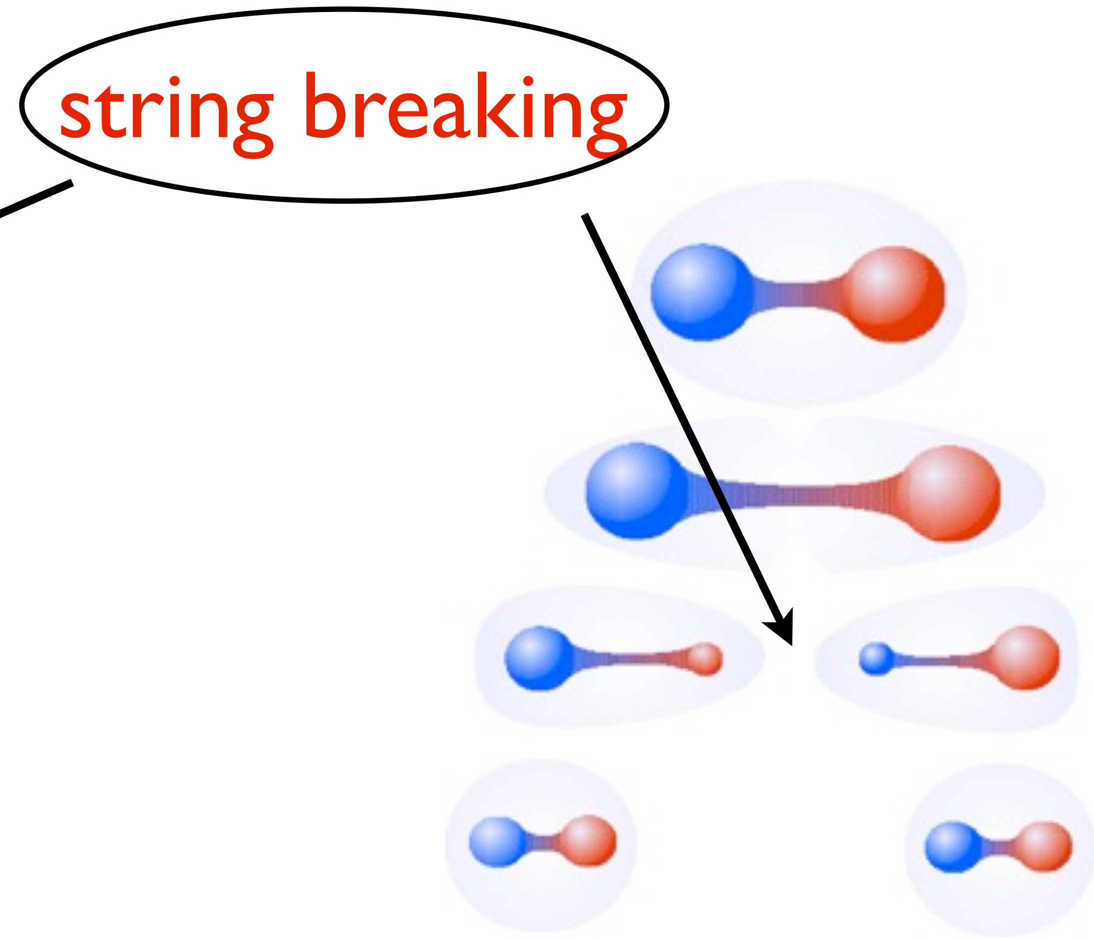



Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513




$$\langle Tr\,\Phi \rangle \sim e^{-F_q/T} \neq 0$$

- String breaking requires presence of dynamical charges in fundamental representation of SU($N_c$)
- Dynamical fundamental charges break center symmetry
- ⇒ QCD is not confining in strict sense



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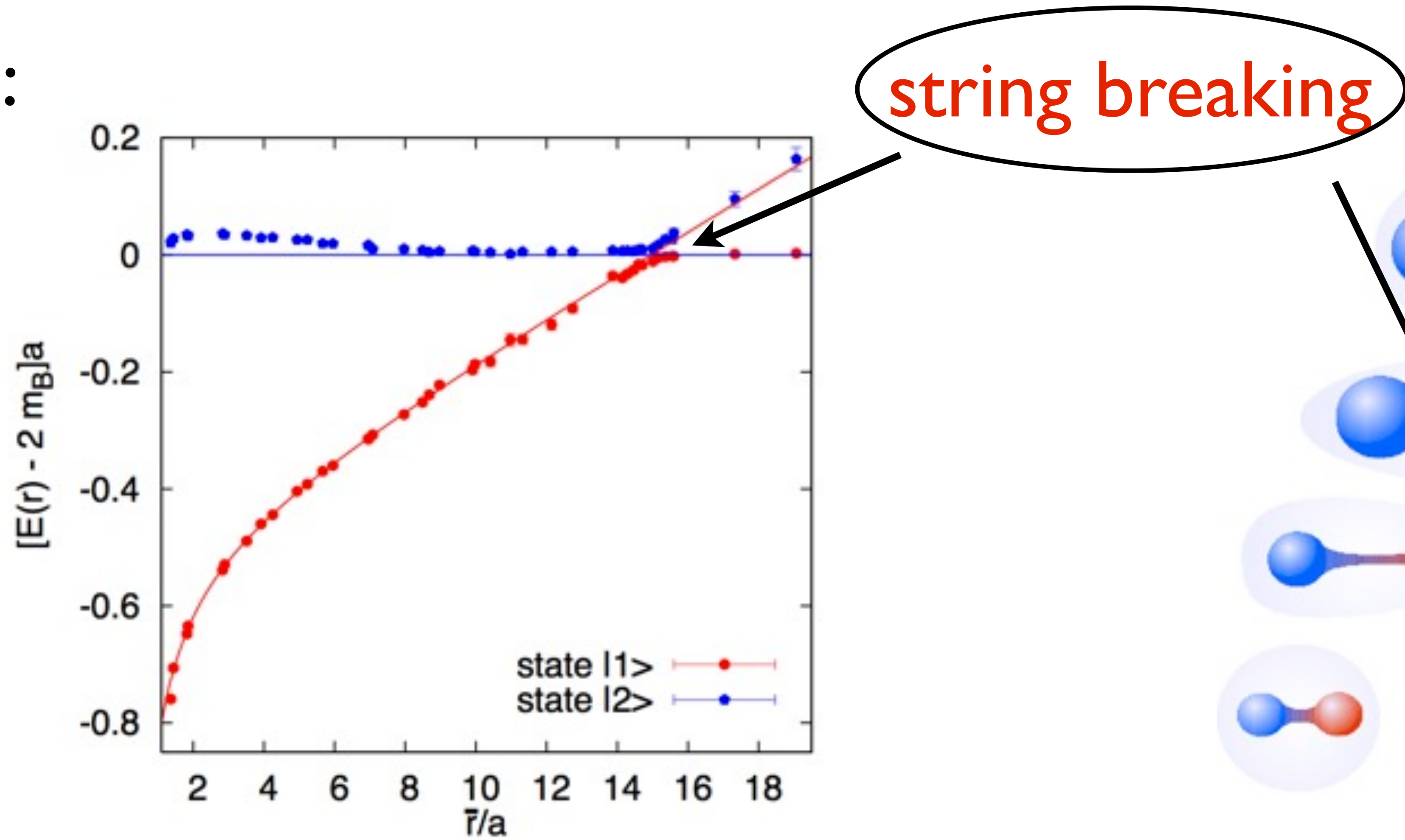
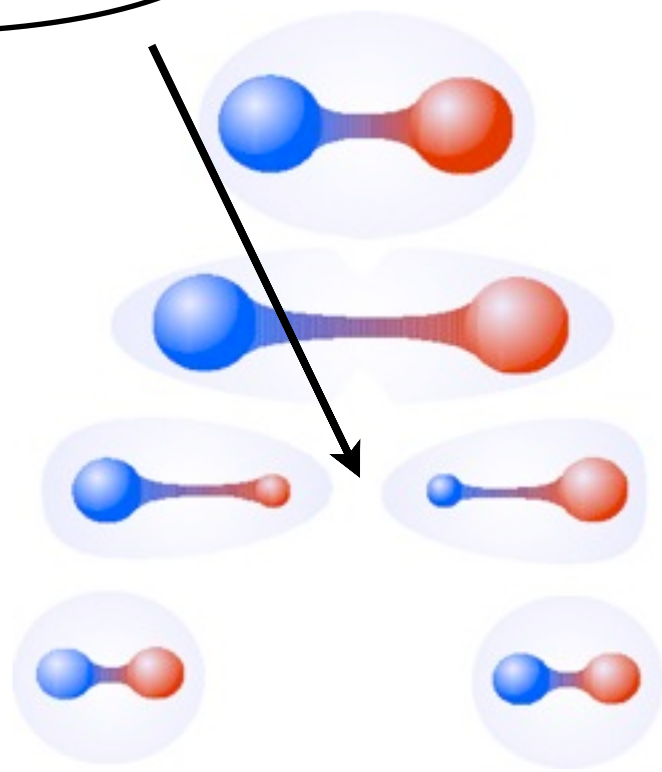
QCD:



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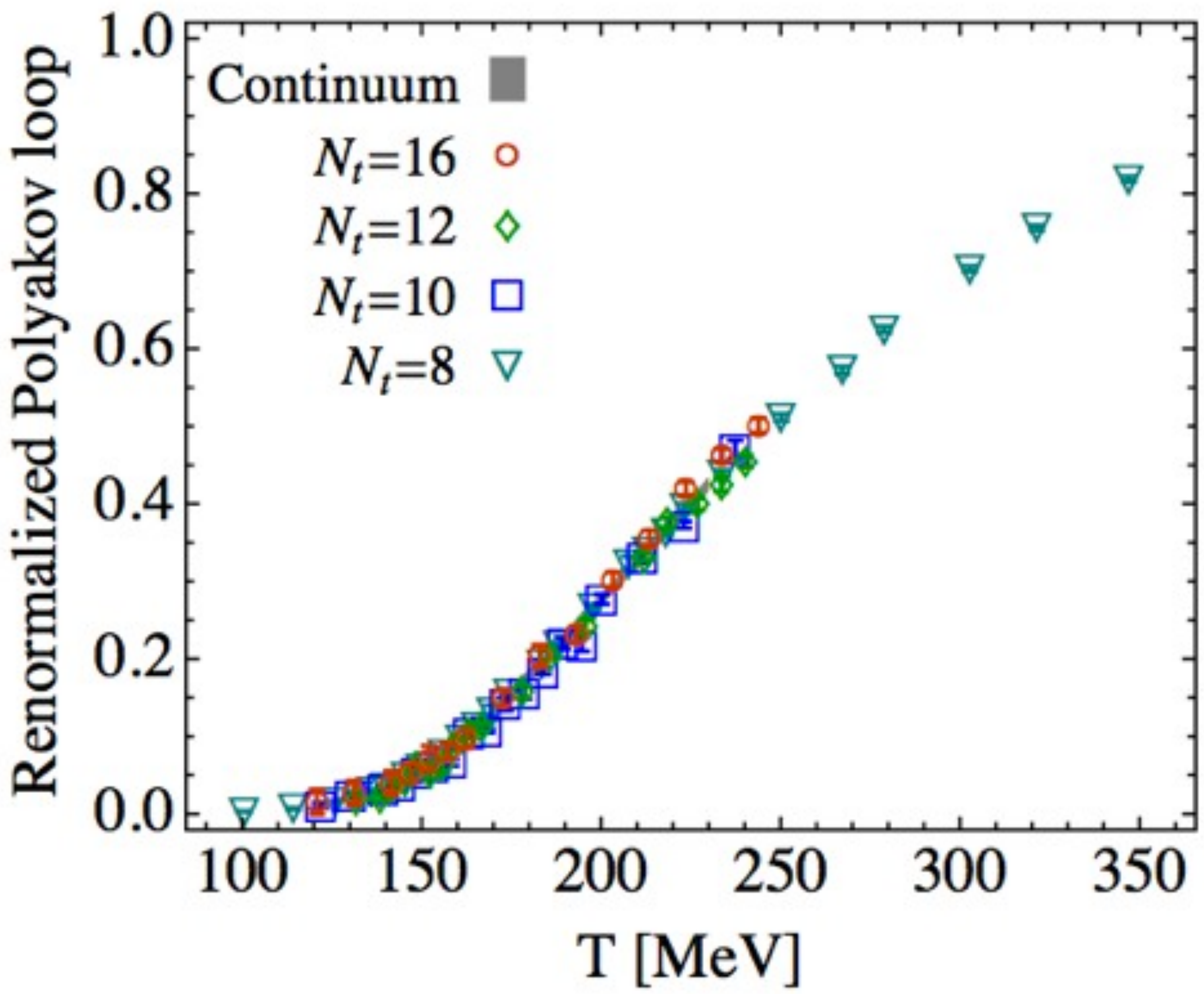
**Confinement ≡ asymptotic string tension**

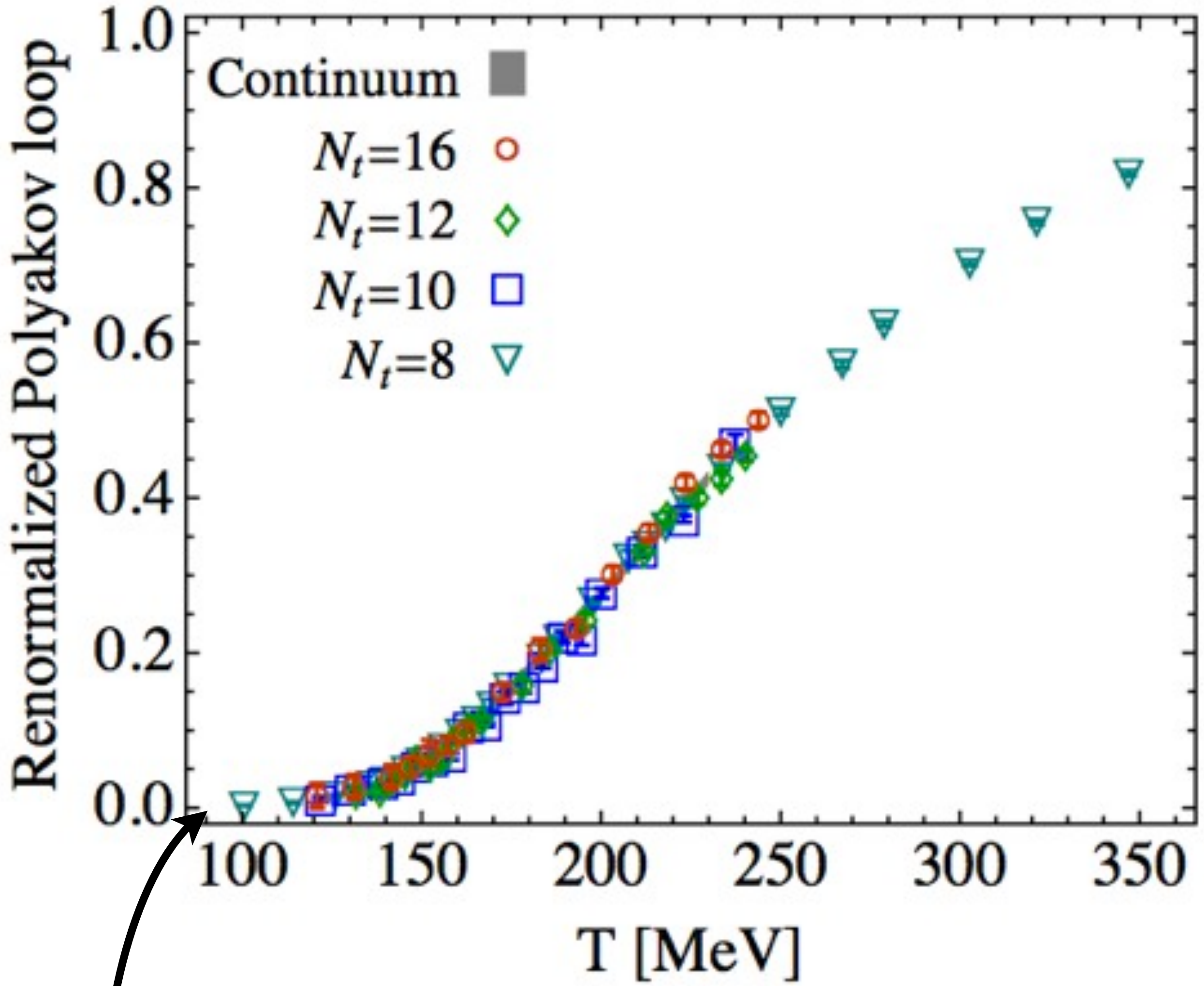
# Polyakov-Loop of QCD




Borsanyi et.al., POS Lattice 2010

- $N_f$=2+1 quark flavors
- Crossover!
- No order parameter in strict sense...

# Polyakov-Loop of QCD




Borsanyi et.al., POS Lattice 2010

Non-zero !

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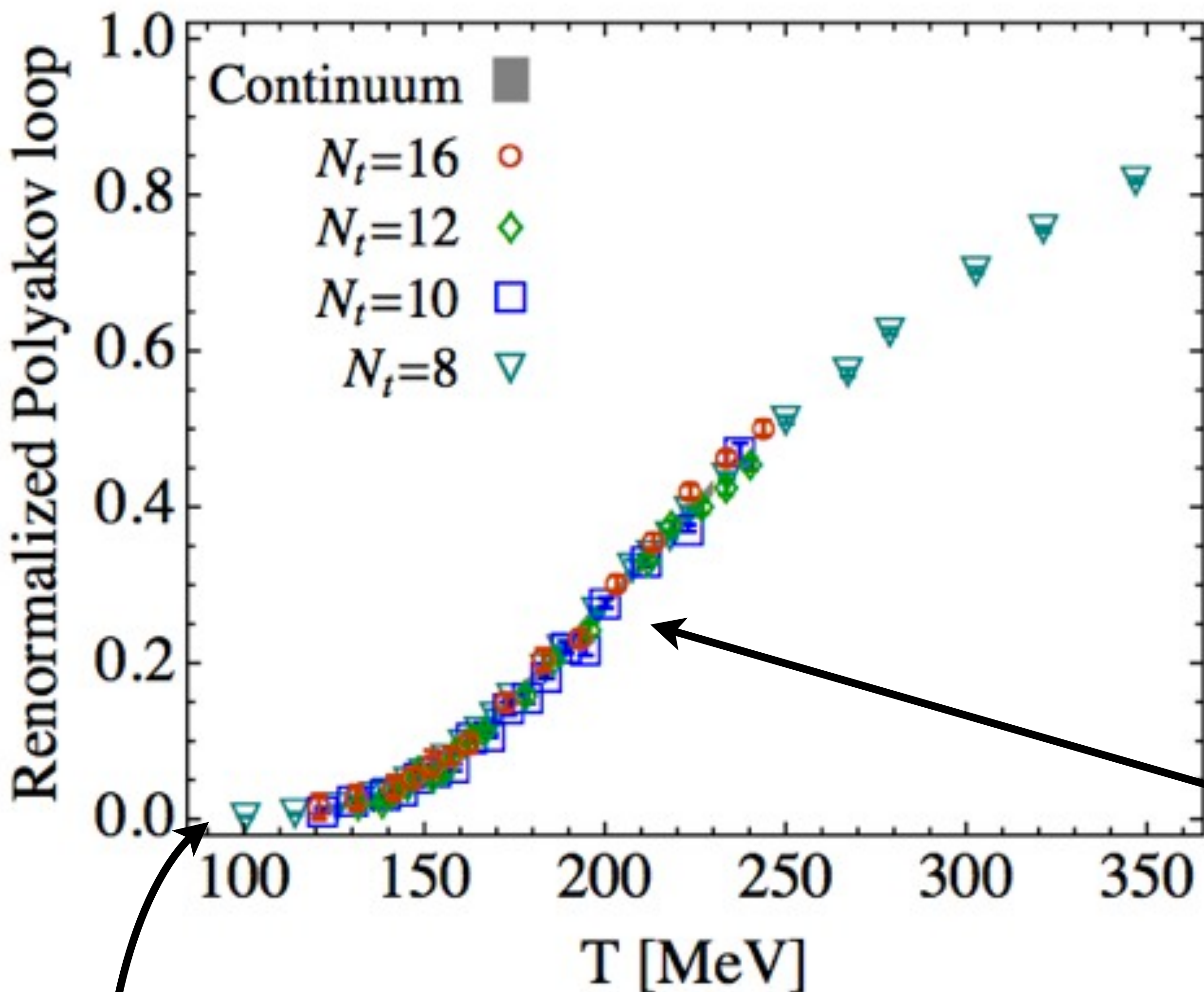
# Polyakov-Loop of QCD




Borsanyi et.al., POS Lattice 2010

- $N_f$=2+1 quark flavors
- Crossover!
- No order parameter in strict sense...

Transition temperature via derivative

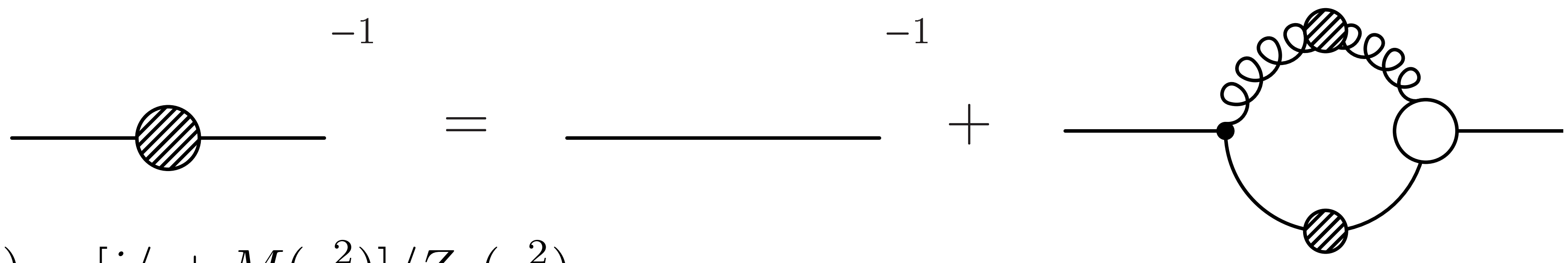
Non-zero !

- $N_f = 0\,(SU(2))\ :\ 305\,\mathrm{MeV}$
- $N_f = 0\,(SU(3))\ :\ 275\,\mathrm{MeV}$
- $N_f = 2+1\ :\ 180\,\mathrm{MeV}$

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

| | u | d | s | c | b | t |
|---|---|---|---|---|---|---|
| $M_{weak}$ $[MeV/c^2]$ | 3 | 5 | 80 | 1200 | 4500 | 176000 |
| $M_{strong}$ $[MeV/c^2]$ | 350 | 350 | 350 | 350 | 350 | 350 |
| $M_{total}$ $[MeV/c^2]$ | 350 | 350 | 450 | 1500 | 4800 | 176000 |



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

# Properties of QCD: Dynamical mass generation

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

Input parameters in $N_f$=2+1 QCD

| | u | d | s | c | b | t |
|---|---|---|---|---|---|---|
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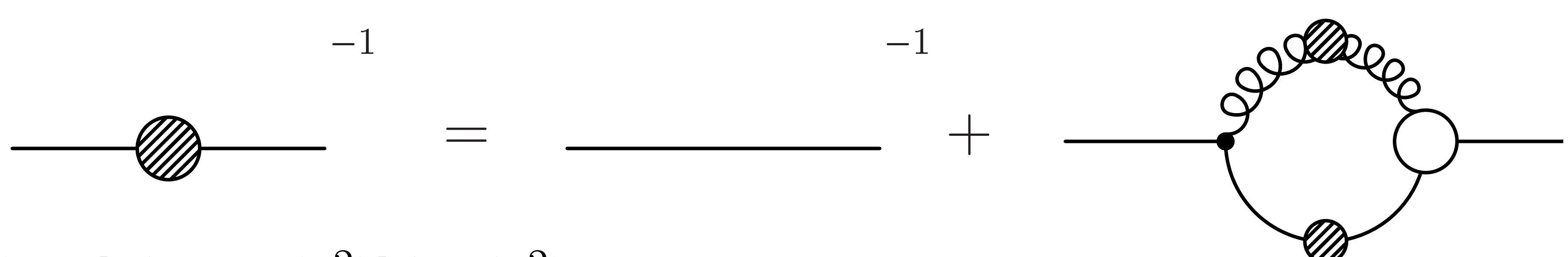


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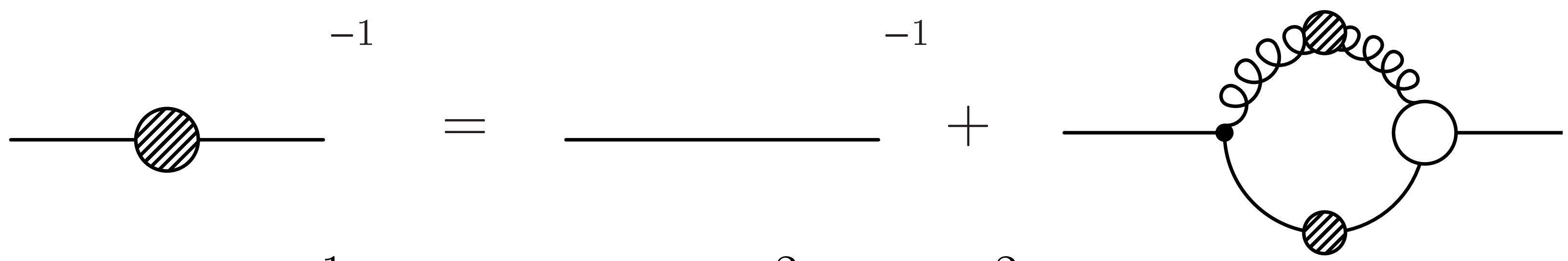


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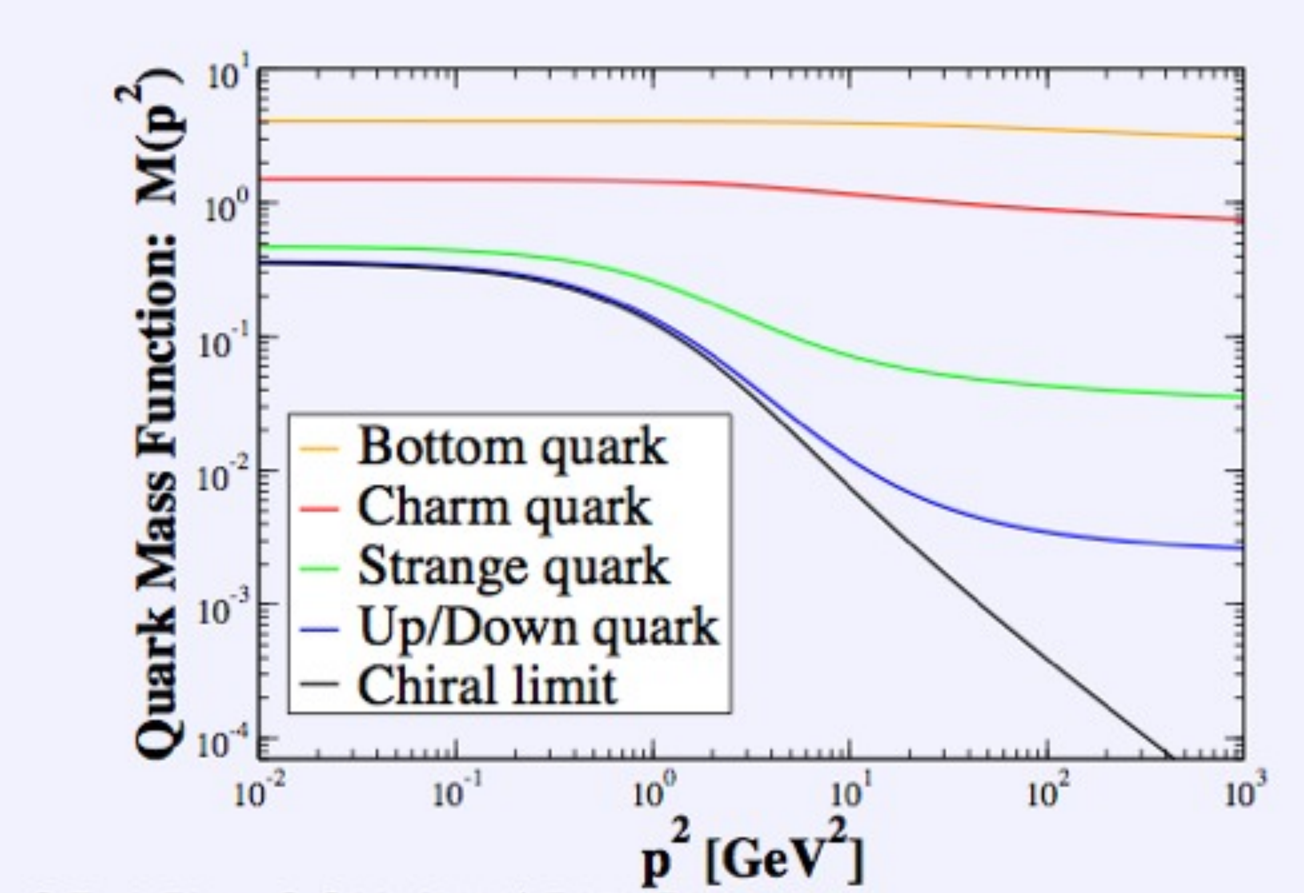
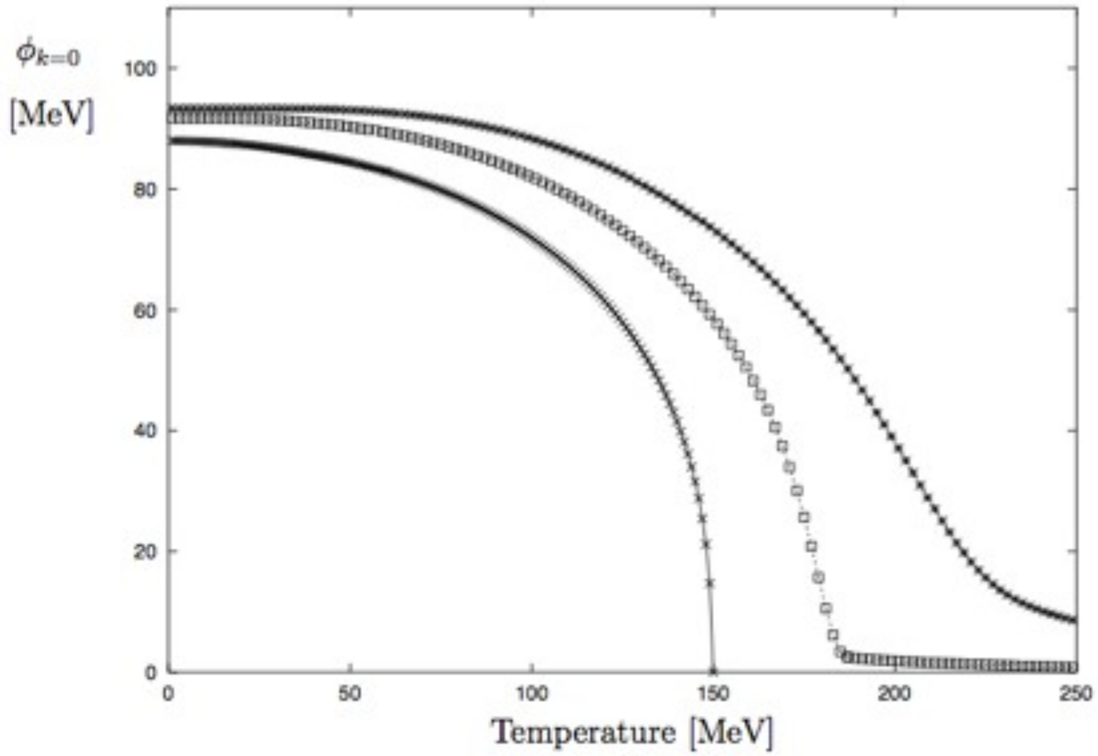


C.F., J. Phys. G **G32** (2006) R253-R291.

- order parameter: chiral condensate
  $$\langle \bar{\psi}\psi \rangle = Z_2 N_c Tr \int \frac{d^4p}{(2\pi)^4} S(p)$$
- dynamical mass $M(p^2)$
- flavor dependence because of $M_{weak}$

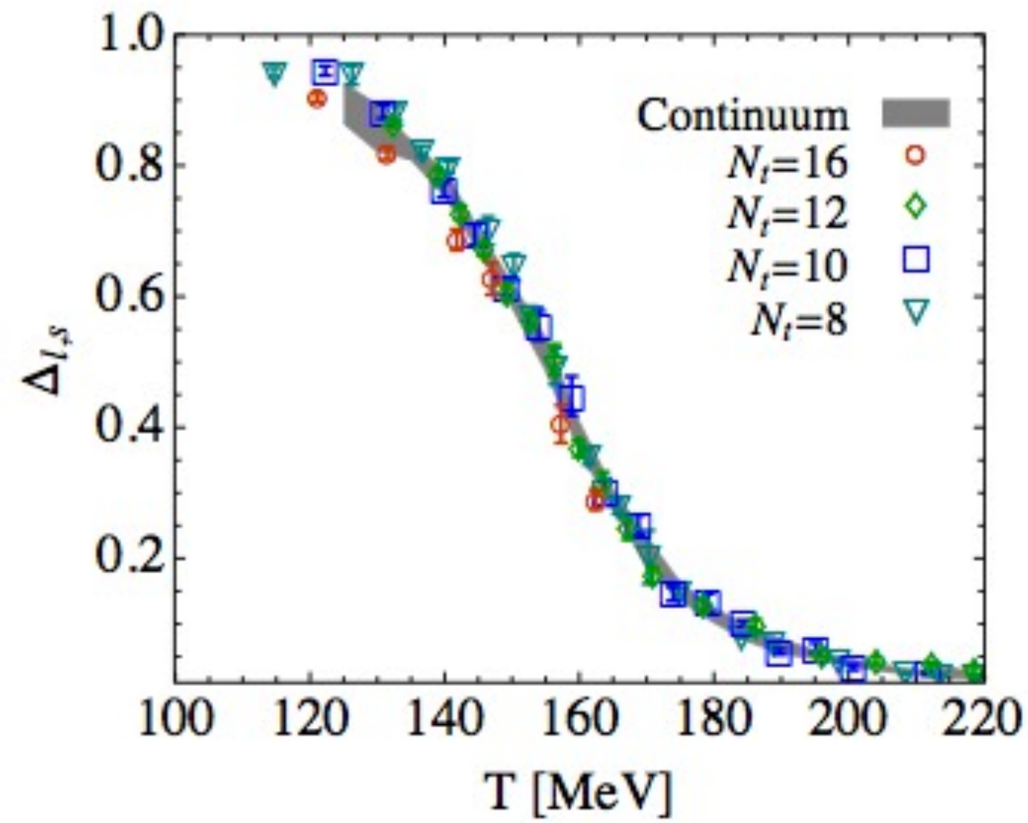
# Quark condensate at finite temperature

QM-model:




Berges, Jungnickel, Wetterich, PRD 59 (1999) 034010
Schaefer, Pirner, NPA 660 (1999) 439

Lattice, $N_f$=2+1:



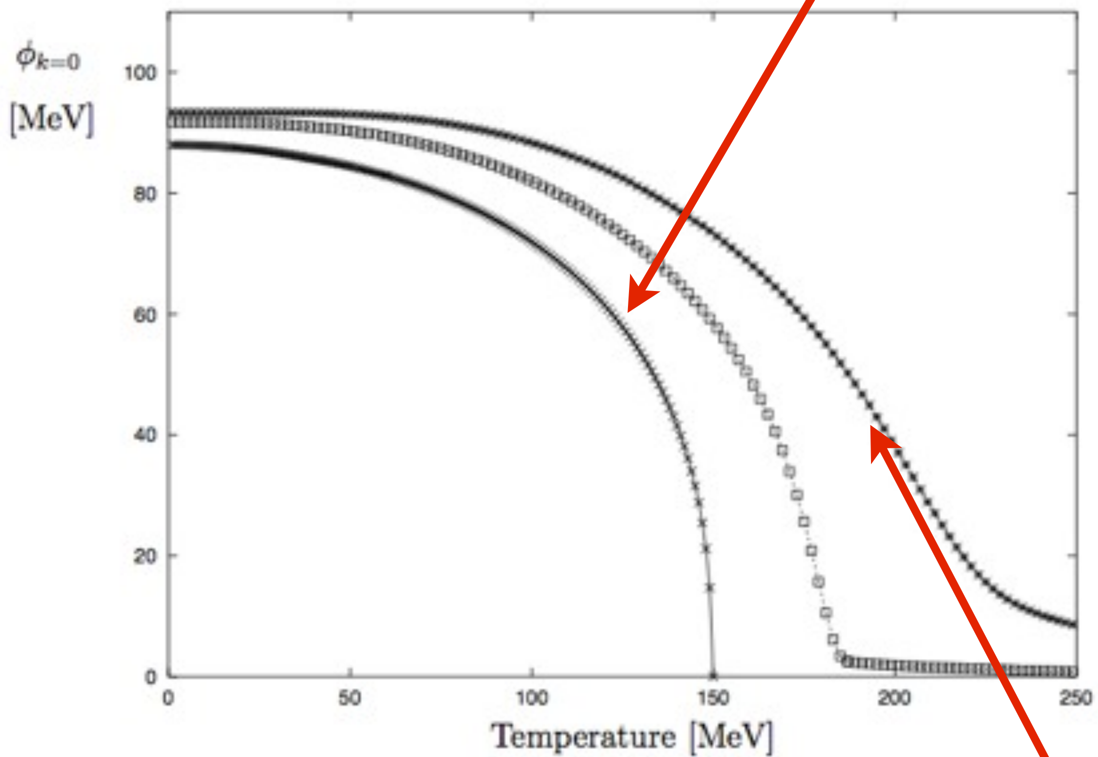

Borsanyi et.al., POS Lattice 2010

- $N_f$=2+1: $T_c$ = 160 MeV

# Quark condensate at finite temperature

$M_{weak}=0$

QM-model:




Berges, Jungnickel, Wetterich, PRD 59 (1999) 034010
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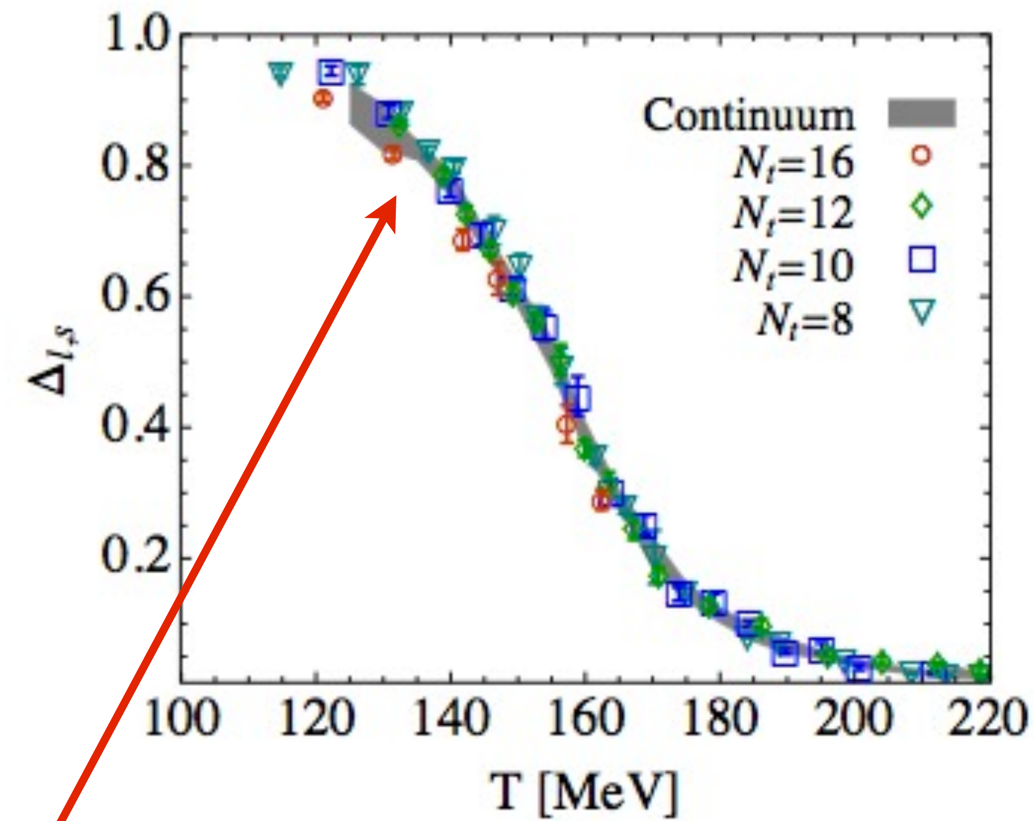
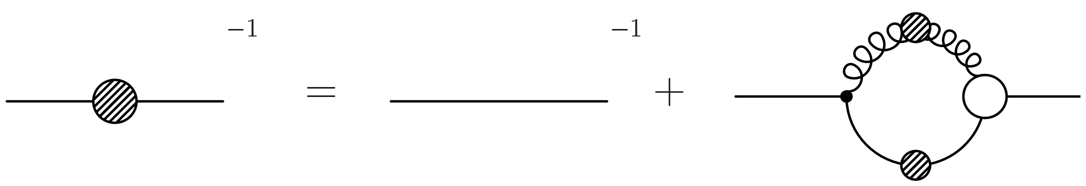
Lattice, $N_f=2+1$:



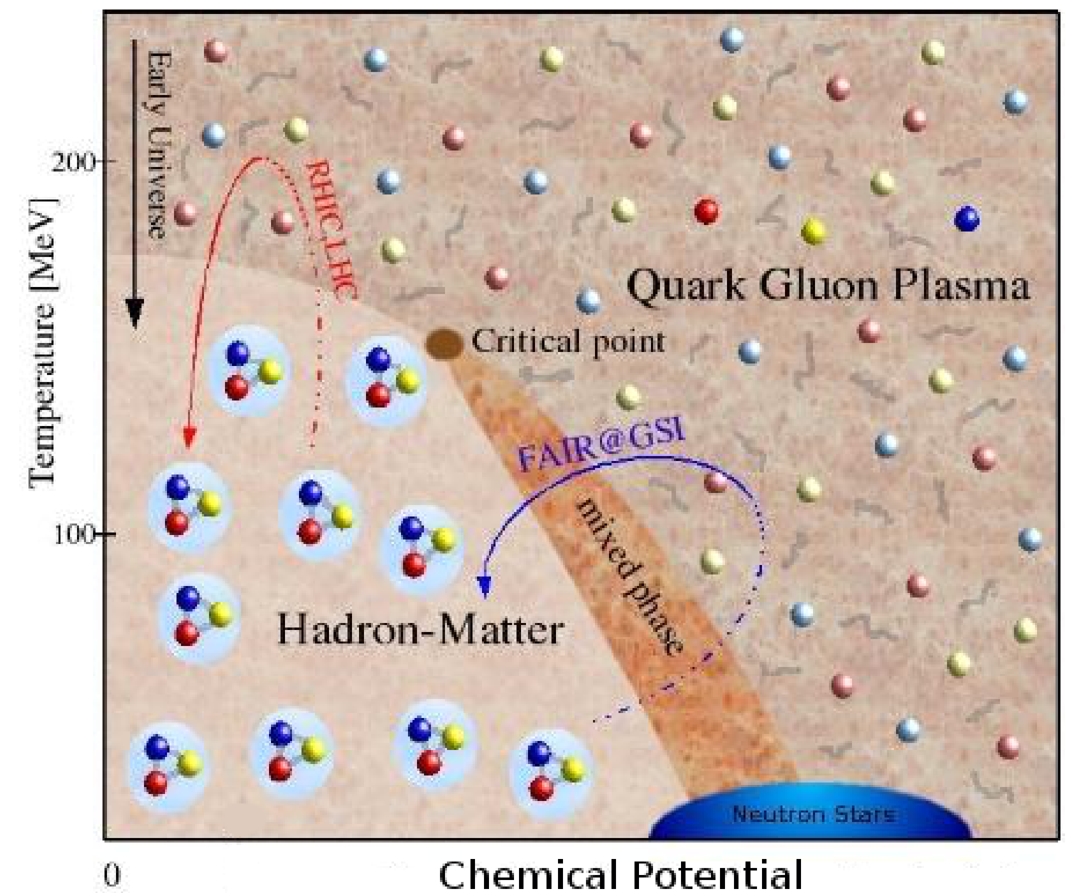

Borsanyi et.al., POS Lattice 2010

$M_{weak} = m_u, m_d, m_s$

- $N_f=2+1$: $T_c$ = 160 MeV



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$



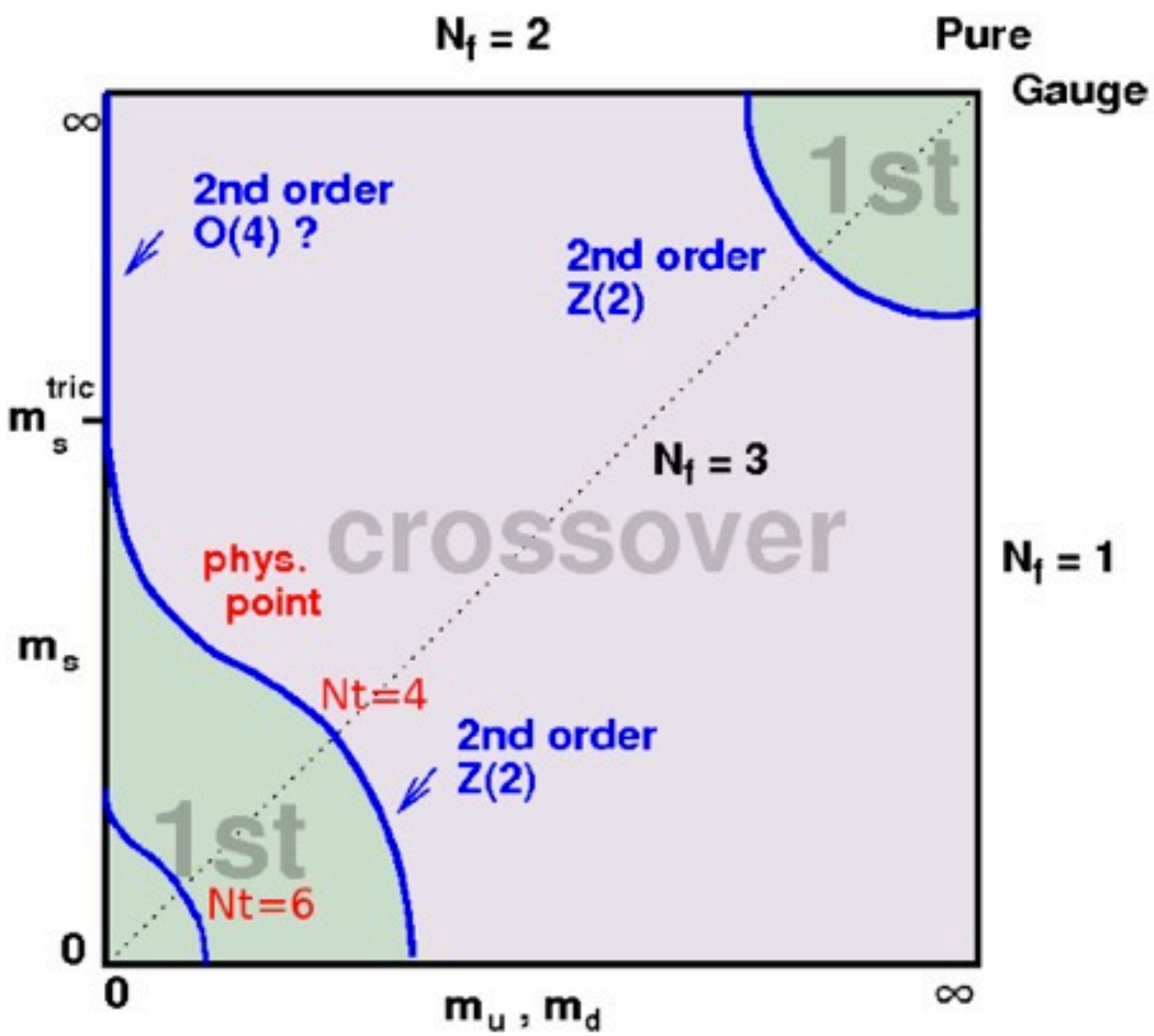

**Phase transitions:**

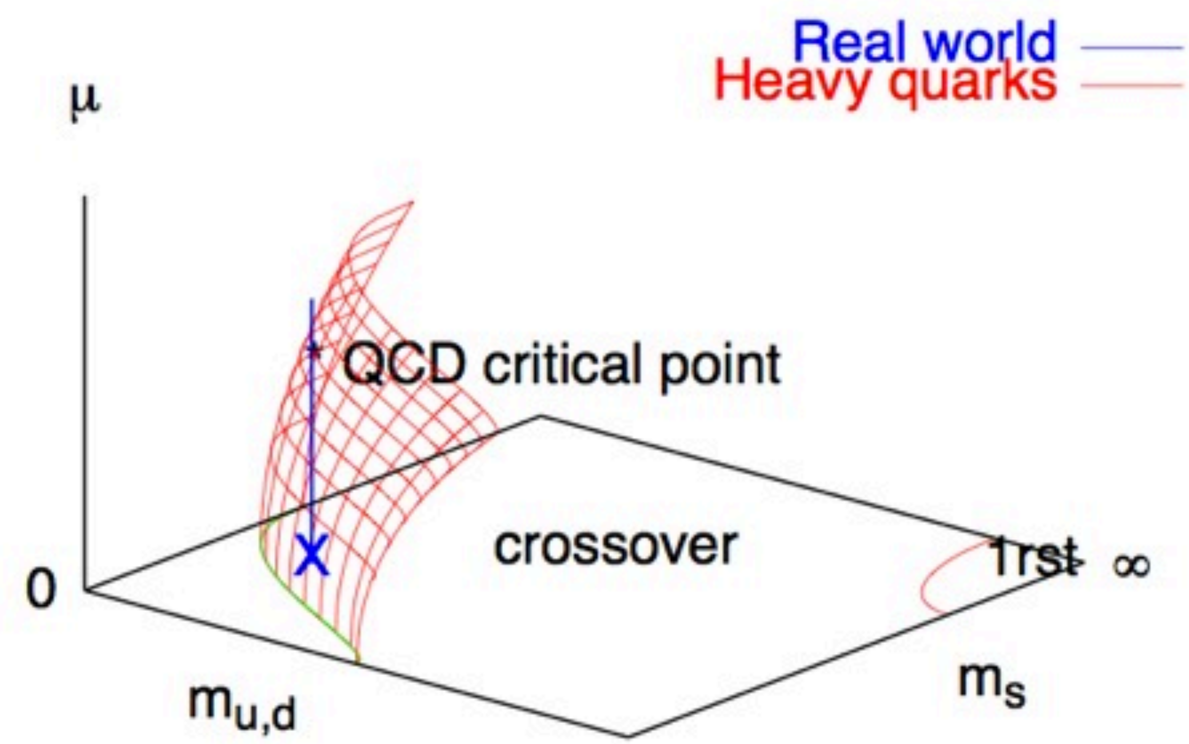
- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c Tr_D \int \frac{d^4p}{(2\pi)^4} S(p)$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

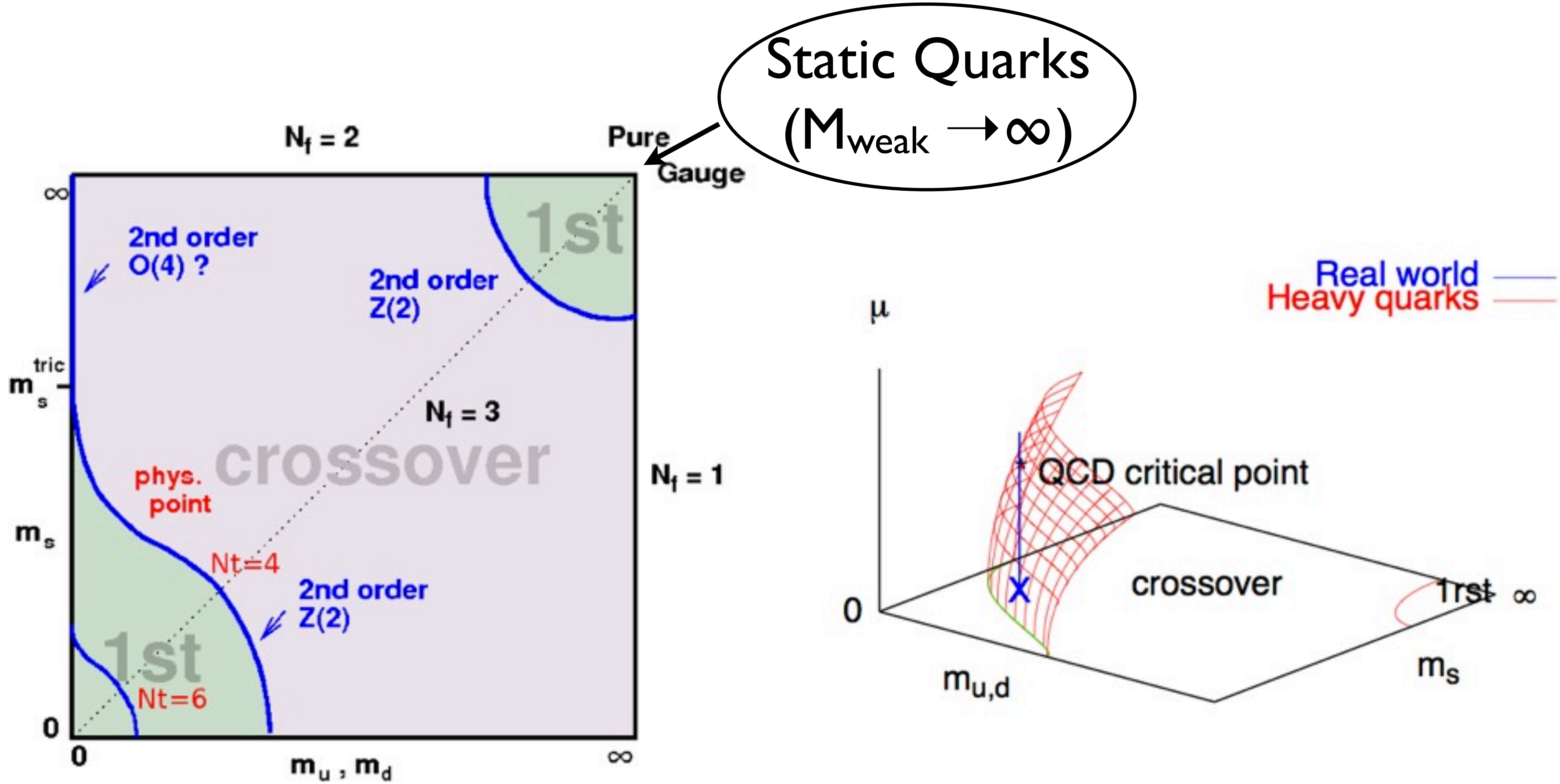
$$\Phi \sim e^{-F_q/T}$$




P. de Forcrand and O. Philipsen, PoS LATTICE 2008 (2008) 208

# QCD phase transitions II




P. de Forcrand and O. Philipsen, PoS LATTICE 2008 (2008) 208

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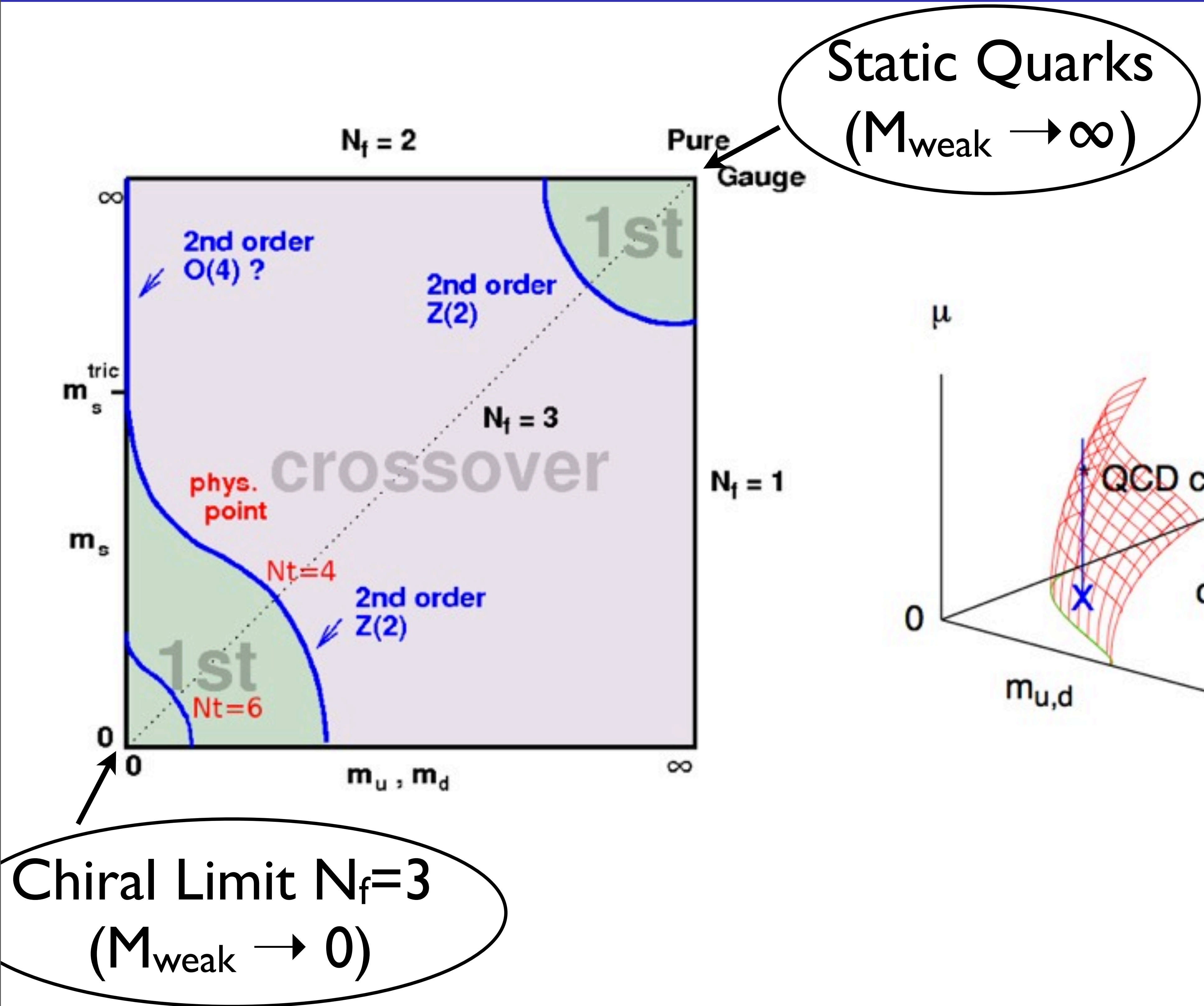


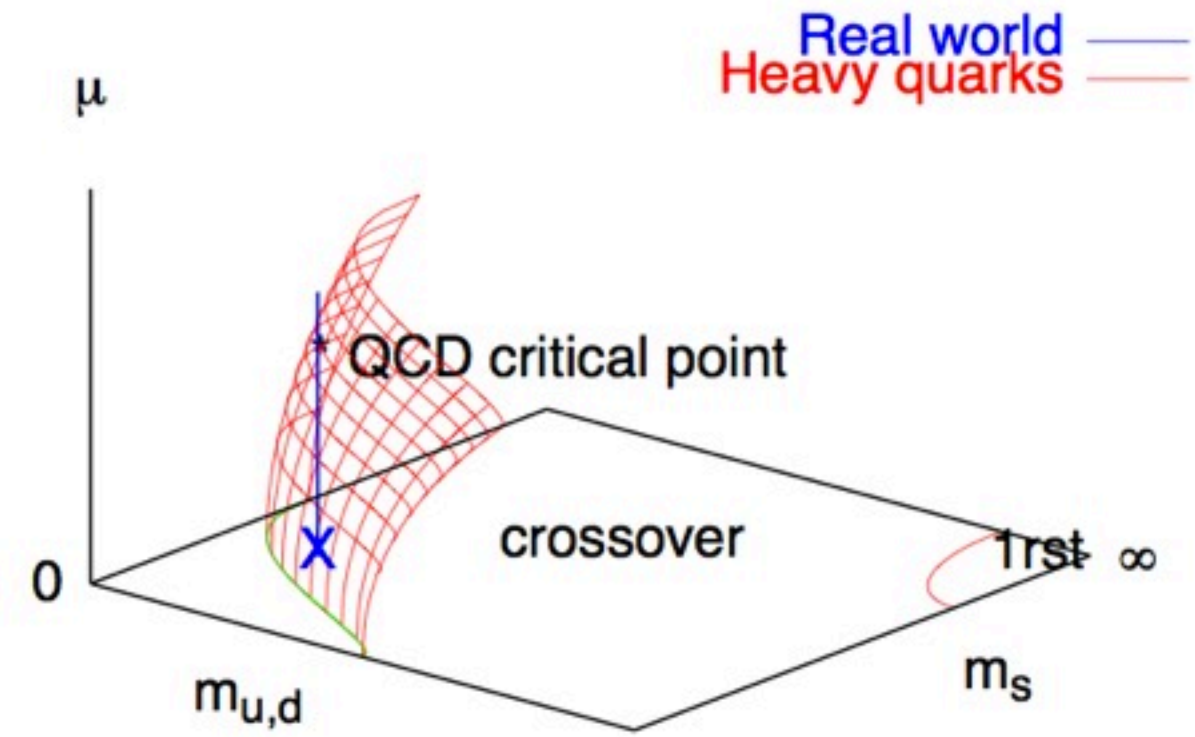


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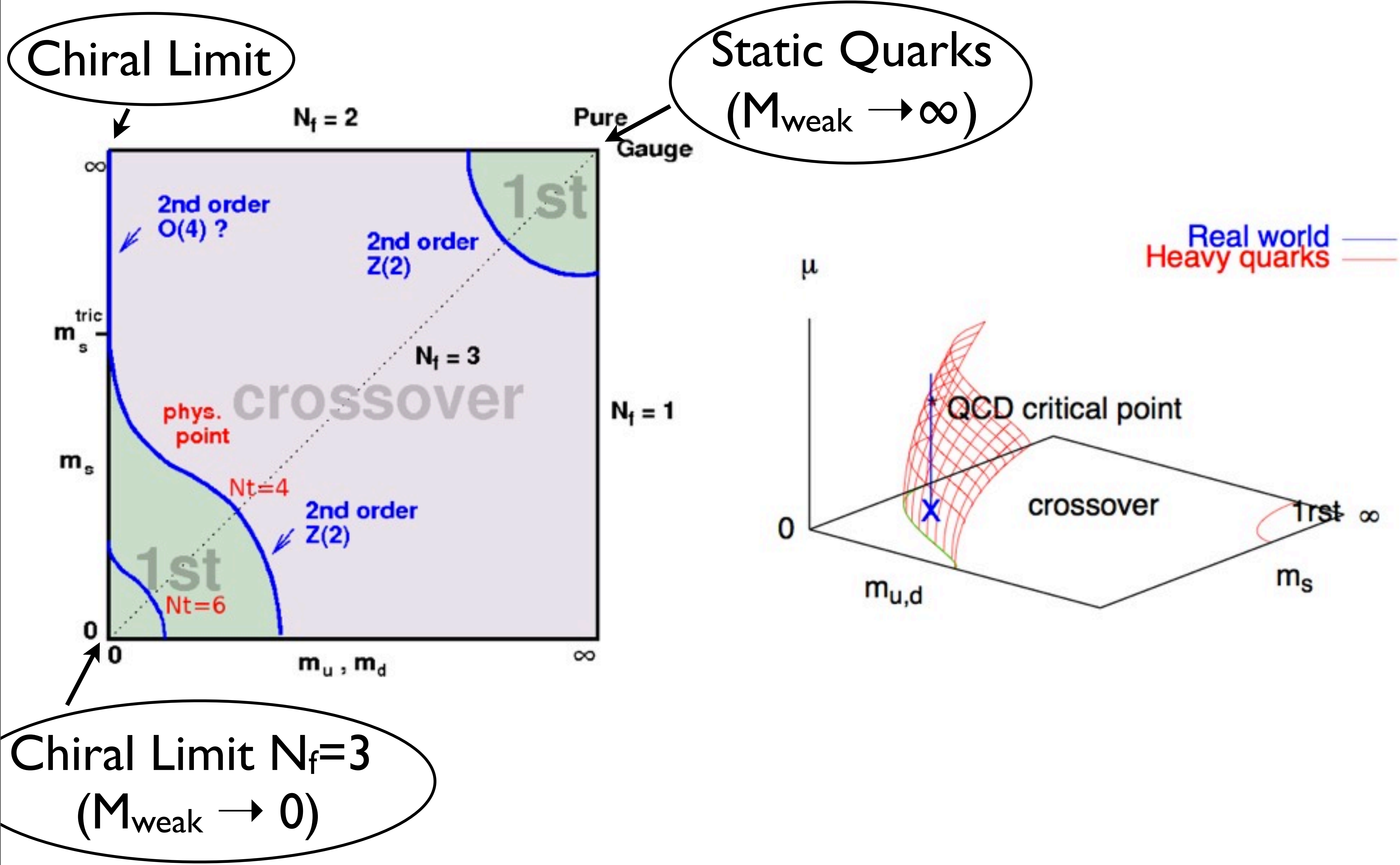


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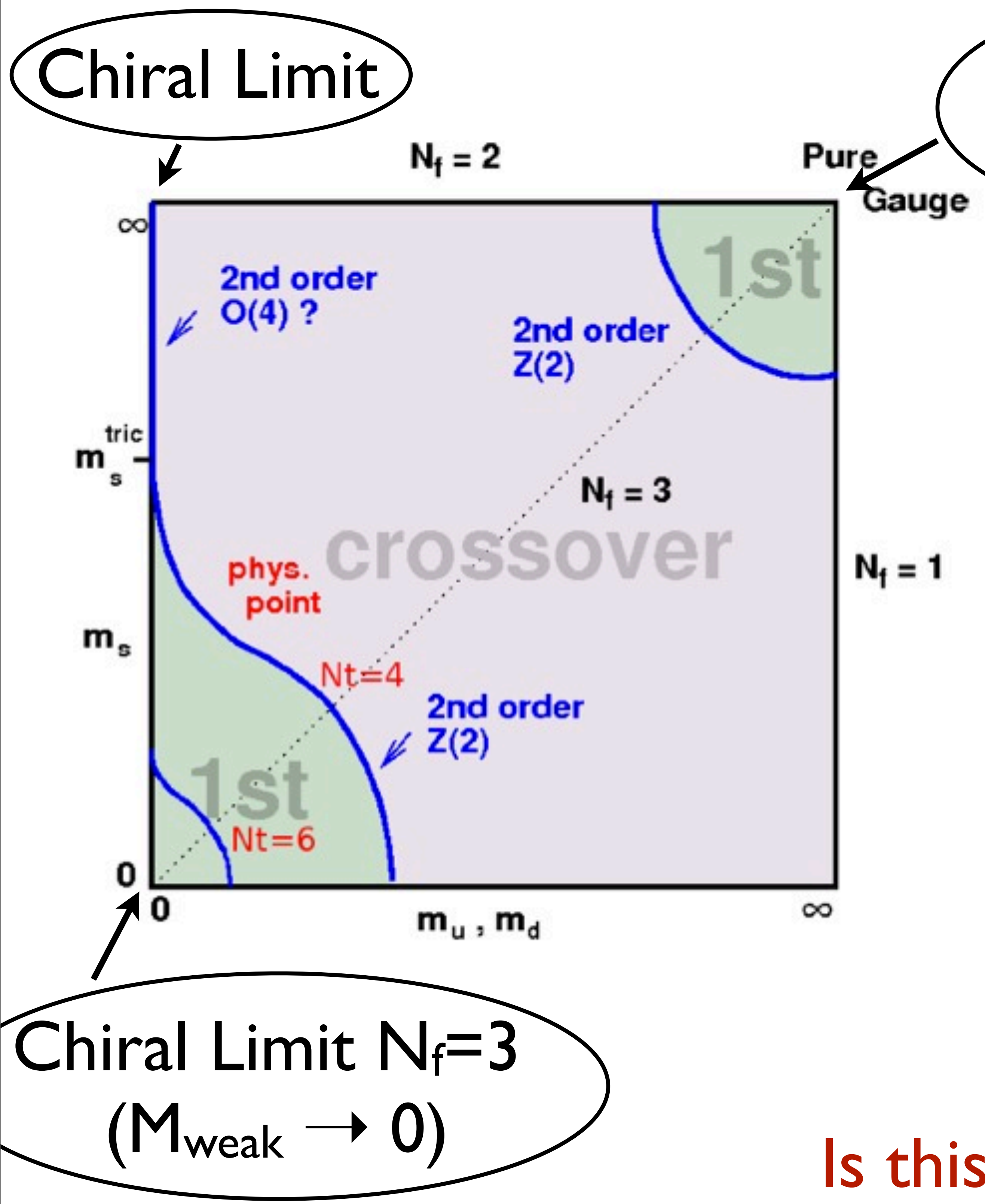
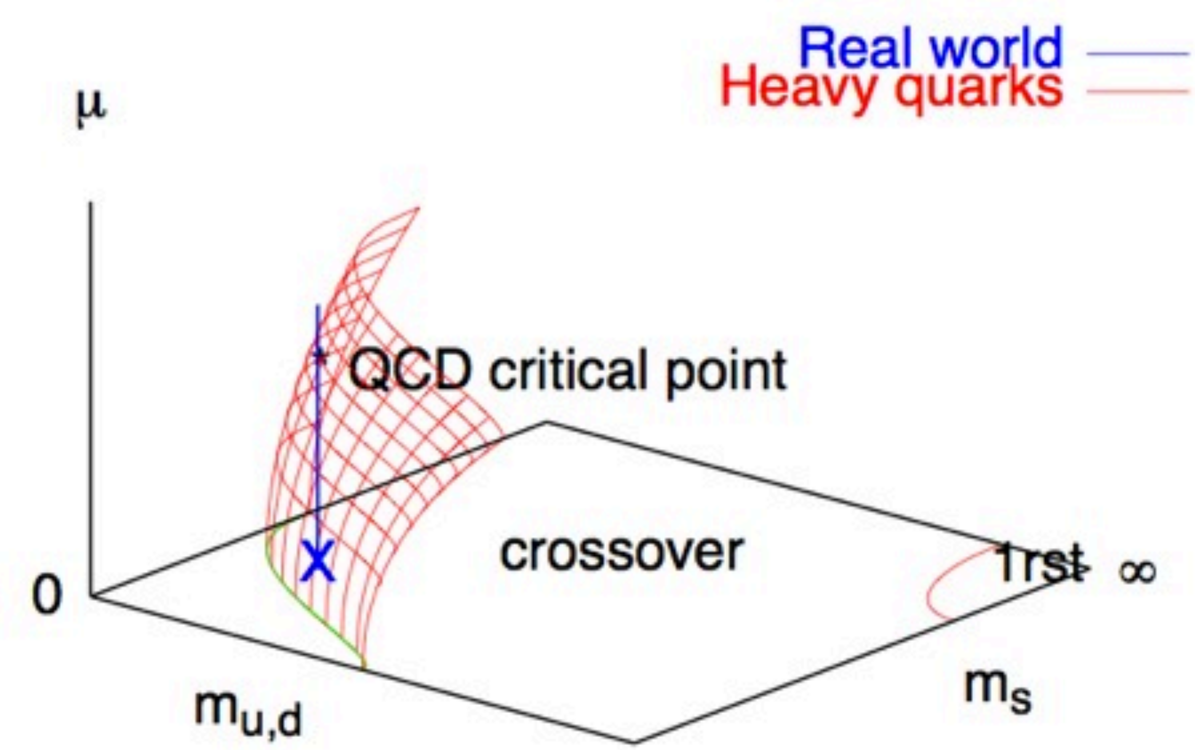
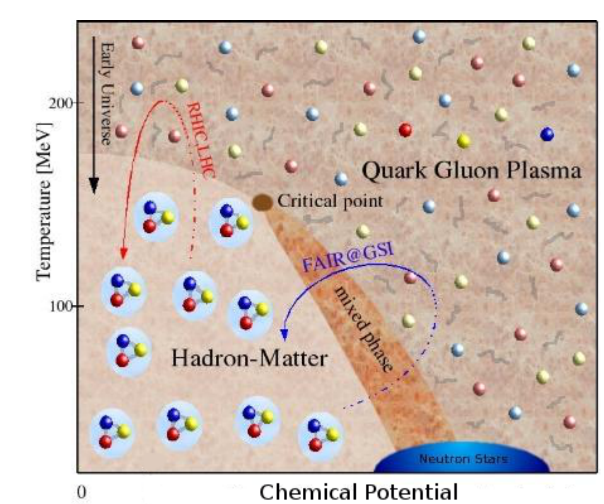


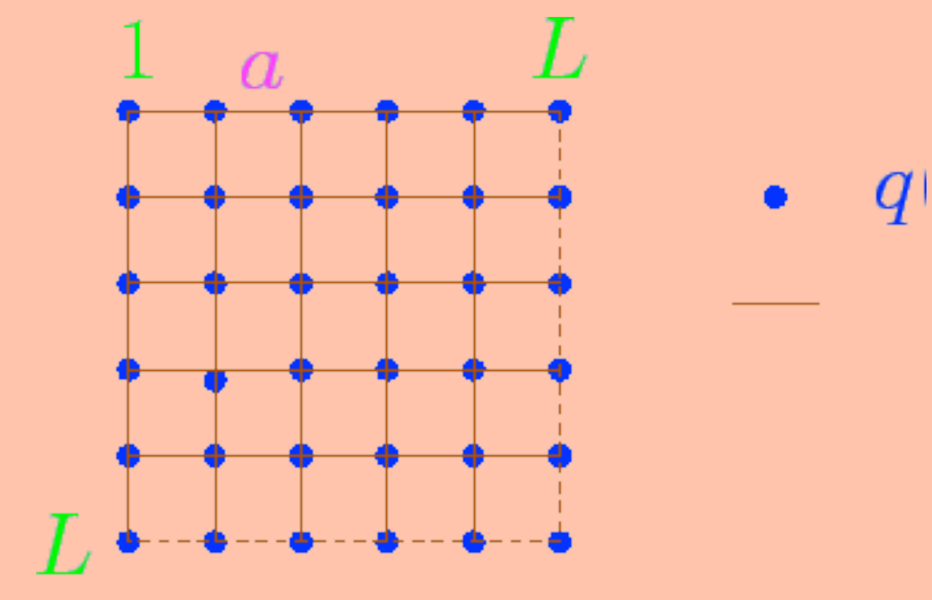




Is this happening ??

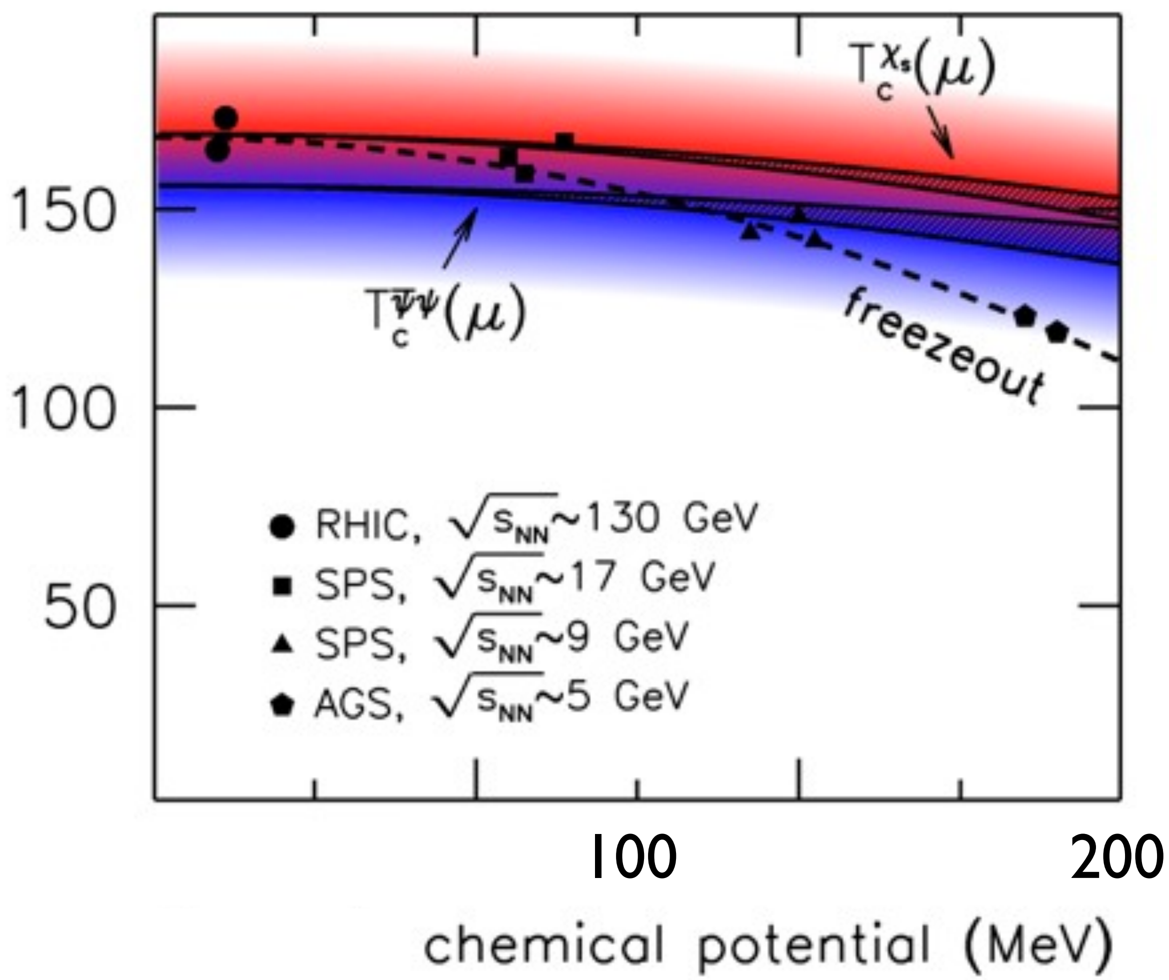
P. de Forcrand and O. Philipsen, PoS LATTICE 2008 (2008) 208

# Lattice QCD vs. DSE/FRG: Complementary!

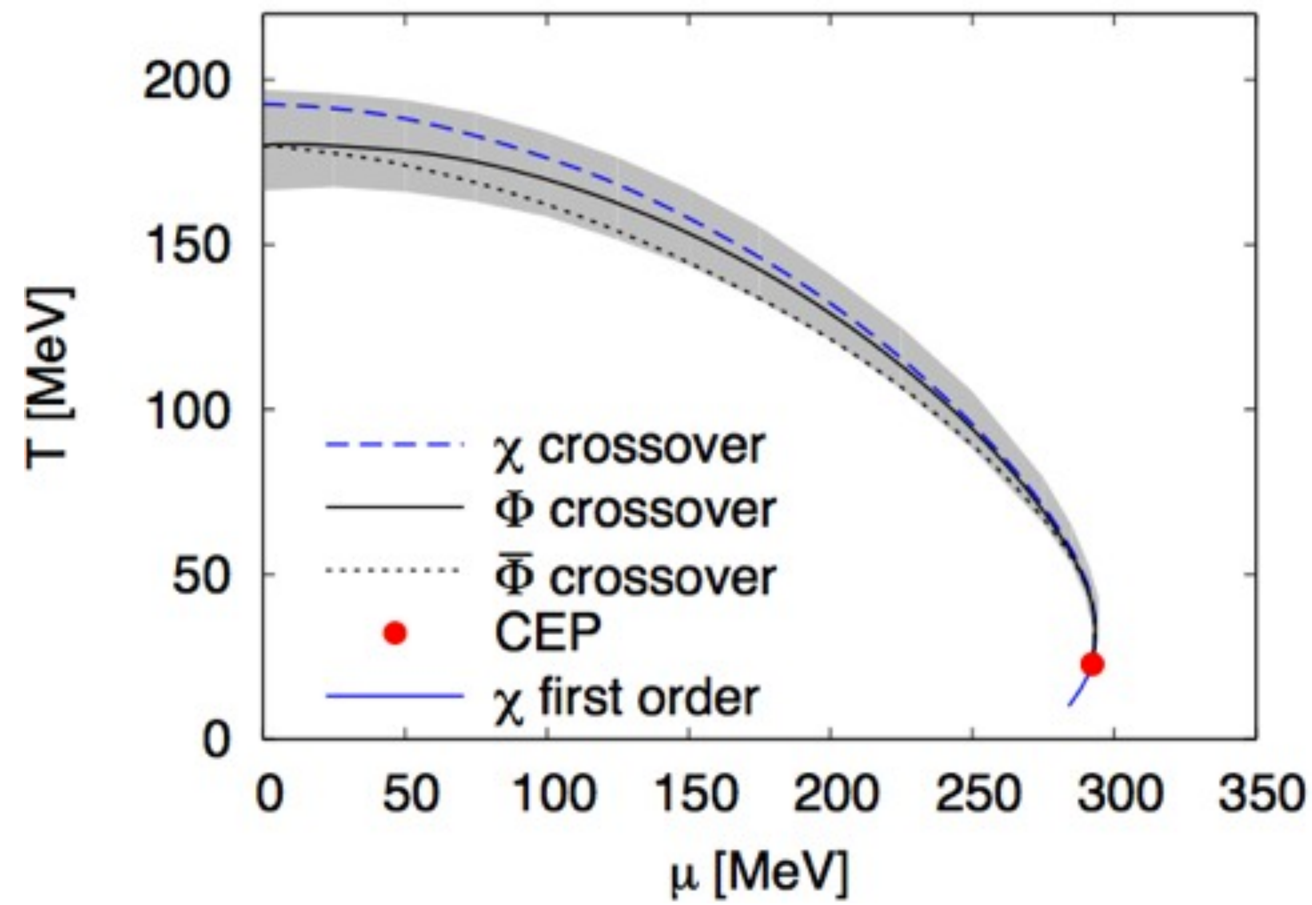



- Lattice simulations
  - Ab initio
  - Gauge invariant
- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
    CF, J. Pawlowski, PRD 80 (2009) 025023
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. $(g\text{-}2)_\mu$
    T. Goecke, C.F., R. Williams, PLB 704 (2011); PRD 83 (2011)
  - Chemical potential: no sign problem

- Models: PNJL, PQM
  - Technically easier
  - Exploratory

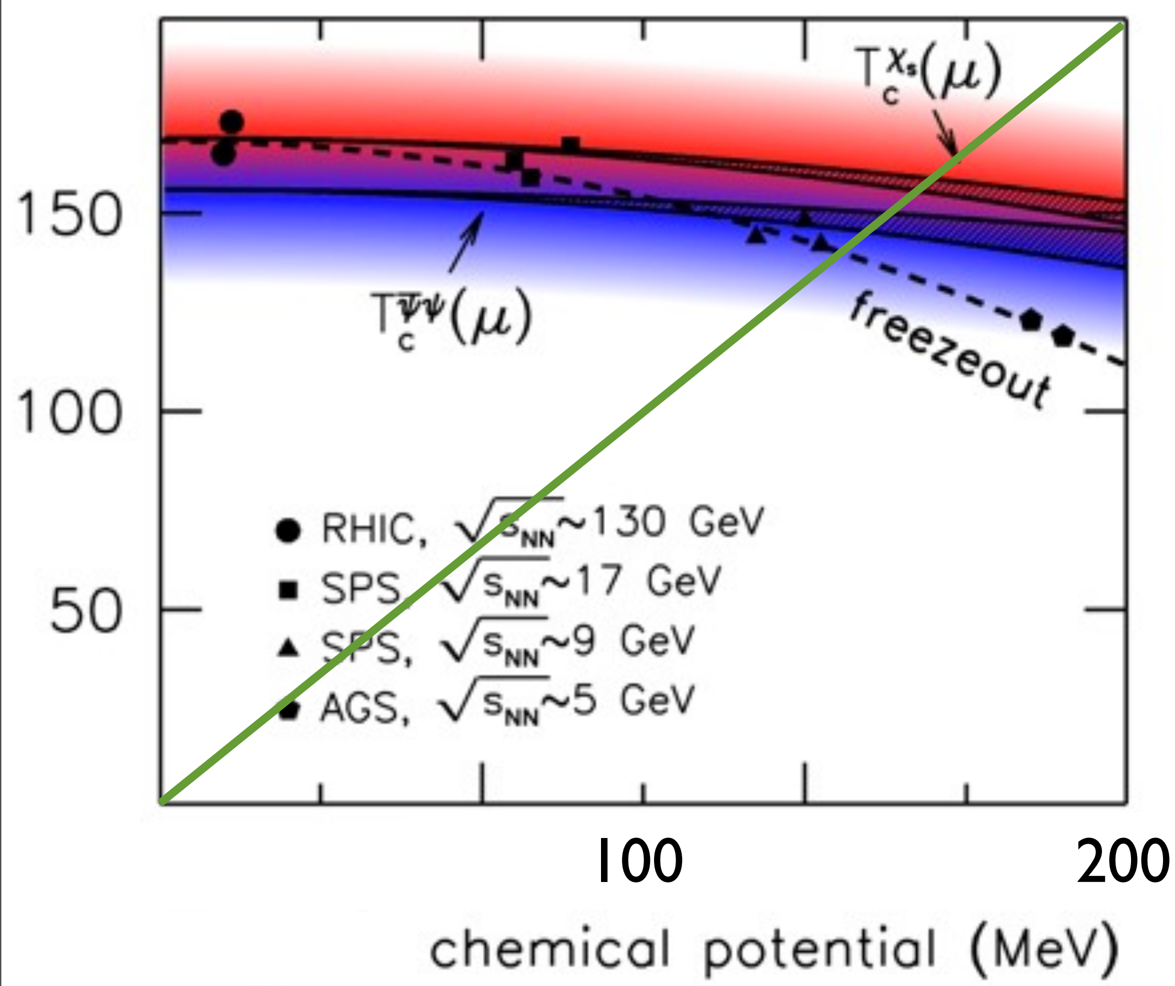



Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001

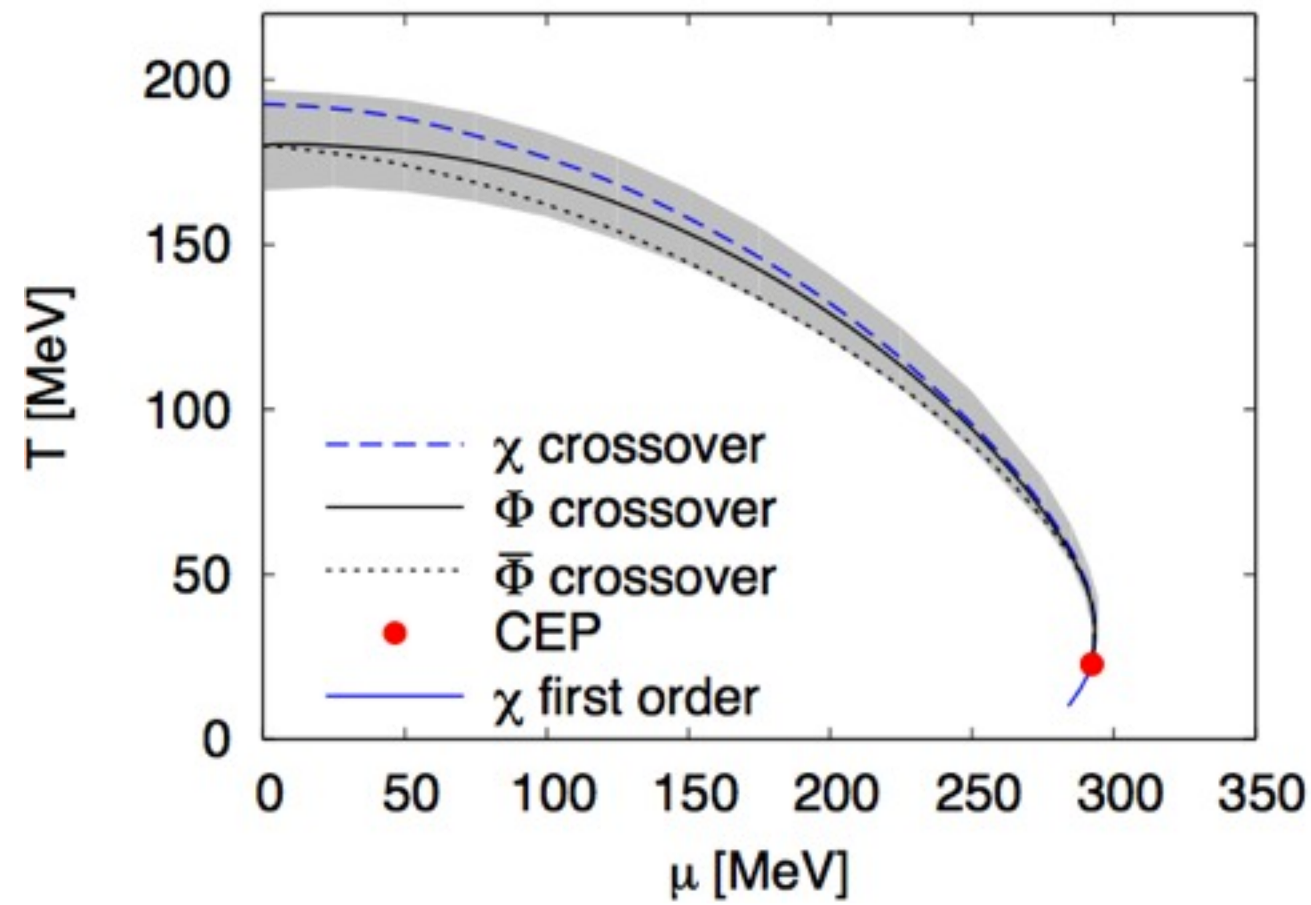



Herbst, Pawlowski, Schaefer, PLB 696 (2011) 58

- Lattice extrapolation reliable for μ/T ≤ 1
- No CEP for small chemical potential
- PQM plus RG-methods (functional methods)




Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001




Herbst, Pawlowski, Schaefer, PLB 696 (2011) 58

- Lattice extrapolation reliable for μ/T ≤ 1
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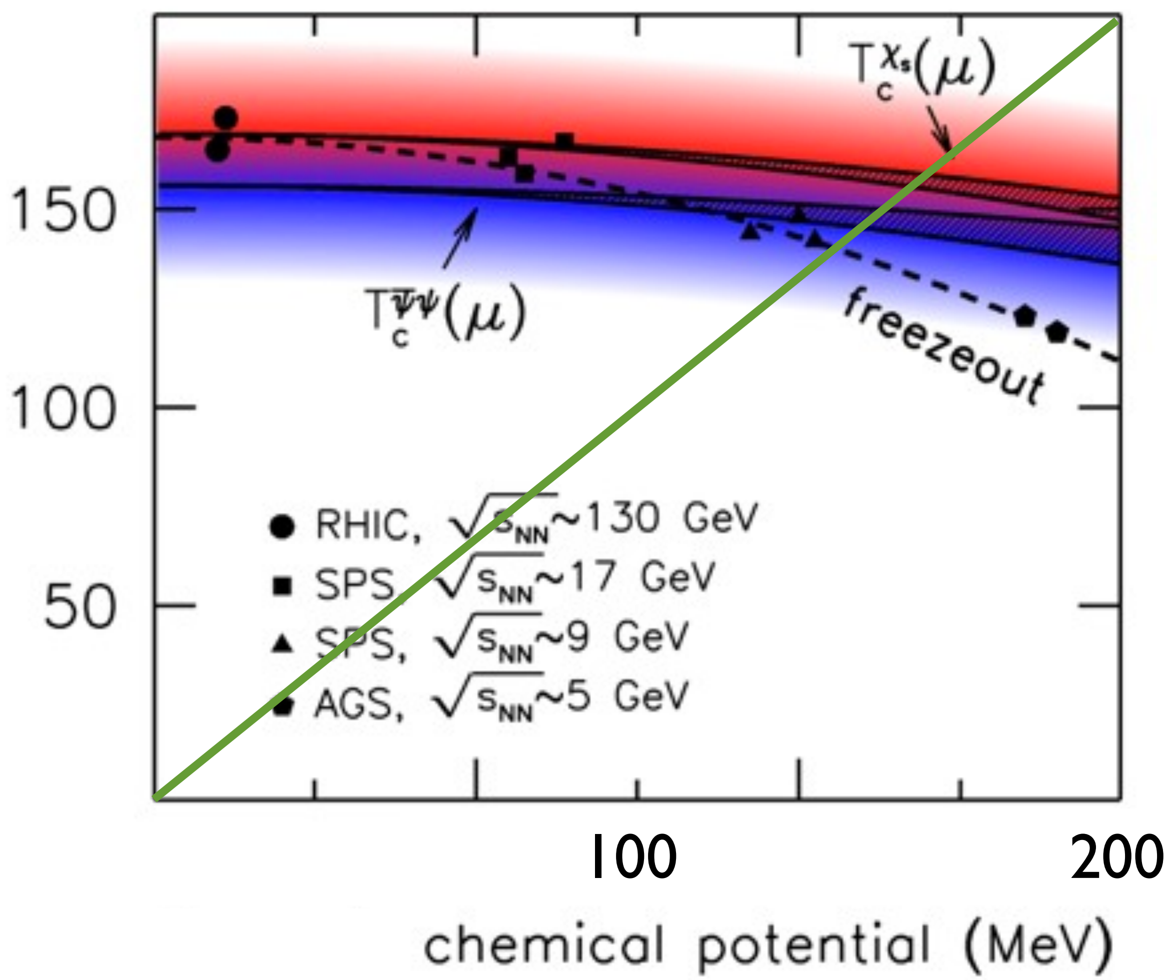
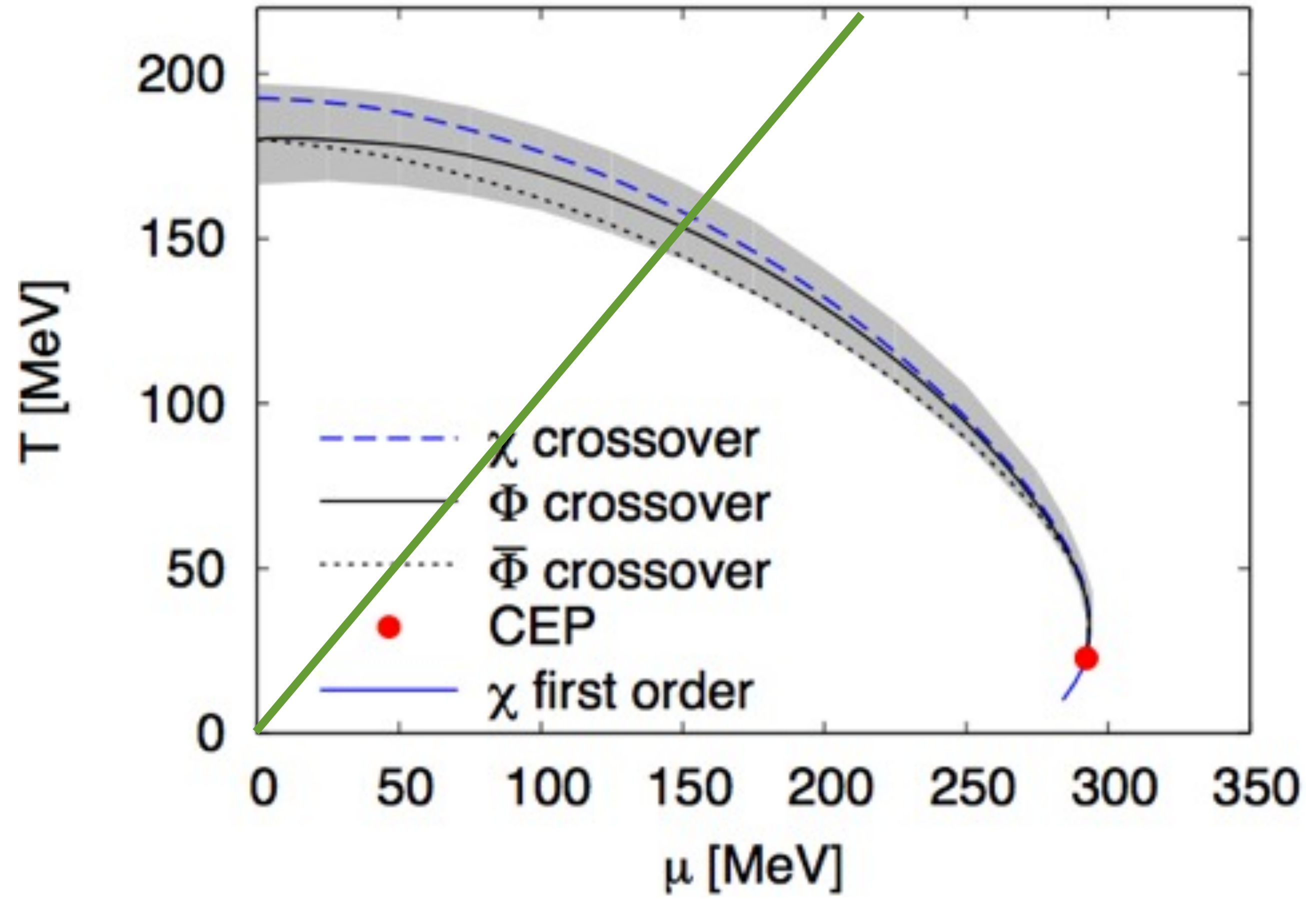
# Phase diagram: Lattice - PQM




Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001




Herbst, Pawlowski, Schaefer, PLB 696 (2011) 58

- Lattice extrapolation reliable for μ/T ≤ 1
- No CEP for small chemical potential
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# Further reading material

- F. Karsch and E. Laermann, ``Thermodynamics and in medium hadron properties from lattice QCD,"  In *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 1-59 [hep-lat/0305025]
- Z.Fodor and S.D.Katz,``The Phase diagram of quantum chromodynamics," arXiv:0908.3341 [hep-ph].
- O. Philipsen,``Status of the QCD phase diagram from lattice calculations," arXiv:1111.5370 [hep-ph]
- B. Friman, (ed.), C. Hohne, (ed.), J. Knoll, (ed.), S. Leupold, (ed.), J.~Randrup, (ed.), R. Rapp, (ed.) and P. Senger, (ed.), ``The CBM physics book: Compressed baryonic matter in laboratory experiments," Lecture Notes in Physics 814 (2011) 1.
- Jeff Greensite,``An introduction to the confinement problem," Lecture Notes in Physics  821 (2011) 1.

# Overview

1. Introduction
   - General
   - Confinement
   - Dynamical chiral symmetry breaking
   - QCD phase diagram

2. QCD with functional methods: Dyson-Schwinger equations
   - Derivation
   - Simple example: pattern of chiral symmetry breaking
   - The gluon propagator
   - Gluons at finite temperature

3. QCD phase diagram
   - Dressed Polyakov-Loops
   - Phase diagram: quenched QCD
   - Transitions of $N_f$=2-QCD, chiral limit
   - Phase diagram: $N_f$=2 vs. $N_f$=2+1

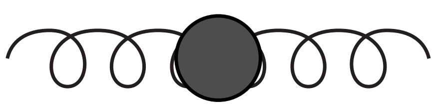
# QCD in covariant gauge

Imaginary time formulation:

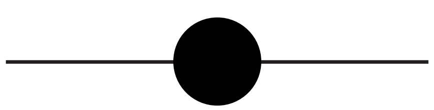
M$_{\text{weak}}$

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \;\exp\left\{ - \int_0^{1/T} dt \int d^3x \Big( \overline{\Psi}\,(i\not{D} + \gamma_4 \mu - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \Big) \right\}$$

Landau gauge propagators in momentum space, $p = (\vec{p}, \omega_p)$ :



$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{Z_T(p)}{p^2} P^T_{\mu\nu}(p) + \frac{Z_L(p)}{p^2} P^L_{\mu\nu}(p)$$



$$S^{\text{Quark}}(p) = Z_f(p)\left[-i\,\vec{\gamma}\vec{p} - i\,\gamma_4 \tilde{\omega}_n\, Z_c(p) + M(p)\right]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.

# Derivation of DSEs

Start from generating functional:

$$\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \;\exp\{-S(\Phi) + j\Phi\} \qquad \text{with} \quad j\Phi = \int d^4x j(x)\Phi(x)$$

The integral of a total derivative vanishes:

$$0 = \frac{\delta}{\delta\Phi(y)}\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \;\left(-\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y)\right) \;\exp\{-S(\Phi) + j\Phi\}$$

$$= \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y) \right\rangle$$

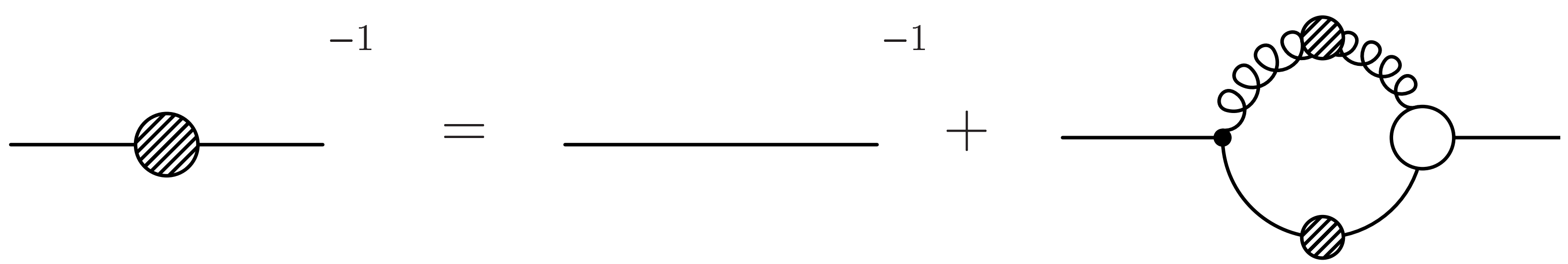
After a further derivative we set j=0 and obtain the DSE for the scalar propagator:

$$0 = \frac{\delta^2}{\delta j(z)\delta\Phi(y)}\mathcal{Z}[j] = \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)}\Phi(z) \right\rangle + \delta(y-z)\mathcal{Z}[0]$$

# The quark DSE

For the DSE of the quark propagator we obtain:

$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4}\, t^a \gamma_\mu\, S(q)\, D^{ab}_{\mu\nu}(q-p)\, \Gamma^b_\nu(q,p)$$

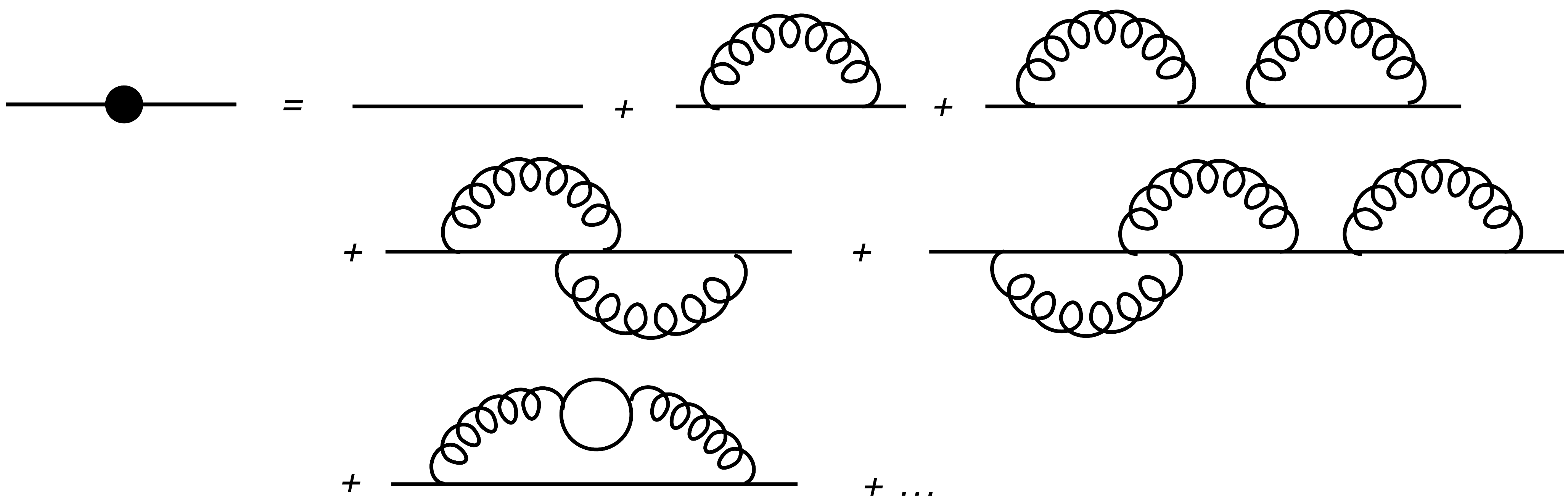



- Tower of DSEs for Euclidean n-point functions
- Similar tower from functional renormalization group (FRG): different structure but similar content !

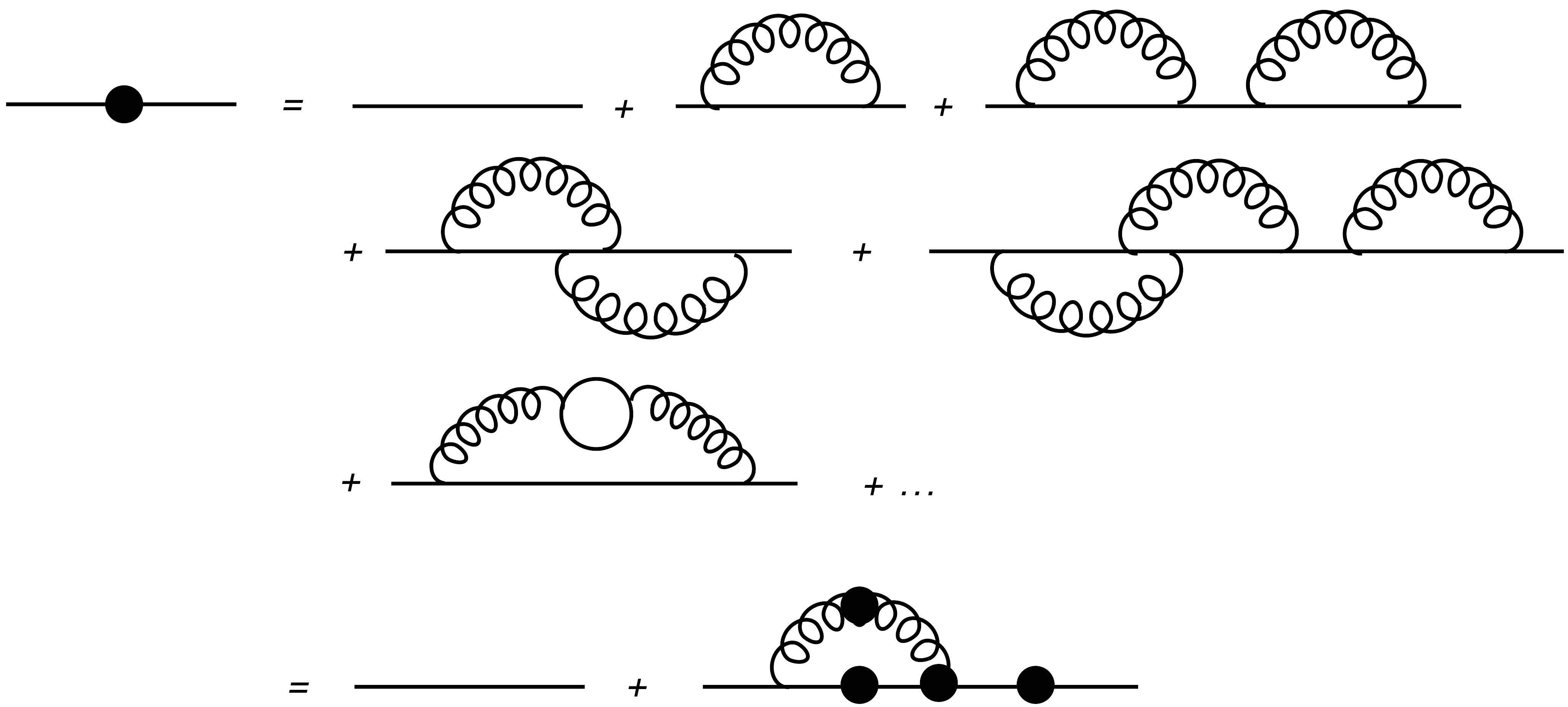

H. Gies,``Introduction to the functional RG and applications to gauge theories,'' hep-ph/0611146.
J.M.Pawlowski,``Aspects of the functional renormalisation group,'' Annals Phys. 322 (2007) 2831 [hep-th/0512261].

# Derivation of DSEs V

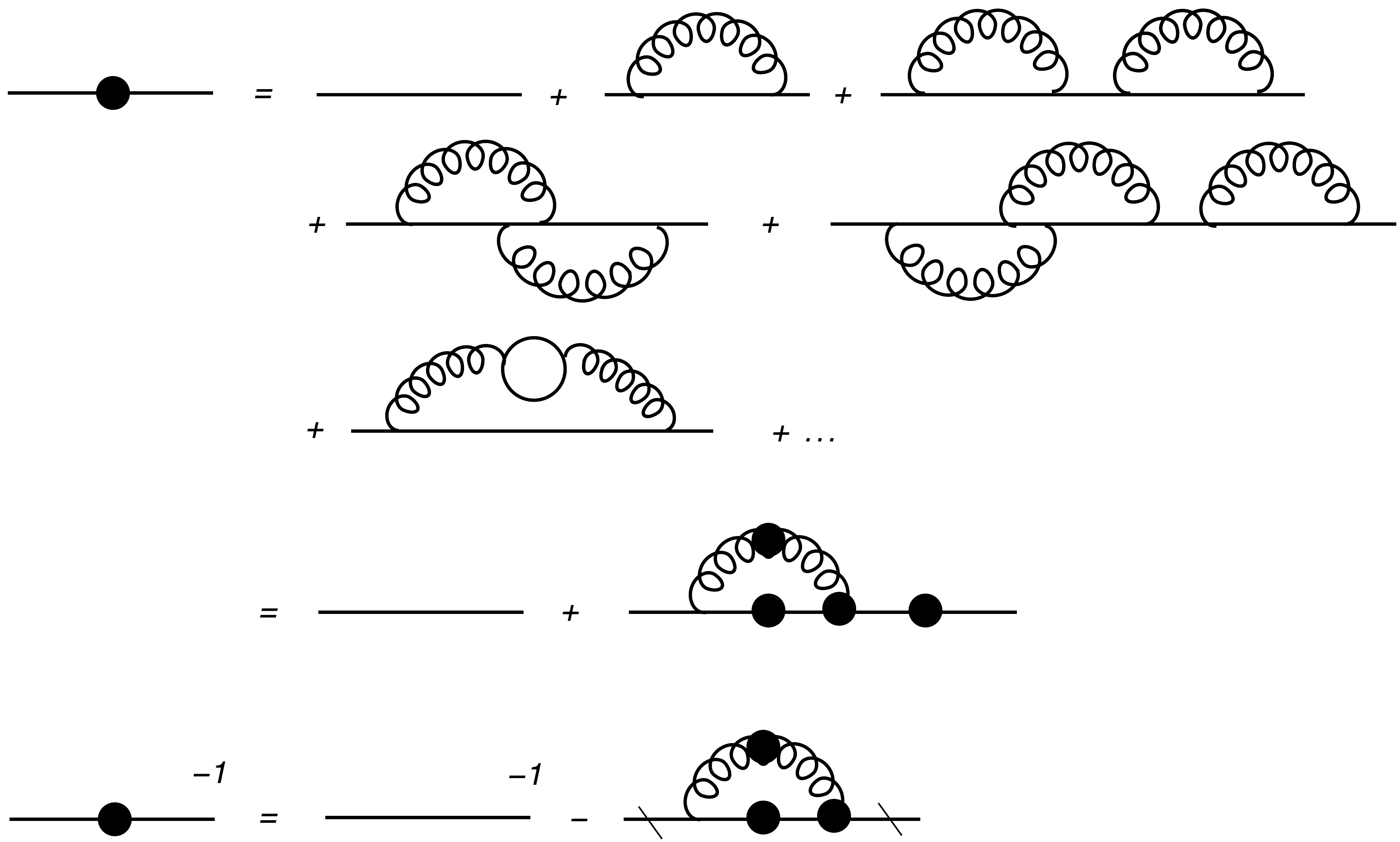
Alternative: start with perturbation theory and resum



## Alternative: start with perturbation theory and resum

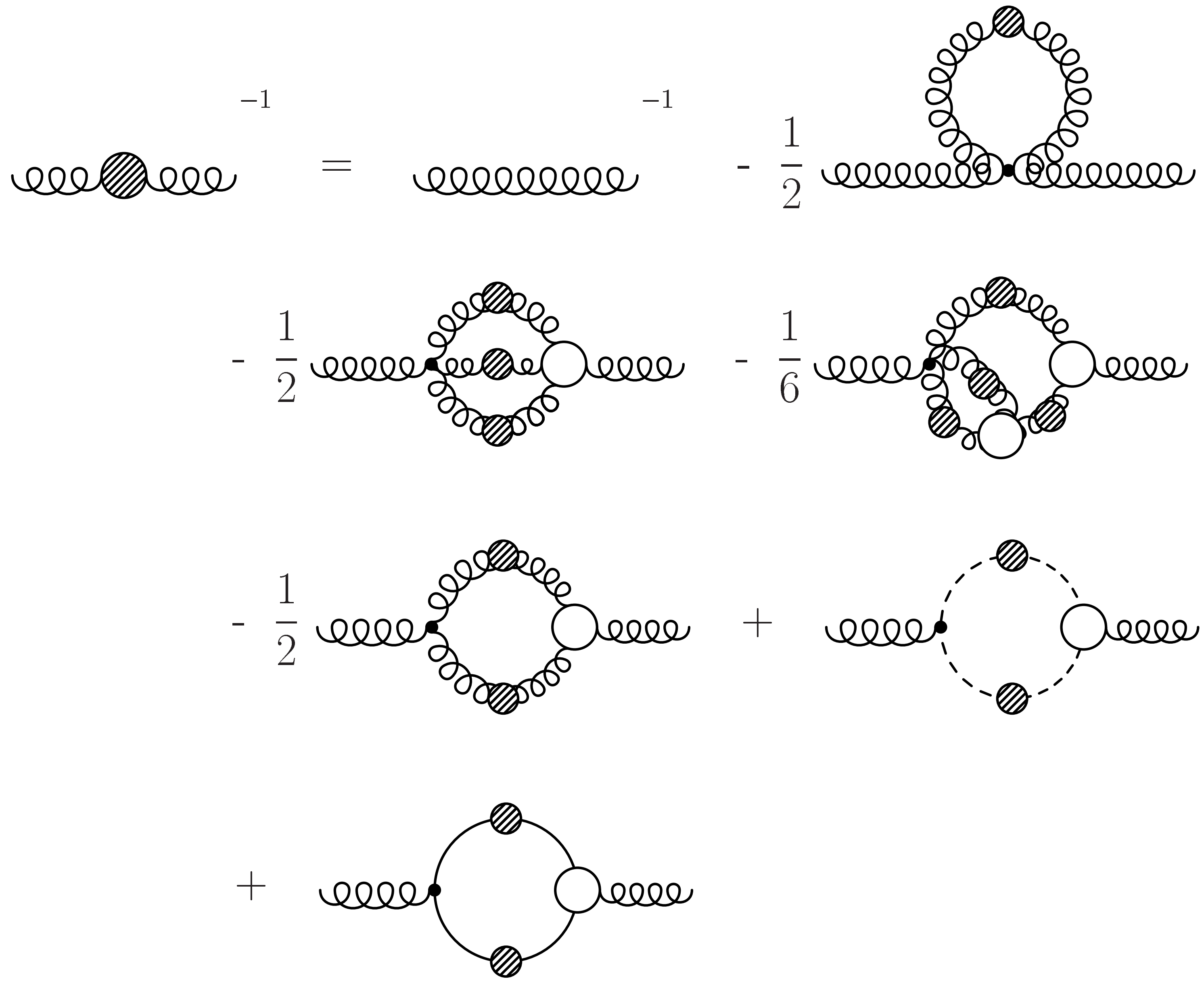


Alternative: start with perturbation theory and resum

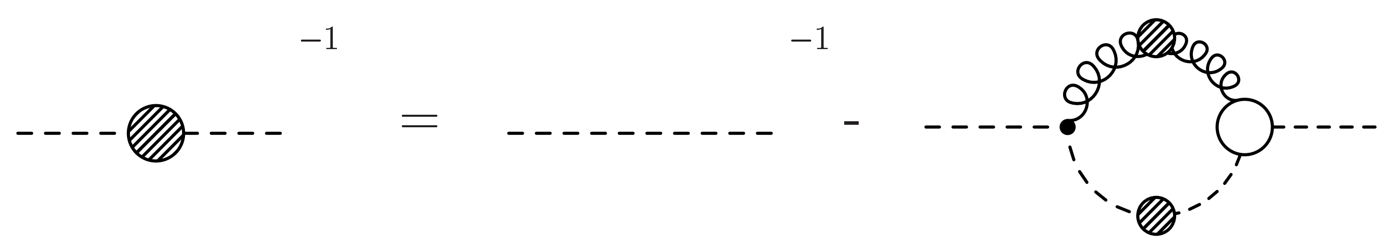


# DSEs: Quark and gluon propagators

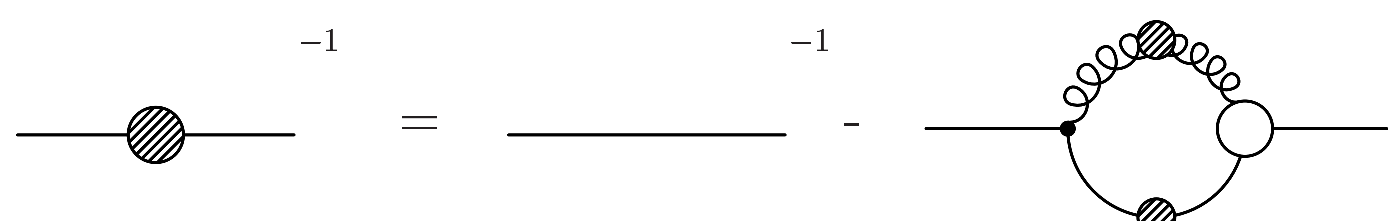
**gluon:**



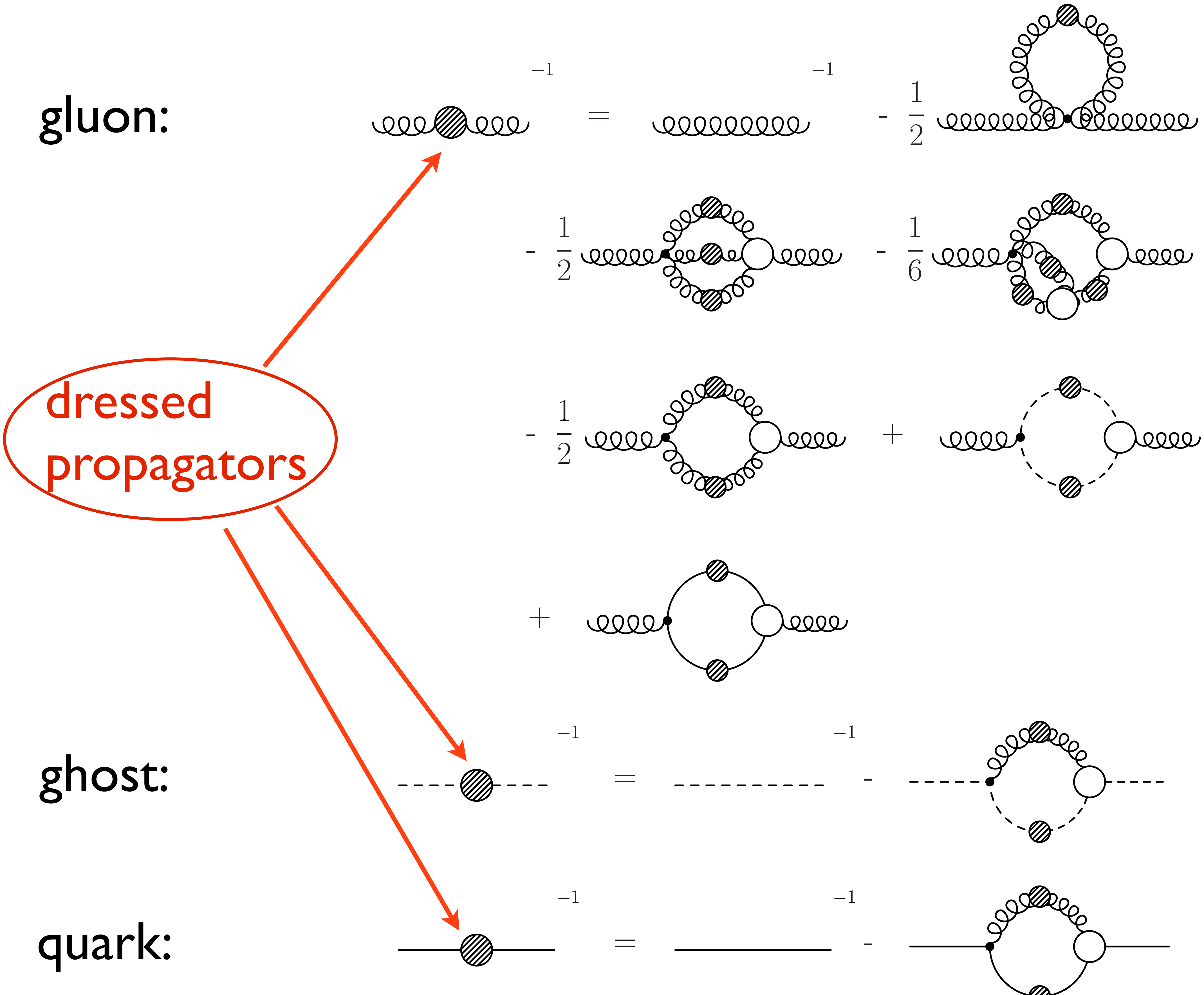
**ghost:**



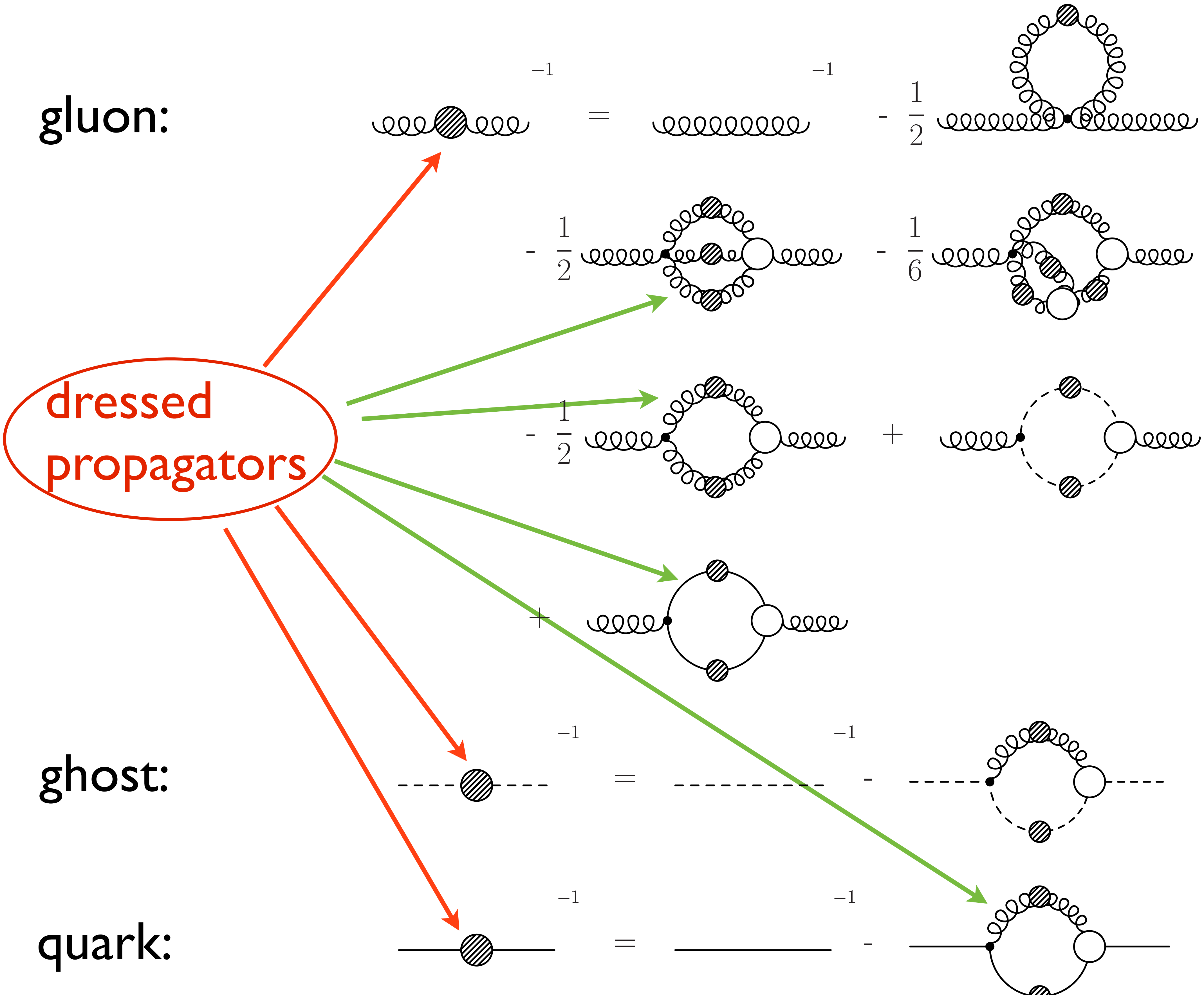
**quark:**



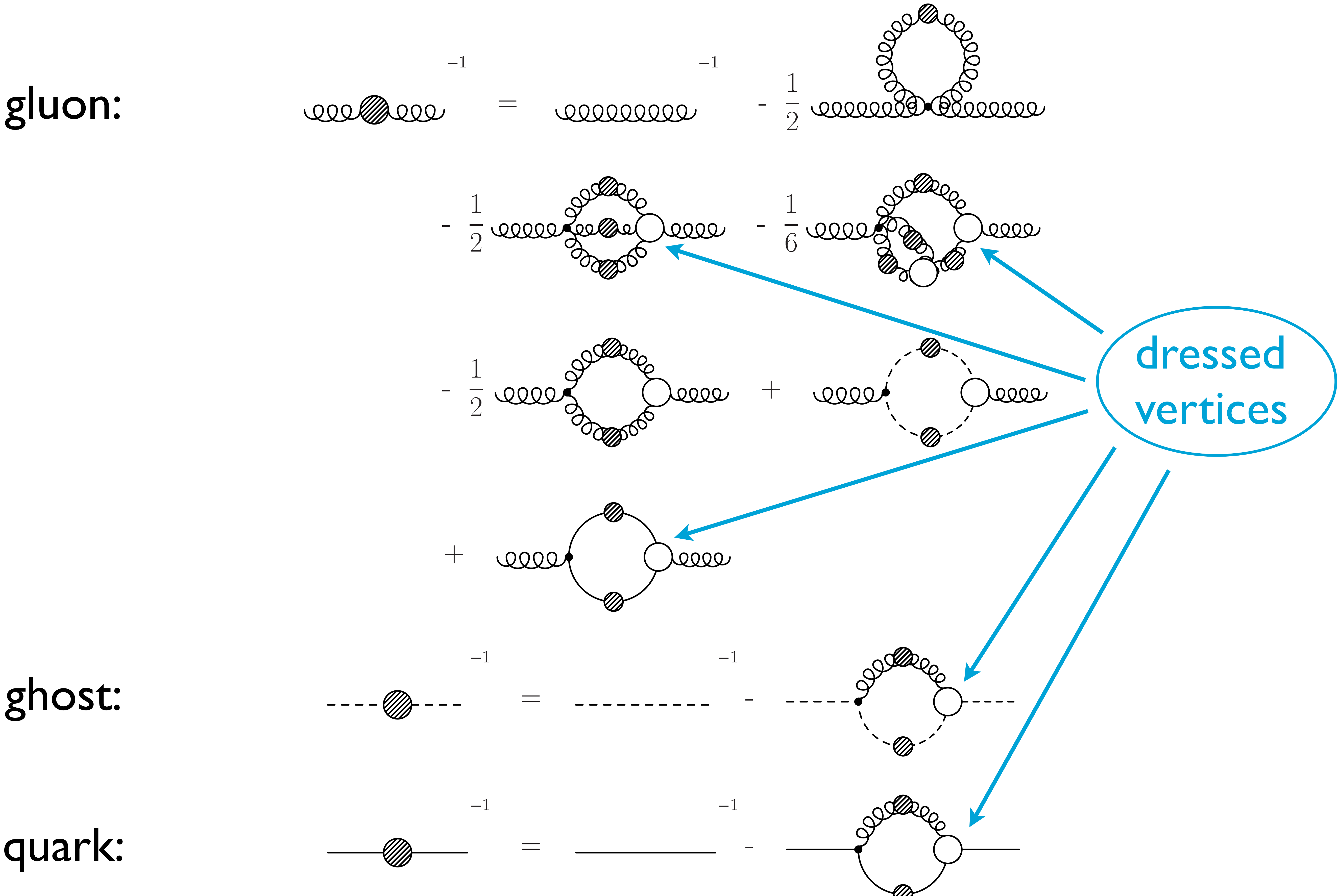
# DSEs: Quark and gluon propagators



# DSEs: Quark and gluon propagators



# DSEs: Quark and gluon propagators



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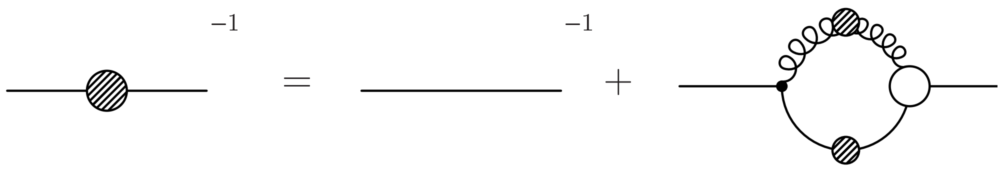
# Dynamical chiral symmetry breaking I

Simple example:




Take bare gluon propagator: $D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}$

and bare quark-gluon-vertex: $\Gamma_\nu(p,q) = i\gamma_\nu$

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^4q}{(2\pi)^4}\, \gamma_\mu D_{\mu\nu}(k)\, S(q)\, \gamma_\nu$$

with $C_F = \frac{N_c^2-1}{2N_c} \to \frac{4}{3}$

$S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$

$S_0^{-1}(p) = i\not{p} + m$ $\quad\longrightarrow$ project onto Dirac structures

$$B(p^2) = m + g^2\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{3B(q^2)}{q^2A^2(q^2)+B^2(q^2)}$$

$$A(p^2) = 1 + g^2\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{A(q^2)}{q^2A^2(q^2)+B^2(q^2)}\left[-\frac{k^2}{p^2}+\frac{p^2+q^2}{2p^2}+\frac{(p^2-q^2)^2}{2p^2k^2}\right]$$



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# Dynamical chiral symmetry breaking II

In our simple example $A \simeq 1$, then:

$$B(p^2) = m + g^2 \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2} \frac{3B(q^2)}{q^2 + B^2(q^2)}$$

Transform $\int d^4q$ in hyperspherical coordinates and perform angular integrals analytically ($\alpha = g^2/4\pi$):

$$B(p^2) = m + \alpha \int_0^{p^2} dq^2 \frac{q^2}{p^2} \frac{B(q^2)}{q^2 + B^2(q^2)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \frac{B(q^2)}{q^2 + B^2(q^2)}$$

This equation for the quark mass function $M(p^2) = B(p^2)/A(p^2)$ has a typical structure.

⟶

# Dynamical chiral symmetry breaking III

Consider chiral limit $m=0$ :

$$B(p) = \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \, \frac{B(q)}{q^2+B^2(q)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q)}{q^2+B^2(q)}$$

Three solutions:

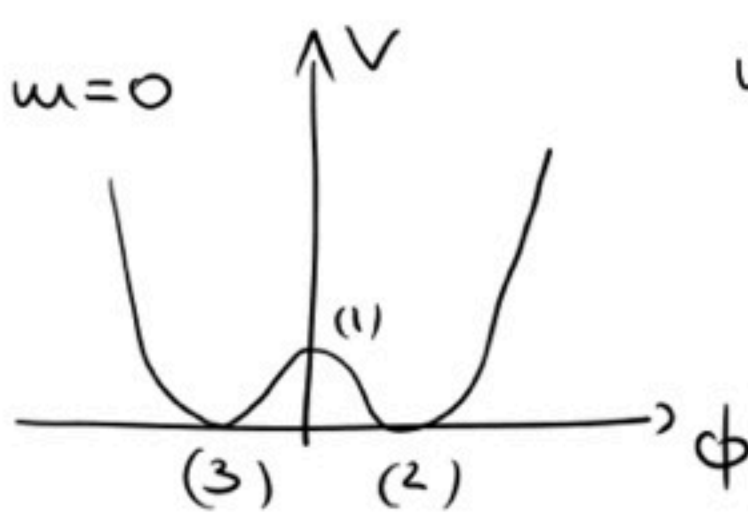
(1) $B(p) \equiv 0$ $\longrightarrow$ chiral symmetric: Wigner-Weyl

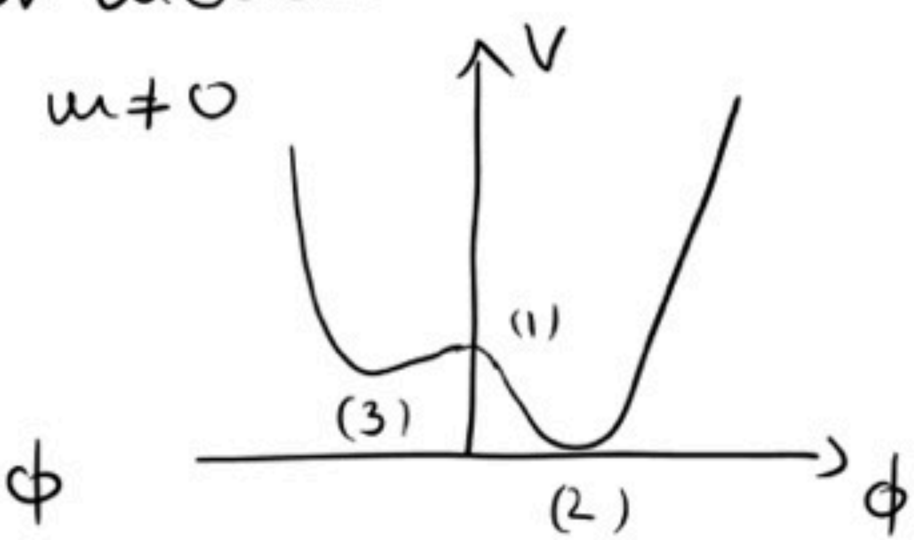
(2,3) $\pm B(p) \neq 0$ $\longrightarrow$ chiral symmetry broken: Nambu-Goldstone

cp. to effective potential in scalar models:

(1) metastable

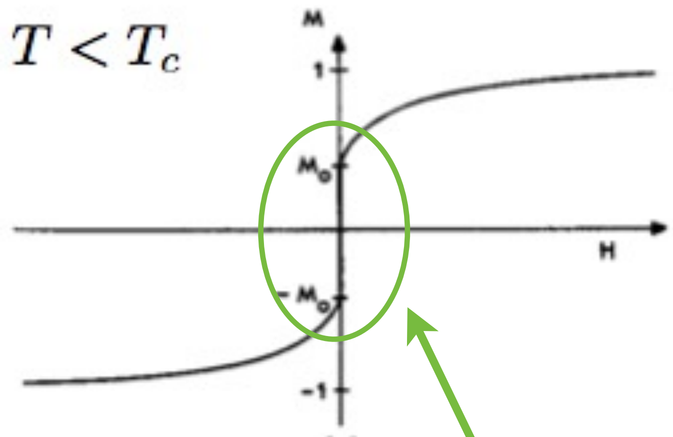
(2,3) stable



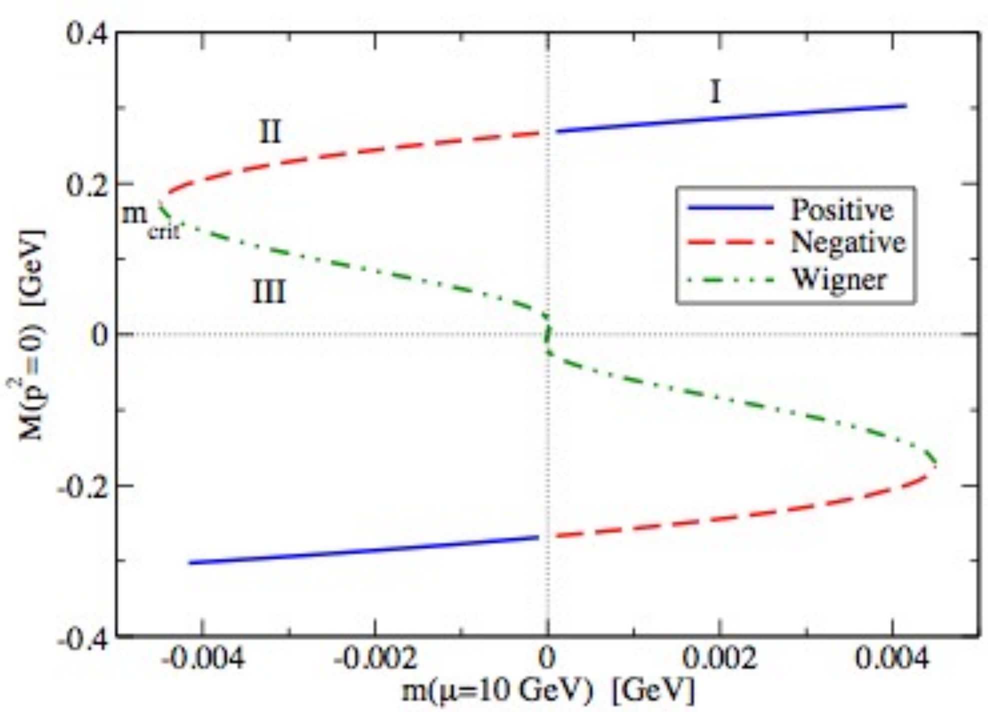
# Compare to Heisenberg ferromagnet

Ferromagnet:




three solutions at H=0: M = 0, M = ±$M_0$

quark-DSE:




CF, Nickel and Williams, EPJC 60 (2009) 47

$$M \leftrightarrow M(0) = \left(\frac{B(p^2)}{A(p^2)}\right)\Big|_{p^2=0}$$

$$H \leftrightarrow m$$



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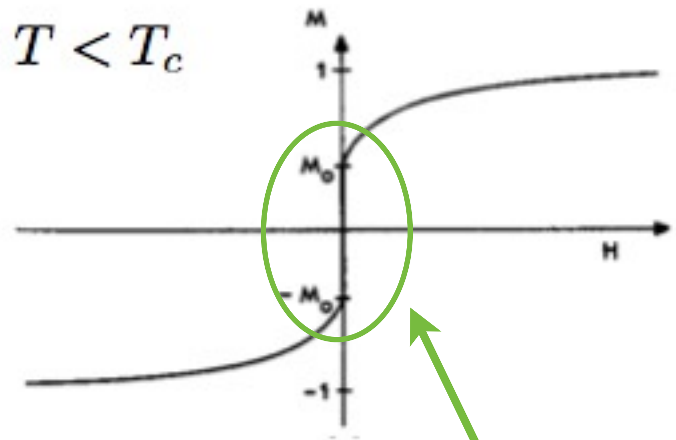
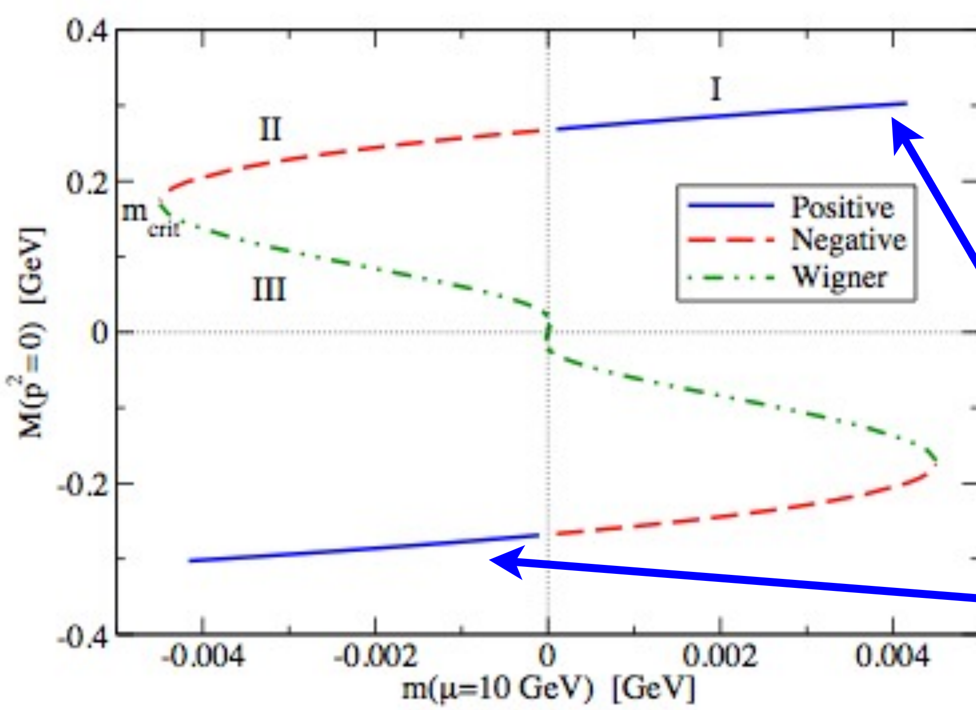
# Compare to Heisenberg ferromagnet

Ferromagnet:




three solutions at H=0: M = 0, M = ±$M_0$

quark-DSE:



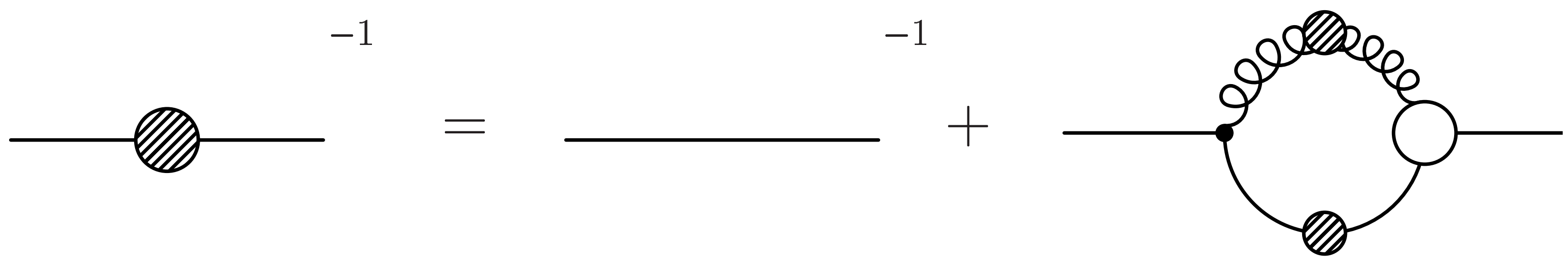

CF, Nickel and Williams, EPJC 60 (2009) 47

$$M \leftrightarrow M(0) = \left(\frac{B(p^2)}{A(p^2)}\right)\Big|_{p^2=0}$$

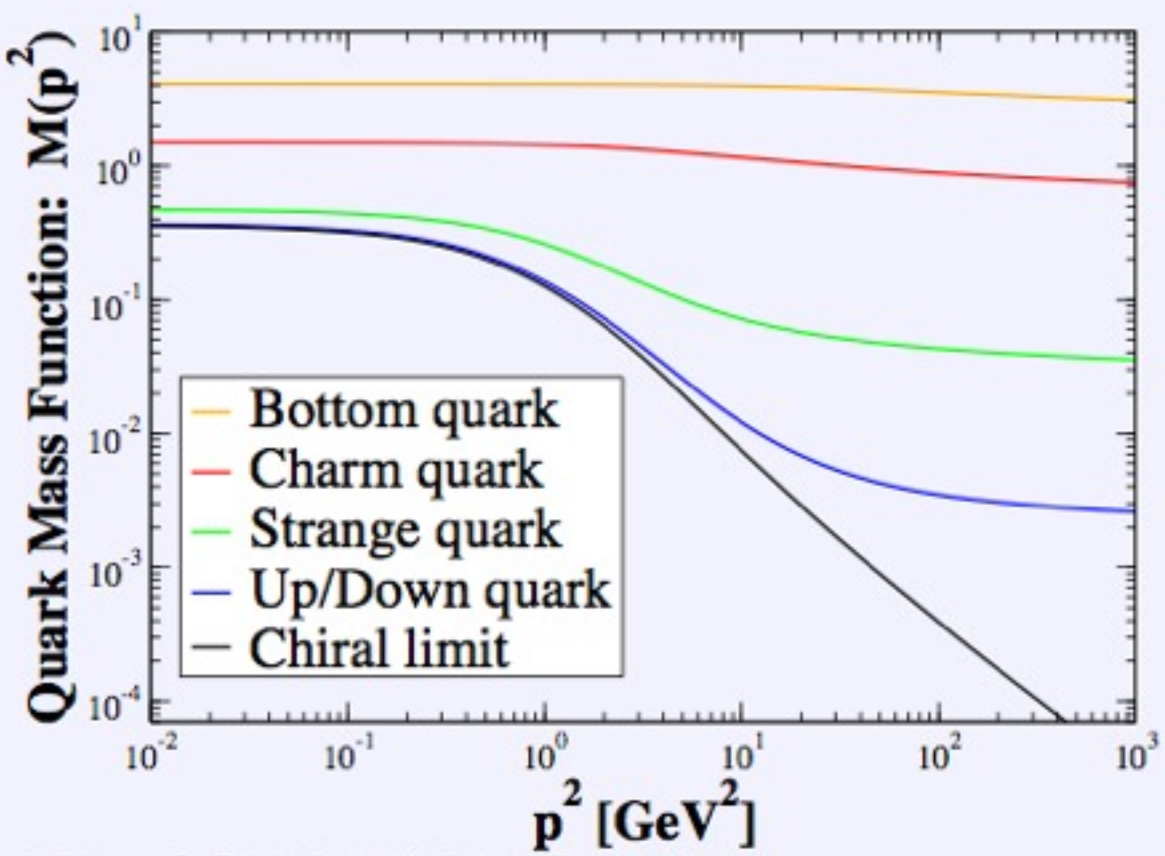
$$H \leftrightarrow m$$

energetically preferred

# Explicit vs. dynamical chiral symmetry breaking




$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$



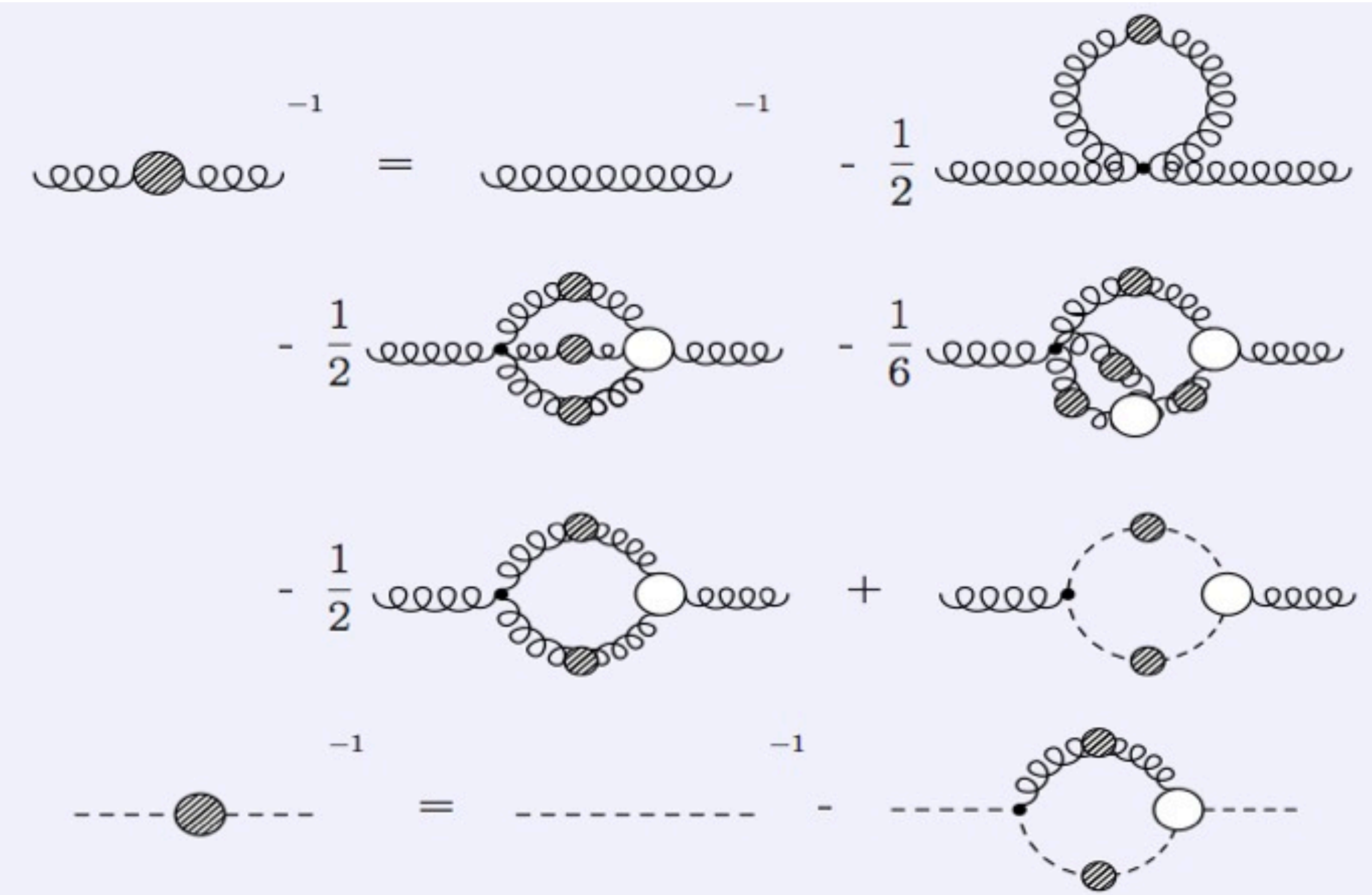

C.F., J. Phys. G **G32** (2006) R253-R291.

- order parameter: chiral condensate

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c Tr \int \frac{d^4p}{(2\pi)^4} S(p)$$

- dynamical mass $M(p^2)$
- flavor dependence because of $M_{weak}$

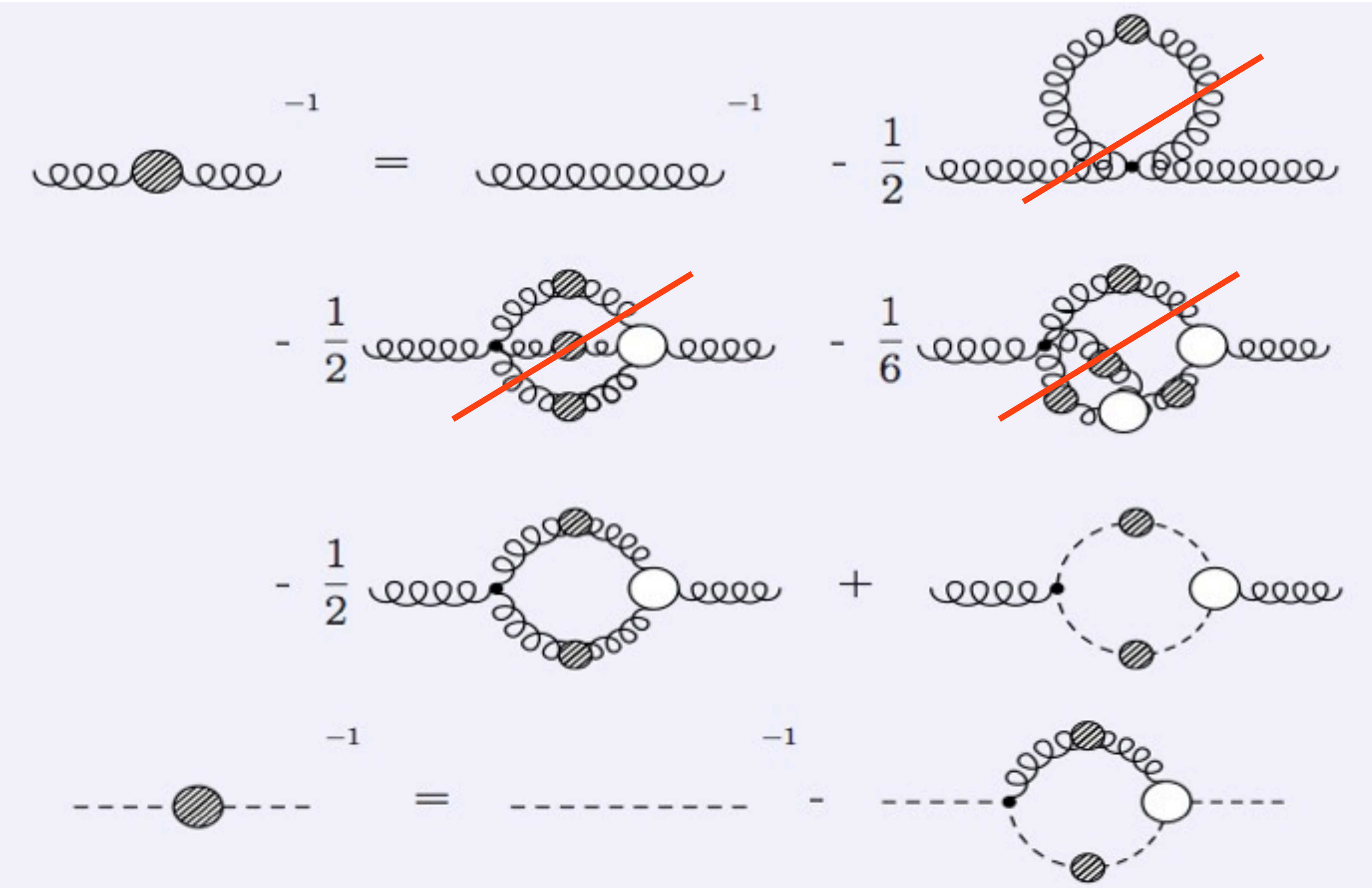
# Gluon propagator




Truncation (=approximation):

- neglect four-gluon interaction
- bare ghost-gluon vertex
- express three-gluon vertex in terms of ghost/glue

# Gluon propagator

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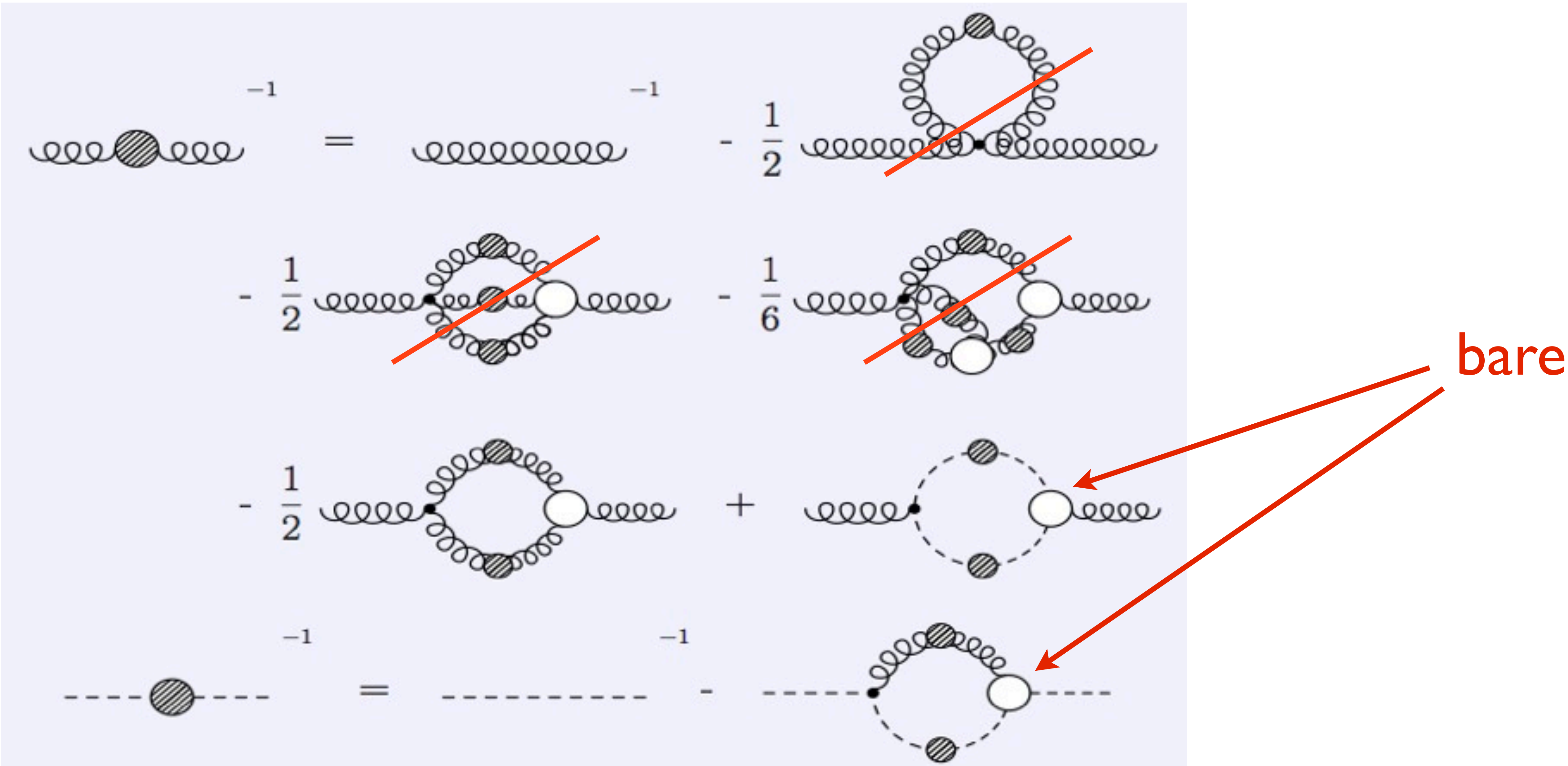
# Gluon propagator

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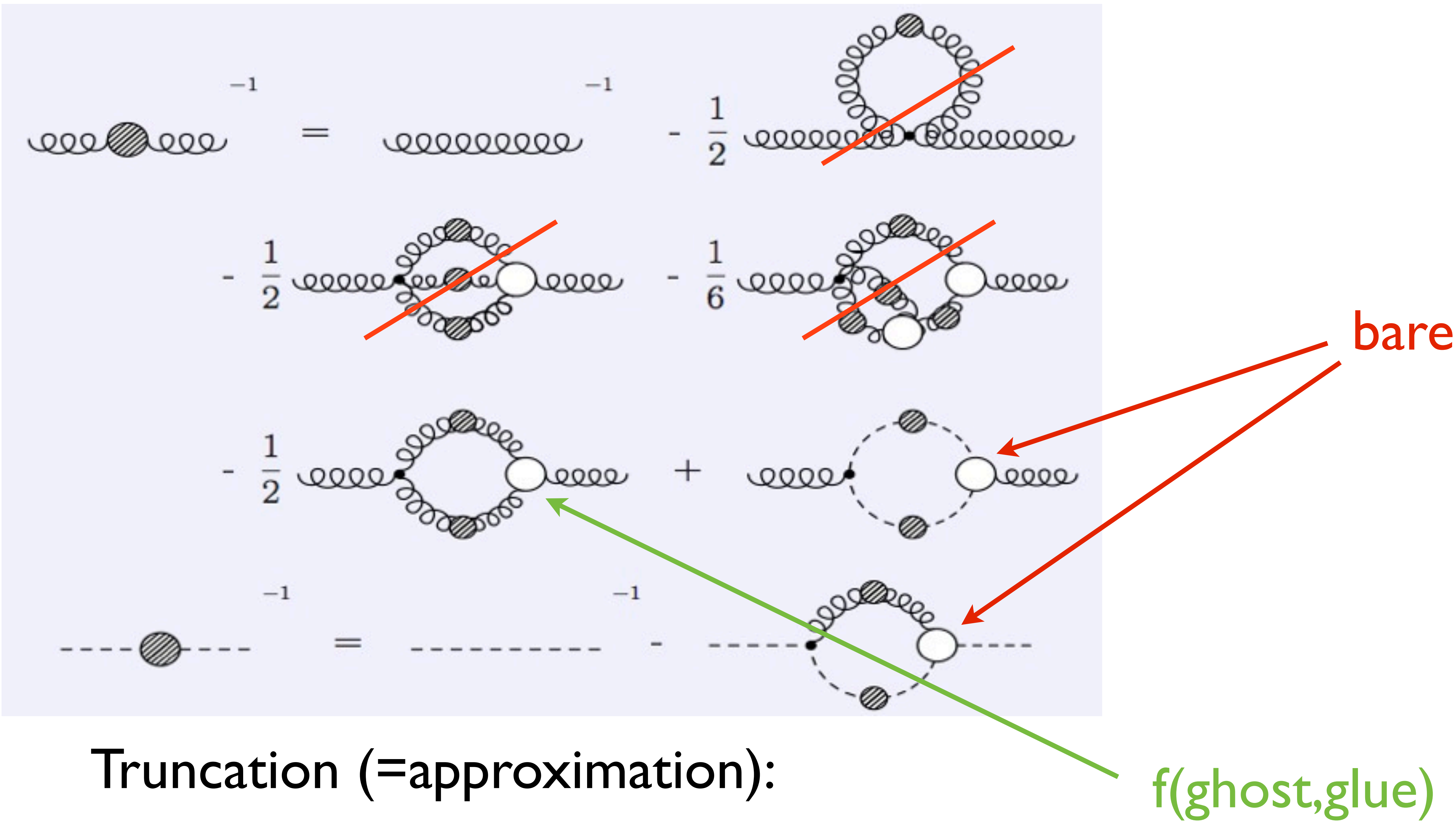
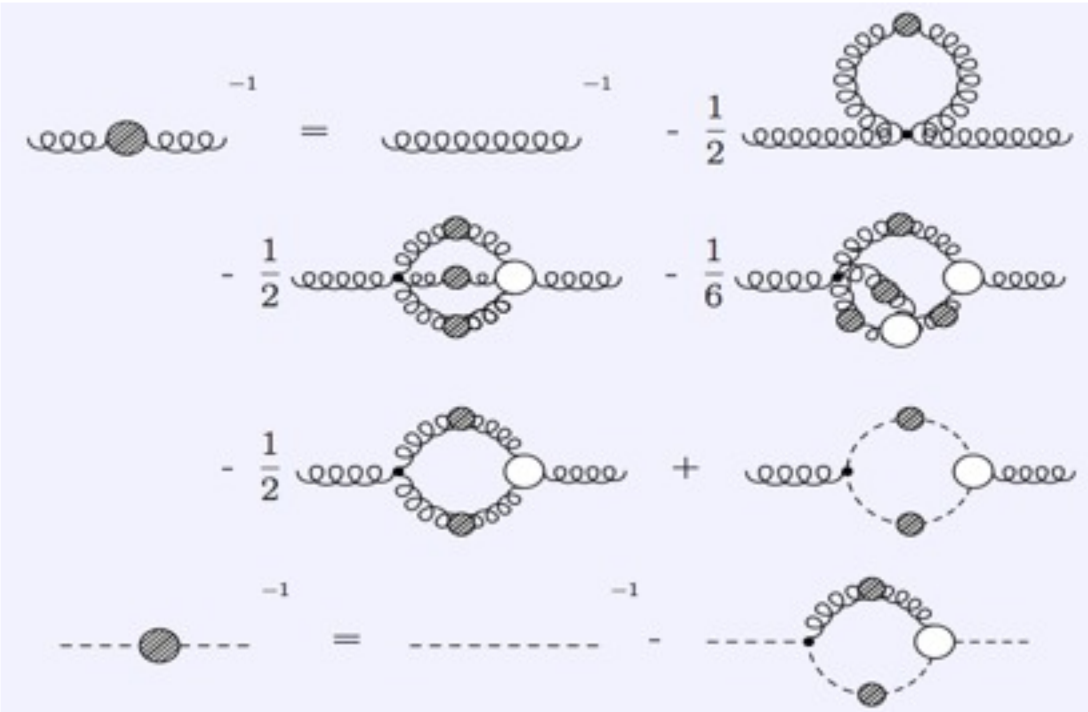
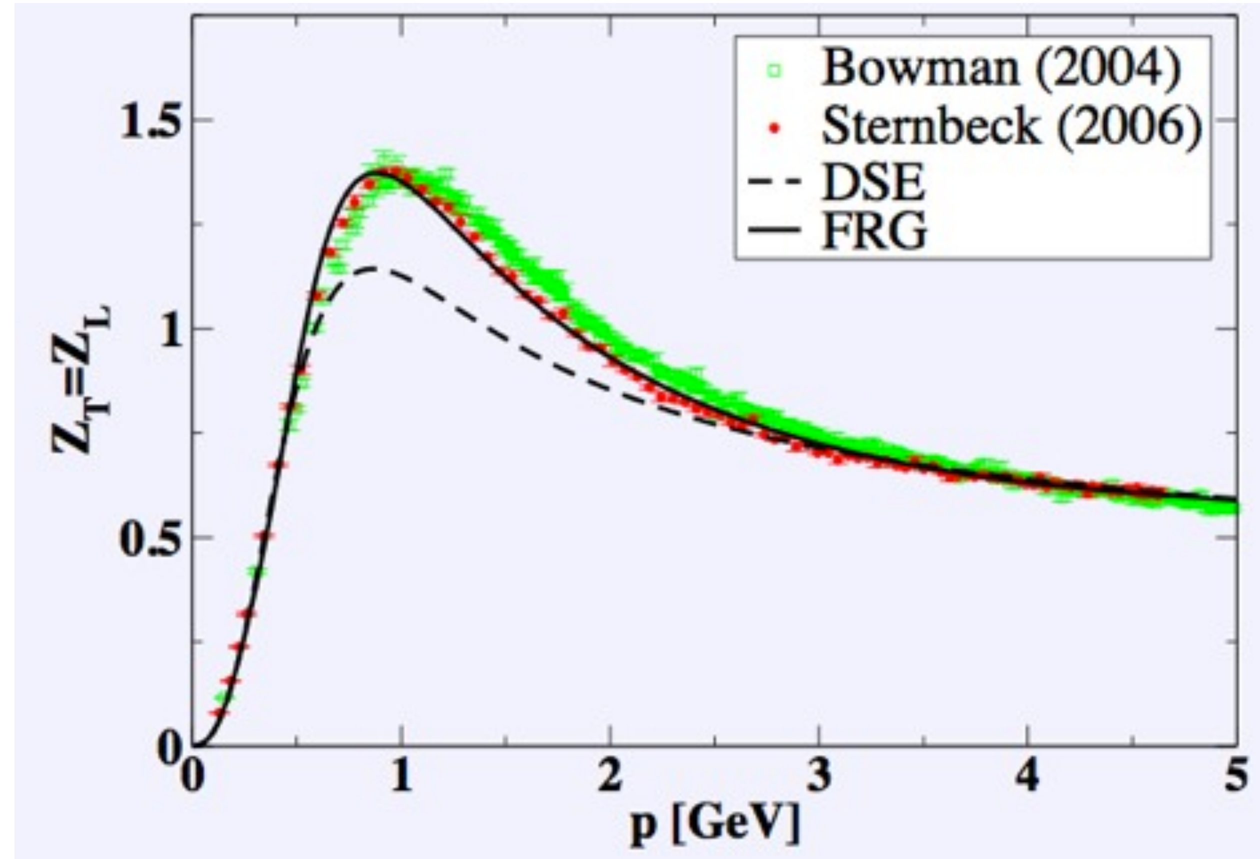



Truncation (=approximation):

- neglect four-gluon interaction
- bare ghost-gluon vertex
- express three-gluon vertex in terms of ghost/glue

# DSE vs. Lattice (T=0)




CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.

- Small momenta: $Z(p^2) \sim p^2$, i.e. gluon mass generation

  Cornwall PRD **26** (1982) 1453; Cucchieri, Mendes, PoS **LAT2007** (2007) 297.
  Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008); Boucaud, et al. JHEP **0806** (2008) 099

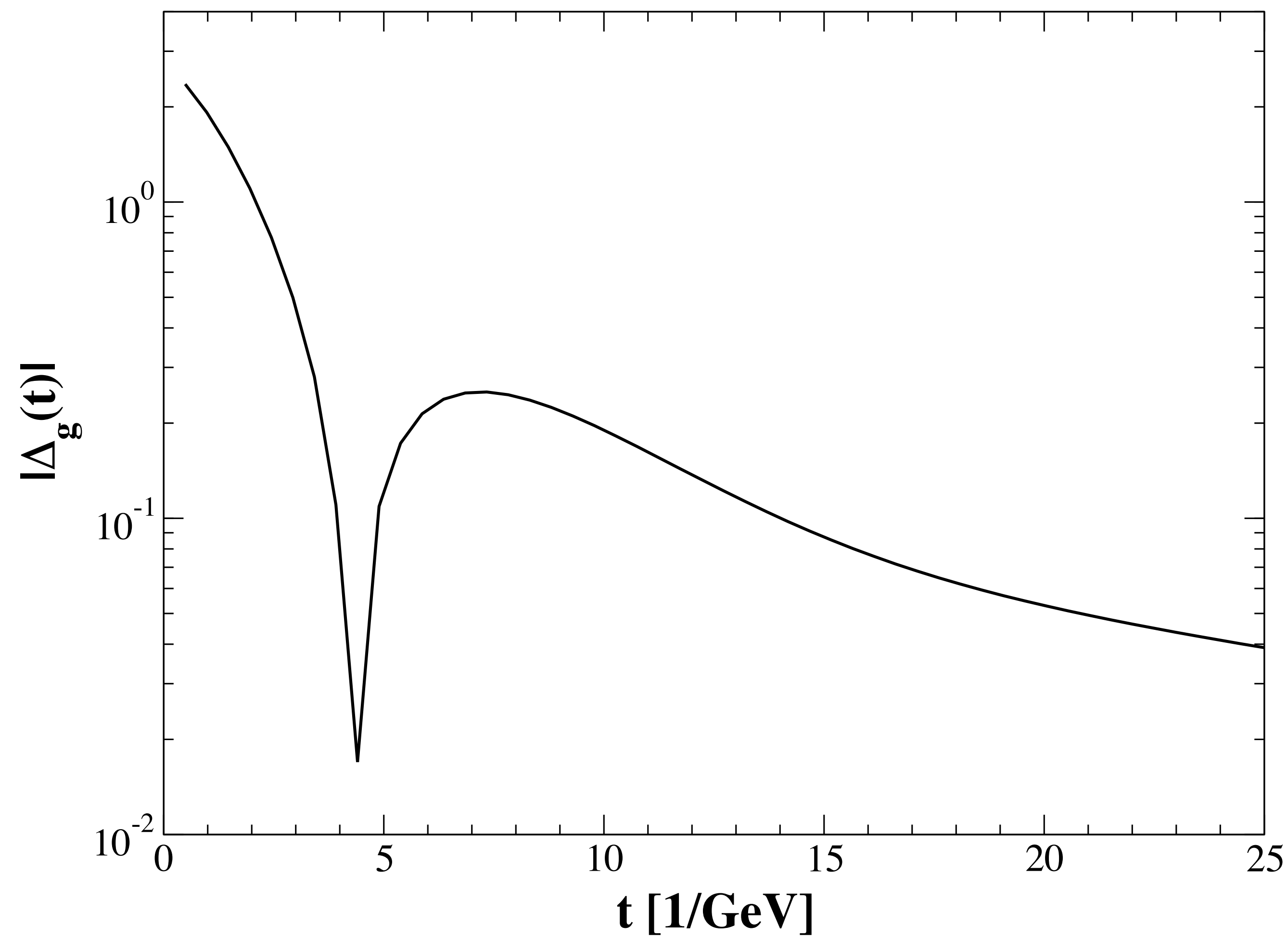
- Deep infrared: subtle questions related to gauge fixing...

  Maas, PLB 689 (2010) 107; Sternbeck, Smekal, EPJC 68 (2010) 487

- Timelike momenta: Positivity violations $\rightarrow$ gluon screening

  Alkofer, Detmold, C.F. and Maris, PRD 70 (2004) 014014

Schwinger function: $$\Delta_g(t) = \int \frac{dp_0}{2\pi}\, e^{itp_0} \left( \frac{Z(p^2)}{p^2} \right)\Bigg|_{\vec{p}=0}$$

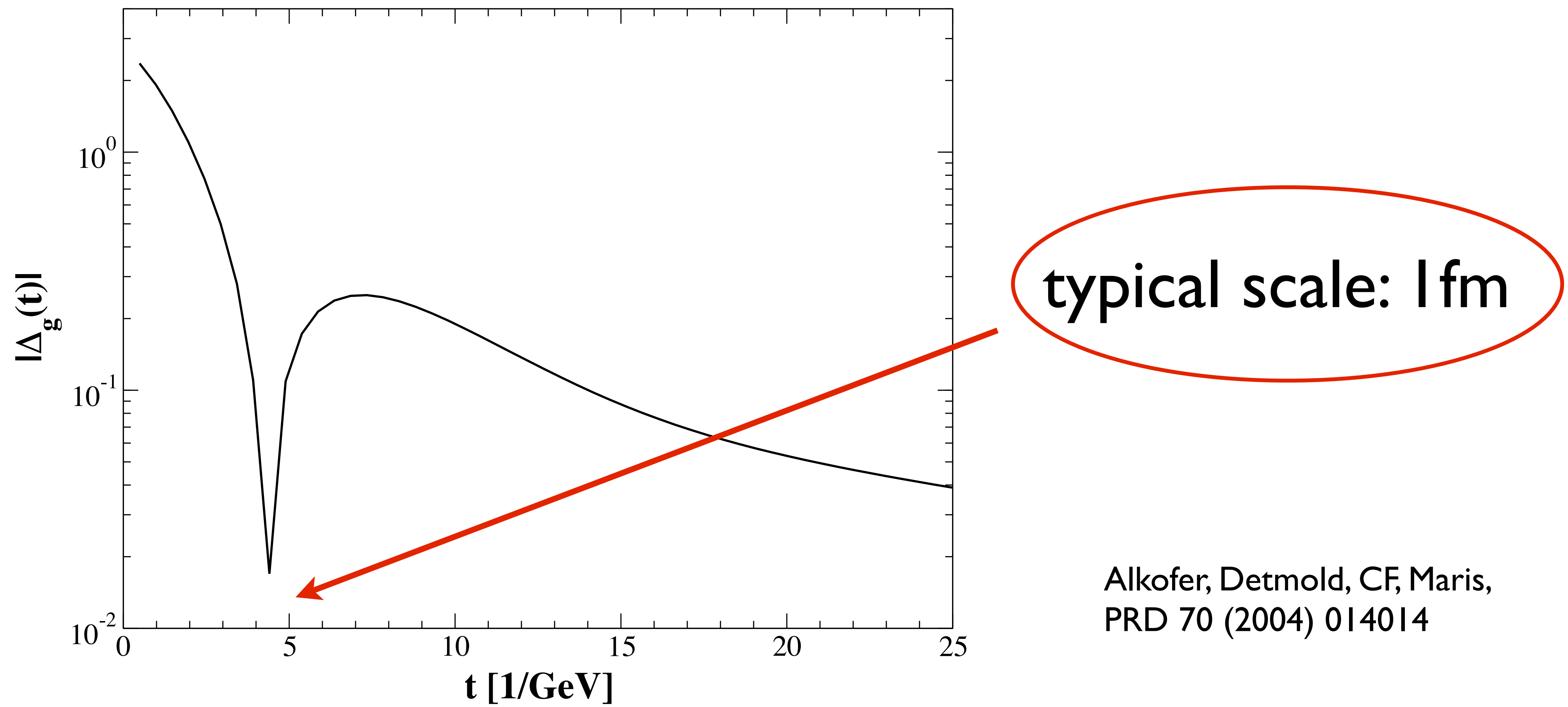



Alkofer, Detmold, CF, Maris,
PRD 70 (2004) 014014

- Violation of positivity: color screening

Gluons cannot exist as asymptotic states

Schwinger function: $$\Delta_g(t) = \int \frac{dp_0}{2\pi}\, e^{itp_0} \left(\frac{Z(p^2)}{p^2}\right)\Bigg|_{\vec{p}=0}$$



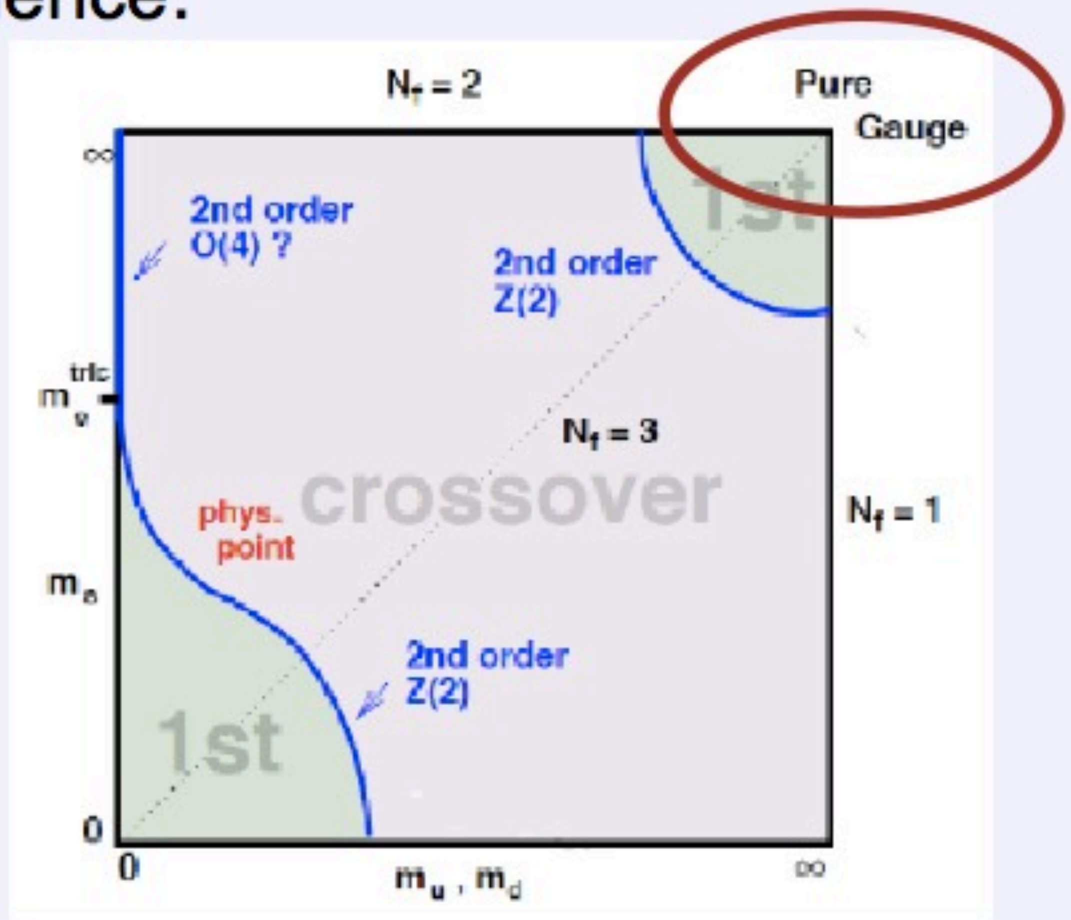

Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014

- Violation of positivity: color screening

Gluons cannot exist as asymptotic states

# QCD phase transition: heavy quark limit/quenched

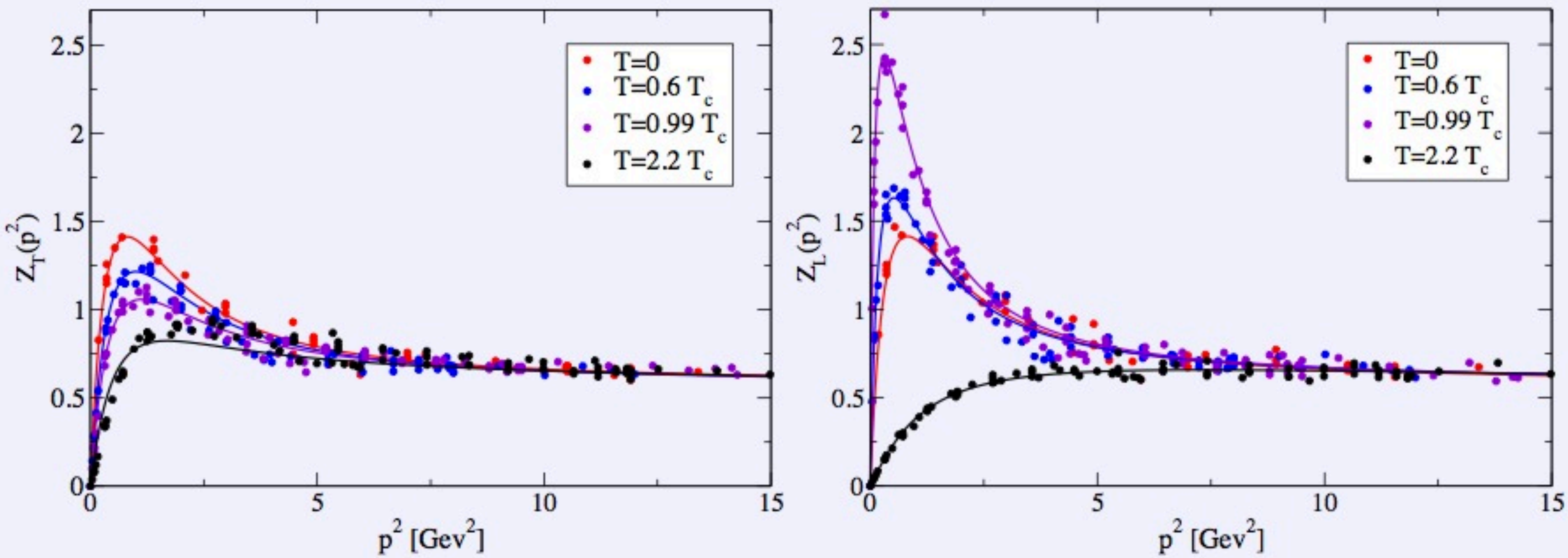
Quark mass dependence:




- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

# Glue at finite temperature (T≠0): Lattice

*T*-dependent gluon propagator from lattice simulations:




- Difference between electric and magnetic gluon
- Maximum of electric gluon around $T_c$

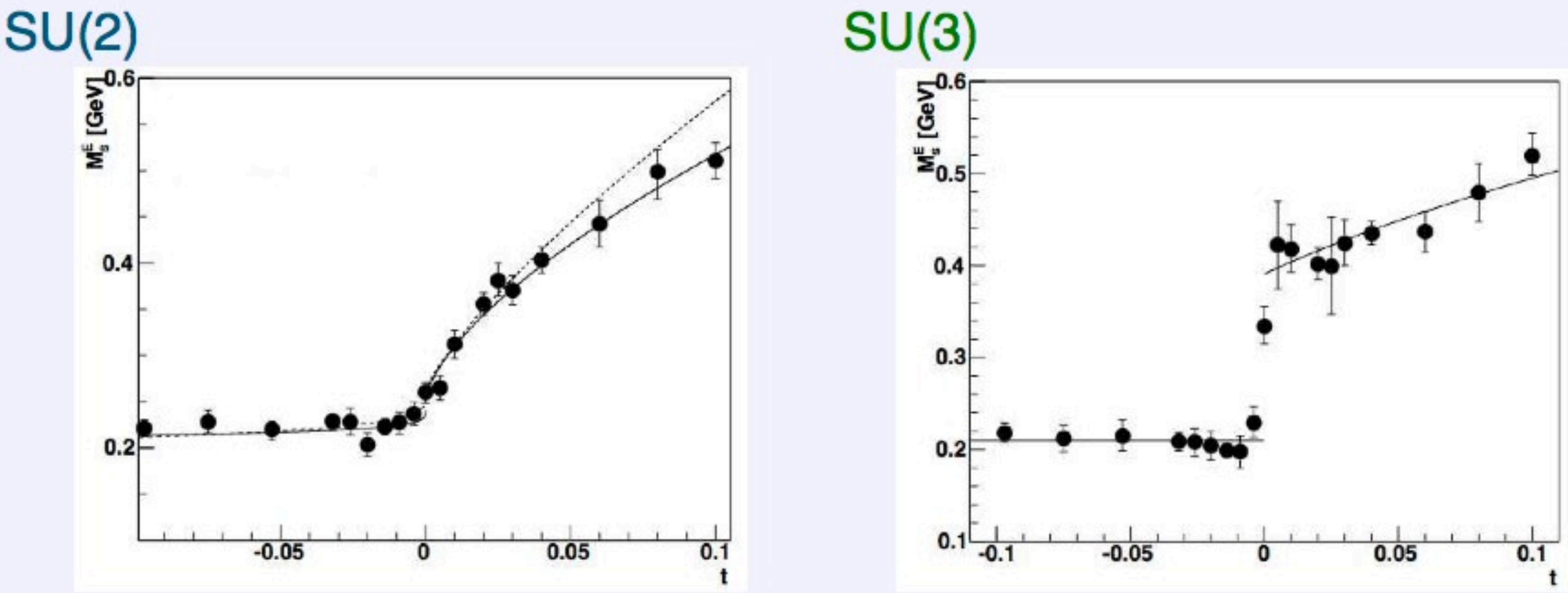

Cucchieri, Maas, Mendes, PRD 75 (2007)

C.F., Maas and Mueller, EPJC 68 (2010)

Cucchieri, Mendes, PoS FACESQCD (2010) 007.

Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker, Sternbeck, [arXiv:1108.1735 [hep-lat]].

# Gluon screening mass: SU(2) vs. SU(3)




$t = (T - T_c)/T_c$

Maas, Pawlowski, Smekal, Spielmann, arXiv:1110.6340.

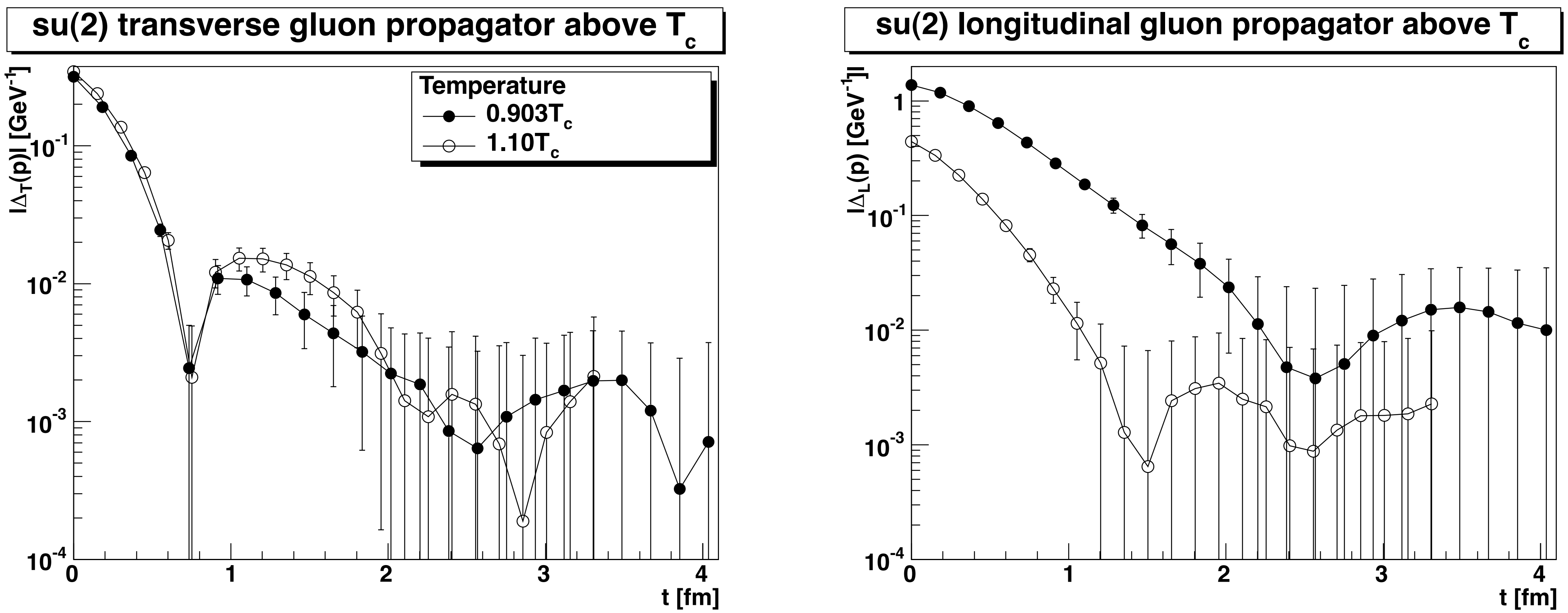
C.F., Maas and Mueller, EPJC 68 (2010)

- phase transition of second and first order clearly visible in electric screening mass

# Positivity violations

Schwinger function:

$$\Delta_g(t) = T \sum_{n_p} e^{it\omega_p} \left( \frac{Z(\omega_p, \vec{p})}{\omega_p^2 + \vec{p}^2} \right) \Bigg|_{\vec{p}=0}$$




A. Maas, arXiv:1106.3942

- transverse gluon violates positivity also above $T_c$
- longitudinal gluon may restore positivity for large T (quasiparticle picture not yet excluded...)



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# Further reading material


- R. Alkofer and L. von Smekal, ``The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states,'' Phys. Rept. 353 (2001) 281 [hep-ph/0007355].
- C. D. Roberts and S. M. Schmidt,``Dyson-Schwinger equations: Density, temperature and continuum strong QCD,'' Prog. Part. Nucl. Phys. 45 (2000) S1 [nucl-th/0005064],
- M.R.Pennington,``Swimming with quarks,'' J. Phys. Conf. Ser. 18 (2005) 1 [hep-ph/0504262].
- C. S. Fischer,``Infrared properties of QCD from Dyson-Schwinger equations,'' J. Phys. G 32 (2006) R253 [hep-ph/0605173].
- A. Maas,``Describing gauge bosons at zero and finite temperature,'' arXiv:1106.3942 [hep-ph].

# Overview

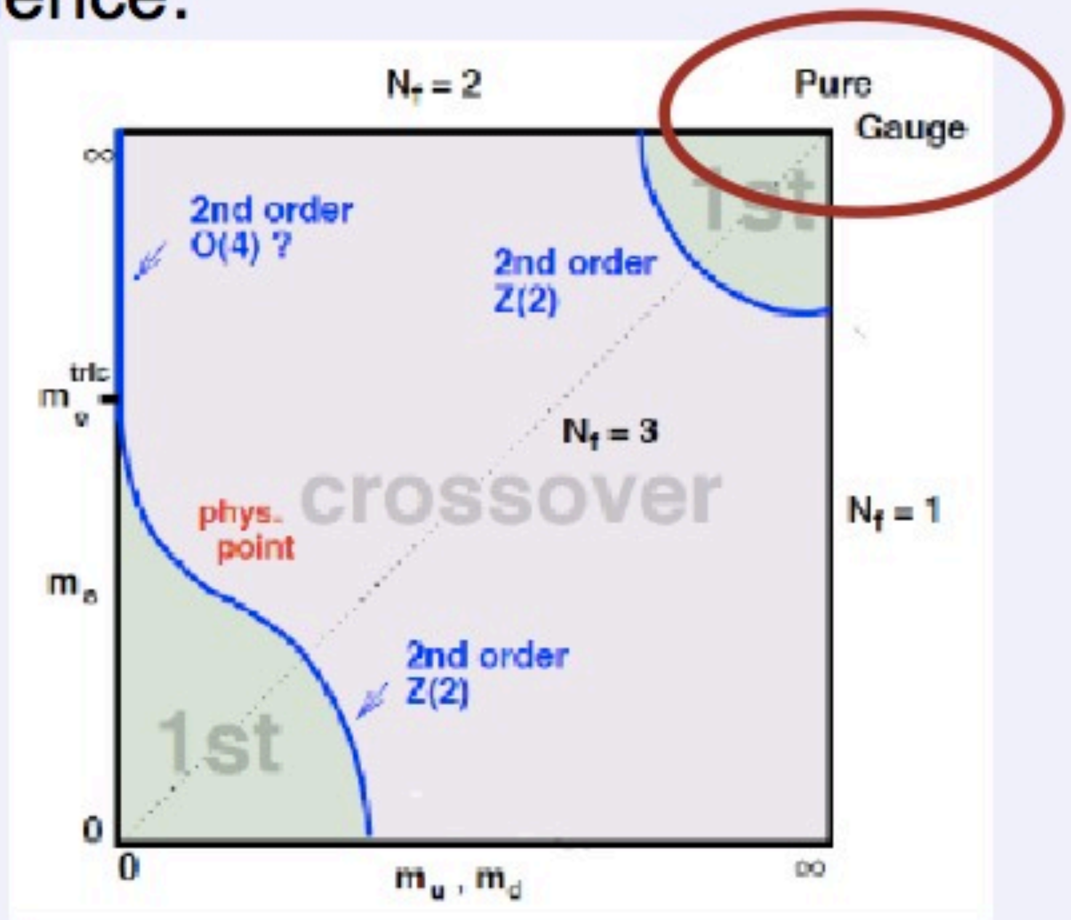
1. Introduction
   - General
   - Confinement
   - Dynamical chiral symmetry breaking
   - QCD phase diagram

2. QCD with functional methods: Dyson-Schwinger equations
   - Derivation
   - Simple example: pattern of chiral symmetry breaking
   - The gluon propagator
   - Gluons at finite temperature

3. QCD phase diagram
   - Dressed Polyakov-Loops
   - Phase diagram: quenched QCD
   - Transitions of $N_f$=2-QCD, chiral limit
   - Phase diagram: $N_f$=2 vs. $N_f$=2+1

# QCD phase transition: heavy quark limit/quenched

Quark mass dependence:

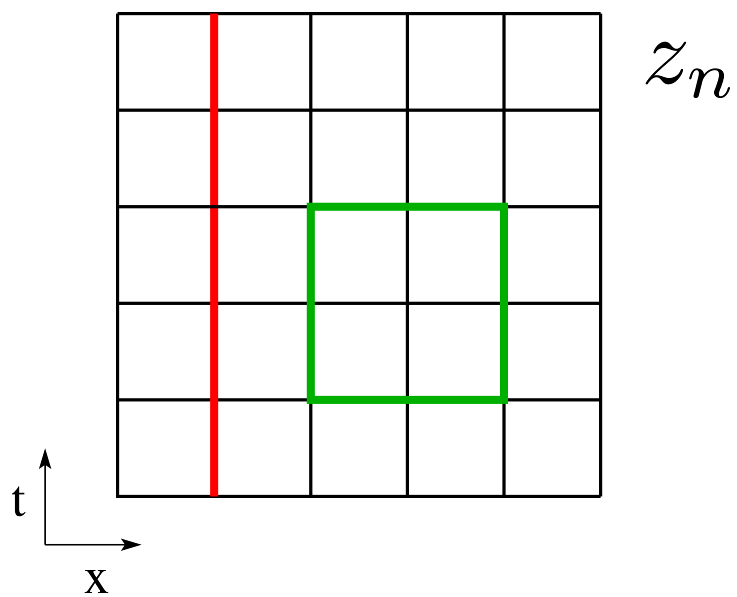



- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

# Order parameter: the dressed Polyakov-loop

ordinary Polyakov-loop:

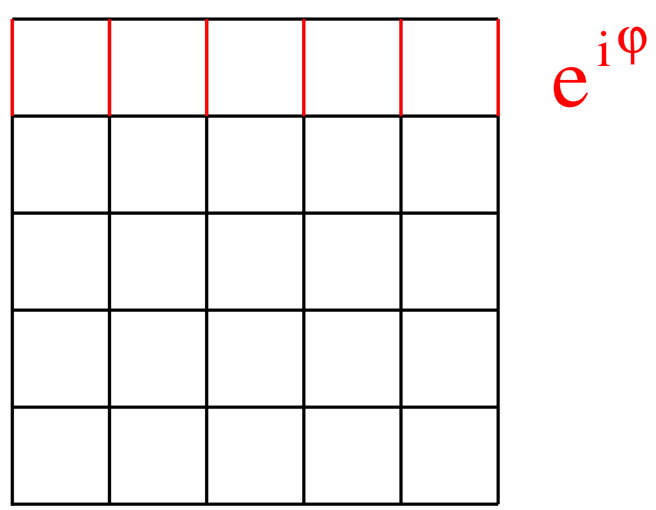
$$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau\, A_4(\tau, \vec{x})\right]$$




sensitive to center transformation

$$z_n = \exp[2\pi i\, n/N_c]\mathbb{1}, \quad n = 0..N_c - 1$$

Now consider general U(1)-valued boundary conditions in temporal direction for quark fields:




$$\psi(\vec{x}, 1/T) = e^{i\varphi}\, \psi(\vec{x}, 0)$$

$$\omega(n_t) = (2\pi T)(n_t + \varphi/2\pi)$$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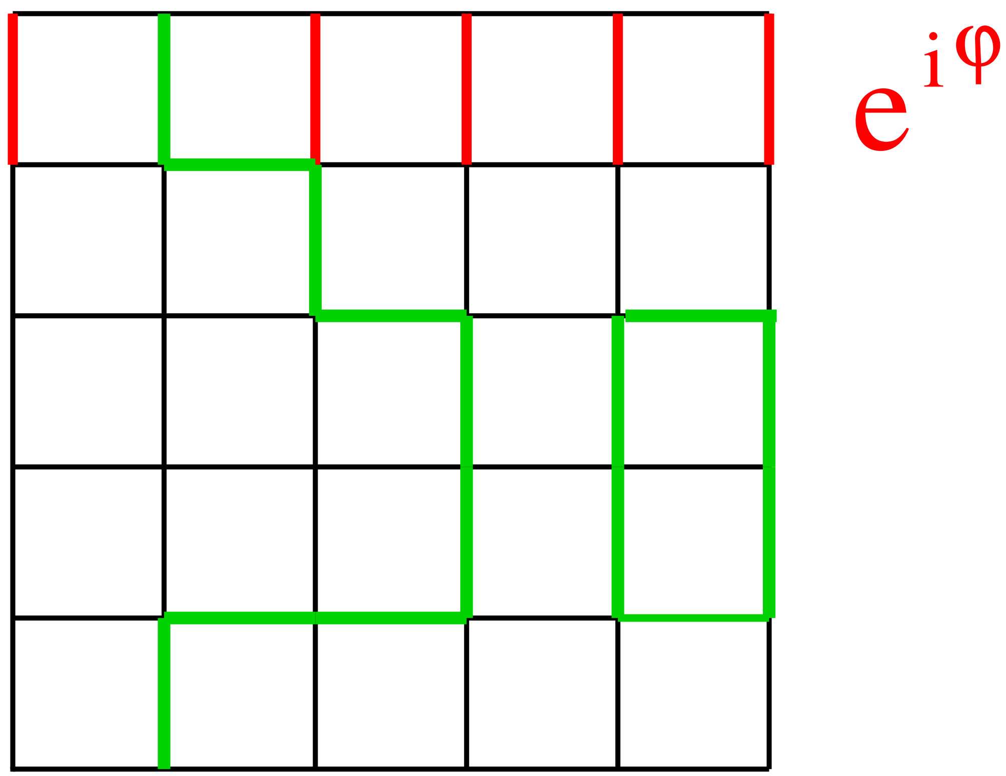
# Order parameter: the dressed Polyakov-loop II




$$\langle \bar{\psi}\psi \rangle_{\varphi} = \frac{1}{Vm} \sum_{l} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c U(l)$$

$m$ :explicit quark mass

$a$ :lattice spacing

$V$ :volume

$|l|$ :Loop length

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

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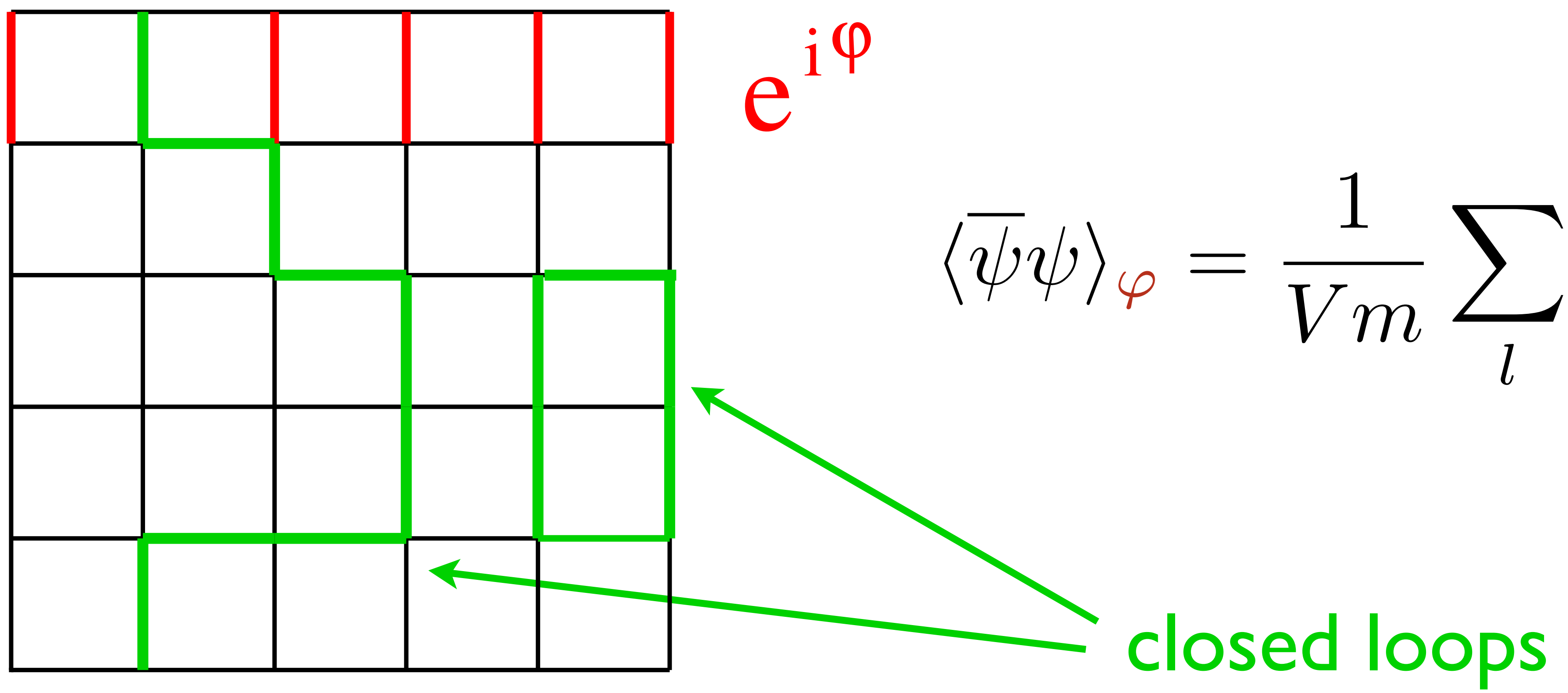


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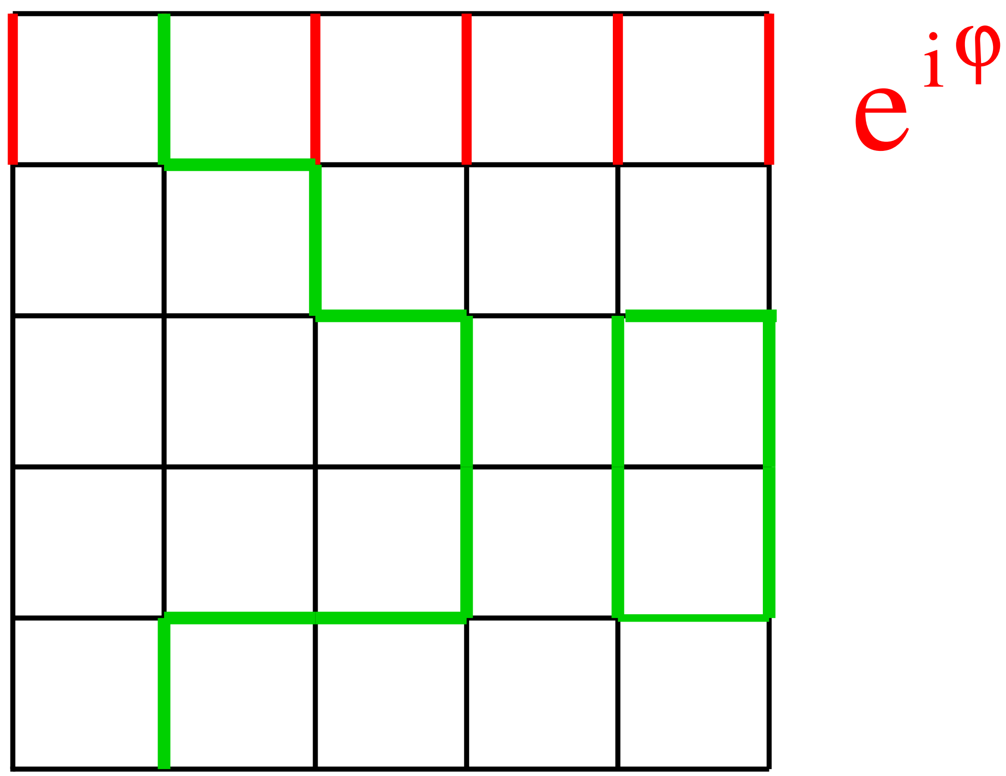


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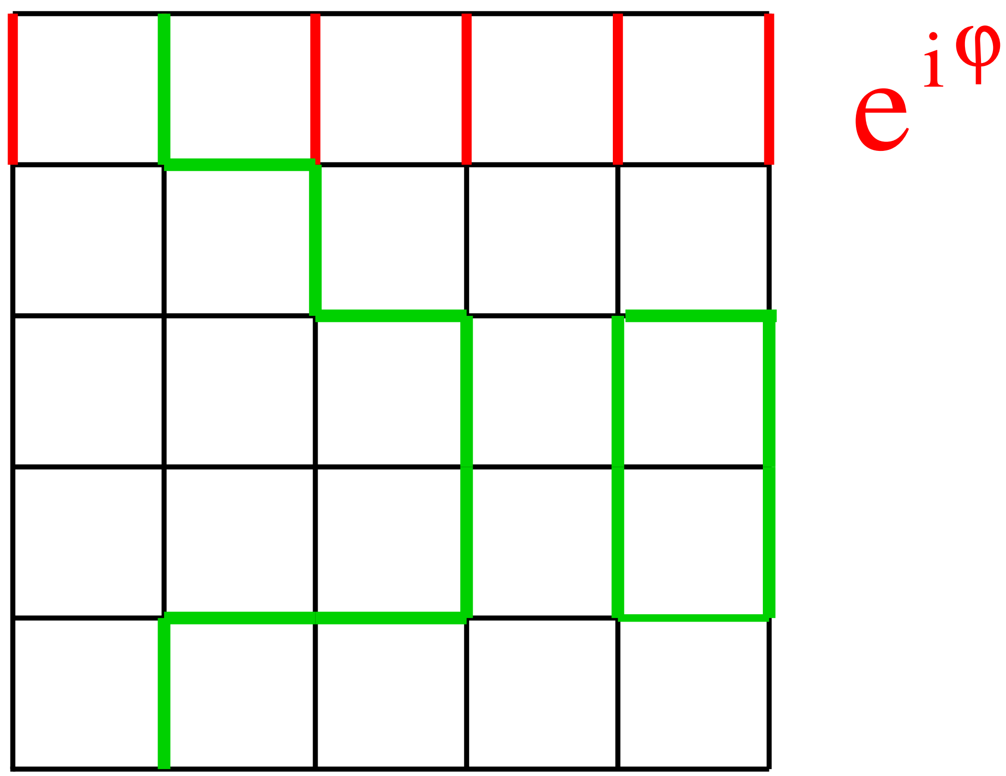

winding number

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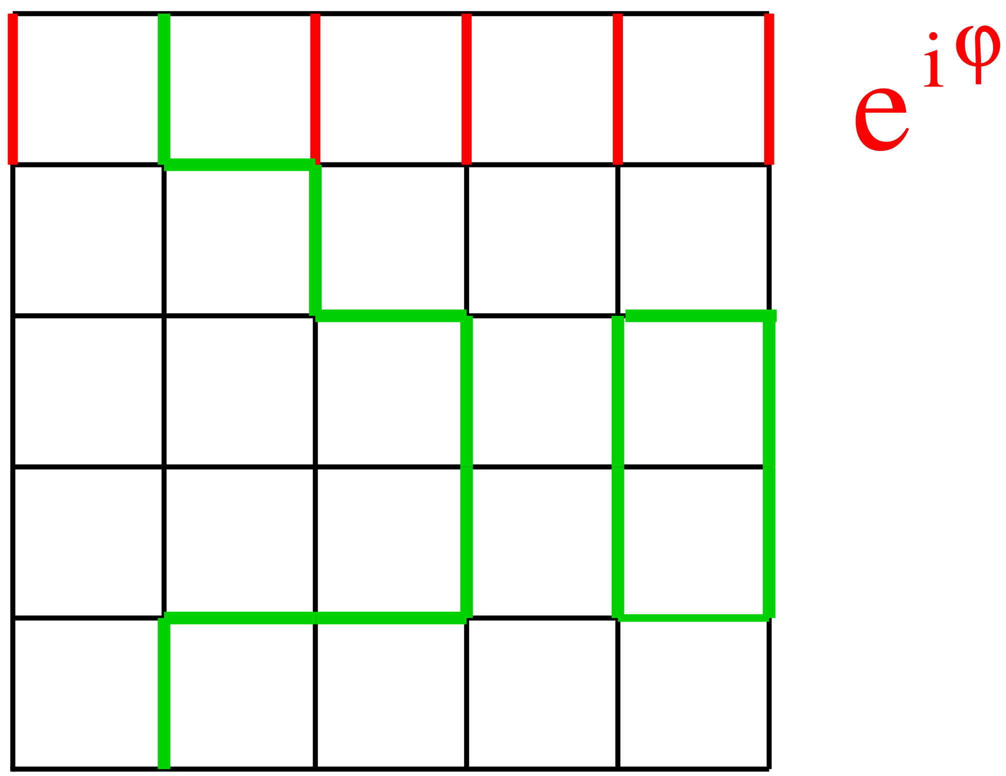


winding number

$$\langle \overline{\psi}\psi \rangle_\varphi = \frac{1}{Vm} \sum_l \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c U(l)$$

$m \to \infty : \quad n(l) = 1$

are ordinary Polyakov-loops

$m$ :explicit quark mass

$a$ :lattice spacing

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$|l|$ :Loop length

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Define dual condensate:

$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle\overline{\psi}\psi\rangle_\varphi$$

- n=1 projects out all loops winding once around the torus: dressed Polyakov-loop
- $\Sigma_1$ transforms under center transformations exactly like ordinary Polyakov-loop:

$$\begin{aligned} {}^z\Sigma_n &= -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle\overline{\psi}\psi\rangle_{\varphi+2\pi k/N_c} \\ &= -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i(\varphi+2\pi k/N_c)n}\, \langle\overline{\psi}\psi\rangle_\varphi \\ &= -z^n \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle\overline{\psi}\psi\rangle_\varphi \end{aligned}$$

# Order parameter: the dressed Polyakov-loop III

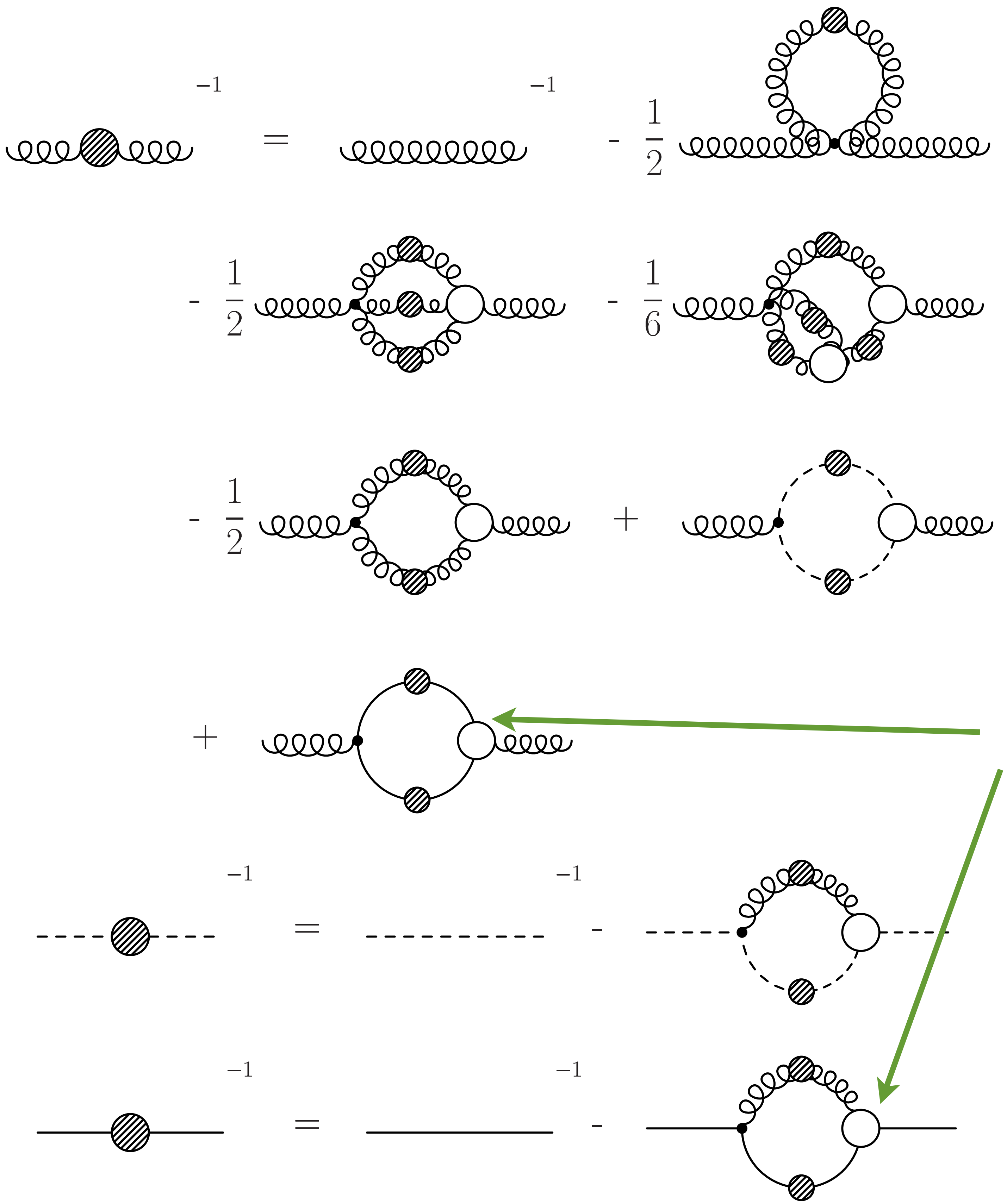
Define dual condensate:

$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle \overline{\psi}\psi \rangle_\varphi$$

- n=1 projects out all loops winding once around the torus: dressed Polyakov-loop
- $\Sigma_1$ is order parameter for center symmetry breaking
- $\Sigma_1$ is accessible with Dyson-Schwinger equations or the functional renormalization group


C. Gattringer, PRL 97, 032002 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD 75, 114003 (2007)
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD 77 094007 (2008)
F. Synatschke, A. Wipf and K. Langfeld, PRD 77, 114018 (2008)
CF, PRL 103 052003 (2009)
CF, J.A.Mueller, PRD 80 (2009) 074029
J. Braun, L. Haas, F. Marhauser, J.M. Pawlowski, PRL 106 022002 (2011)

# DSEs of QCD




quark gluon vertex

- much studied at T=0

Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.
C.F, R. Williams, PRL **103** (2009) 122001

- T≠0: ansatz,
  T,m,μ dependent

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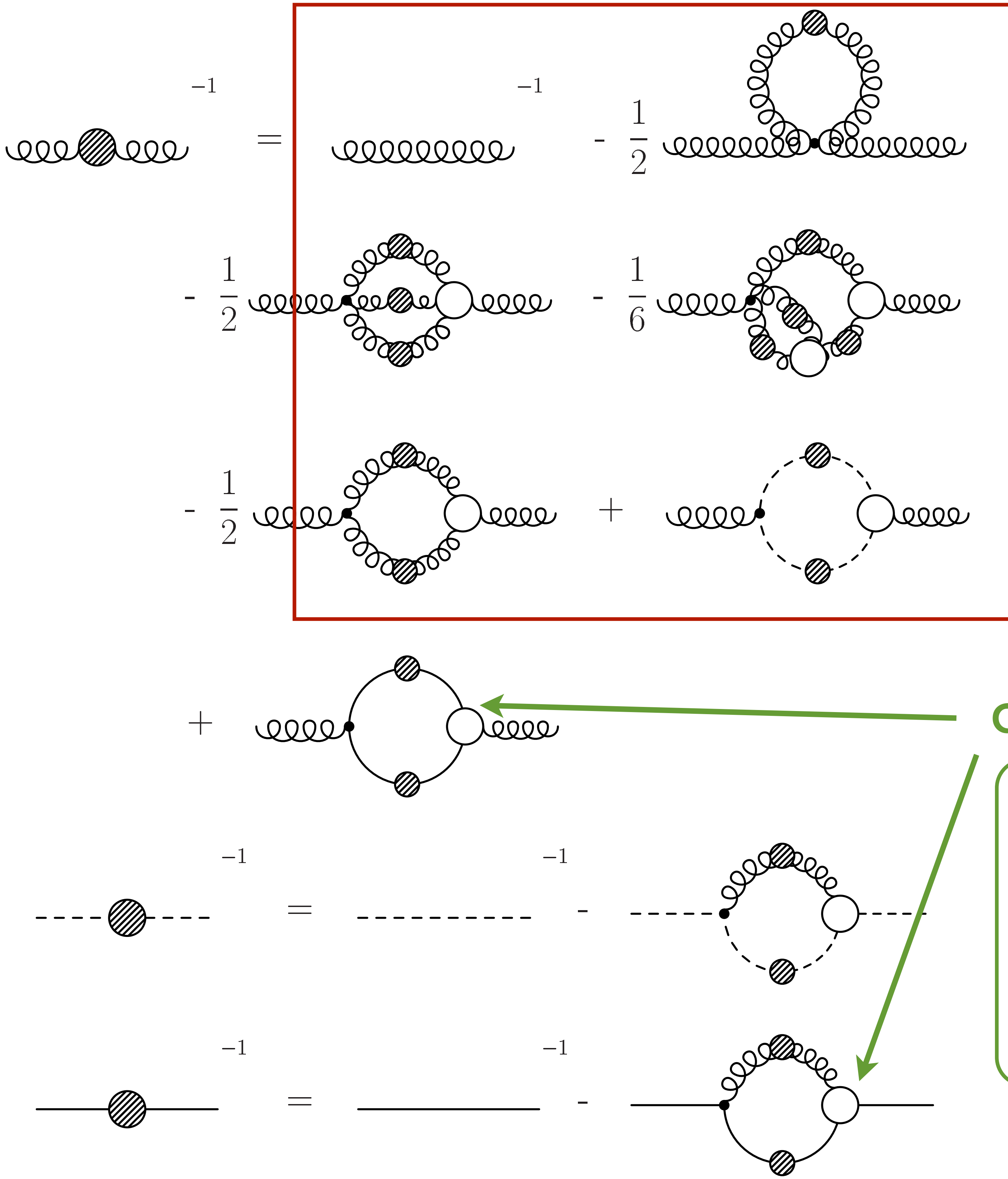

quenched lattice propagator

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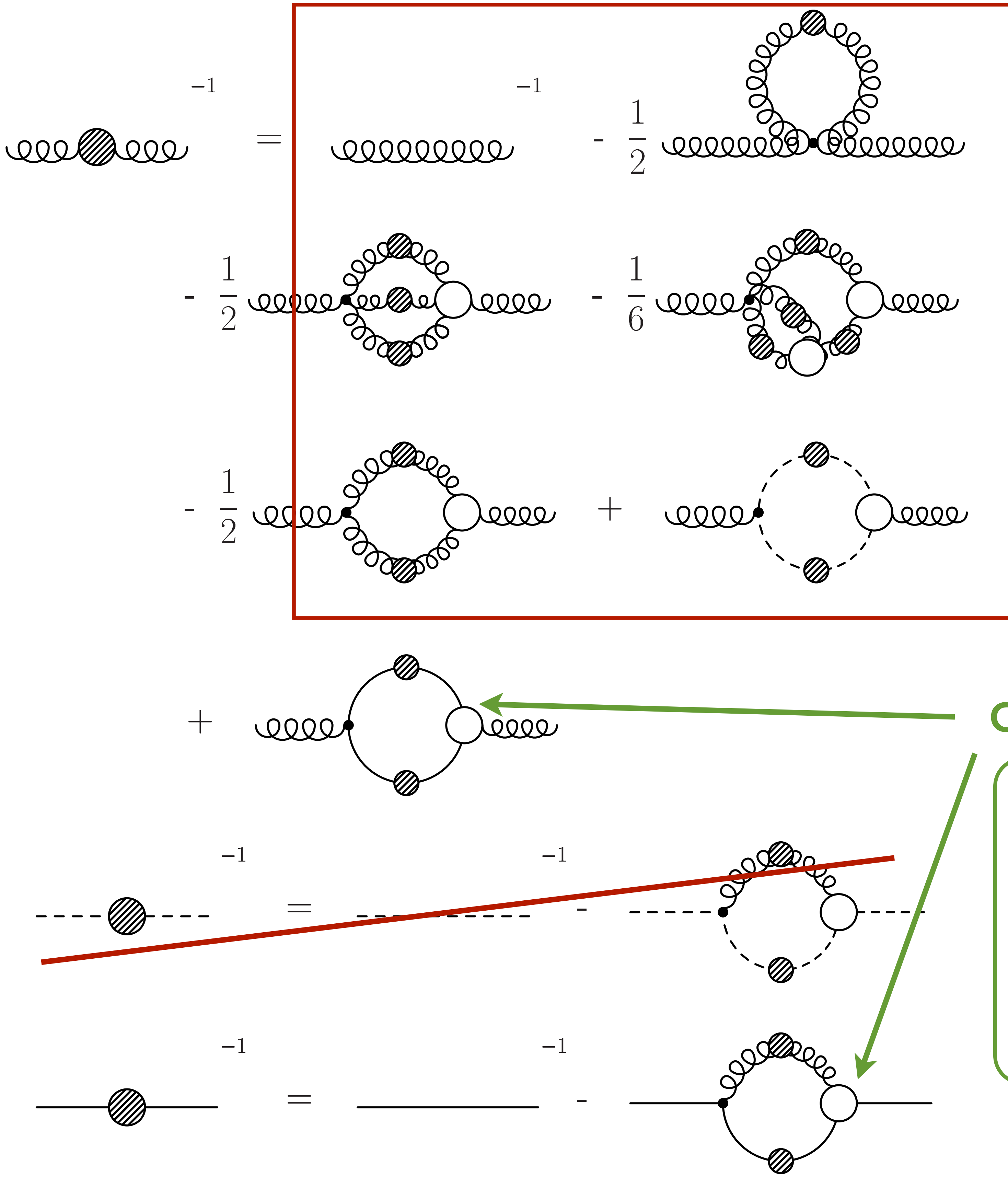

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- T≠0: ansatz, T,m,μ dependent



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# The quark-gluon interaction

Vertex ansatz:

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left( \delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$
$$\times \left( \frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

- UV: correct RG running of vertex
- IR: interaction strength
- satisfies Slavnov-Taylor identity approximately

$$q_\nu \Gamma_\nu(q,k,p) = [S^{-1}(k)\, H(k,p) - H(k,p)\, S^{-1}(p)]\; G(q)$$

- Scales $\Lambda$, $d_2$ adjusted to Yang-Mills sector, strength $d_1$ to $f_\pi$

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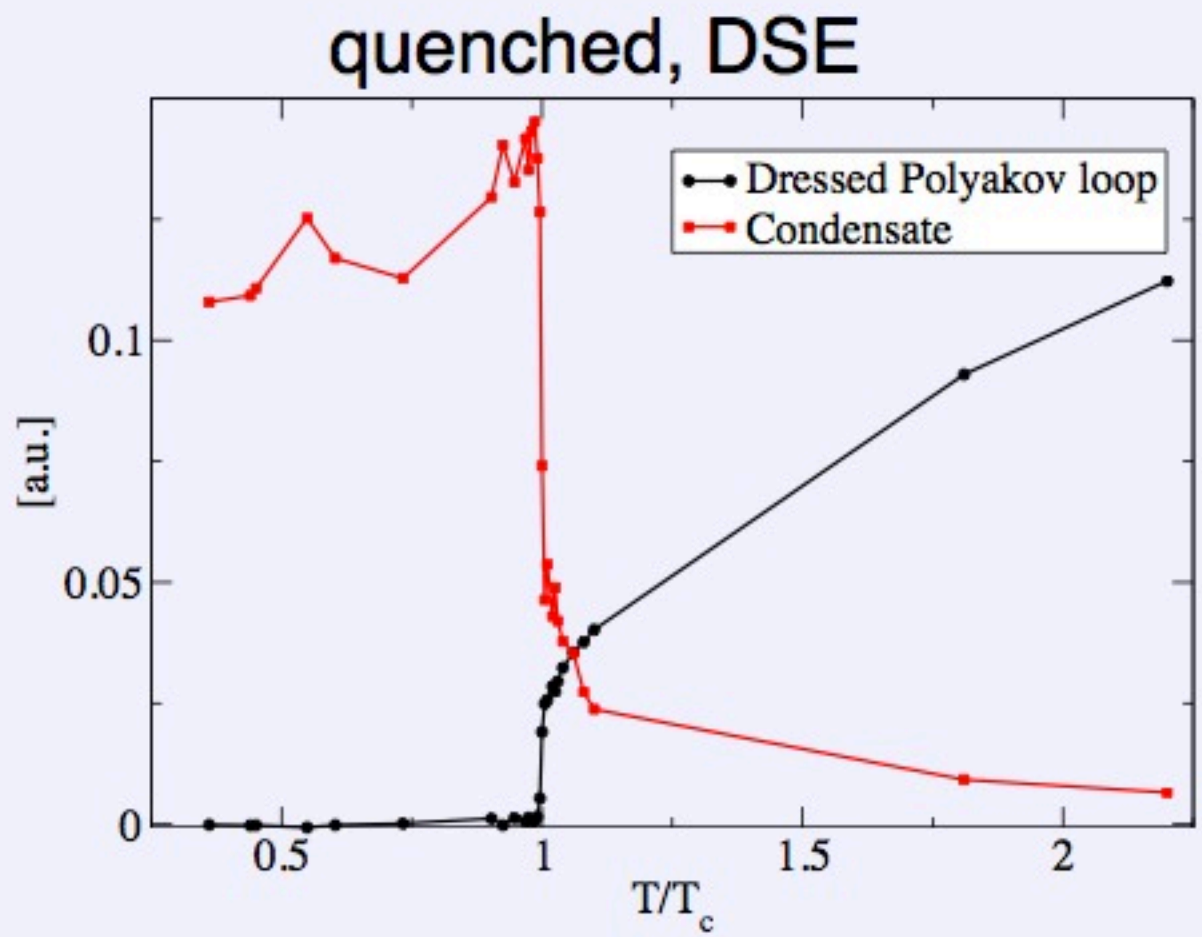
# Quenched QCD: (De-)Confinement




Luecker, C.F., arXiv:1111.0180; C.F., Maas, Mueller, EPJC 68 (2010).

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- $T \leq T_c$: increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.
cf. Braun, Gies, Pawlowski, PLB 684 (2010) 262-267.



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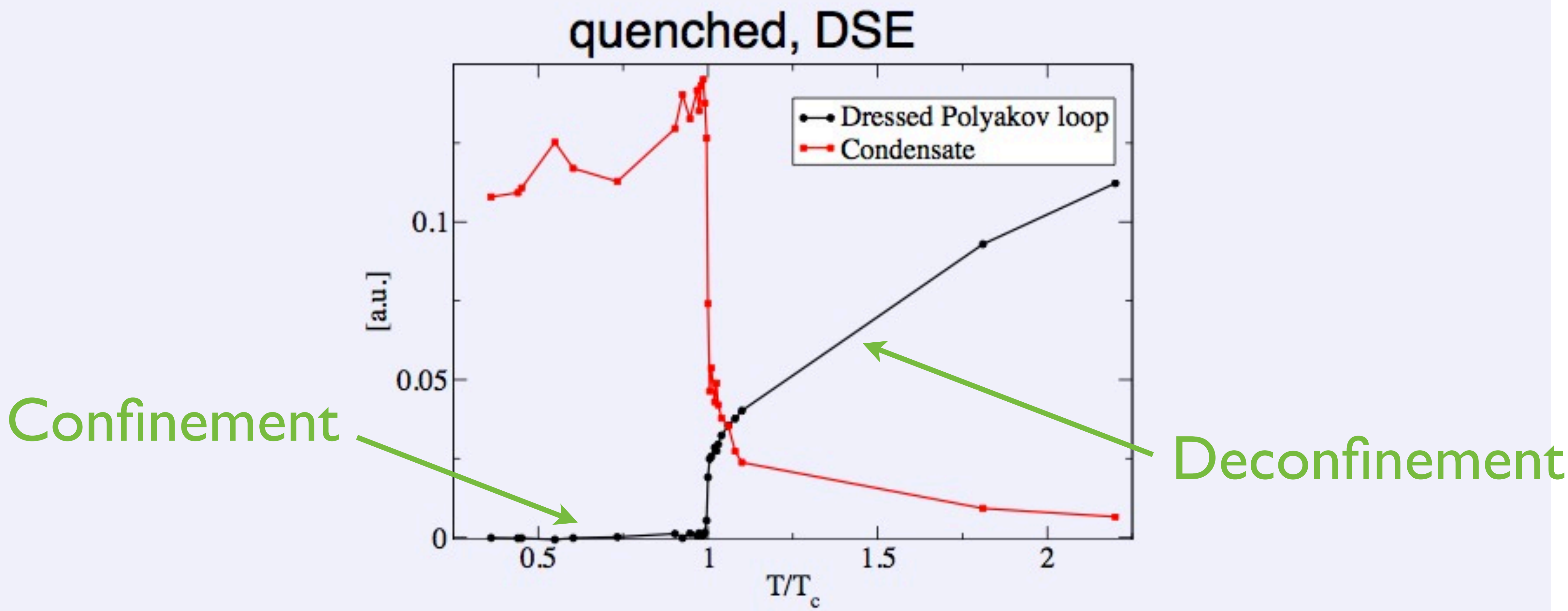


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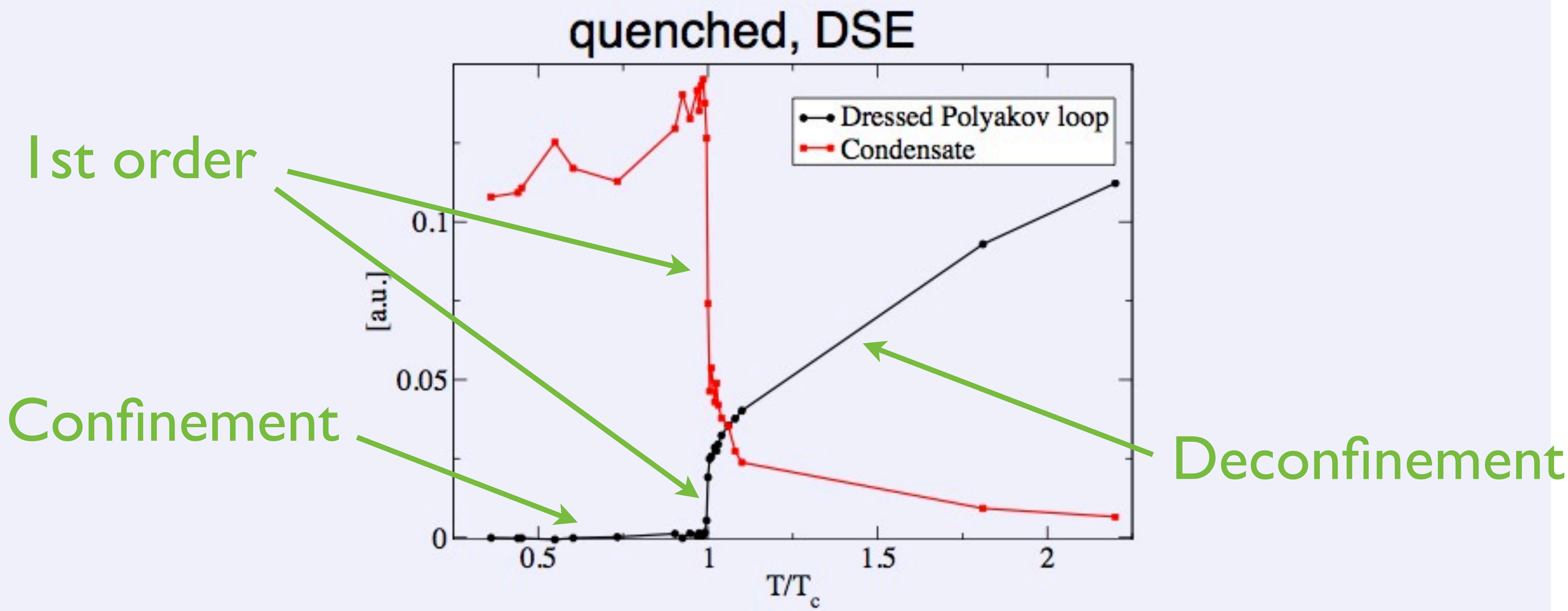


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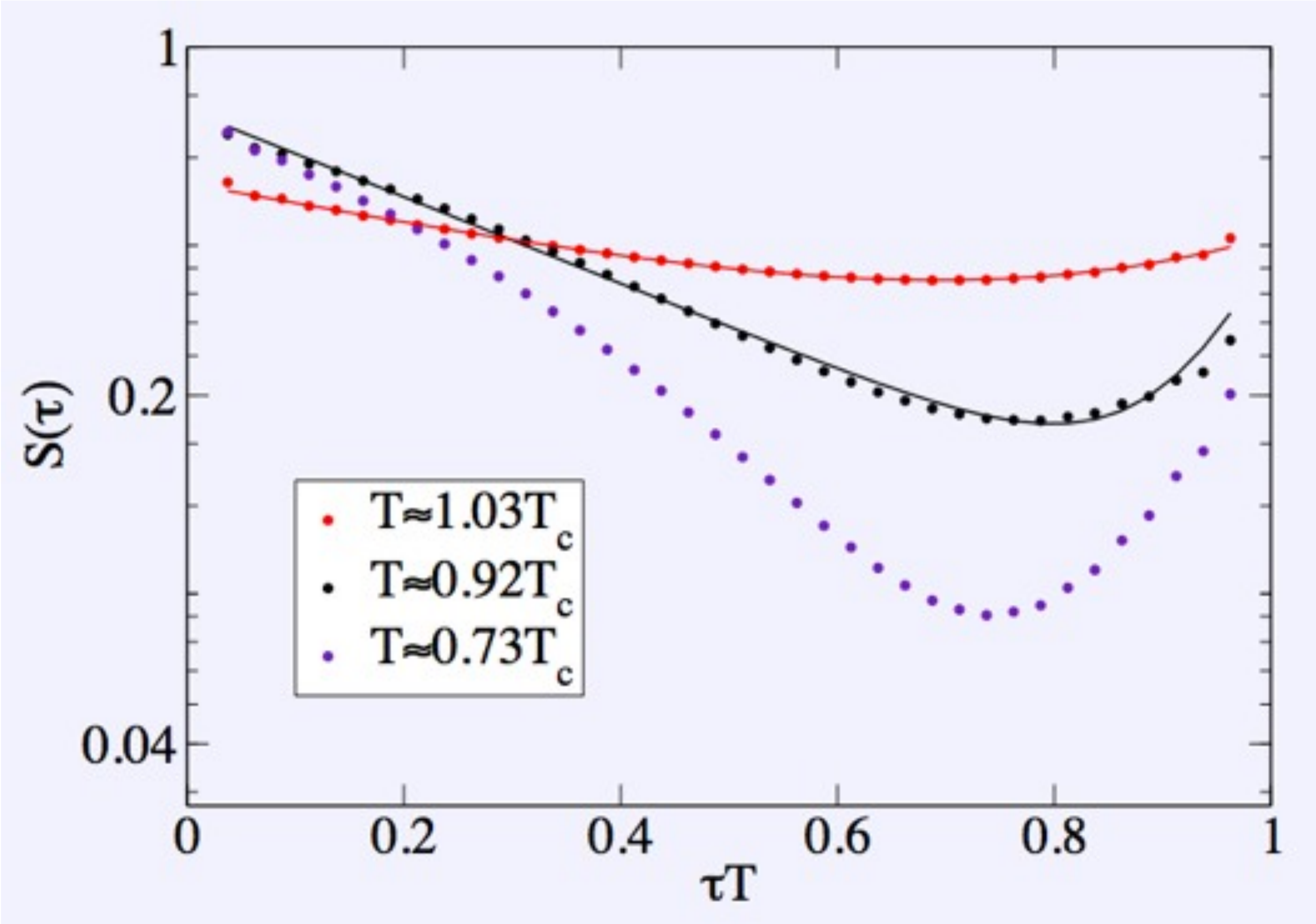
# Quenched QCD: Positivity violations I

Schwinger function: $$S(\tau) = T\sum_{n_p} e^{i\tau\omega_p}\left(\frac{i\omega_n C(i\omega_n) + B(i\omega_n)}{\omega_n^2 C^2(i\omega_n) + B^2(i\omega_n)}\right)$$




- $T>T_c$: positive curvature - quasiparticle picture possible
- $T<T_c$: negative curvature - positivity violations

Karsch and Kitazawa, PRD 80, 056001 (2009)
Mueller, CF, Nickel, EPJC 70 (2010) 1037-1049

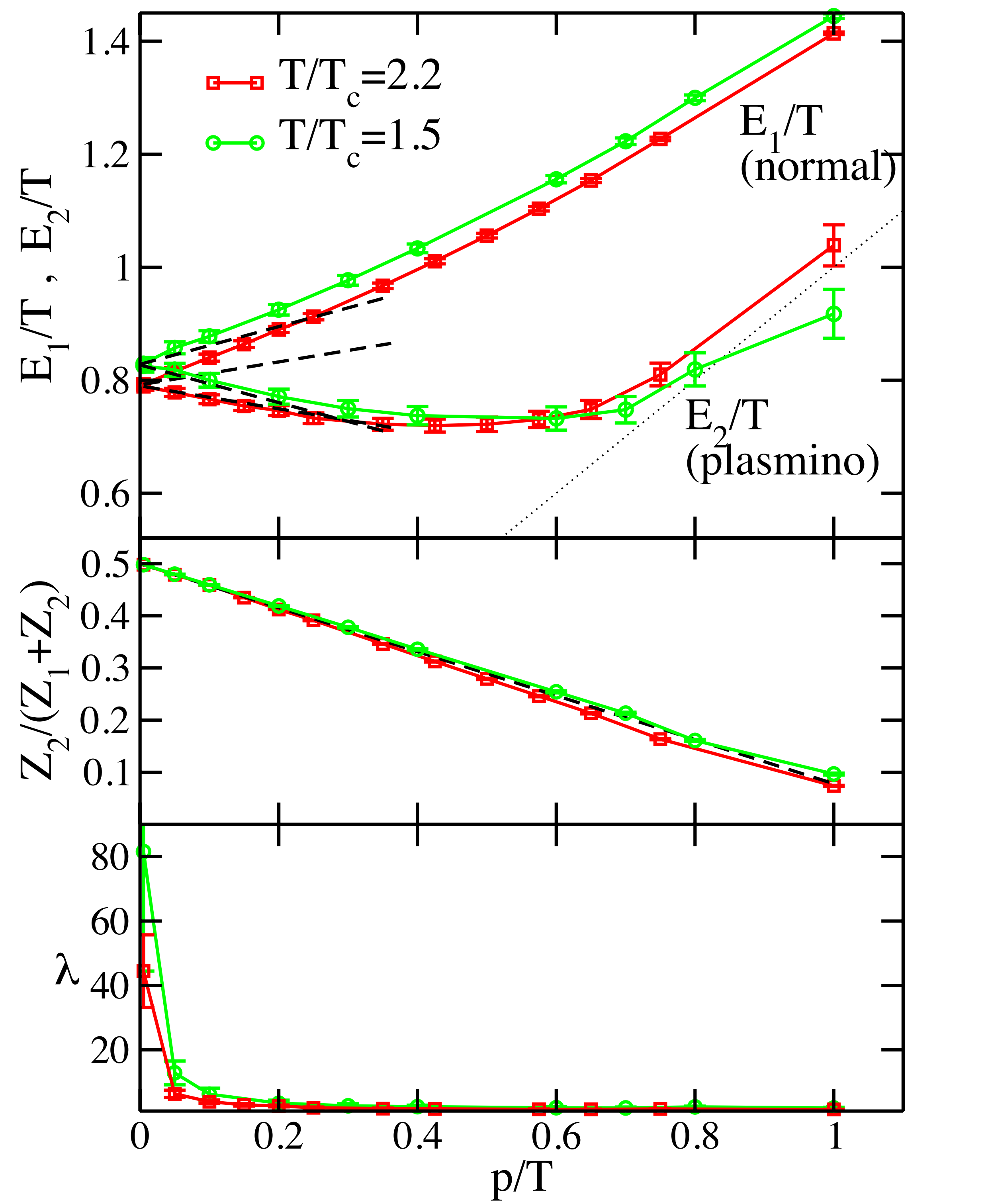
Idea: Fit spectral representation to quark propagator

Karsch and Kitazawa, PRD 80, 056001 (2009)

$$S(p_0,\vec{p}) = \int dp_0' \frac{\rho(p_0',\vec{p})}{p_0-\omega'}$$

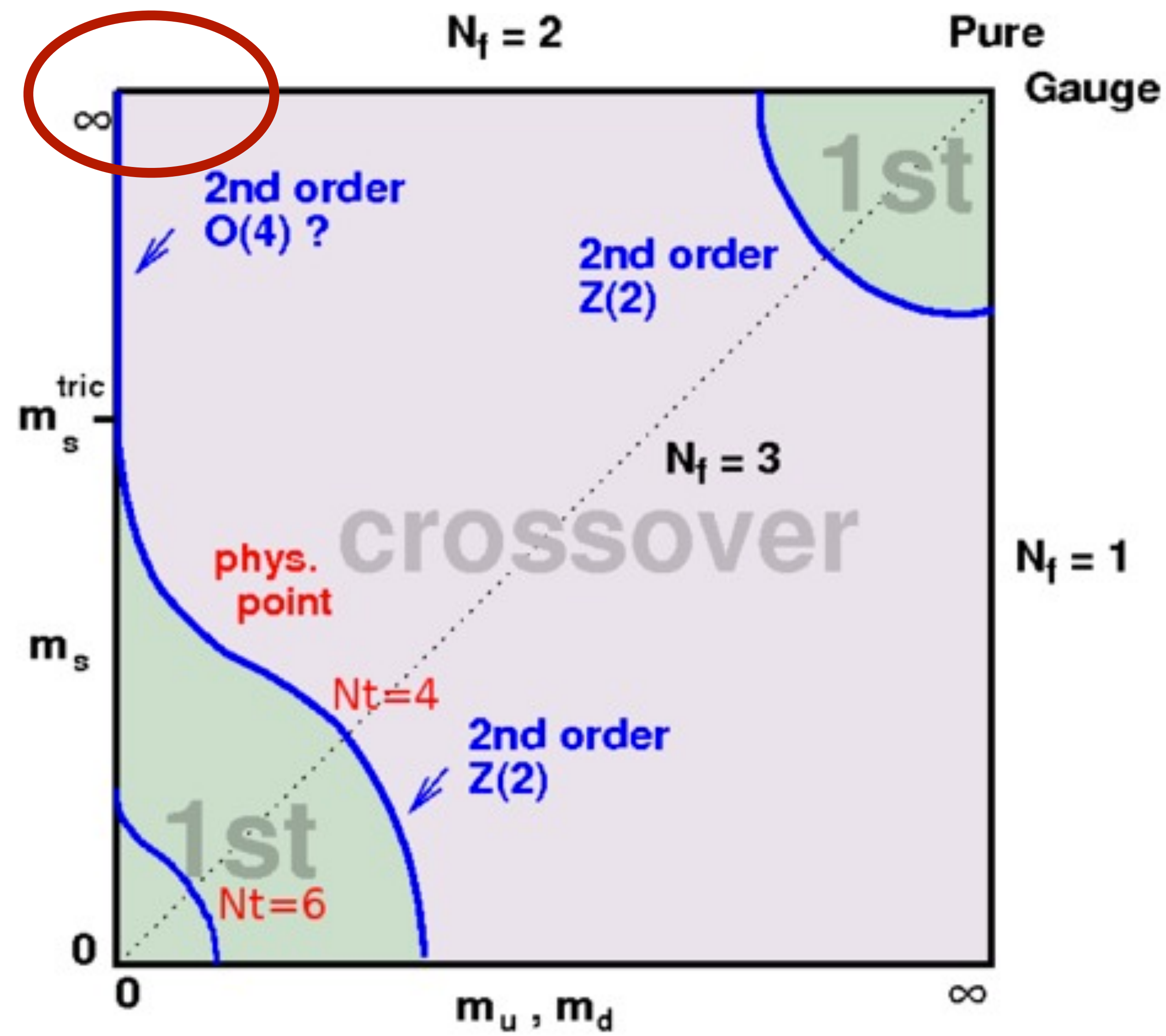
$$\rho_\pm(p_0,p) = 2\pi\left[Z_1\delta(p_0\mp E_1) + Z_2\delta(p_0\pm E_2)\right] + \lambda\left(1-\frac{p_0^2}{p^2}\right)e^{-p_0^2}\,\Theta\left(1-\frac{p_0^2}{p^2}\right)$$

- Quark, plasmino and continuum (Landau damping)
- agreement with HTL at p=0




Mueller, CF, Nickel, EPJC 70 (2010) 1037-1049

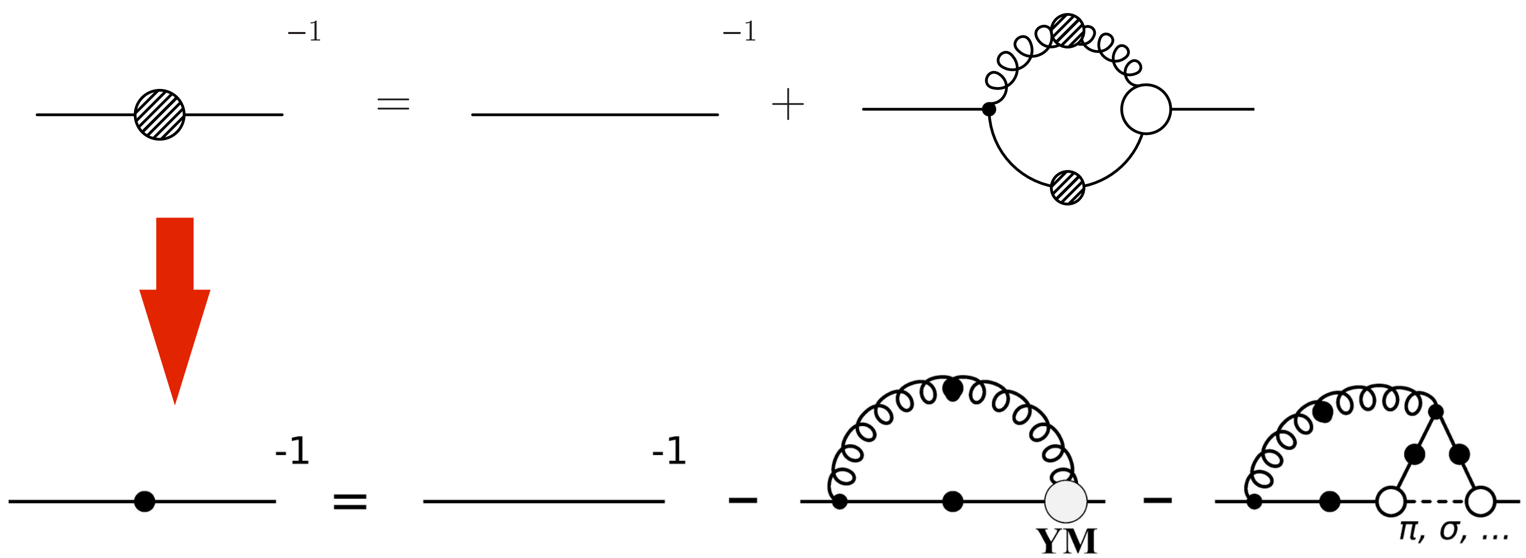
# QCD phase transitions: chiral limit

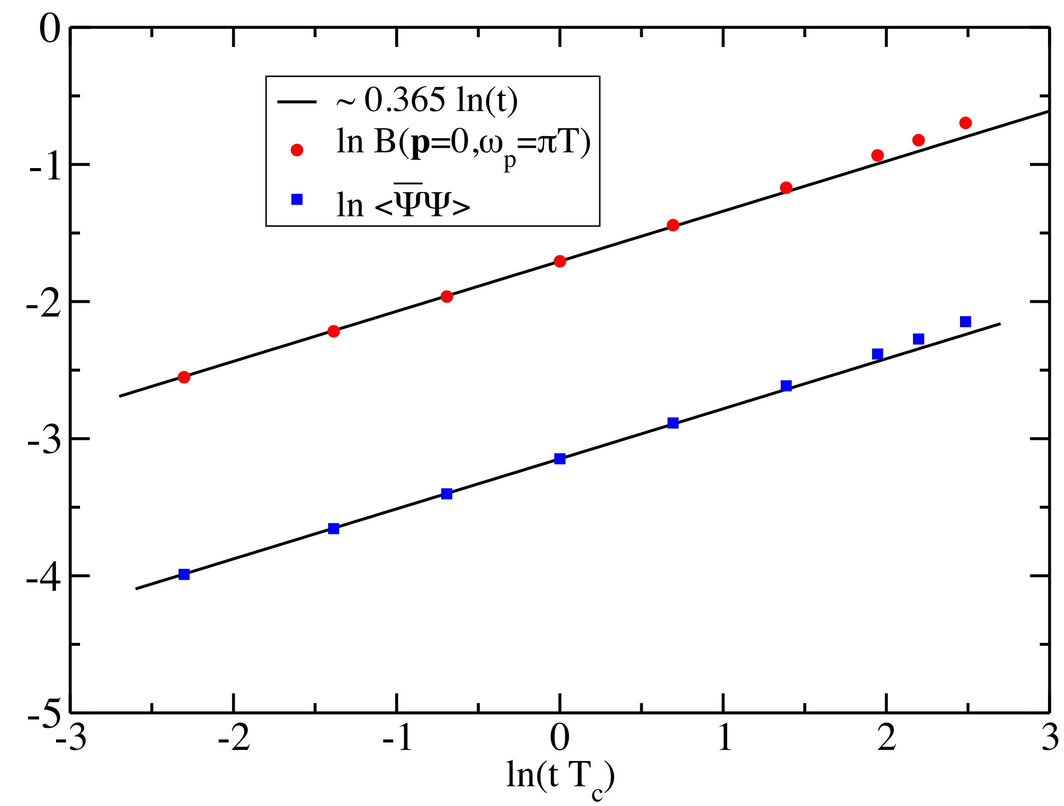



- $N_f$=2, chiral limit: phase transition dominated by Goldstone boson physics → Quark-Meson (QM) model
- SU(2)×SU(2) ≅ O(4)-second order vs. O(2)×O(4)-first order

Pisarski and Wilczek, PRD 29 (1984) 338

# Critical scaling from DSEs



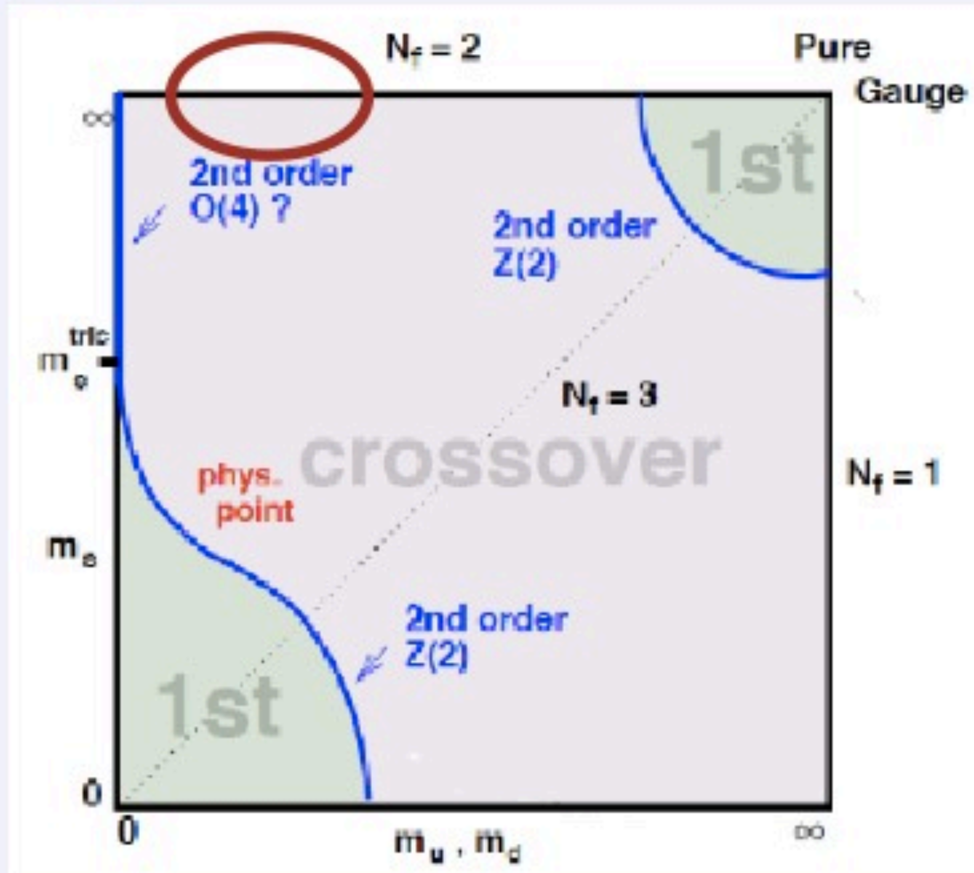

- Need to take meson part of vertex explicitly into account
- T=0: meson cloud corrections of order of 10-20 %
  CF, Williams, PRD 78 (2008) 074006
- T=$T_c$: meson corrections are dominant !
- Critical scaling: $\langle\bar\Psi\Psi\rangle(t) \sim B(t) \sim t^{\nu/2}$

$$f_s^2 \sim t^\nu \qquad (t = (T_c - T)/T_c)$$

CF and Mueller, PRD 84 (2011) 054013

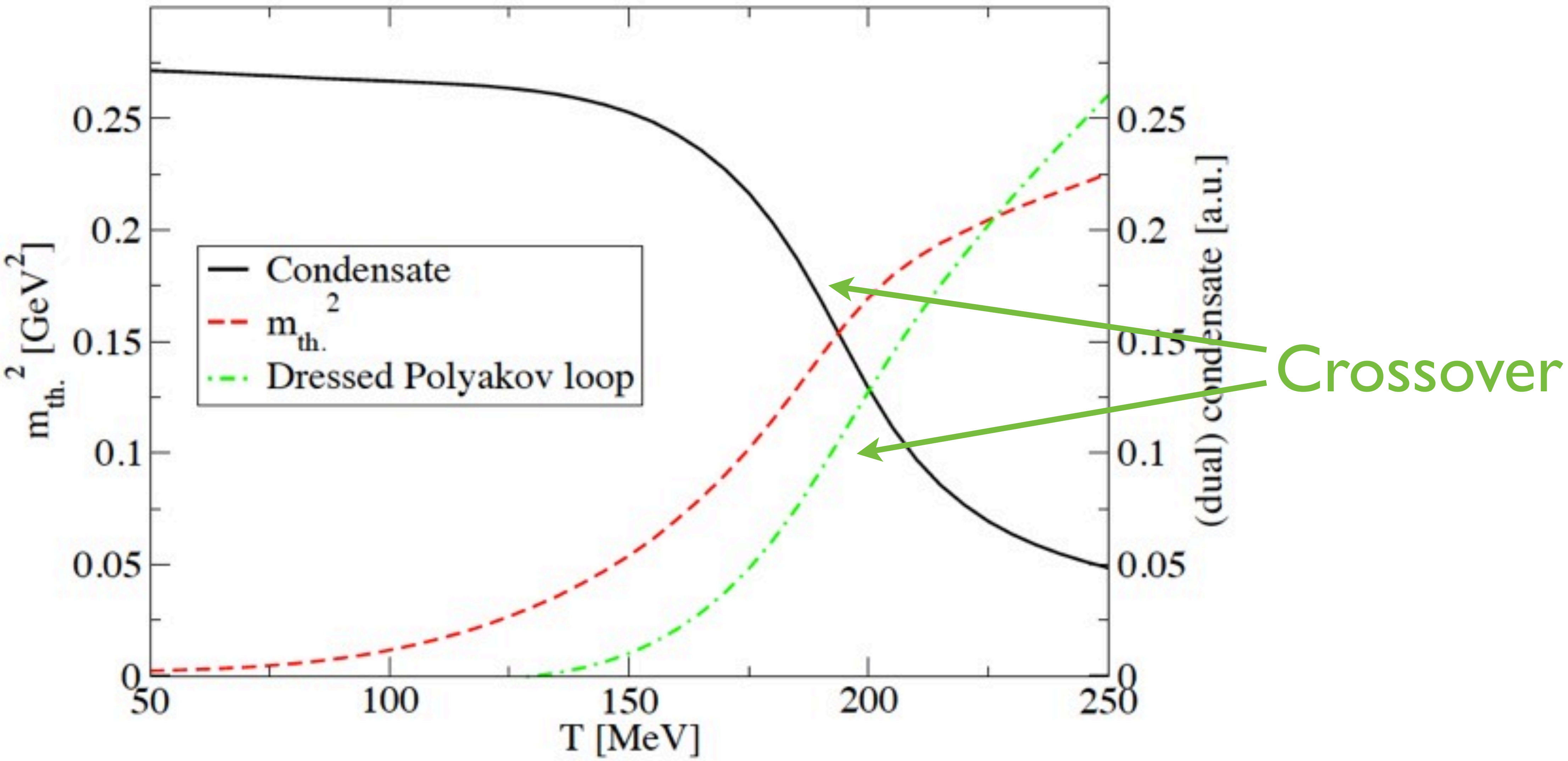
# QCD phase transitions: $N_f$=2

Quark mass dependence:




- $N_f = 2$, physical up/down quark masses
- Transition controlled by chiral dynamics

# $N_f$=2: Transition temperatures at μ=0



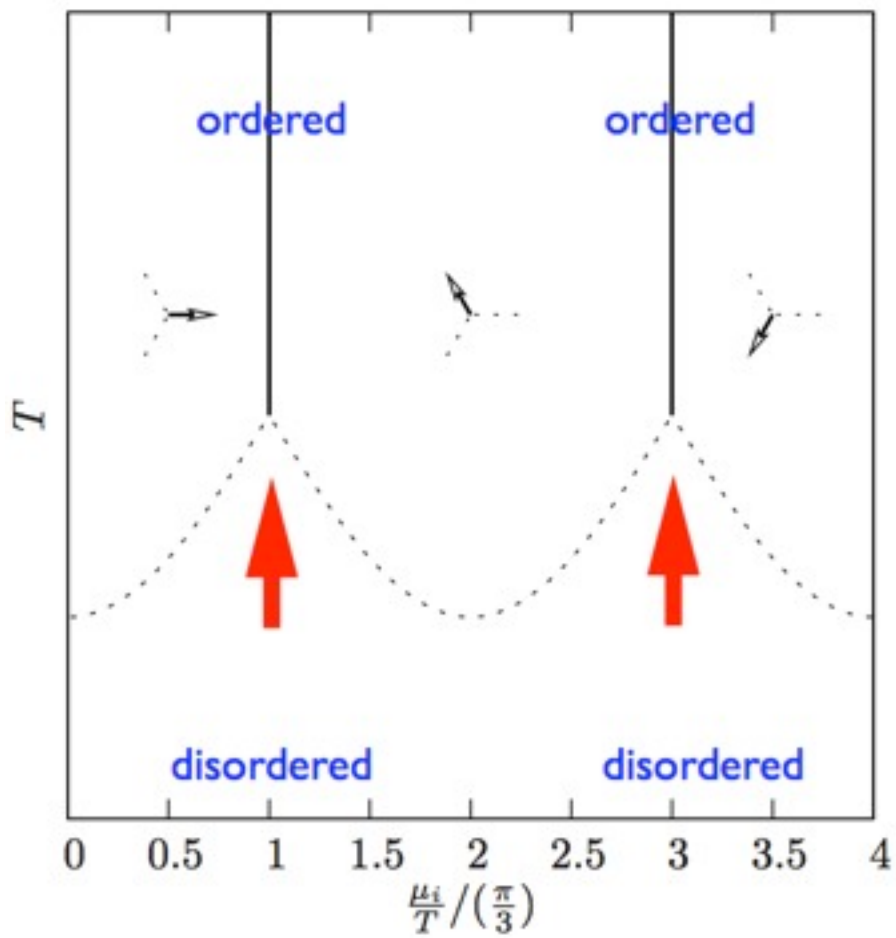

CF, Luecker, Mueller, PLB 702 (2011) 438-441
CF, Luecker, arXiv:...

- $T_\chi \approx 185$ MeV
- $T_{conf} \approx 195$ MeV
- similar results in FRG-approach

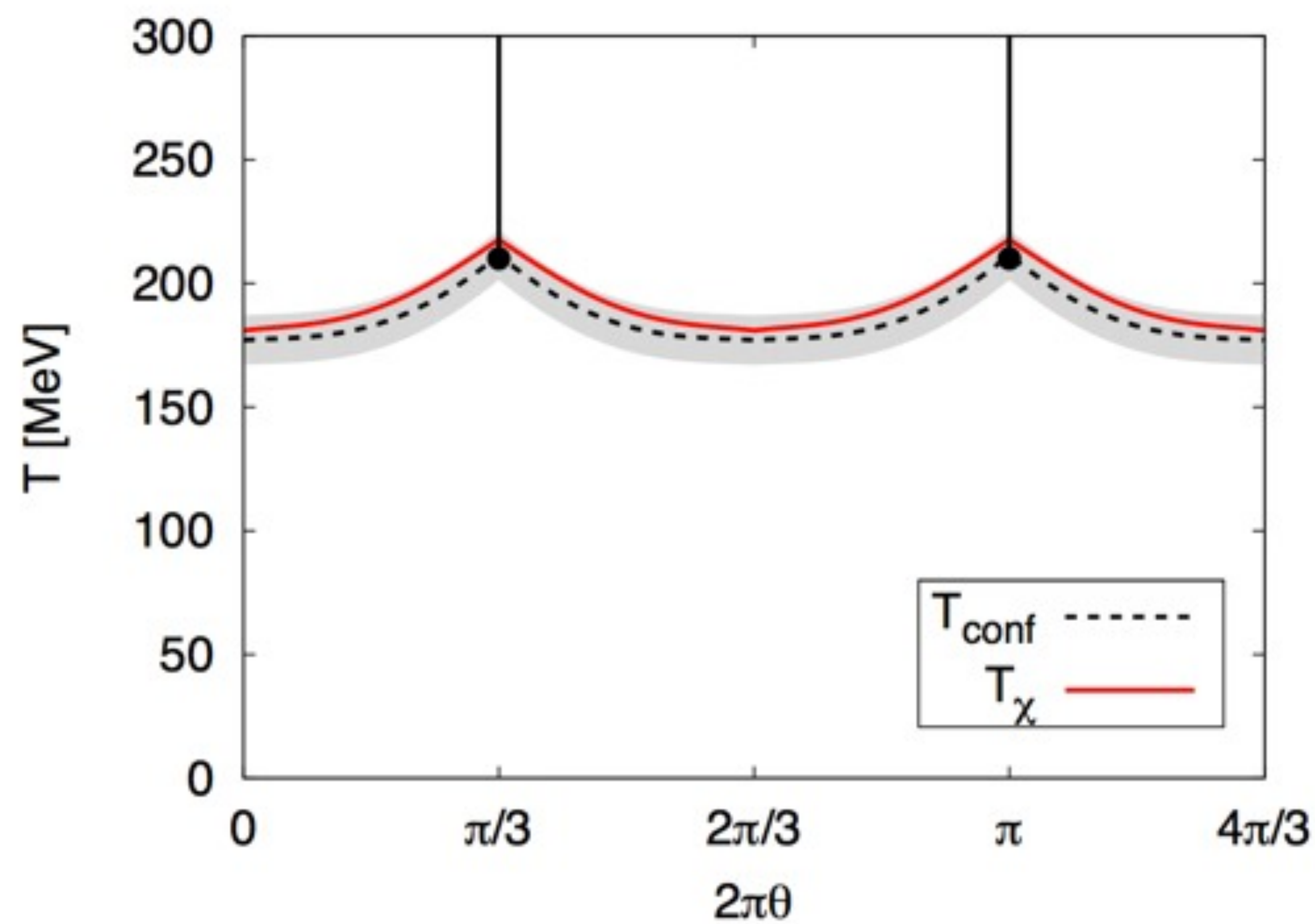
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011) 022002

# $N_f$=2: Imaginary chemical potential

No sign problem: comparison with lattice QCD possible



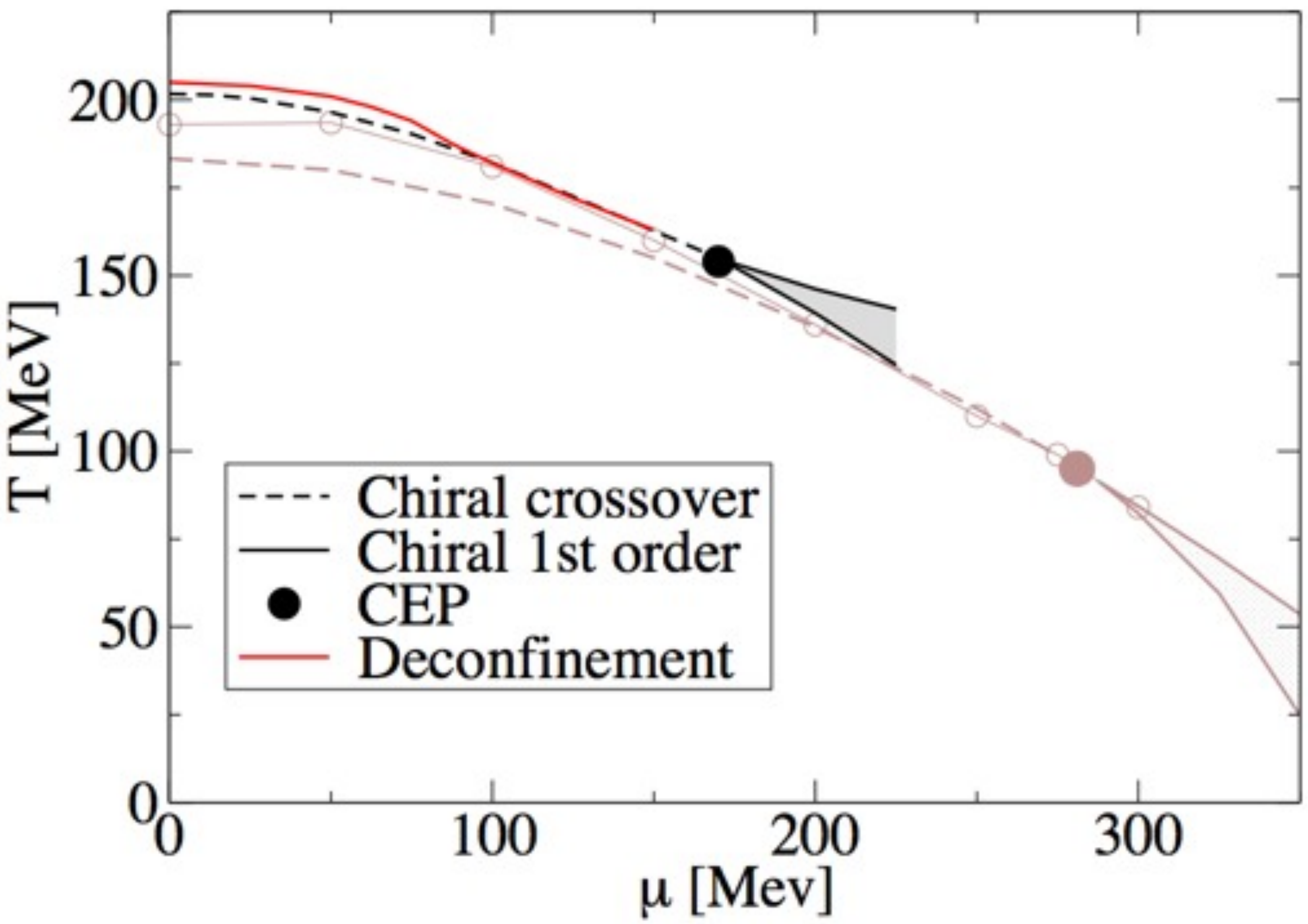

de Forcrand and Philipsen,PRL 105 (2010) 152001

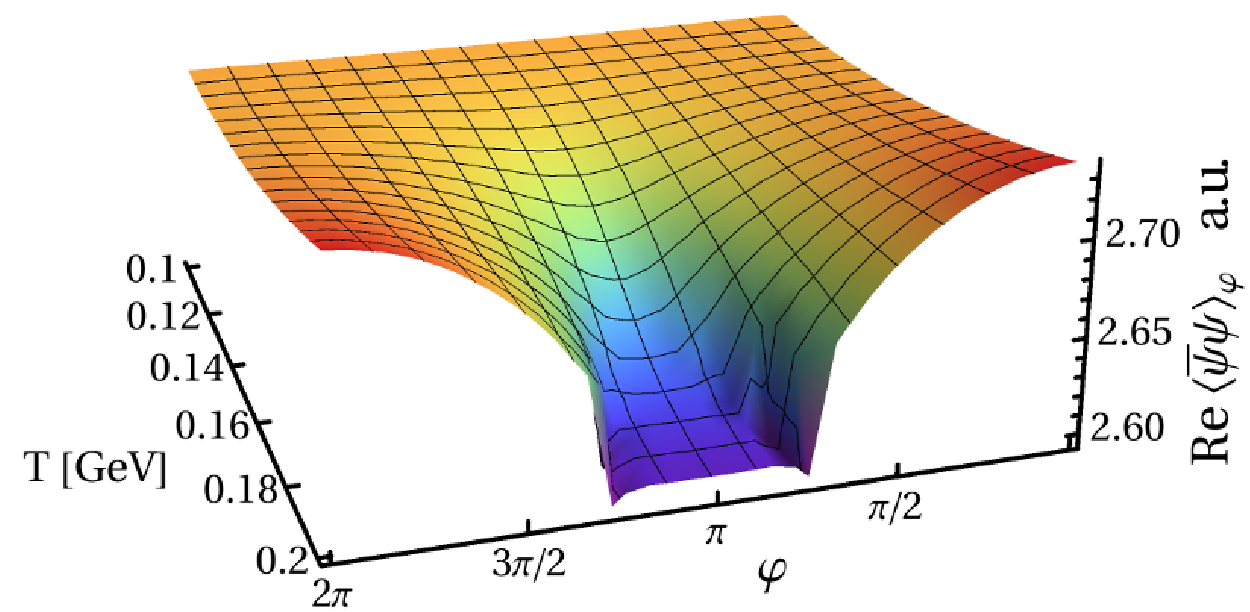



Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011) 022002

- Z(3)-symmetry of QCD with imaginary μ
- above $T_c$: order parameter Im(Polaykov-loop)
- functional RG results agree with lattice QCD

# $N_f$=2: QCD phase diagram

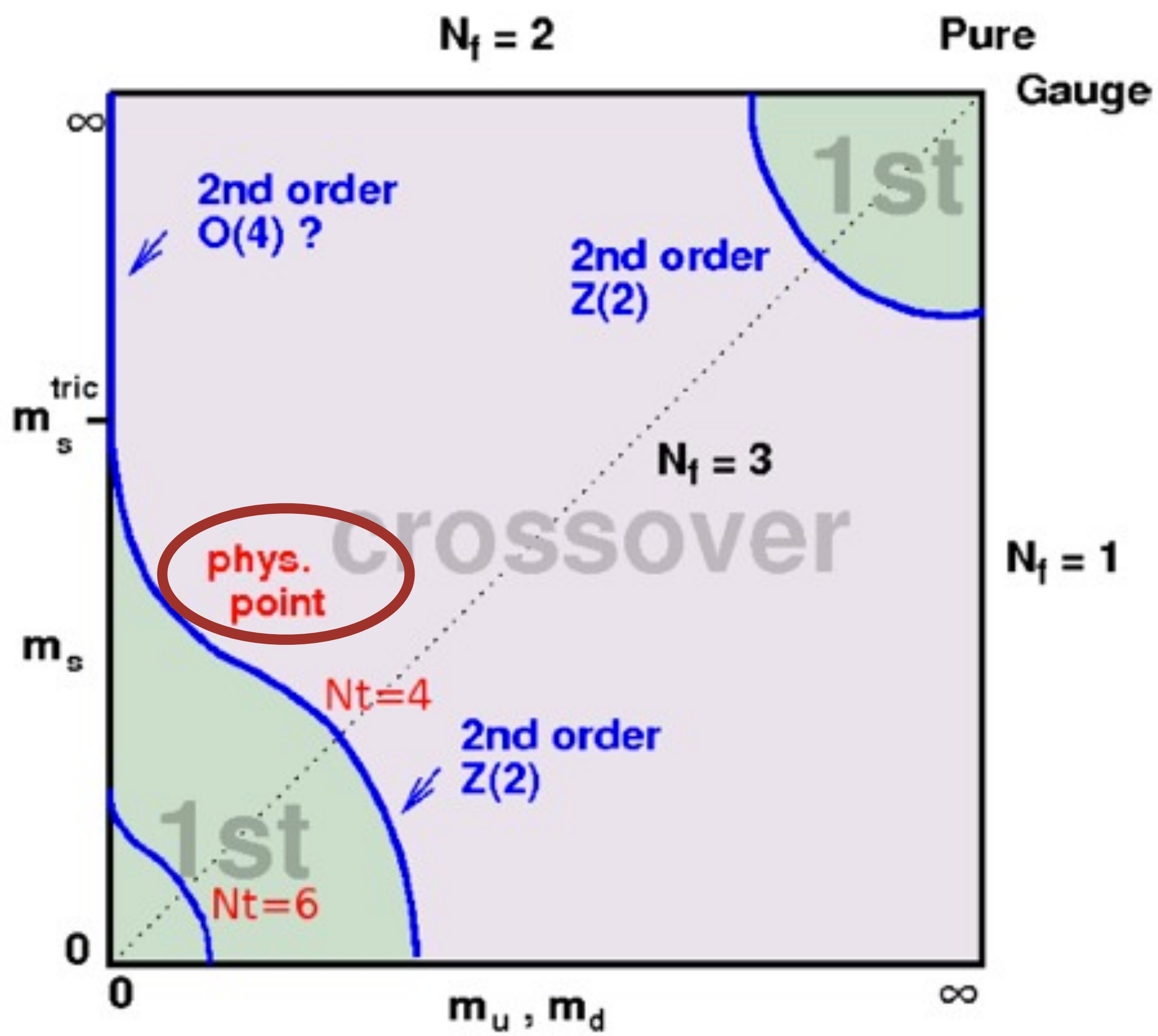



CF., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441
CF., J Luecker, arXiv...

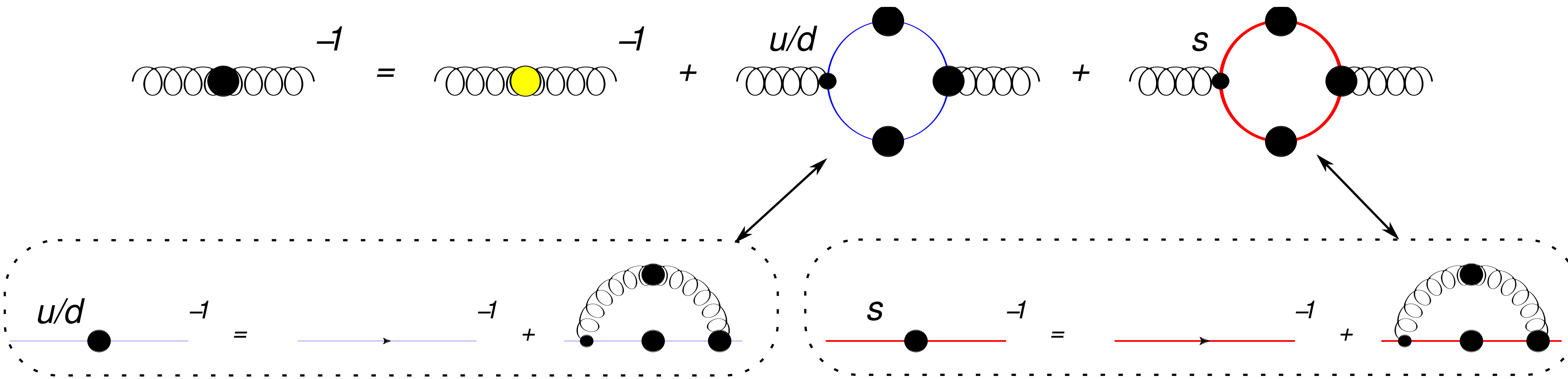
- chiral CEP
- crucial: backreaction of quark onto gluon
- qualitative agreement with RG-improved PQM model

Herbst, Pawlowski, Schaefer, PLB 696 (2011)



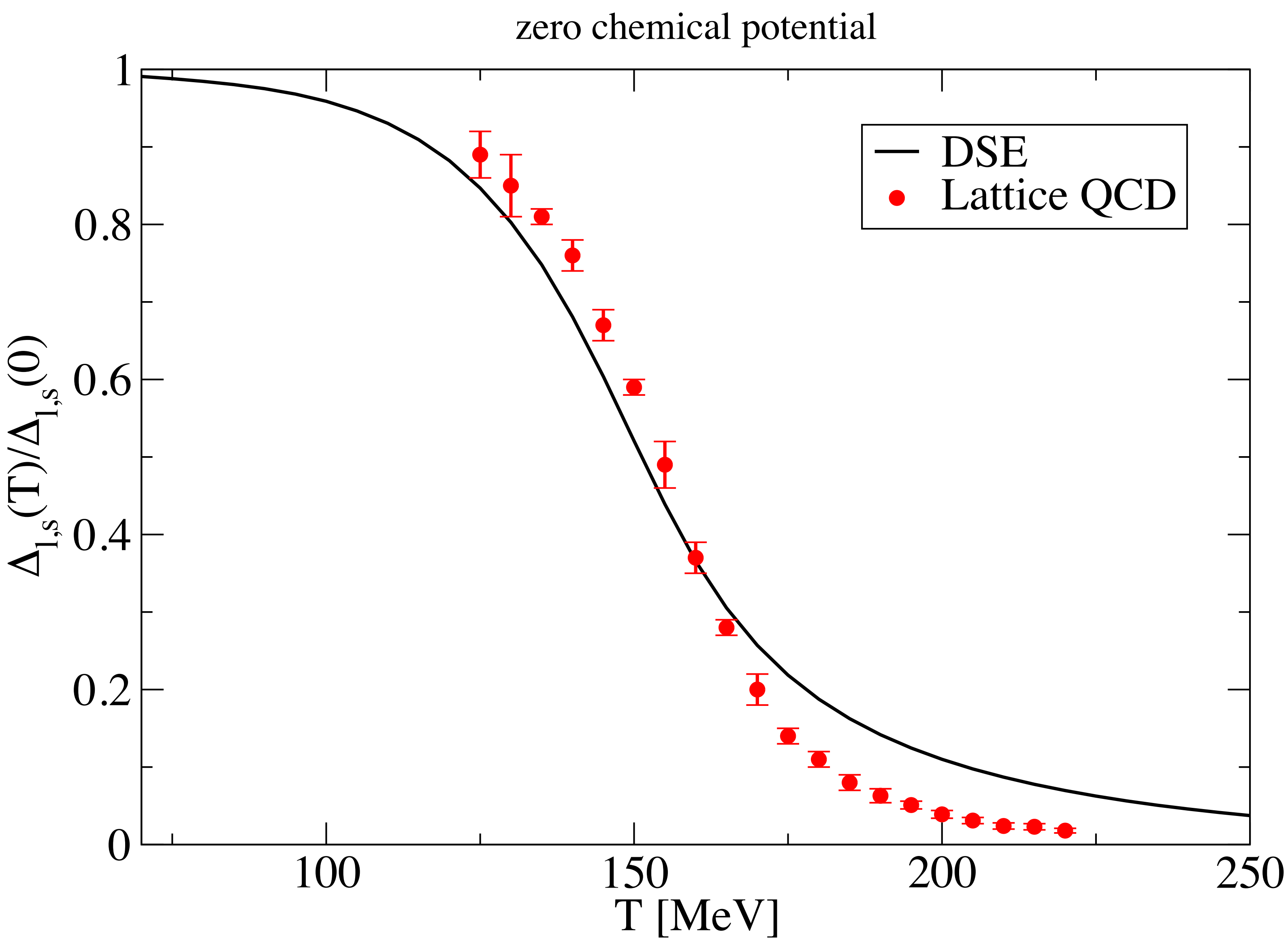

- Physical up/down and strange quark masses
- Transition controlled by chiral dynamics
- at μ=0: compare to available lattice results

# DSEs with $N_f$=2+1




- solve coupled system of three equations

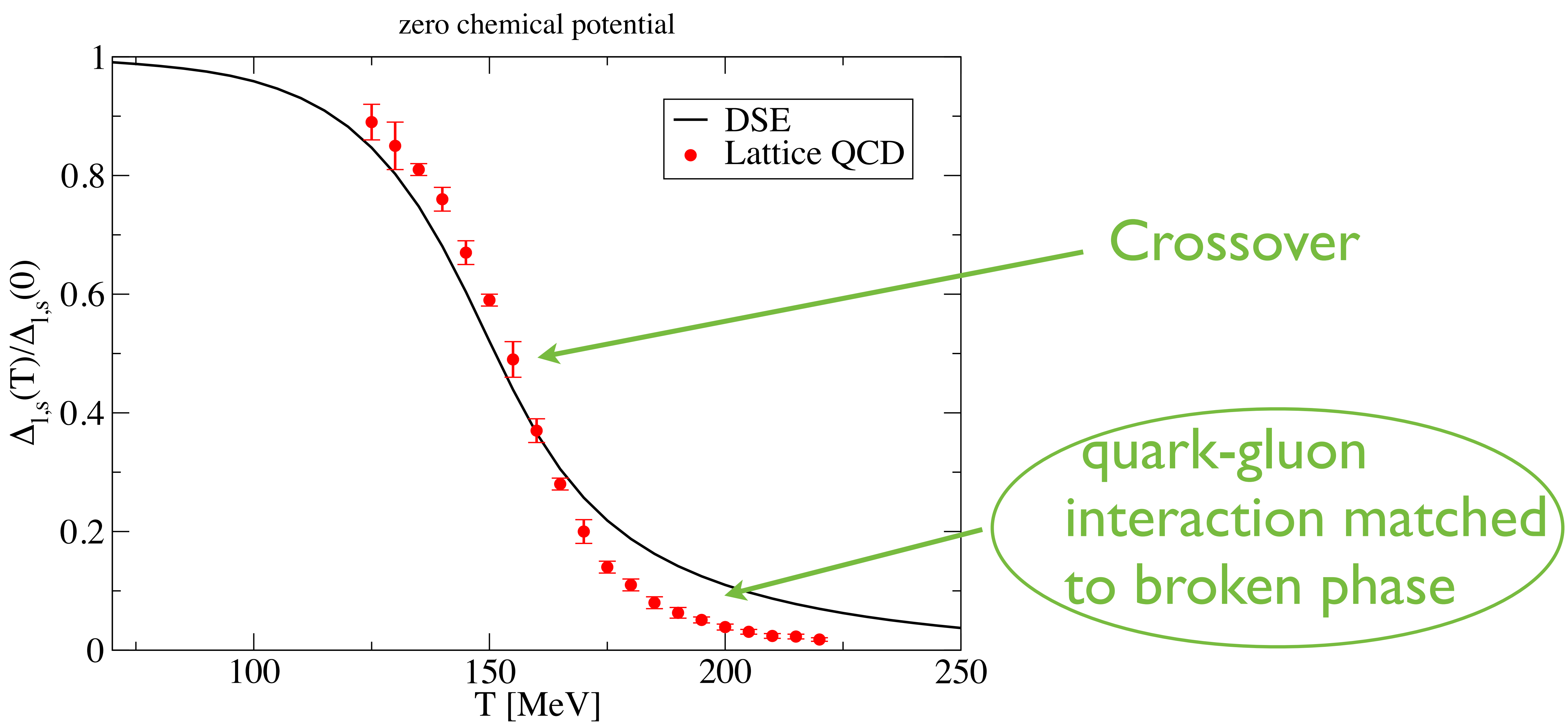
# $N_f$=2+1, zero chemical potential




Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: Lücker, CF, in preparation

- semi-quantitative agreement

# $N_f$=2+1, zero chemical potential




Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: Lücker, CF, in preparation

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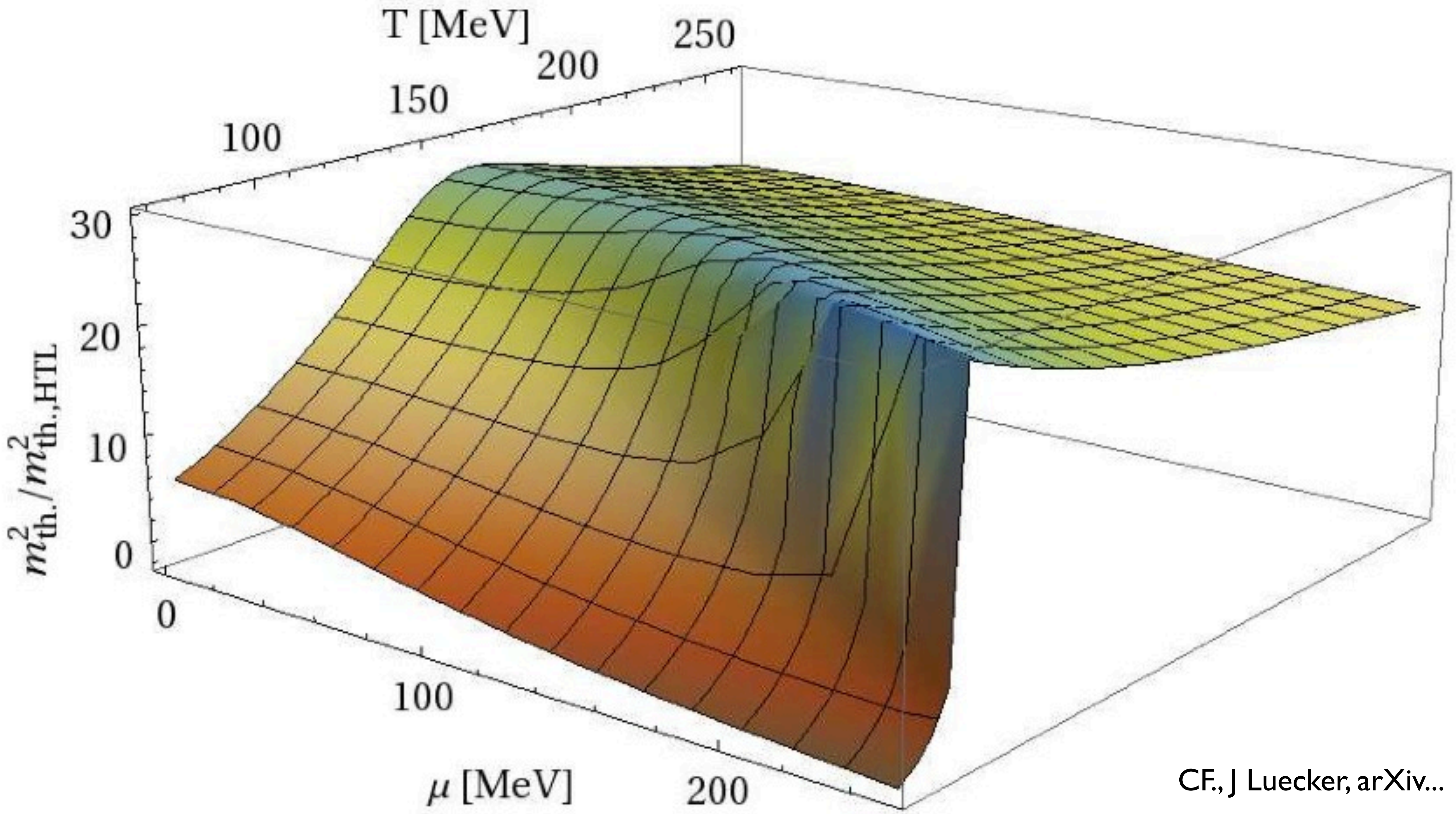
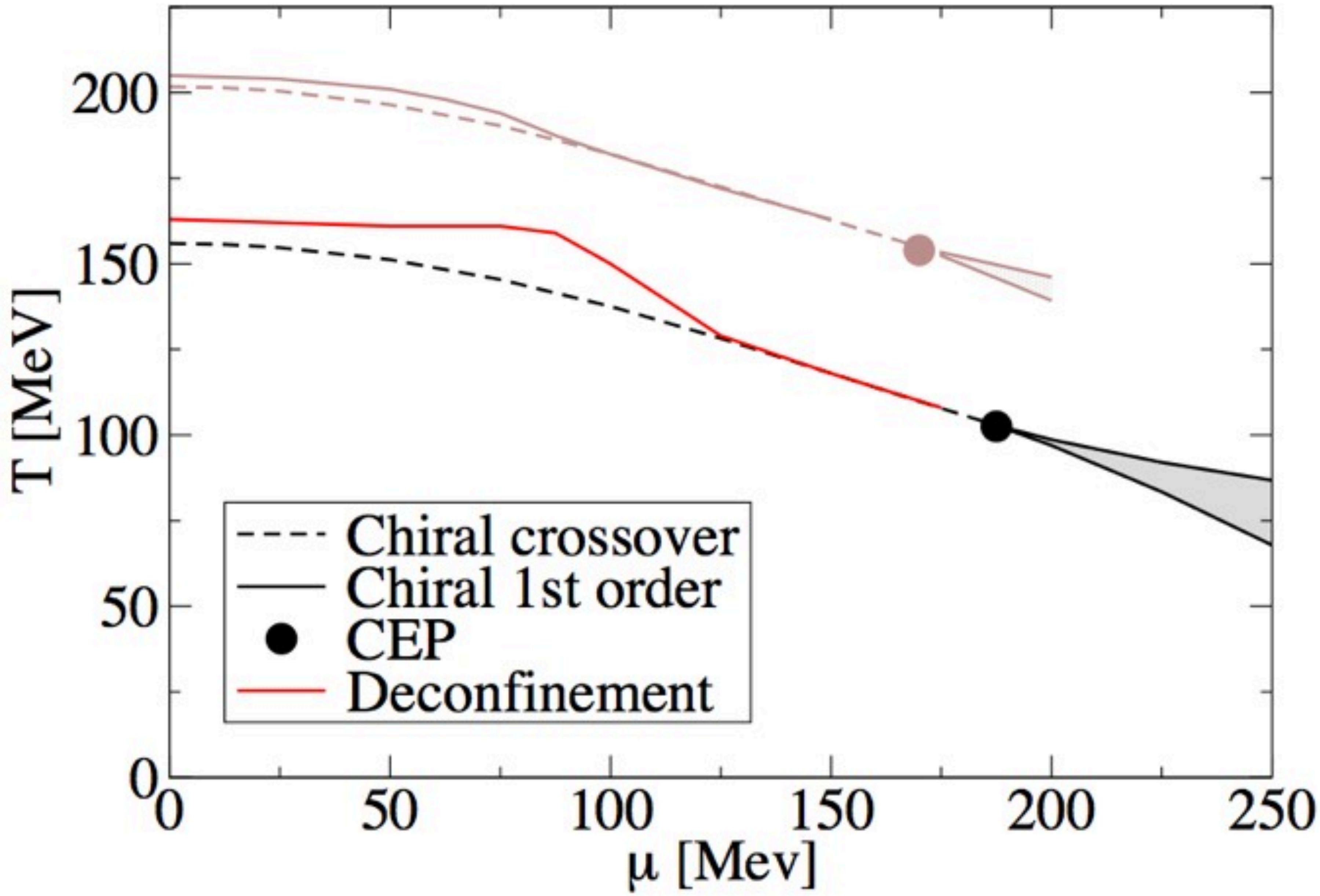
# Nf=2+1: thermal electric gluon mass




CF., J Luecker, arXiv...

- large temperatures: behavior as expected from HTL
- first order transition at large chemical potential

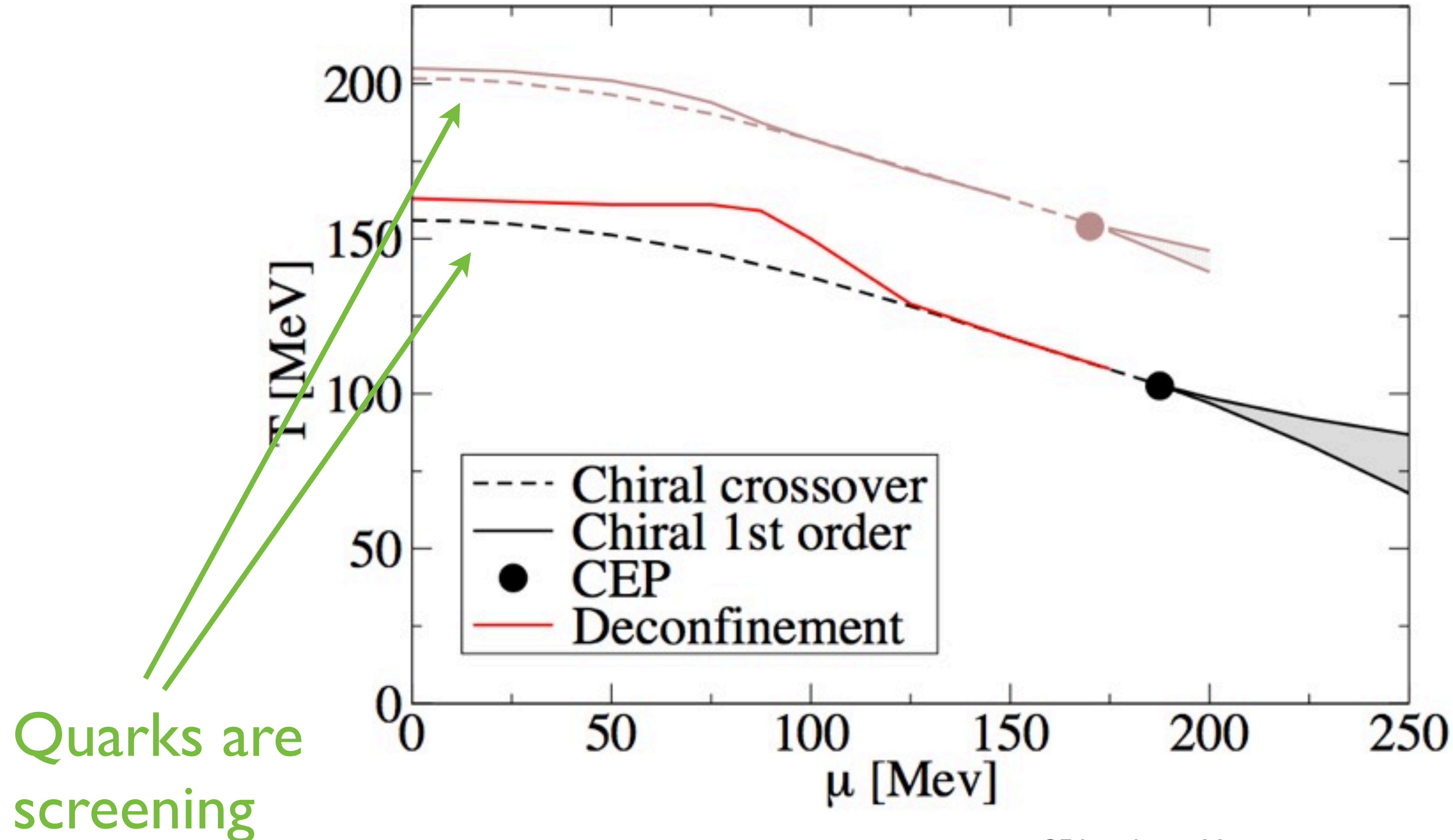



CF, Luecker, arXiv...

- no quarkyonic region
- no CEP at $\mu_c/T_c < 1$ in agreement with lattice

de Forcrand, Philipsen, JHEP 0811 (2008) 012; Nucl Phys. B642 (2002) 290-306
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001.

# $N_f$=2+1: phase diagram




CF, Luecker, arXiv...

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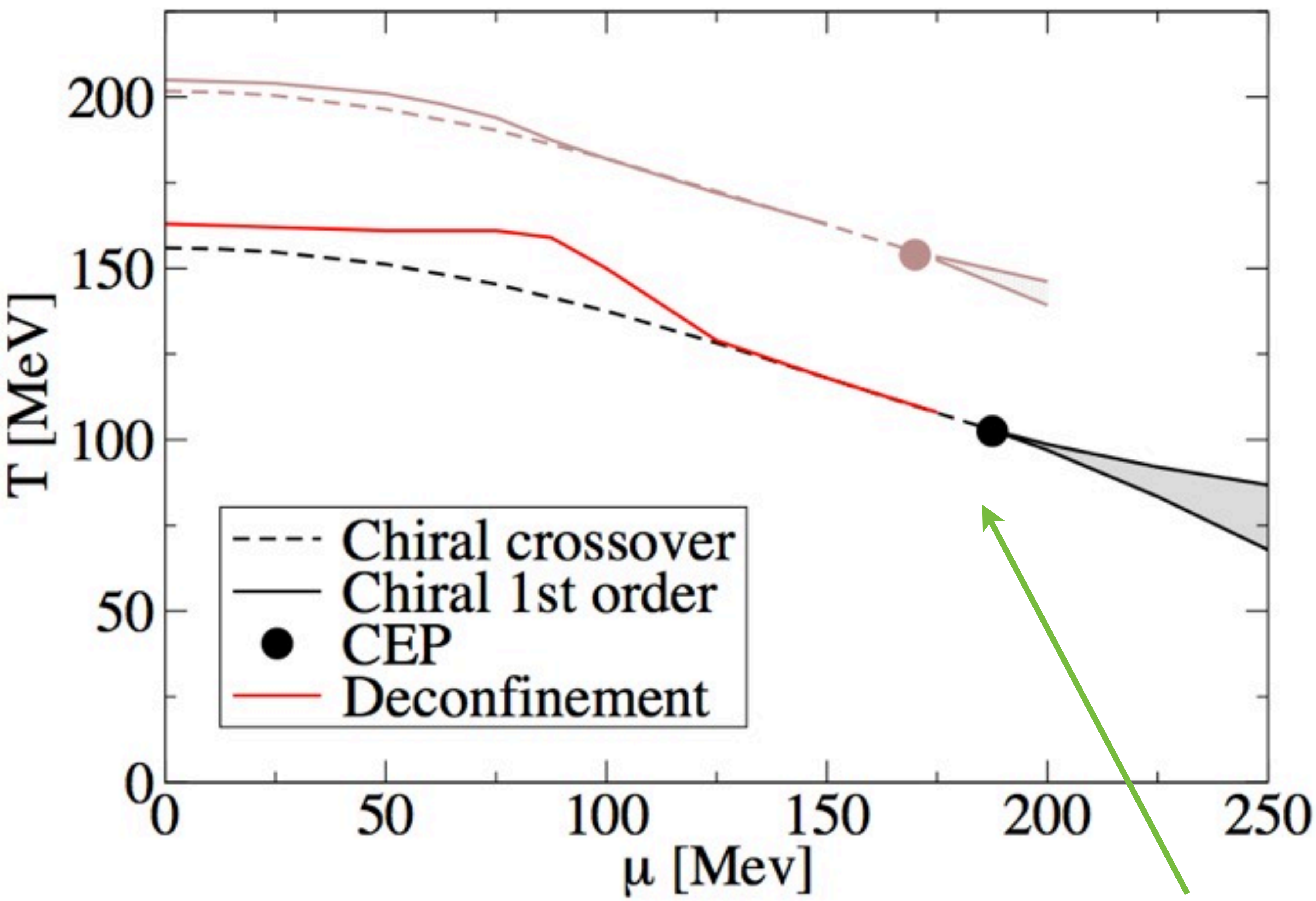


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CEP at large μ

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de Forcrand, Philipsen, JHEP 0811 (2008) 012; Nucl Phys. B642 (2002) 290-306
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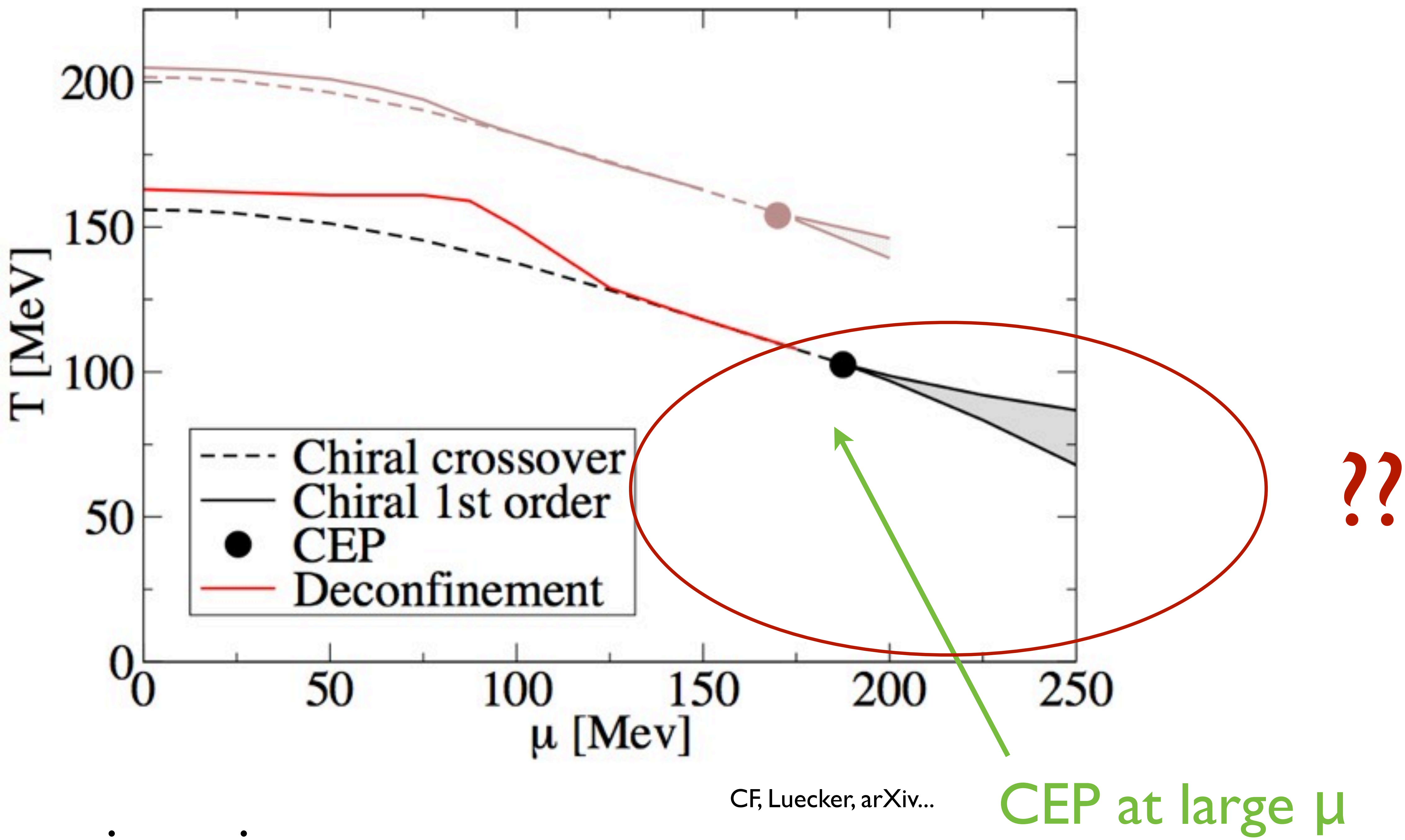


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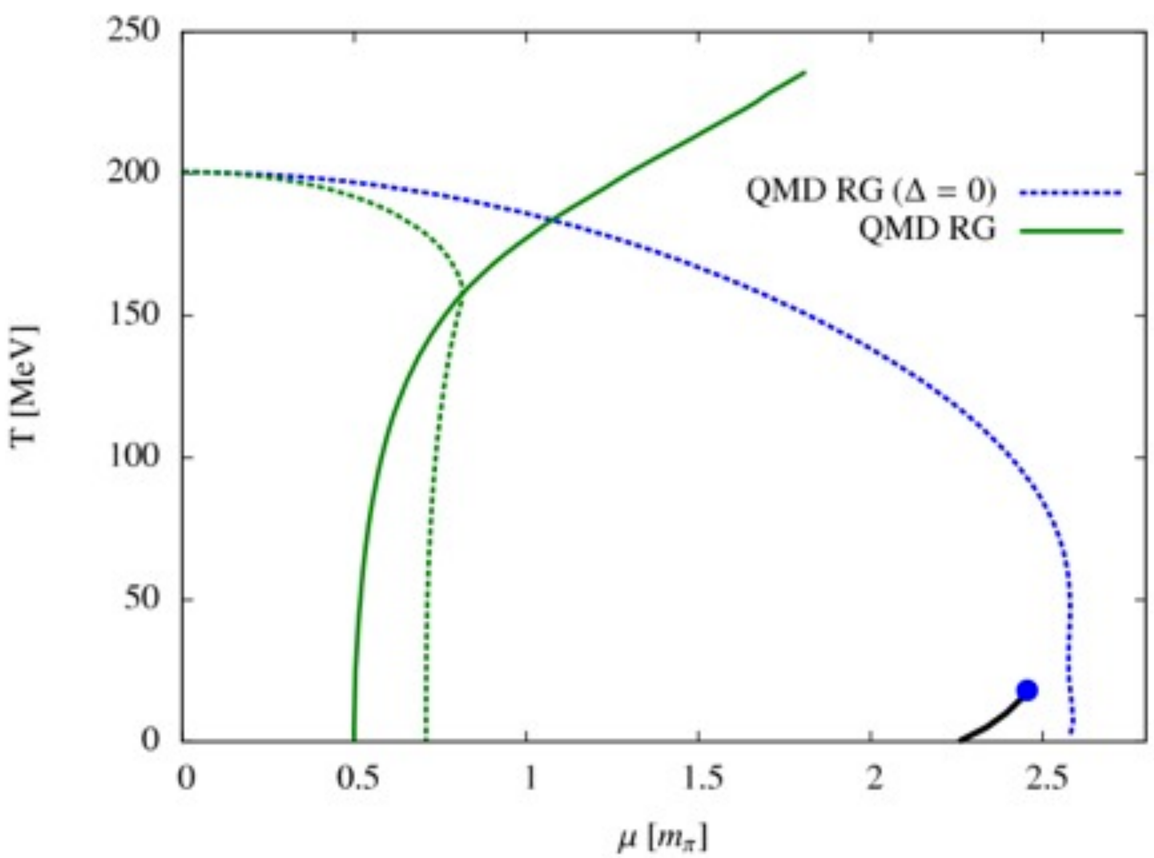
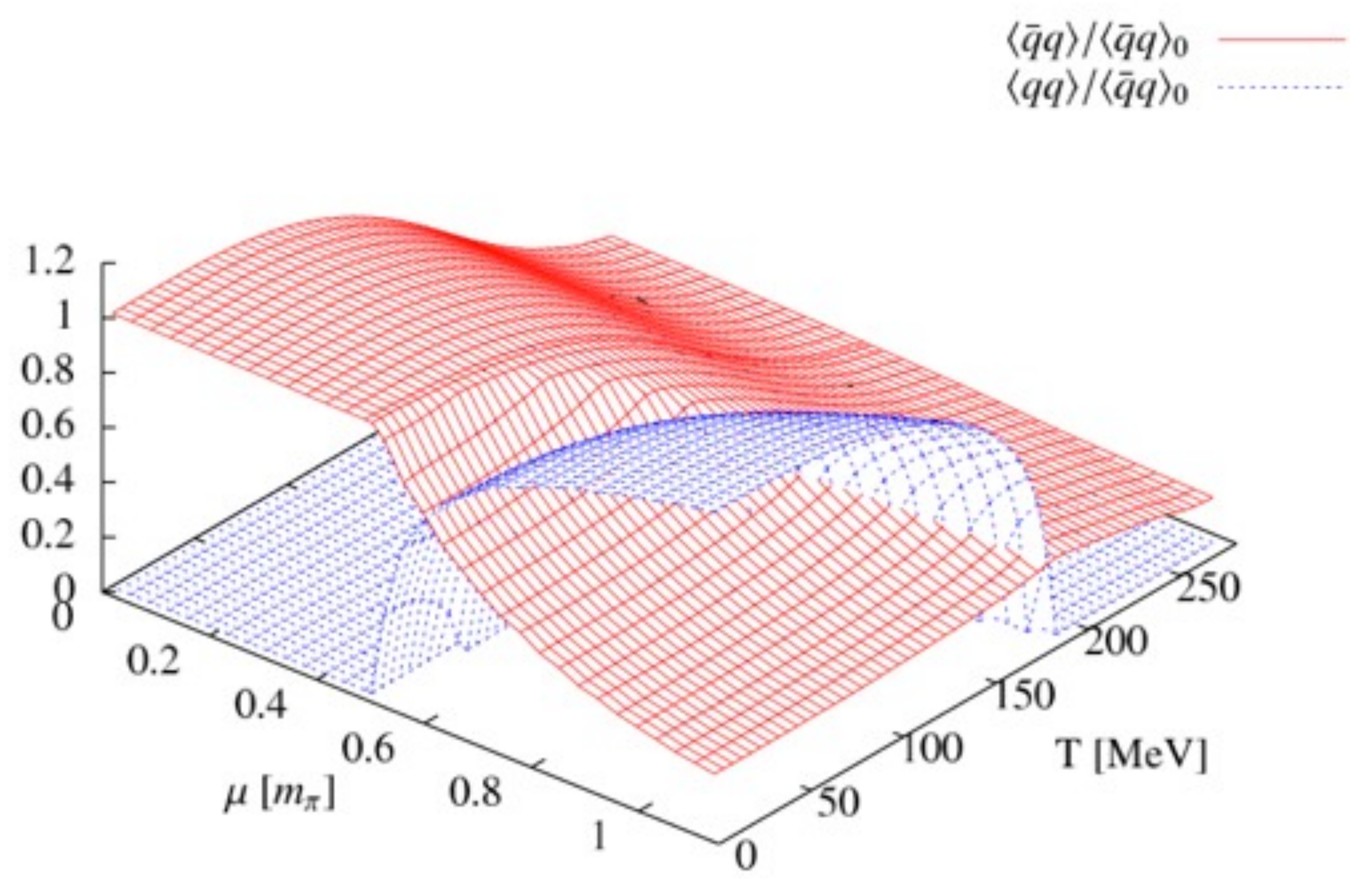
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001.



Monday, June 25, 2012

# Two-color QCD: baryon condensate




- QCD$_2$: diquarks are baryons
- Lattice QCD at finite μ works
- good agreement with PQM-model
- appearance of baryon condensate kills CEP

N.Strodthoff, B.-J.Schaefer and L. von Smekal, PRD 85 (2012) 074007

# Summary

QCD phase diagram:

- Temperature dependent gluon propagator
  - characteristic behavior of electric gluon
  - ‘melting’ of magnetic gluon with temperature
- Deconfinement $T_c$ from dressed Polyakov-loop via DSEs
- QCD with finite chemical potential (beyond mean field)
  - backreaction of quarks onto gluons important
  - $N_f$=2+1: CEP at $\mu_c/T_c > 1$

Other topics:

- Meson structure (pion cloud, form factors etc.)
  CF and R. Williams, PRL 103, 122001 (2009)
- Baryon structure (3-body problem, form factors etc.)
  G. Eichmann and CF, PRD 85 (2012) 034015, EPJA 48 (2012) 9
- Hadronic contributions to $(g-2)_\mu$
  T. Goecke, CF, R. Williams, PLB 704 (2011); PRD 83 (2011)