New Effective Model of the Polyakov Loop for the Deconfinement Phase Transition

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Content

Aspects of deconfinement phase transition

- Spontaneous breaking of center symmetry
- Effective models for phase transition
- New lattice data on Polyakov loop susceptibility

Content

Polyakov loop effective potentials

- Order parameter and target region
- Polyakov loop susceptibility
- Extraction from lattice

Content

Results and discussions

- Model Vs lattice
- Fluctuations and the width of phase transition
- Field theoretical issues

Aspects of deconfinement phase transition

Deconfinement phase transition

Spontaneous breaking of Z_3 center symmetry

- Limit of exact symmetry: no explicit breaking pure gauge theory
- Confining potential for quarks $V_{q\bar{q}}$

Effective model approach

Intuitive picture of phase transition

- Non-trivial vacuum

$$l = < \hat{l} >' = \frac{1}{\beta V} \frac{\partial ln Z[h]}{\partial h} \Big|_{h \to 0}$$

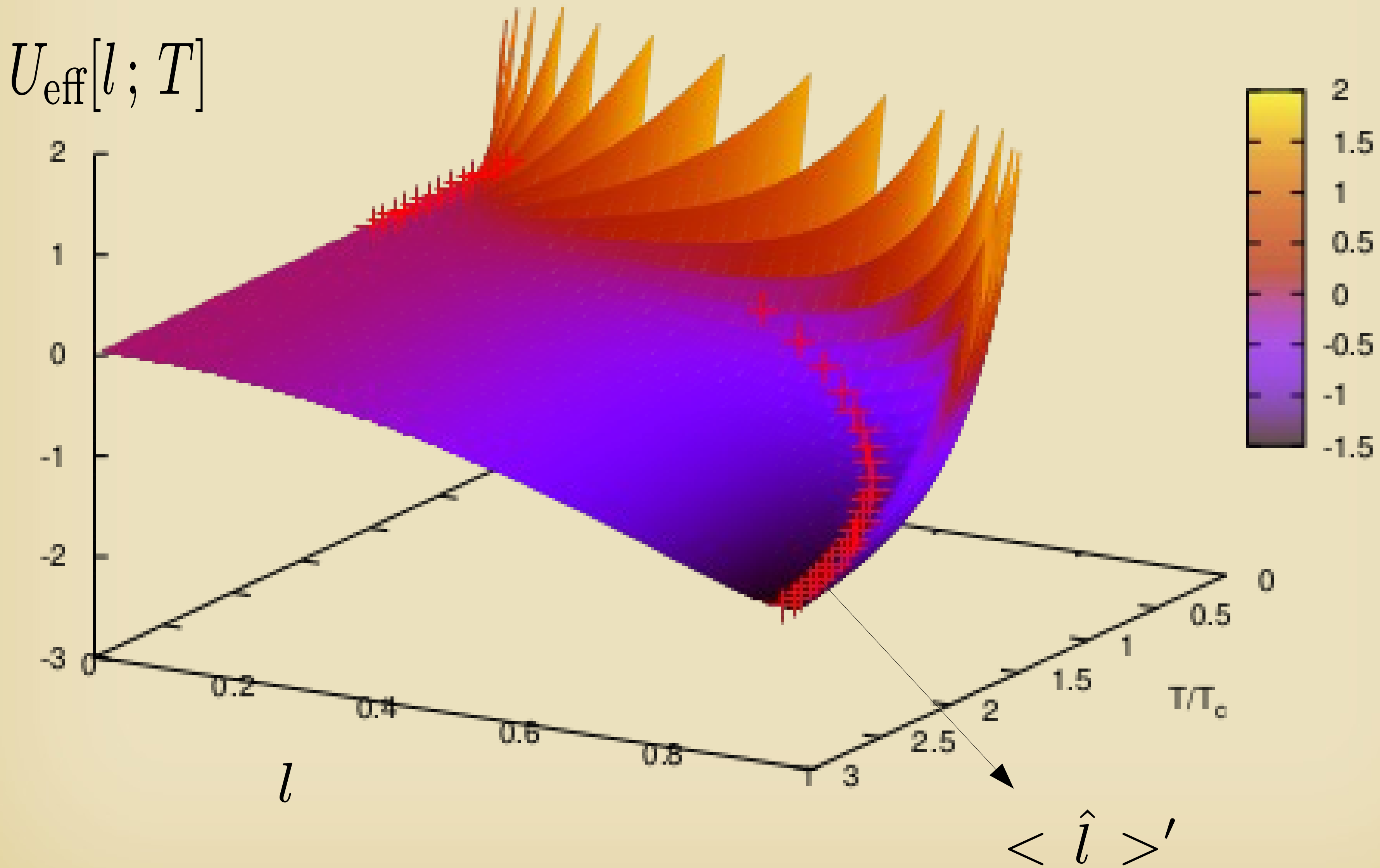
does not respect the symmetry

symmetric phase: $l = 0$
broken phase: $l \neq 0$

Effective model approach

- Order parameter as the degree of freedom
- Effective potential $U_{\rm eff}[l\,;\,T]$
  - possess the same symmetry
  - competing vacua

Effective model approach



Effective model approach

- Order parameter **characterizes** the state of the system
- Fluctuations
  - Sensitive to **transition** of phases: e.g width
  - Susceptibility: $\chi = < \hat{l}\,\hat{l} >'_c = \frac{1}{\beta V}\frac{\partial^2 lnZ[h]}{\partial h\,\partial h}\Big|_{h\to 0}$
  - Effective potential: inverse of curvature

Goals of current study

- To understand the new lattice data on Polyakov loop susceptibility
- To further constraint the model
  - Previous PQM models only fit to order parameter and thermodynamic quantities, not fluctuations
- To examine various fluctuation-related quantities
  - width: $1.4 - 1.6\,T_c$

Polyakov loop effective potentials

Model definitions

- W.Weise, C.Ratti and S.Roessner:

$$\bar{U}_G = -\frac{a[T]}{2}\,\bar{l}l + b[T]\ln f_{\mathrm{Haar}}$$

$$f_{\mathrm{Haar}} = 1. - 6.\bar{l}l + 4.(\bar{l}^3 + l^3) - 3.(\bar{l}l)^2$$

- C.Sasaki and K.Redlich
  M.Ruggieri *et al.*

$$\bar{U}_G = \bar{U}_{SC}[l\,,\,\bar{l}\,;\,T, m_G] + b[T]\ln f_{\mathrm{Haar}}$$

$$\bar{U}_{SC} = 2. \int \frac{d^3x}{(2\pi)^3} \ln\{1. + e^{-8\bar{E_G}} + \sum_{n=1}^{7} C_n e^{-n\bar{E_G}}\}$$

$$C_1 = 1. - N_c^2 \bar{l}l$$

$$C_2 = 1. - 3.N_c^2 \bar{l}l + N_c^3(\bar{l}^3 + l^3)$$

$$C_3 = -2. + 3.N_c^2 \bar{l}l - N_c^4(\bar{l}l)^2$$

$$C_4 = -2. + 2.N_c^2 \bar{l}l - 2.N_c^3(\bar{l}^3 + l^3) + 2.N_c^4(\bar{l}l)^2$$

$$C_5 = C_3$$

$$C_6 = C_2$$

$$C_7 = C_1$$

$$E_G = \sqrt{x^2 + \bar{m_G}^2} \qquad \bar{m_G} = \frac{m_G}{T}$$

General features

- Target region

  Polyakov gauge:

$$\hat{l} = \frac{1}{N_c} Tr \mathcal{P} e^{ig\int_0^\beta d\tau\, A_4} = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i(\phi_1+\phi_2)} \end{pmatrix}$$

$$l = \frac{1}{N_c} Tr \hat{l}.$$

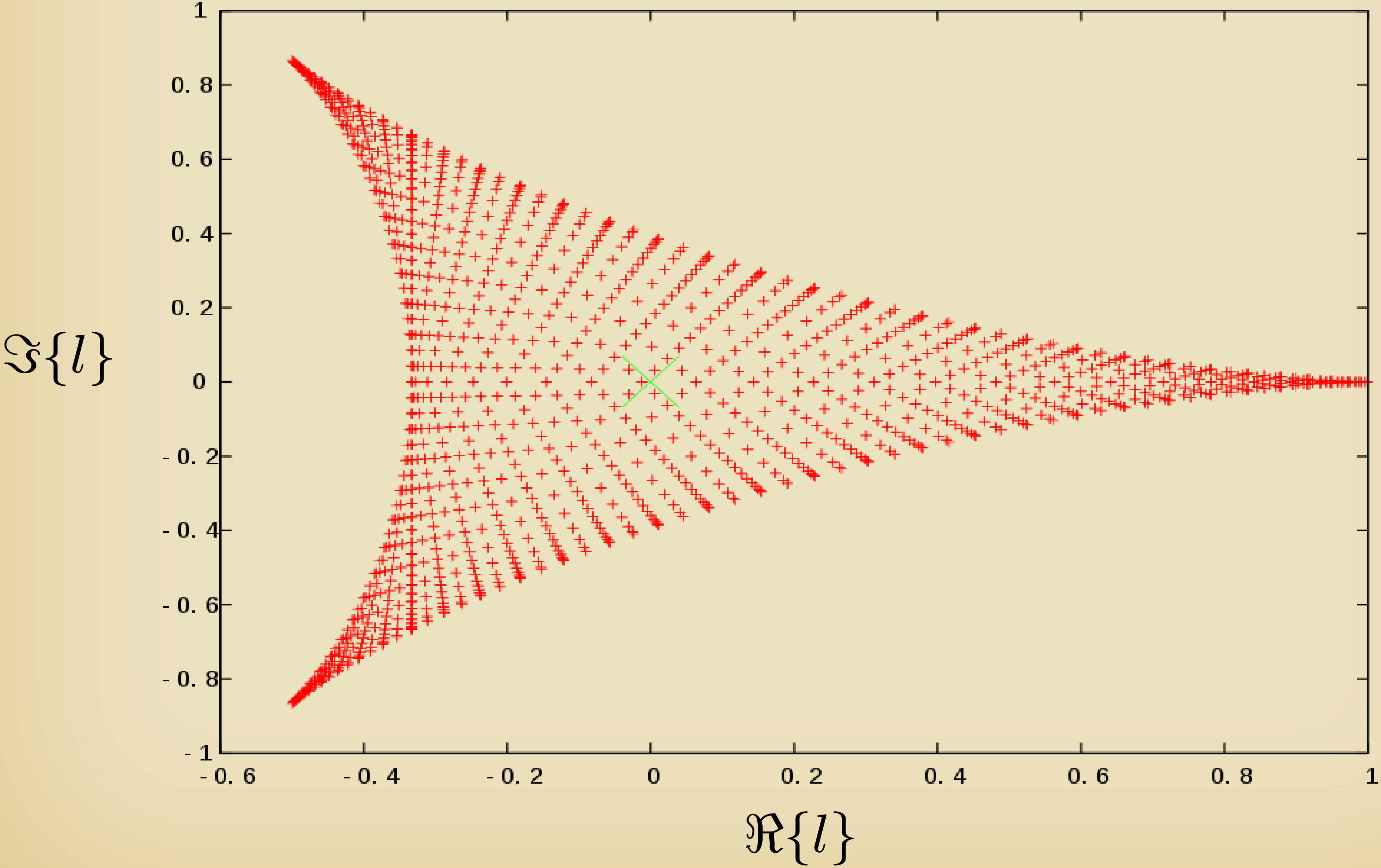
Target region

- Natural restriction on the range of complex values of l

$$\Re\{l\} = \frac{1}{3}(cos(\phi_1) + cos(\phi_2) + cos(\phi_1 + \phi_2))$$

$$\Im\{l\} = \frac{1}{3}(sin(\phi_1) + sin(\phi_2) - sin(\phi_1 + \phi_2))$$

Target region



Target region

- $\bar{U}_{SC}$ and Haar potential naturally comply to this restriction.
- The renormalized Polyakov loop:

$$l_R \approx 1 + \frac{N_c^2 - 1}{4N_c}\,\alpha_s\,\frac{m_D}{T}$$

violates the $SU(3)$ matrix structure

- Lattice data shows that $l_R > 1$ at around $3\,T_c$ and approaches unity at high temperature from **above**.

Polyakov loop susceptibility

- Within the effective model:

$$\chi_{IJ} =< l_I l_J >_c = \left(\frac{\partial^2 U_G}{\partial l_I \partial l_J} \right)^{-1}$$

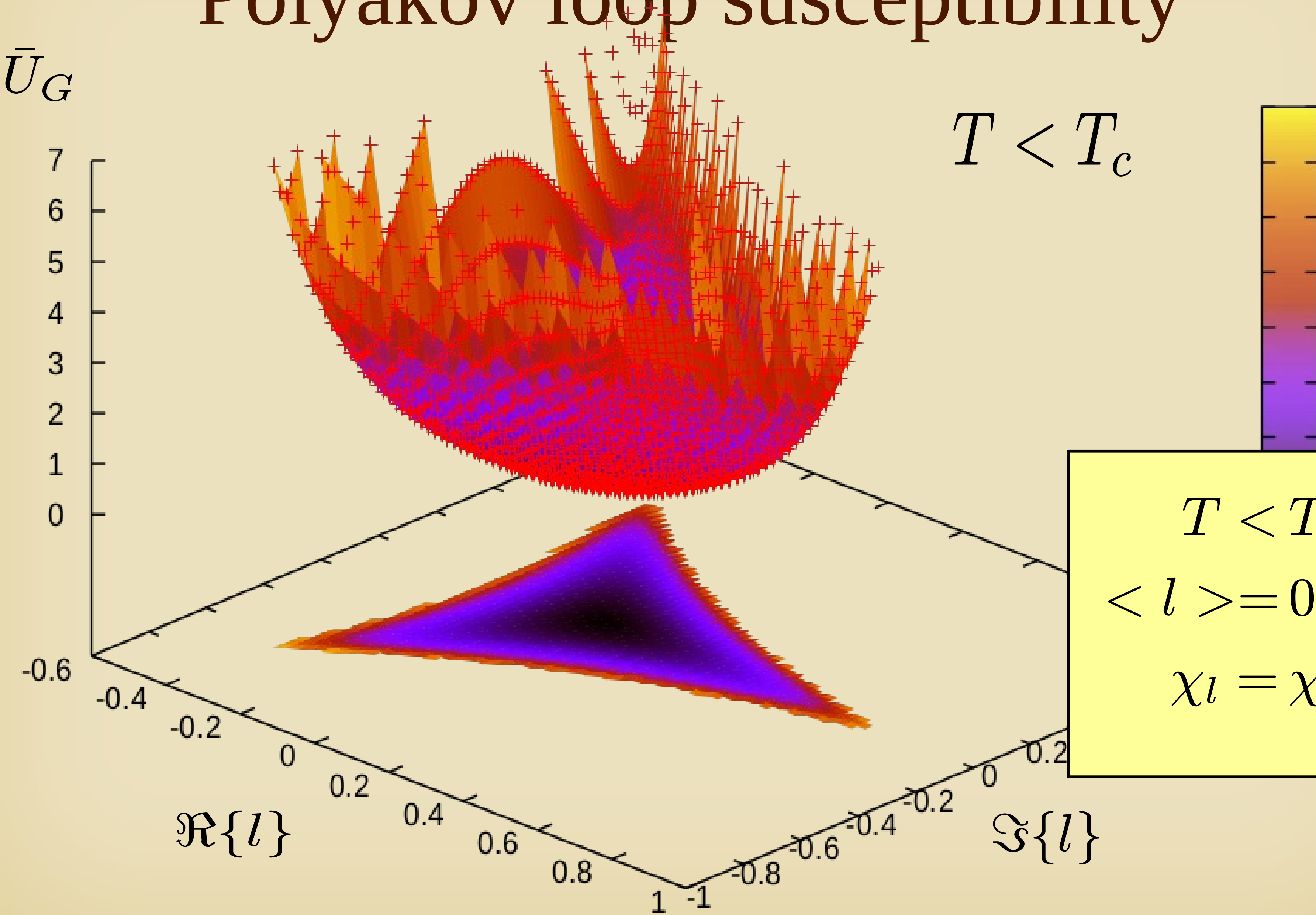
$$l_I = \{l, \bar{l}\}$$

- Physical meaning of χ :

  inverse of curvatures

  longitudinal and transverse

Polyakov loop susceptibility

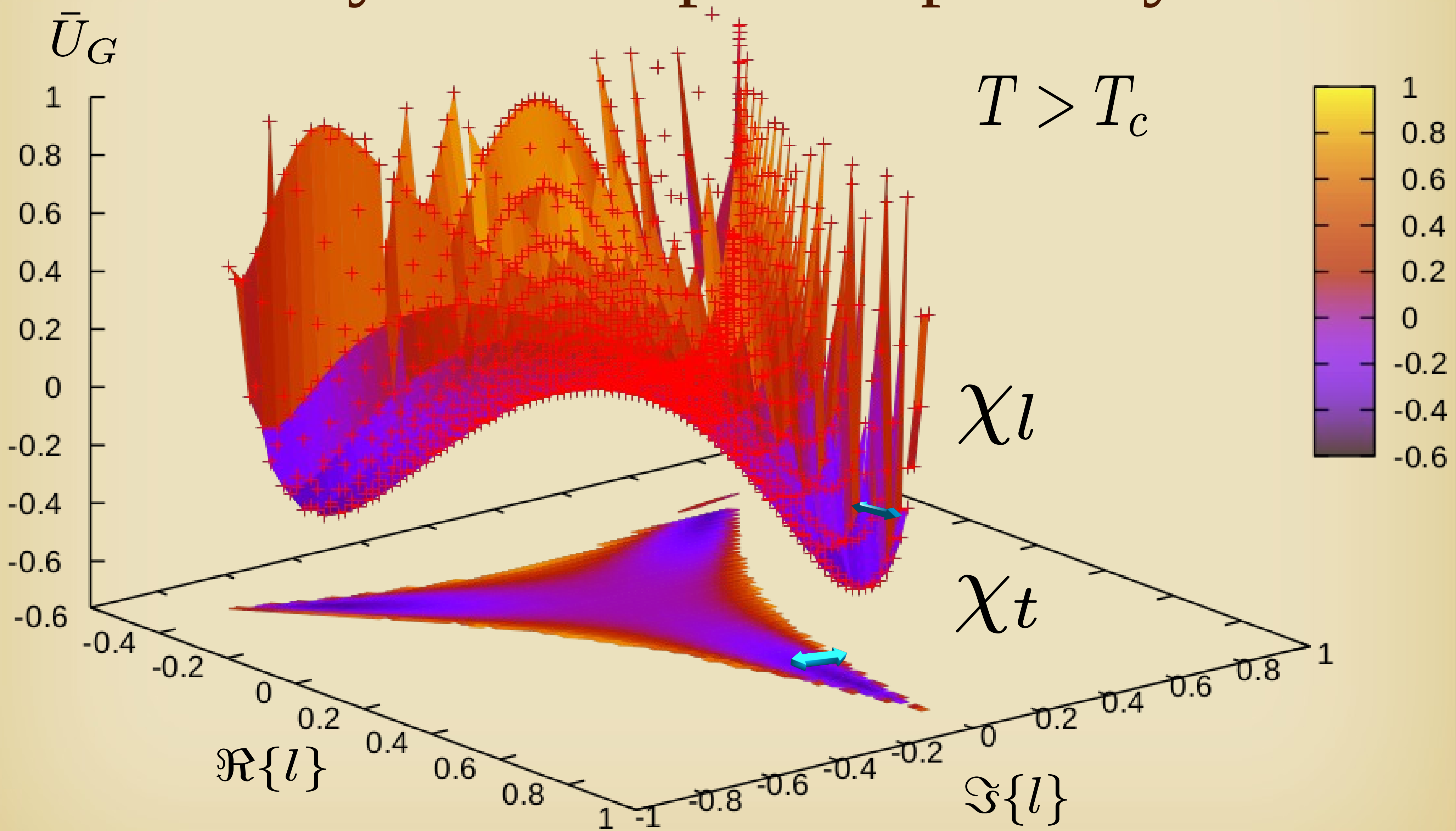


$T < T_c$

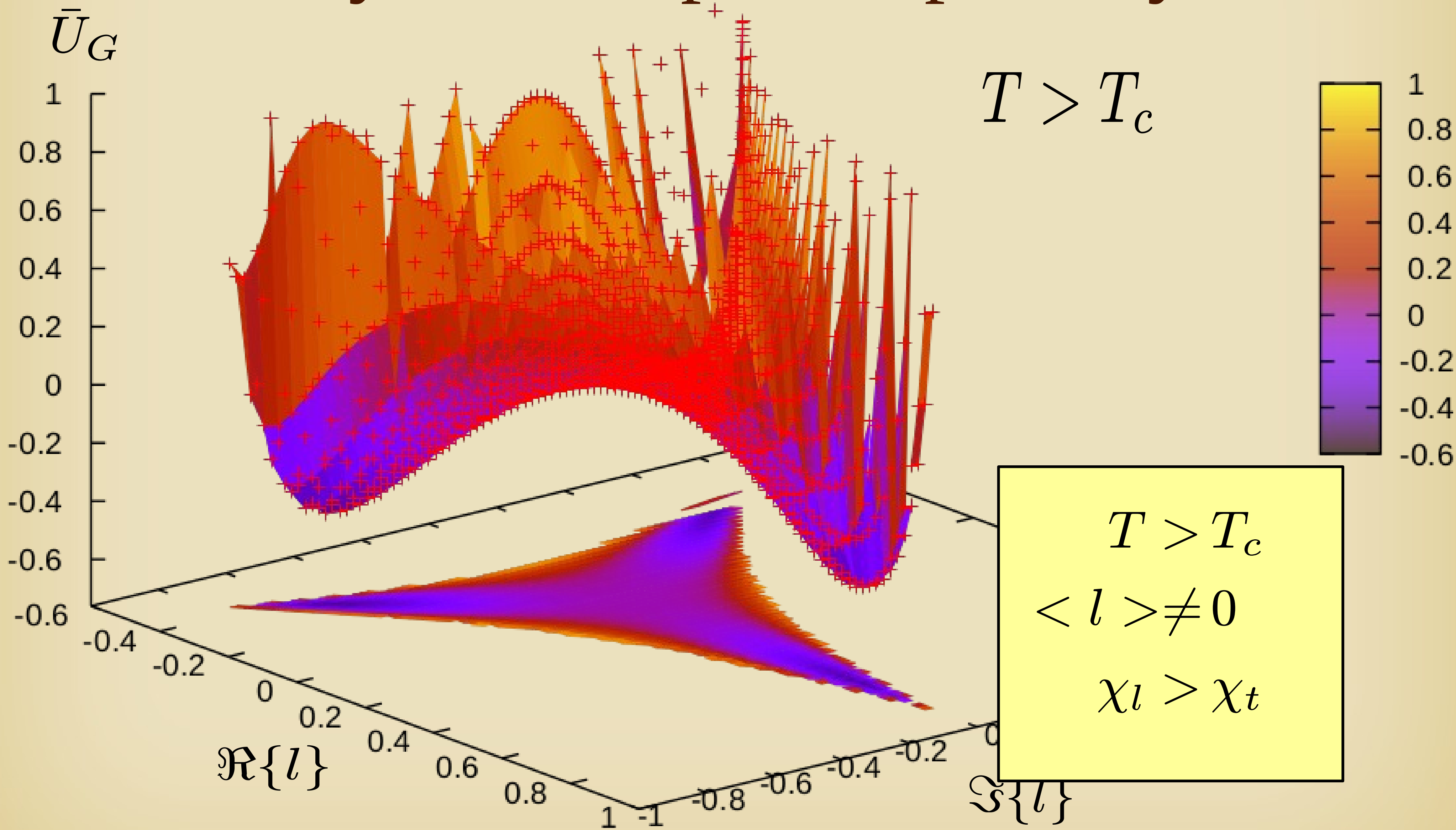
$< l >= 0$

$\chi_l = \chi_t$

Polyakov loop susceptibility



Polyakov loop susceptibility

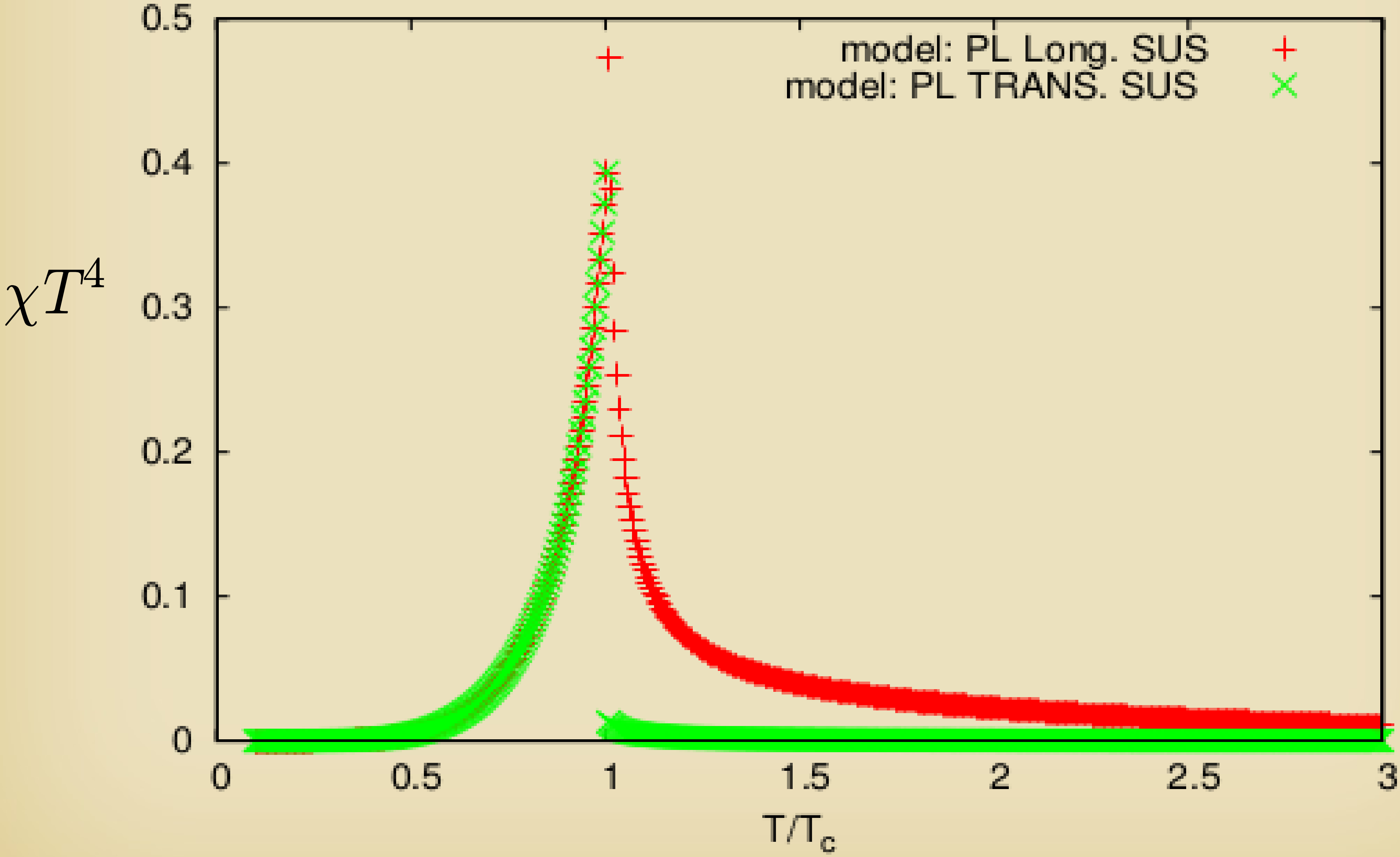


$T > T_c$

$< l > \neq 0$

$\chi_l > \chi_t$

Polyakov loop susceptibility



Lattice data on susceptibility

- To match lattice quantities to those obtained in a continuum approach

$$V = N_\sigma^3 a^3$$

$$\beta = N_\tau a,$$

- Continuum limit $a \to 0$
- Thermodynamic limit $V \to \infty$
  only intensive quantities survive this limit
  e.g pressure, energy density

Lattice data on susceptibility

$$\chi_1 = N_\sigma^3 < \sum_{i,j} \frac{1}{N_\sigma^3} L_i \frac{1}{N_\sigma^3} L_j >_c$$

$$= \frac{1}{N_\sigma^3} < \sum_{i,j} L_i L_j >_c$$

$$L_i = N_\sigma^3 \sum_{\vec{n}} \frac{1}{N_c} Tr \prod_i^{N_\tau} U_{[\vec{n};i];\hat{0}}$$

$$< L_i L_j >_c \leftrightarrow G_c[x-y]$$

$$a^3 \sum_i \leftrightarrow \int d^3x$$

$$\sum_i 1 = N_\sigma^3.$$

$$\chi = \beta \int d^3x\, G_c[x]$$

$$= \beta^4 \frac{\chi_1}{N_\tau^3}$$

Other fluctuations

- Start with the partition function...

$$Z[h,T] = e^{-\beta V f[h,T]} \qquad < l >= -\frac{\partial f}{\partial h}$$

- Fluctuations:

$$\chi = -\frac{\partial^2 f}{\partial h \partial h}$$

$$c_V = -T\frac{\partial^2 f}{\partial T \partial T}$$

$$\frac{\partial}{\partial T} < l > = -\frac{\partial^2 f}{\partial h \partial T}$$

All three naturally display a peak at T_c

Results and discussions

Strategy in solving effective potential

- Gap equation

$$\frac{\partial U_G[l\,,\,T]}{\partial l} = 0 \quad \Rightarrow \quad < \ l \ >$$

- Thermodynamics quantities

$$U_G[l =< l >, T] = f(T) \quad \Rightarrow$$

$$P = -f$$

$$s = -\frac{d}{dT} f$$

$$\epsilon = f + sT = (I - T\frac{d}{dT})f$$

$$\Delta = \epsilon - 3P$$

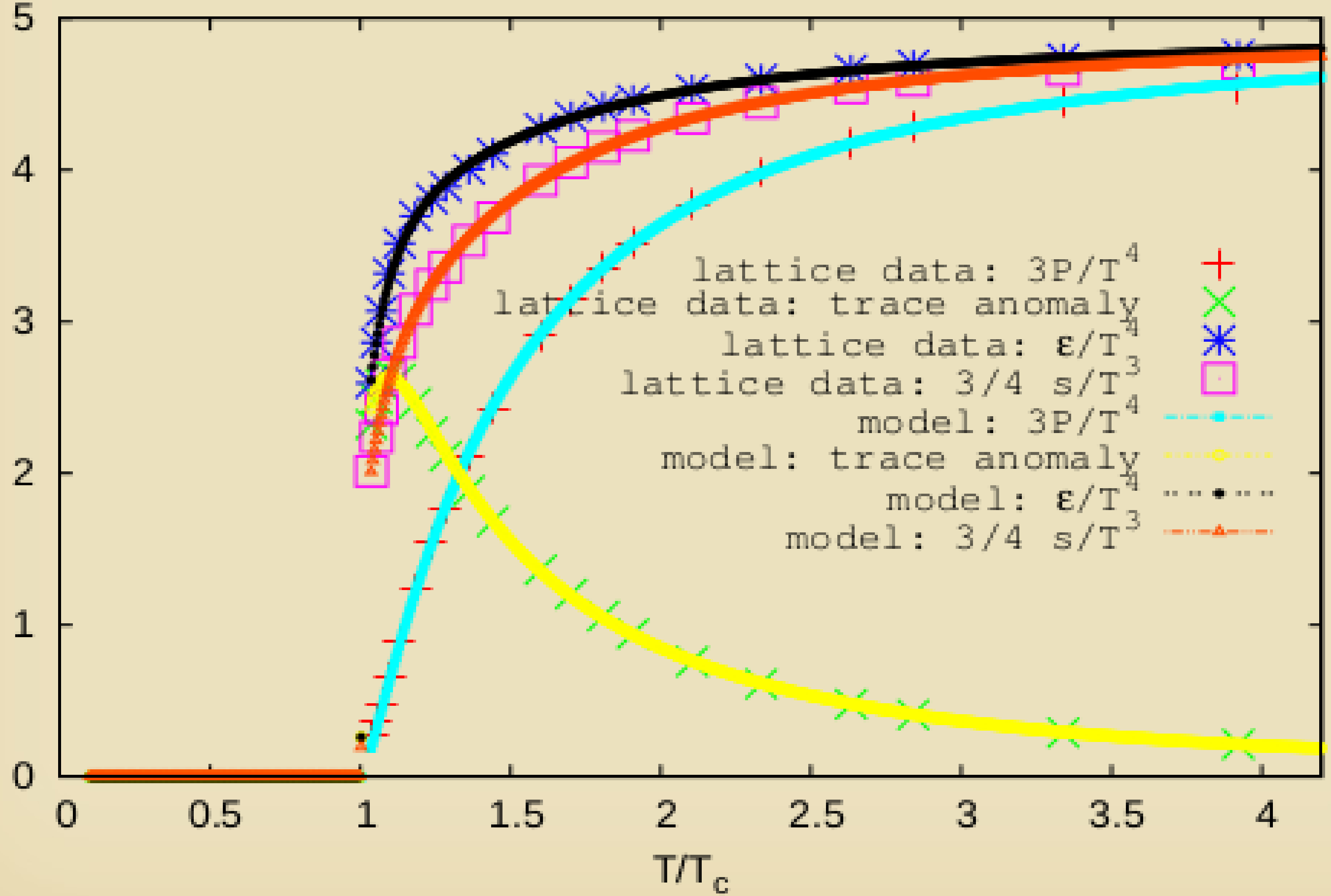
Strategy in solving effective potential

- Fluctuations

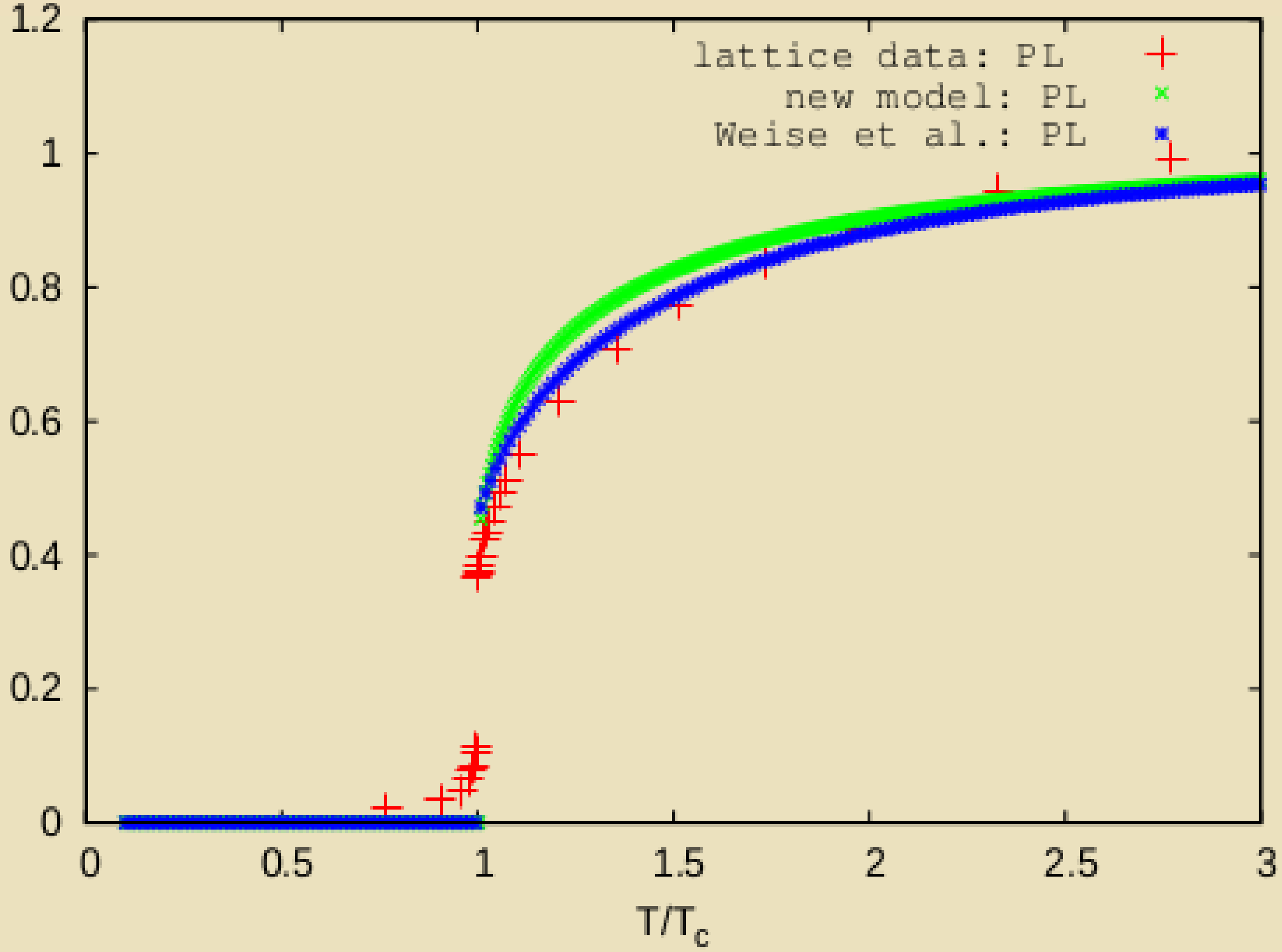
$$\chi_l ,\ \chi_t ,\ c_V ,\ \frac{\partial < l >}{\partial T}$$

- Compare with lattice and fit the potential

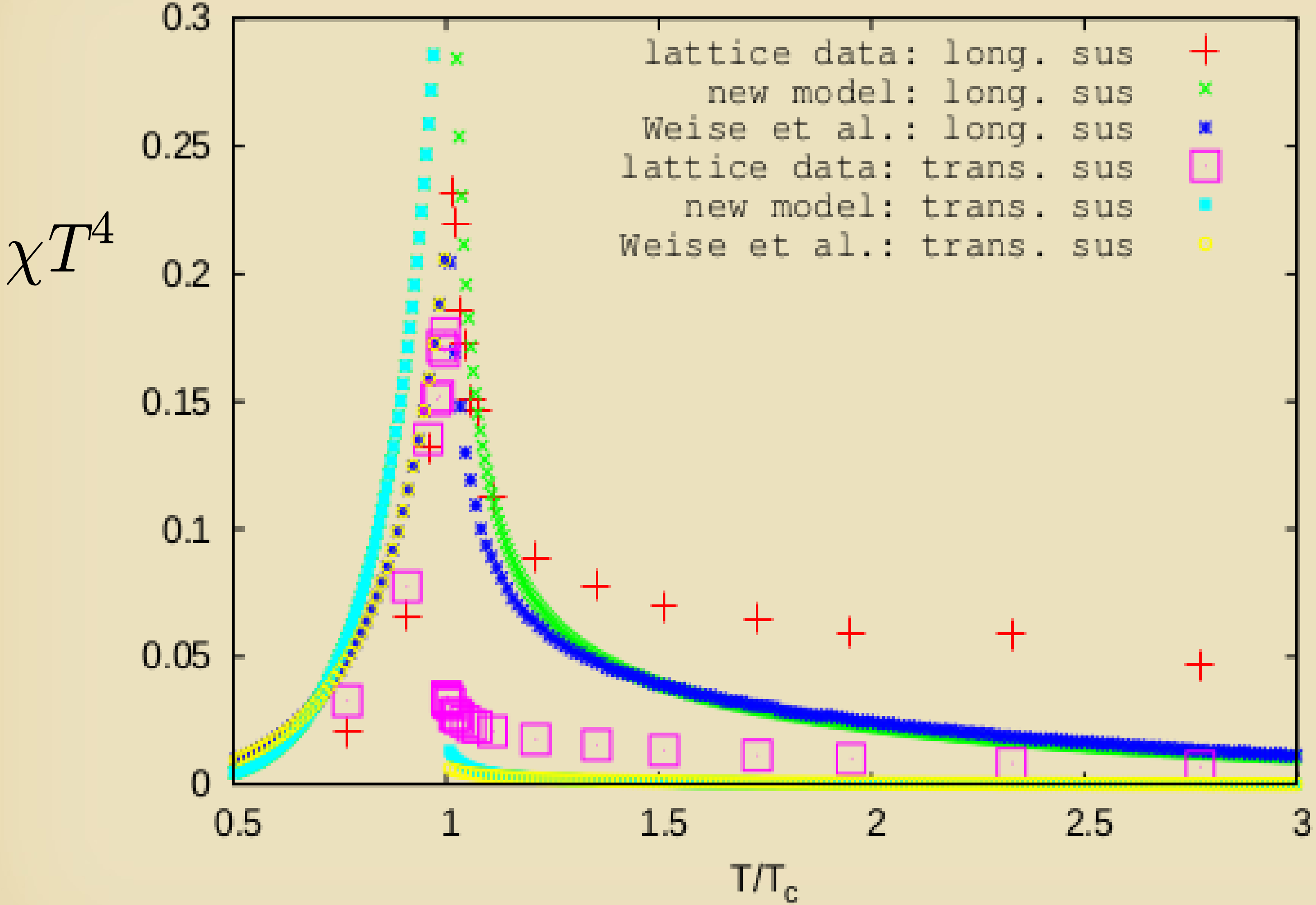
Model 1



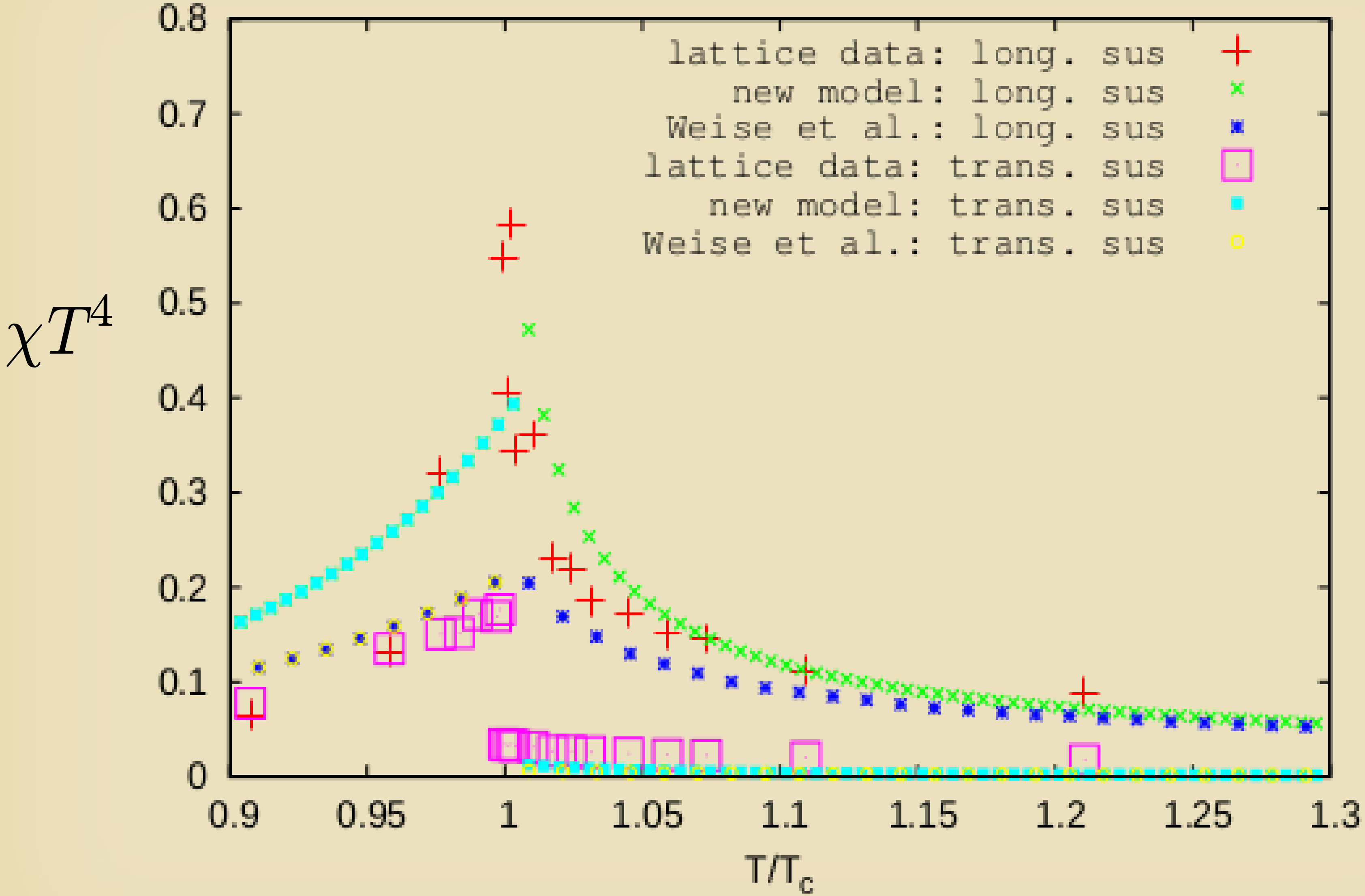
lattice data: 3P/T^4
lattice data: trace anomaly
lattice data: ε/T^4
lattice data: 3/4 s/T^3
model: 3P/T^4
model: trace anomaly
model: ε/T^4
model: 3/4 s/T^3
0
1
2
3
4
5
0
0.5
1
1.5
2
2.5
3
3.5
4
T/T_c



lattice data: PL
new model: PL
Weise et al.: PL
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
T/T$_c$

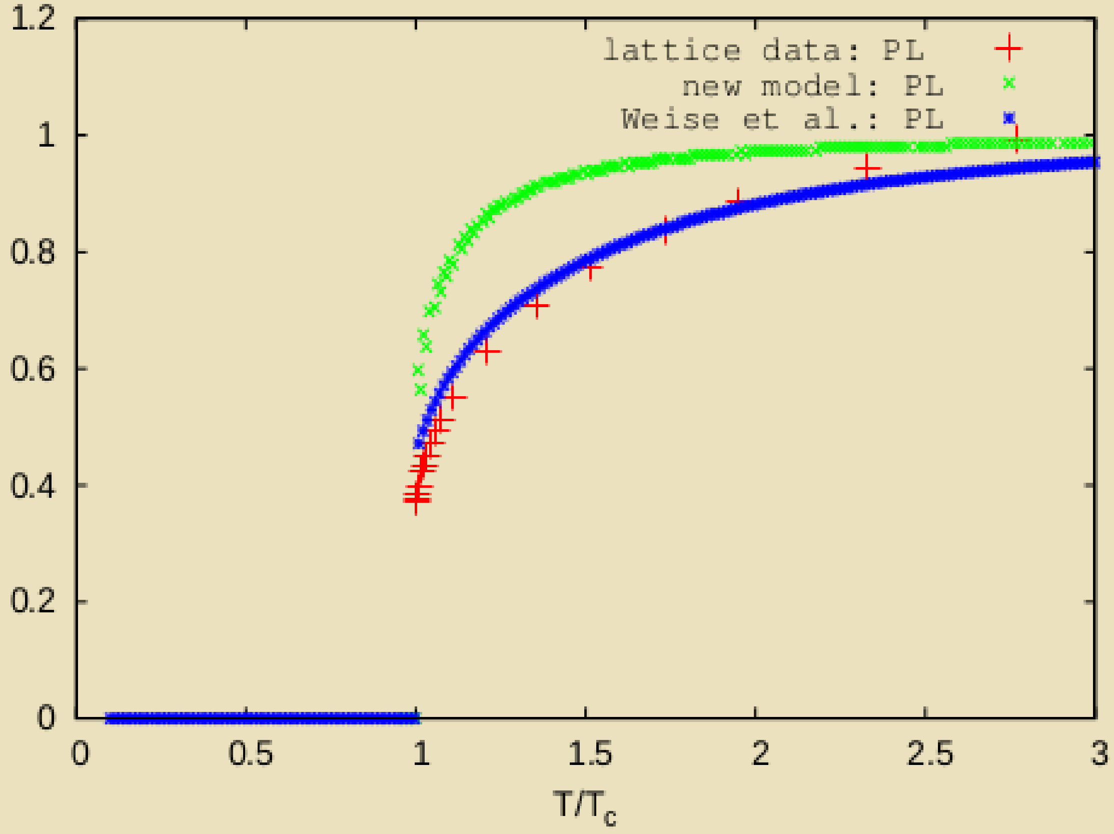


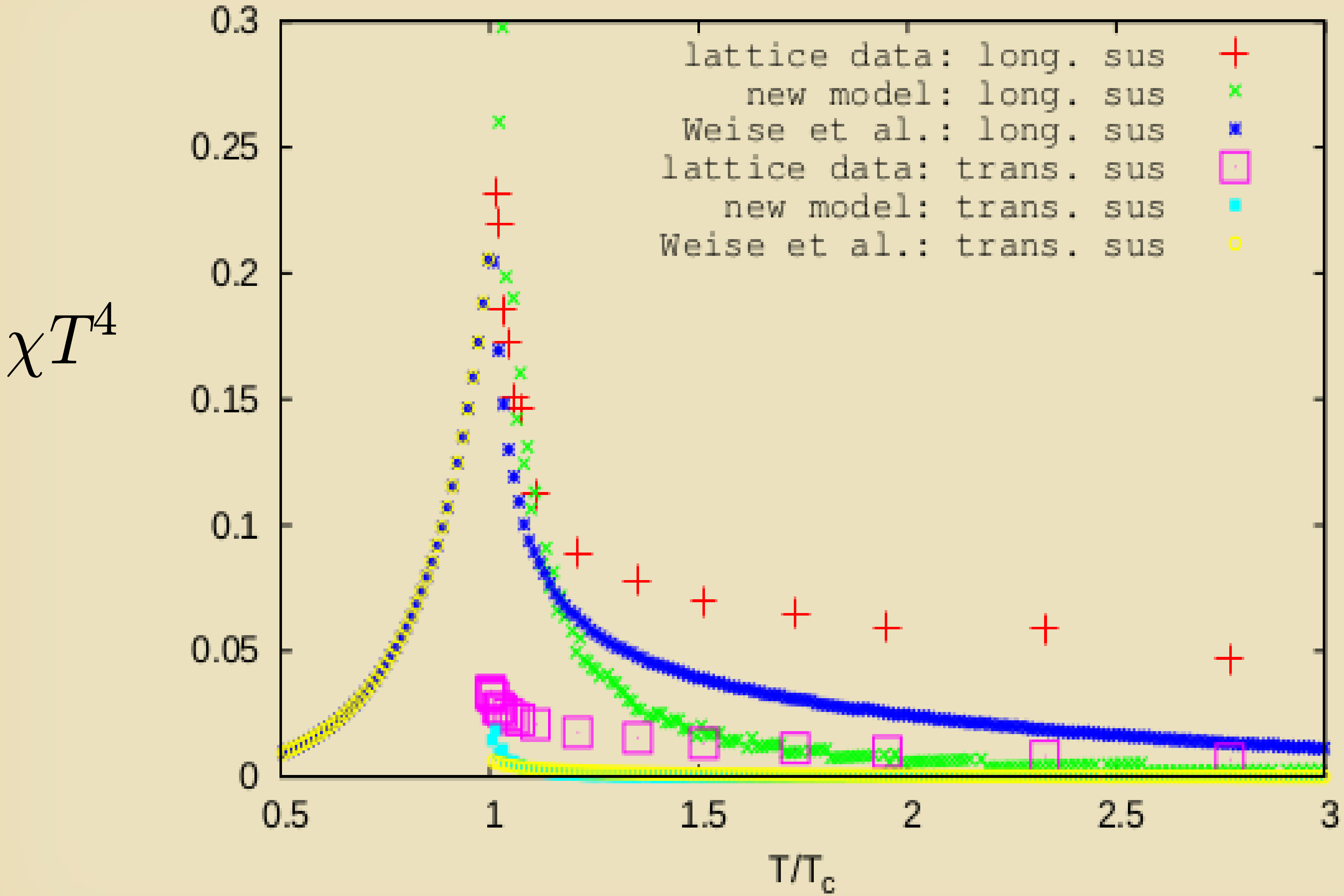
χT^4
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3
T/T$_c$
lattice data: long. sus
new model: long. sus
Weise et al.: long. sus
lattice data: trans. sus
new model: trans. sus
Weise et al.: trans. sus

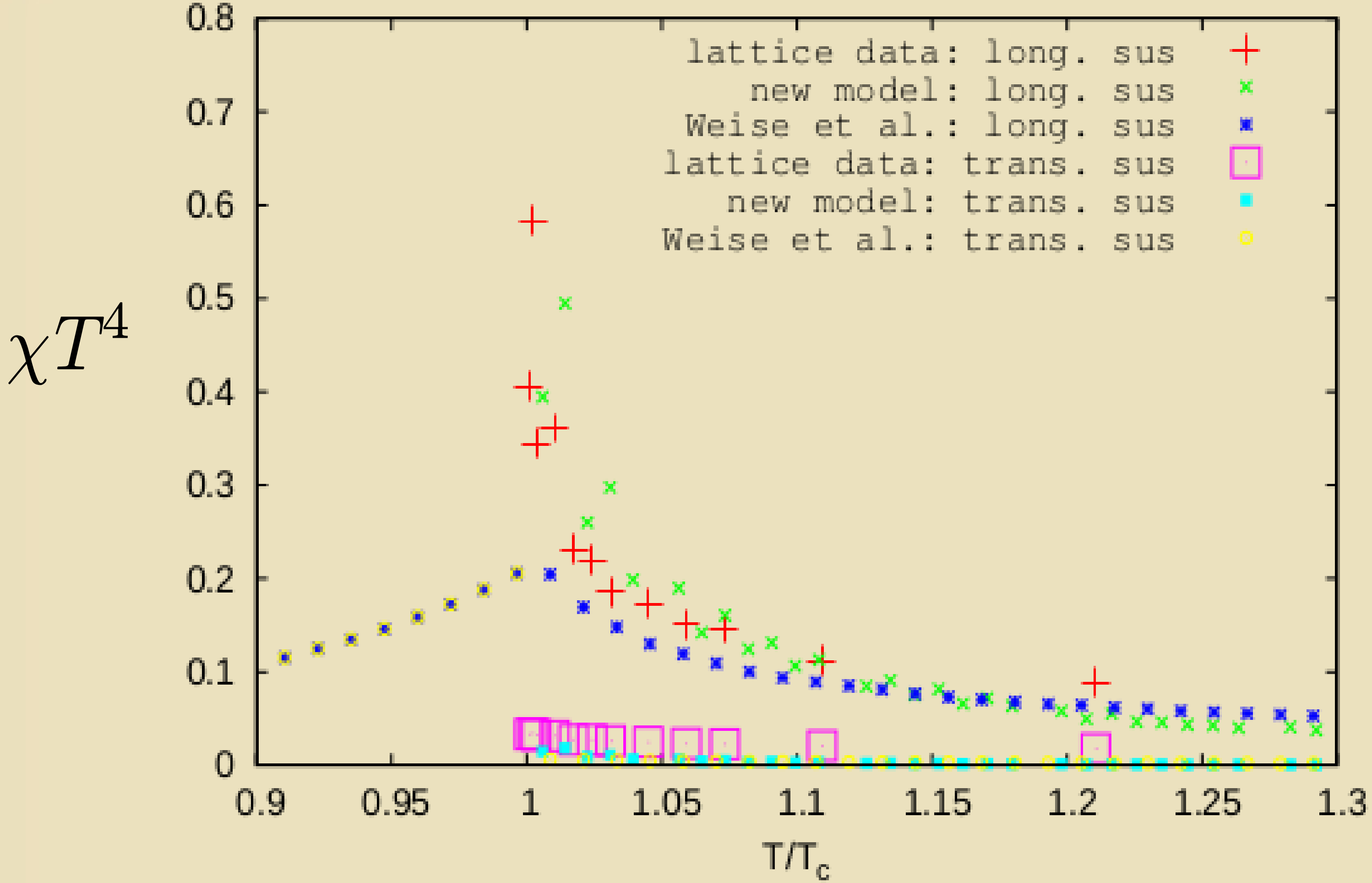


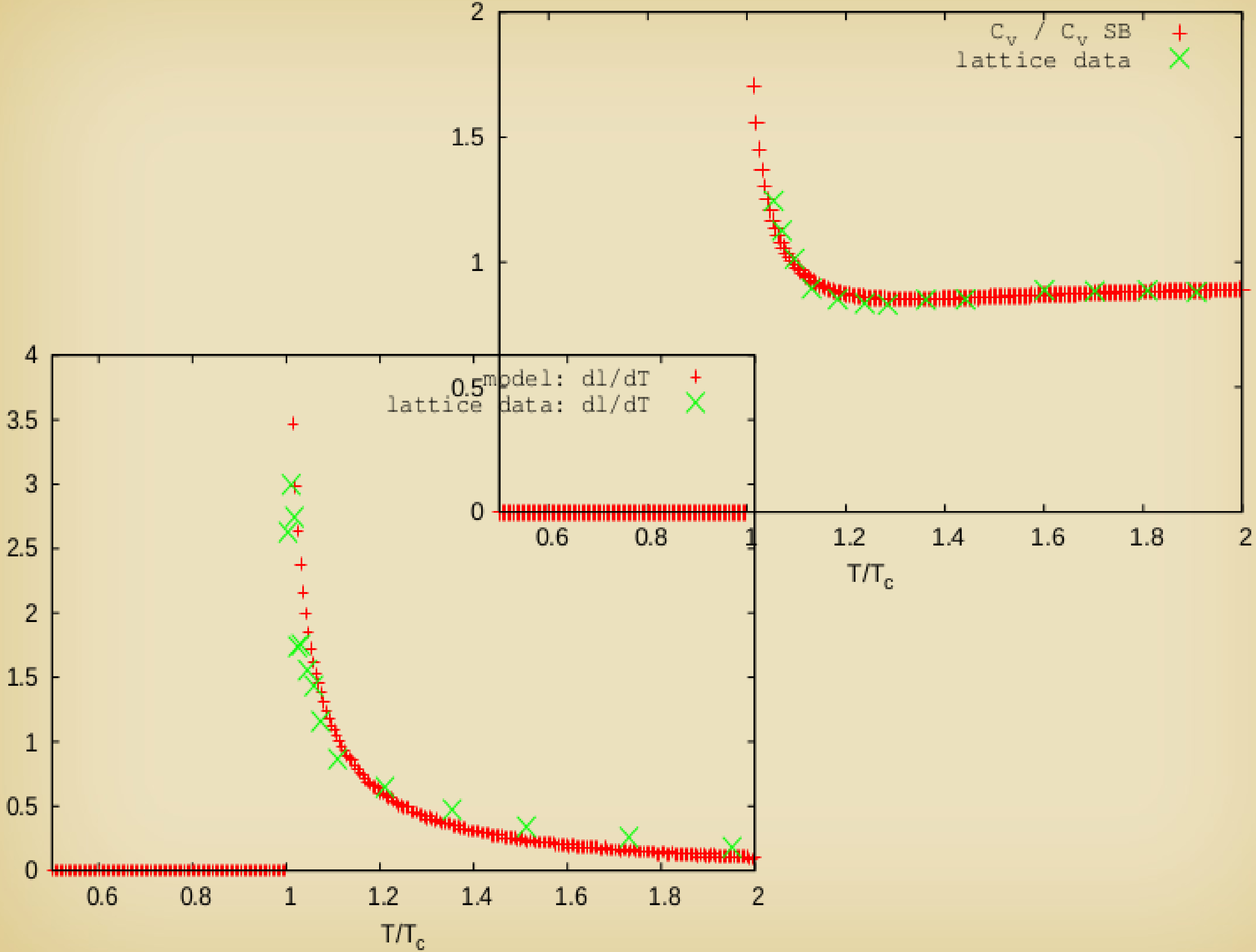
χT^4
lattice data: long. sus
new model: long. sus
Weise et al.: long. sus
lattice data: trans. sus
new model: trans. sus
Weise et al.: trans. sus
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25
1.3
T/T_c

Model 2


1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
T/T_c
lattice data: PL
new model: PL
Weise et al.: PL


χT^4
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3
T/T$_c$
lattice data: long. sus
new model: long. sus
Weise et al.: long. sus
lattice data: trans. sus
new model: trans. sus
Weise et al.: trans. sus


χT^4
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
lattice data: long. sus
new model: long. sus
Weise et al.: long. sus
lattice data: trans. sus
new model: trans. sus
Weise et al.: trans. sus
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25
1.3
T/T$_c$



C_v / C_v SB
lattice data
model: dl/dT
lattice data: dl/dT
T/T_c
T/T_c
0
0.5
1
1.5
2
0
0.5
1
1.5
2
2.5
3
3.5
4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Field theoretical issues

- Composite operators

$$l_{\vec{x}} = < \frac{1}{N_c} Tr \mathcal{P} e^{ig \int_0^\beta d\tau \, A^4[\tau, \vec{x}]} >$$

$$C(\vec{x}) = < l_{\vec{x}} l_{\vec{0}} >$$

$$\chi = \beta \int d^3x \, C(r)$$

- Effective field theory: expansion in $< A^4 A^4 >_c , < A^4 A^4 A^4 A^4 >_c$ ...

Field theoretical issues

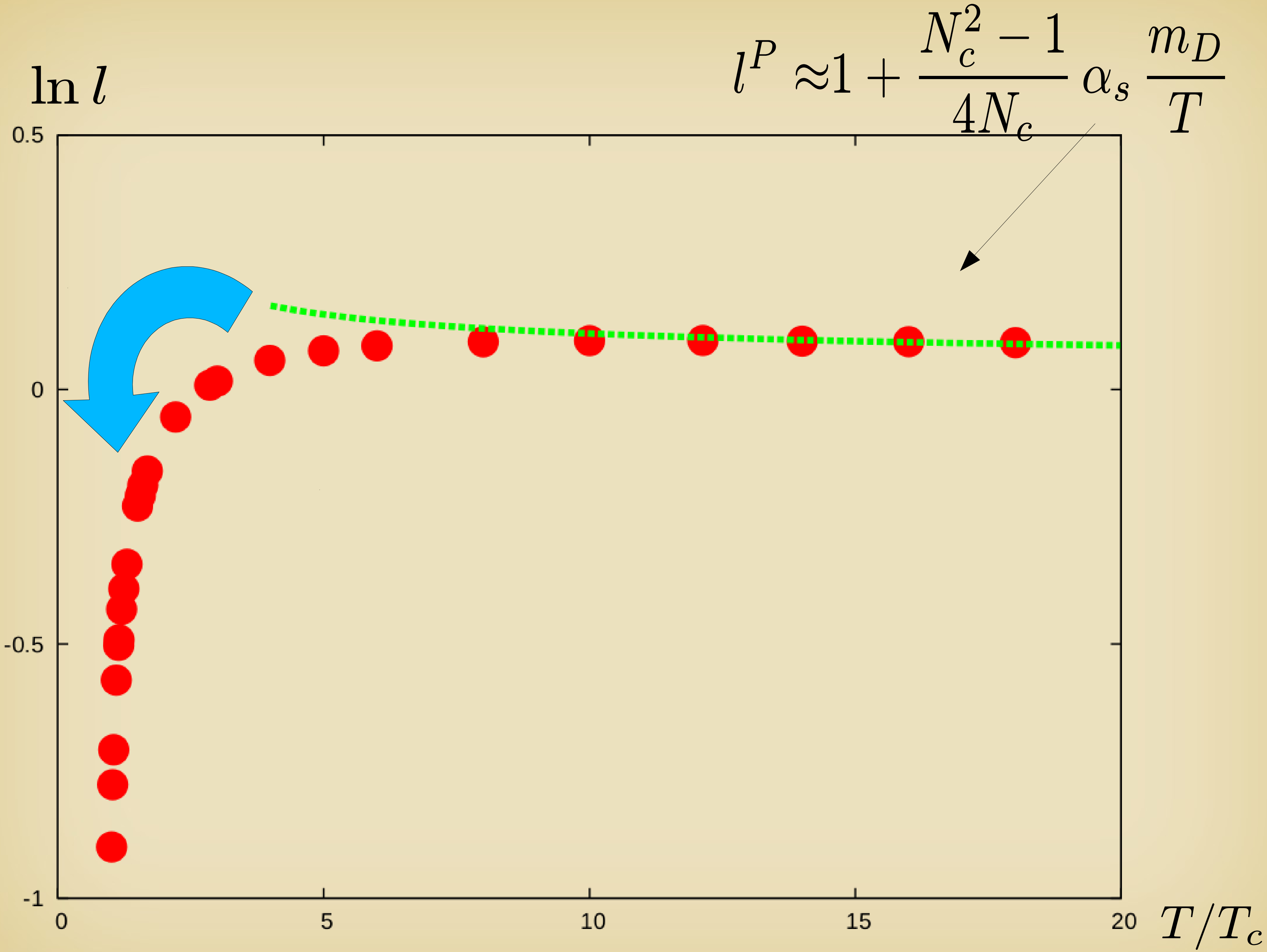
- Perturbation is **not** sufficient...

$$l = < \frac{1}{N_c} Tr \left(I_3 + ig\beta A^4 + \frac{1}{2} ig\beta ig\beta A^4[x] A^4[x] + ... \right) >$$

$$\approx 1. - \frac{1}{2N_c} g^2 \beta^2 Tr(T^a T^b) < A^4_a[x] A^4_b[x] > .$$

$$< A^4_a[k] A^4_b[0] >^P = \delta^{ab} \frac{1}{\beta} \frac{1}{k^2 + m_D^2}$$

$$l^P = 1 + \frac{N_c^2 - 1}{4N_c} \alpha_s \frac{m_D}{T}$$



$\ln l$
$l^P \approx 1 + \frac{N_c^2 - 1}{4N_c} \alpha_s \frac{m_D}{T}$
0.5
0
-0.5
-1
0
5
10
15
20
T/T_c

Field theoretical issues

- Ansatz for non-perturbative propagator (Megias *et al.)*

$$< A_a^4[x] A_b^4[0] > = \delta^{ab} \left(D_{44}^P[\vec{x}] + D_{44}^{NP}[\vec{x}] \right)$$

$$D_{44}^P[\vec{x}] = \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m_D^2} e^{i\vec{k}\cdot\vec{x}}$$

$$D_{44}^{NP}[\vec{x}] = \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \frac{m_G^2}{(k^2 + m_D^2)^2} e^{i\vec{k}\cdot\vec{x}}$$

Field theoretical issues

- Zero temperature limit

$$g^2\,(D_{44}^{P}[\vec{x}] + D_{44}^{NP}[\vec{x}]) = T\,(\frac{g^2}{4\pi|\vec{x}|}\,e^{-m_D|\vec{x}|} + \frac{g^2 m_G^2}{8\pi}\,\frac{e^{-m_D|\vec{x}|}}{m_D})$$

$$\xrightarrow{m_D\to 0.} \delta(\tau)(\frac{g^2}{4\pi}\frac{1}{r} + -\frac{g^2\,m_G^2}{8\pi} r + \mathrm{Const.})$$

- Effective string tension

$$b' = \frac{g^2 m_G^2}{8\pi}.$$

Field theoretical issues

To leading order...

$$l \approx 1. - \frac{1}{2N_c} g^2 \beta^2 Tr(T^a T^b) < A_a^4[x] A_b^4[x] >$$

$$C[r;T] \approx \frac{1}{4} \frac{1}{N_c^2} g^4 \beta^4 \; < Tr(A_4^2[x]) \, Tr(A_4^2[0]) >_c$$

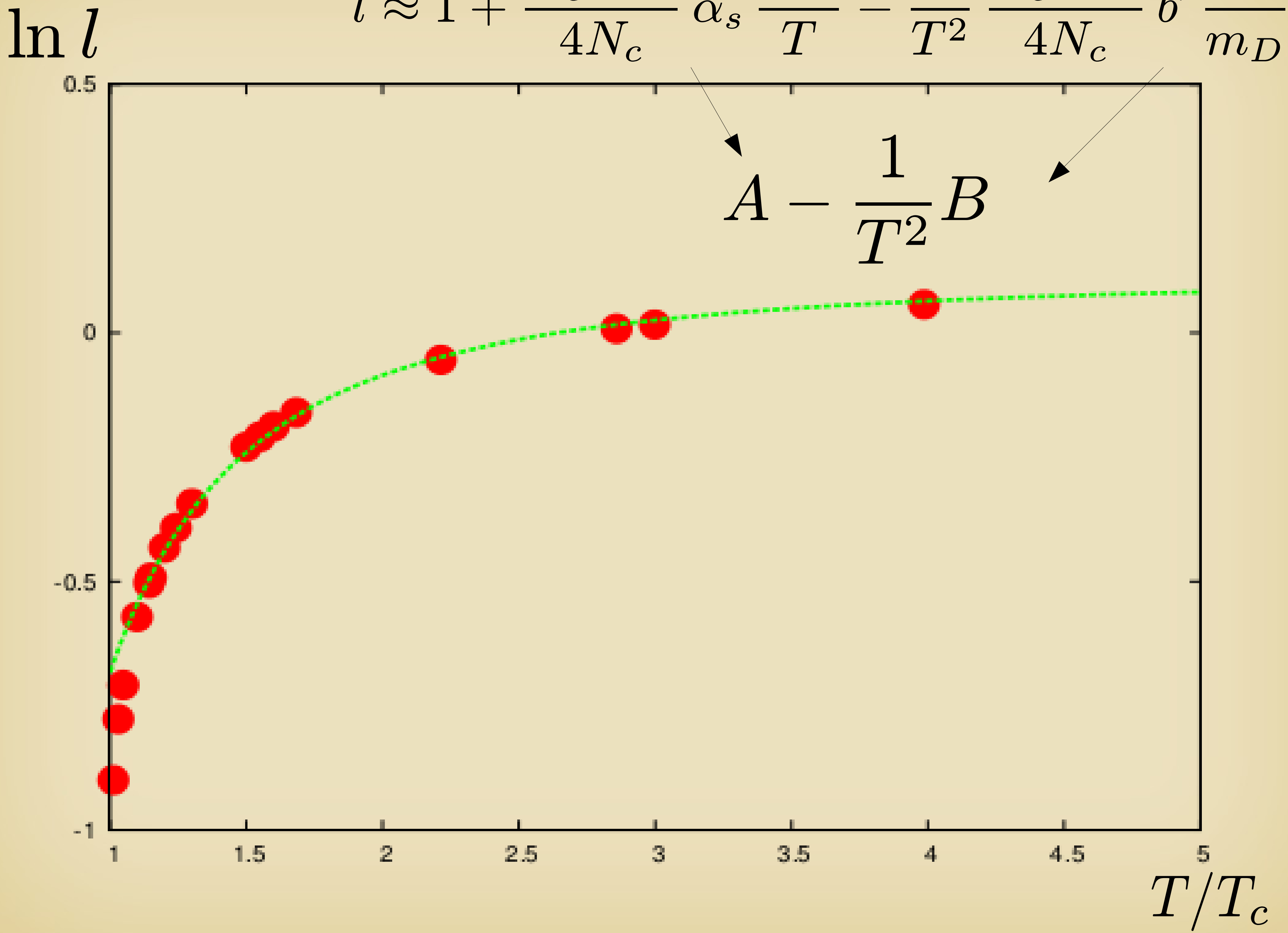
$$\chi_l = \beta \int d^3x \, C(x)$$

$$
\begin{aligned}
l =& l^P + l^{NP} \\
=& 1 + \frac{N_c^2 - 1}{4N_c}\,\alpha_s\,\frac{m_D}{T} - \frac{1}{T^2}\,\frac{N_c^2 - 1}{4N_c}\,b'\,\frac{T}{m_D}
\end{aligned}
$$

$$
\begin{aligned}
C[r;T] =& C^P[r;T] + C^{NP}[r;T] \\
=& \frac{N_c^2 - 1}{8N_c^2}\alpha_s^2\frac{e^{-2rm_D}}{(rT)^2} + \frac{N_c^2 - 1}{8N_c^2}\,b'^2\,\frac{1}{m_D^2}\,\frac{e^{-2m_D r}}{T^2}
\end{aligned}
$$

$$
\begin{aligned}
\chi_l =& \chi_l^P + \chi_l^{NP} \\
=& \frac{N_c^2 - 1}{8N_c^2}\,\alpha_s^2\,\frac{2\pi}{m_D T^3} + \frac{N_c^2 - 1}{8N_c^2}\,\pi\,b'^2\,\frac{1}{m_D^5 T^3}
\end{aligned}
$$

$$l \approx 1 + \frac{N_c^2 - 1}{4N_c}\,\alpha_s\,\frac{m_D}{T} - \frac{1}{T^2}\,\frac{N_c^2-1}{4N_c}\,b'\,\frac{T}{m_D}$$

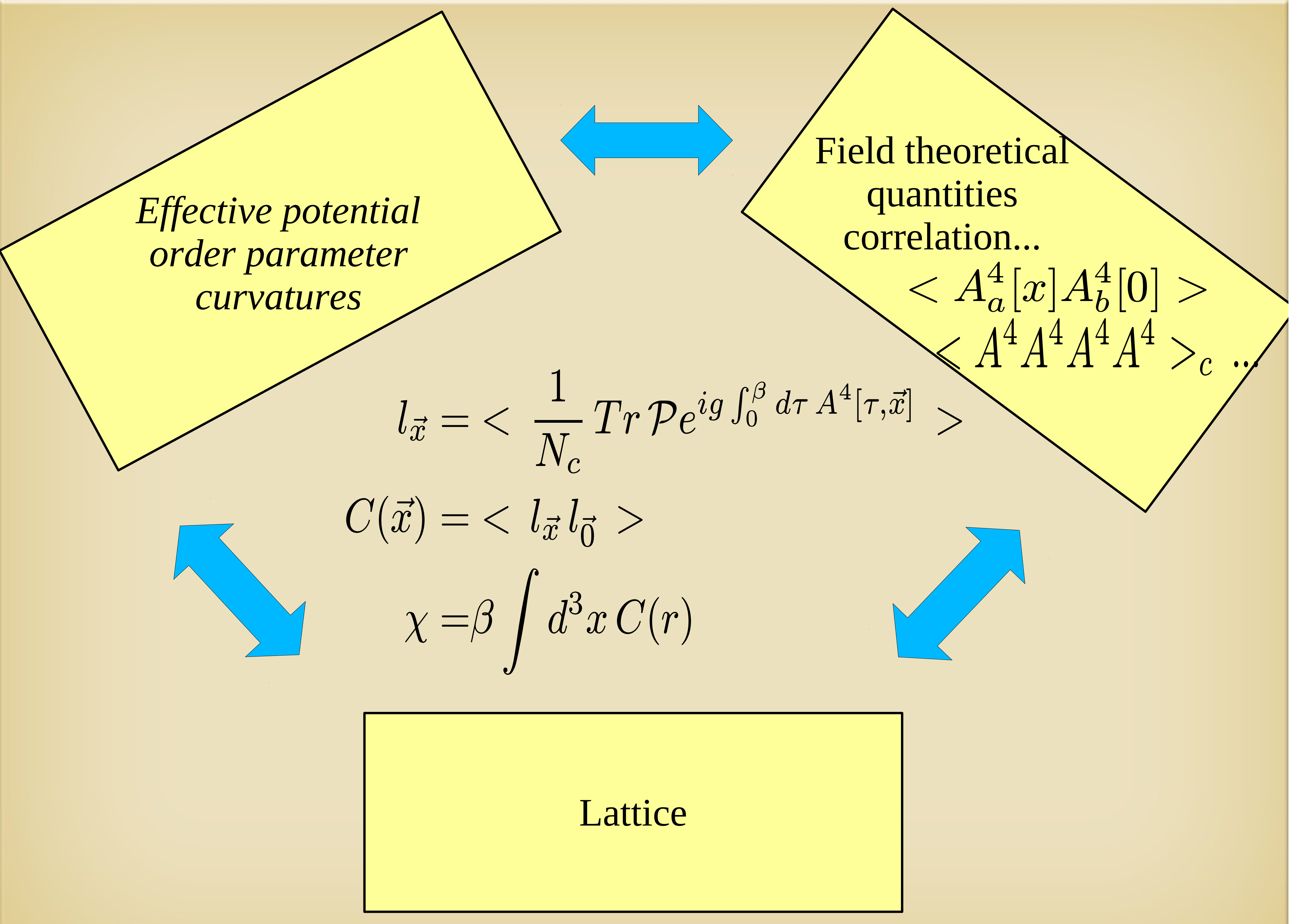


Field theoretical issues

- Overall consistent description of lattice data in temperature range $1.1\,T_c - 4\,T_c$ with

$$g^2 < A_{0,a}^2 >^{NP} = \frac{g^2(N_c^2-1)Tm_G^2}{8\pi m_D} = 0.96\,\mathrm{GeV}^2 = 13.2\,T_c^2$$

- Similar analysis for trace anomaly



Effective potential
order parameter
curvatures
Field theoretical
quantities
correlation...
$< A_a^4[x] A_b^4[0] >$
$< A^4 A^4 A^4 A^4 >_c$ ...
$l_{\vec{x}} = < \frac{1}{N_c} Tr \mathcal{P} e^{ig \int_0^\beta d\tau A^4[\tau,\vec{x}]} >$
$C(\vec{x}) = < l_{\vec{x}} l_{\vec{0}} >$
$\chi = \beta \int d^3x\, C(r)$
Lattice

Conclusions

- New lattice data for Polyakov loop and intensive definition to match to continuum calculation
- New fit taking into account the fluctuation
- Fluctuations: width of phase transition: $1.4\,T_c - 1.6\,T_c$
- Field-theoretical issues

Thank you!