Non-equilibrium Dynamics, Hersonissos, Crete, 25-30 June, 2012

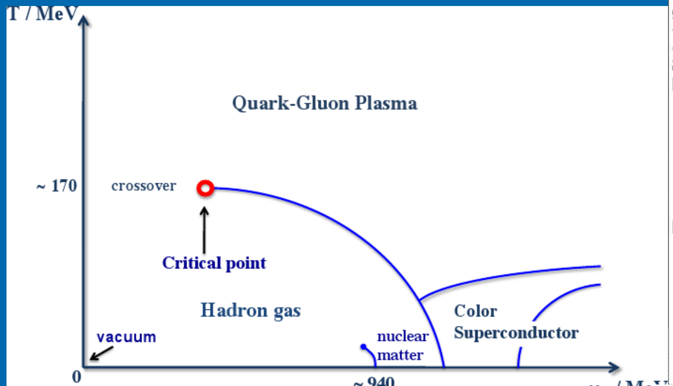
Non-equilibrium phase transitions in expanding matter

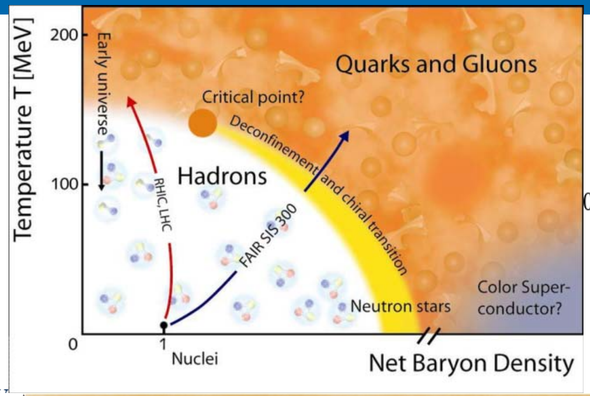
Igor Mishustin*)

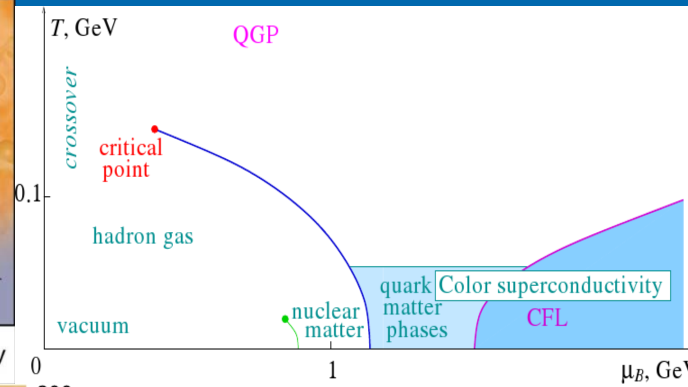
Frankfurt Institute for Advanced Studies,
J.W. Goethe Universität, Frankfurt am Main
National Research Centre, "Kurchatov Institute"
Moscow

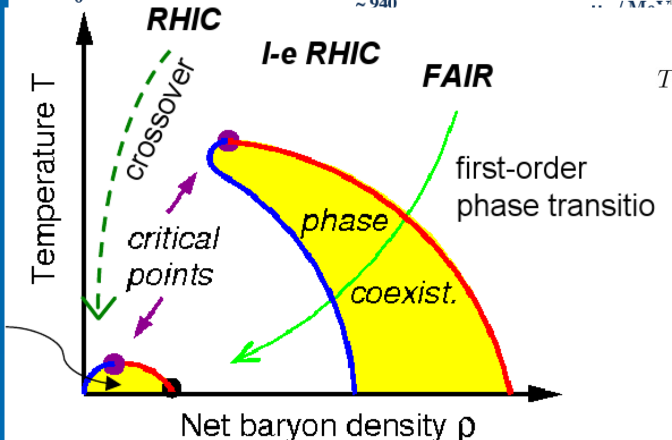
*) in collaboration with Marlene Nahrgang and Marcus Bleicher

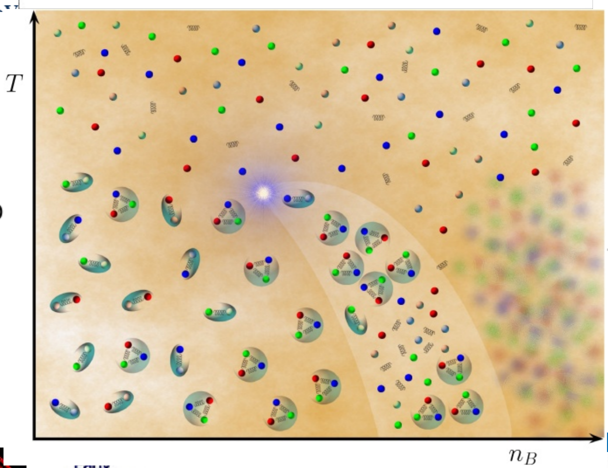
"QCD" phase diagram

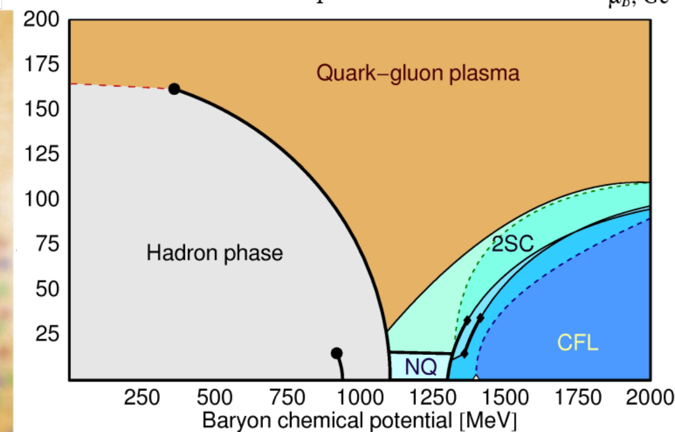


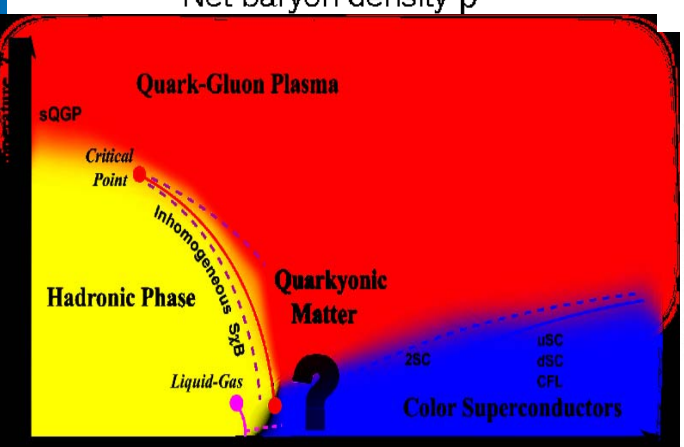


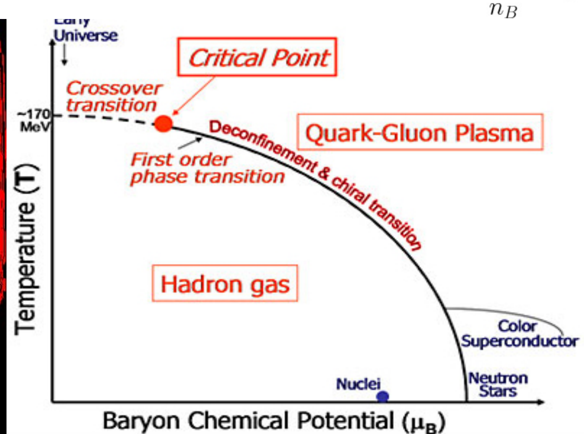


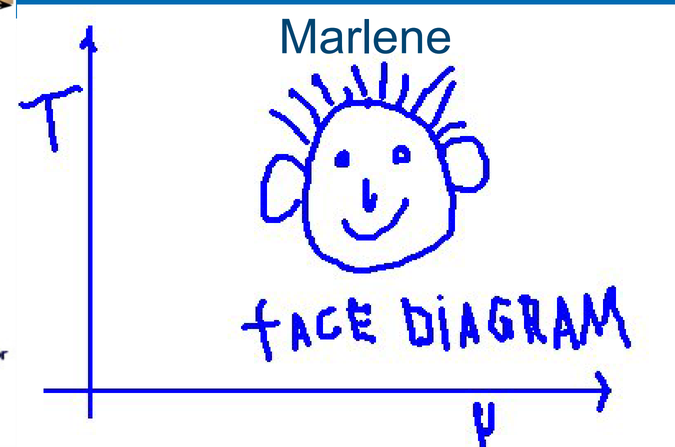












Contents

- **Introduction: Effects of fast dynamics**
- **Fluctuations of the order parameter**
- **Chiral fluid dynamics with damping and noise**
- **Critical slowing down and supercooling**
- **Observable signatures**
- **Conclusions**

This talk is based on recent works:

M. Nahrgang, M. Bleicher, S. Leupold and I. Mishustin, The impact of dissipation and noise on fluctuations in chiral fluid dynamics, arXiv:1105.1962 [nucl-th];

M. Nahrgang, I.Mishustin, M. Bleicher, Dynamical domain formation in expanding chiral fluid underegoing 1st order phase transition, in preparation

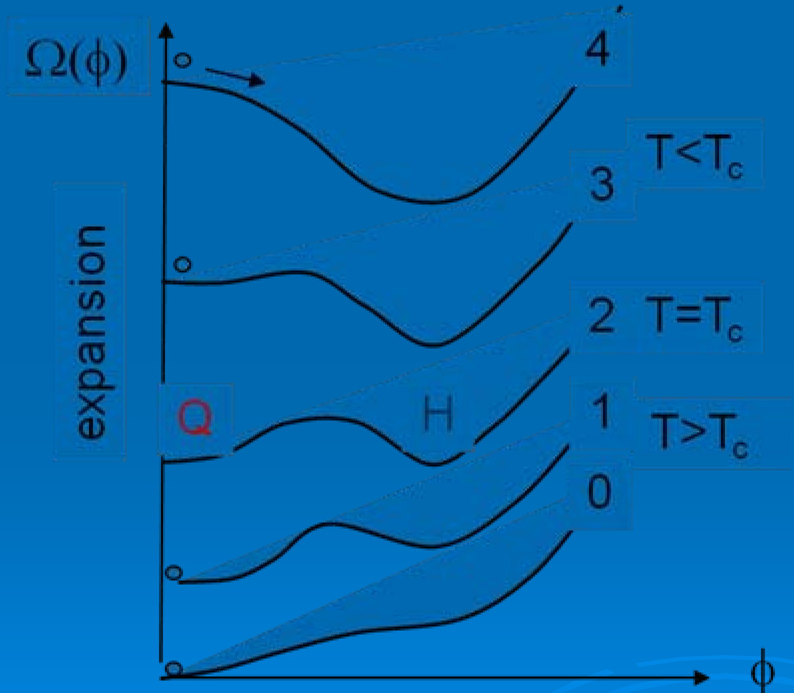
Effects of fast dynamics

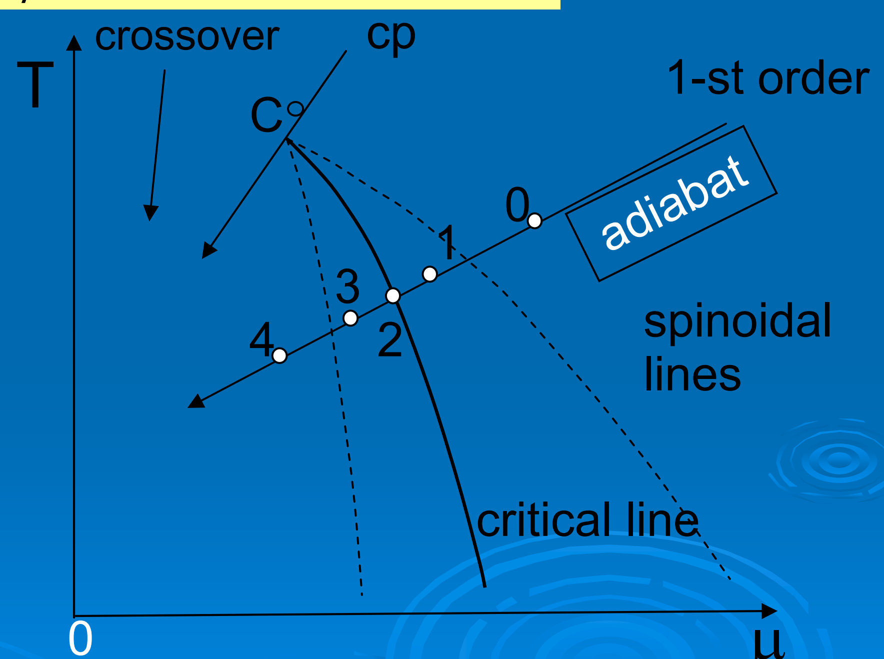
Effective thermodynamic potential for a 1st order transition

$$\Omega(\phi;T,\mu)=\Omega_0(T,\mu)+\frac{a}{2}\phi^2+\frac{b}{4}\phi^4+\frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by $\dfrac{\partial\Omega}{\partial\phi}=0\Rightarrow P=-\Omega(\phi_{eq})$





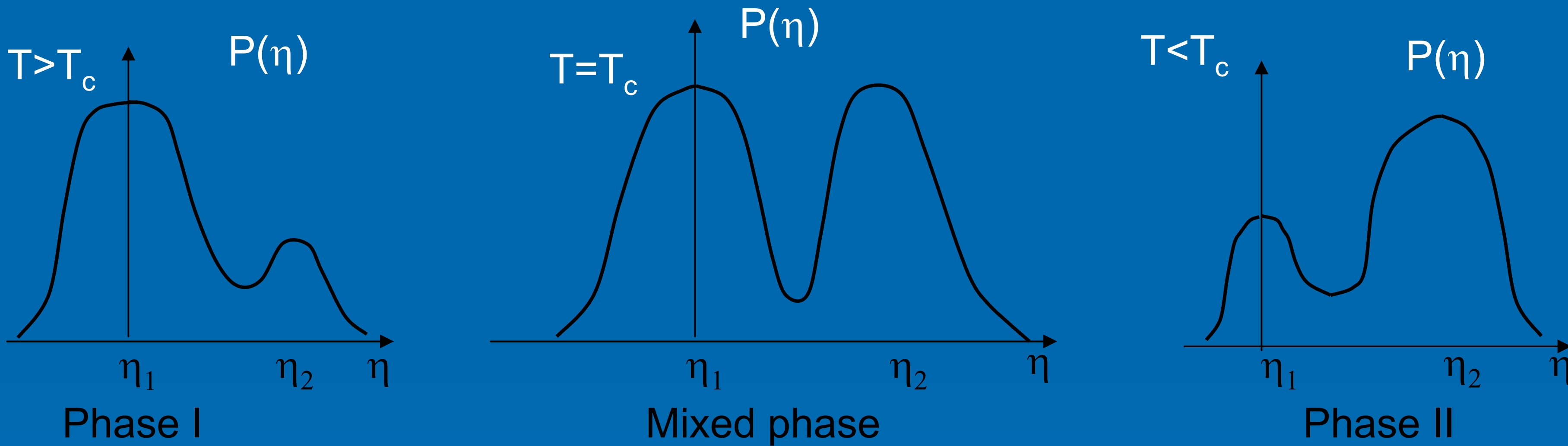
In rapidly expanding system 1-st order transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I.N. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Equilibrium fluctuations of order parameter in 1^{st} order phase transition

Probability distribution for fluctuations

$$P(\delta\phi) \propto \exp\left(-\frac{\Delta\Omega(\delta\phi)V}{T}\right), \quad \delta\phi = \phi - \langle\phi\rangle$$



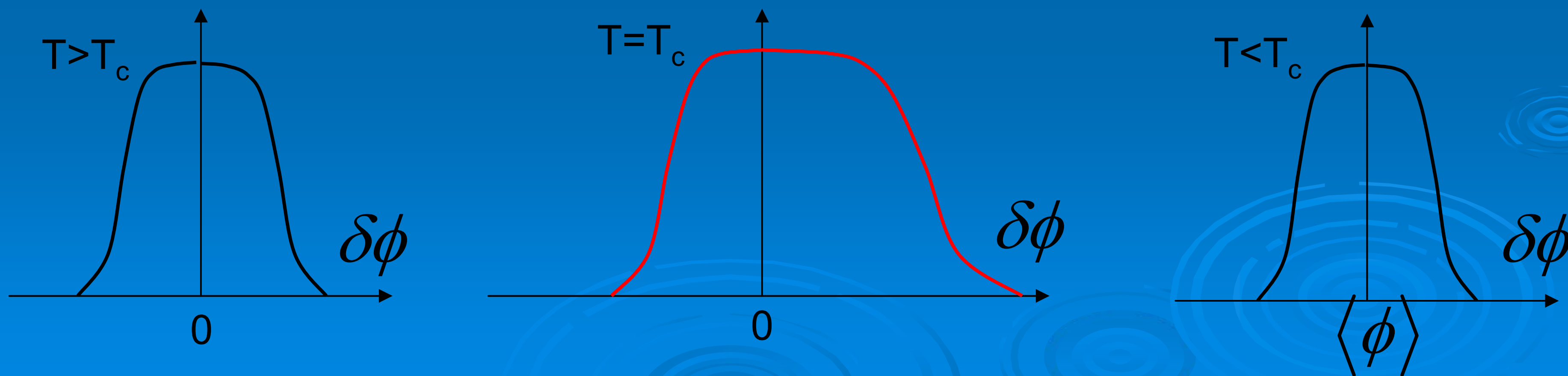
- Bi-modality in distribution of fluctuations/observables, **e.g. Z_{max} distributions**
- **In rapid process fluctuations of the order parameter are out of equilibrium**
- **During supercooling process strong fluctuations may develop in the form of droplets** of metastable phase

Evolution of equilibrium fluctuations in 2nd order phase transition

$$\Omega(\phi) = \frac{1}{2}a(T)\phi^2 + \frac{1}{2}b(\nabla\phi)^2 + \frac{\lambda}{4}\phi^4, \quad a(T) = a_0(T - T_c)$$

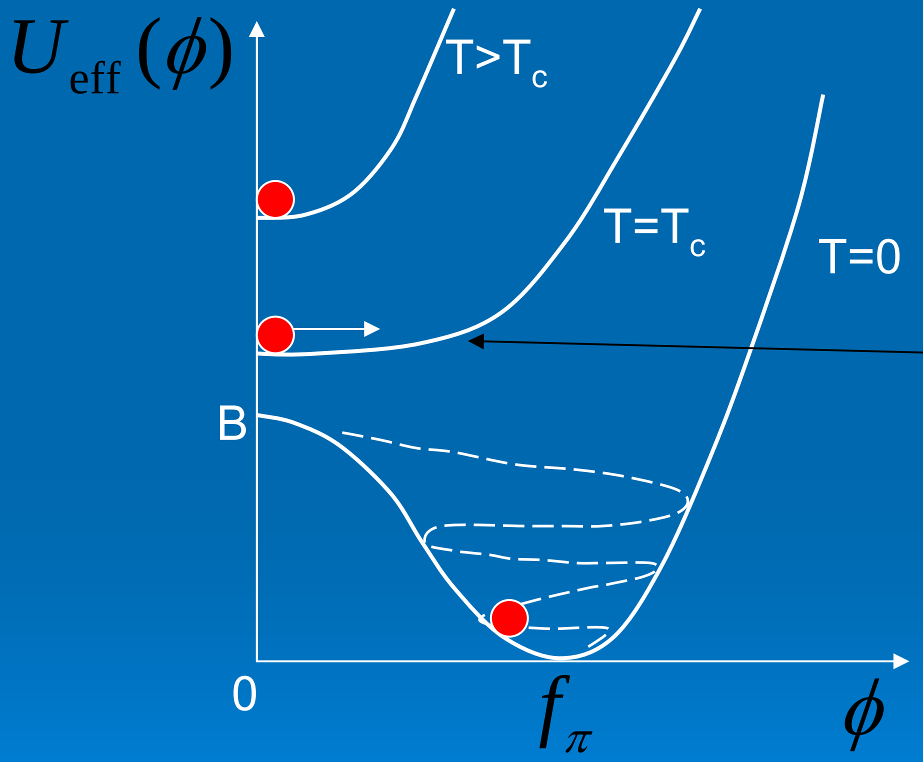
$$\langle\phi\rangle = \frac{a(T)}{\lambda},\ T < T_c \text{ and } \langle\phi\rangle = 0,\ T > T_c, \quad \delta\phi = \phi - \langle\phi\rangle$$

$$\text{Distribution of fluctuations} \quad P(\delta\phi) \sim \exp\left[-\frac{\Delta\Omega(\delta\phi)V}{T}\right]$$



In rapidly expanding system critical fluctuations have not sufficient time to develop

Critical slowing down in the 2nd order phase transition



"Rolling down" from the top of the potential is similar to spinodal decomposition (Csernai&Mishustin 1995)

Fluctuations of the order parameter evolve according to the equation

$$\frac{d\delta\phi}{dt} = -\gamma\frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\mathrm{rel}}}$$

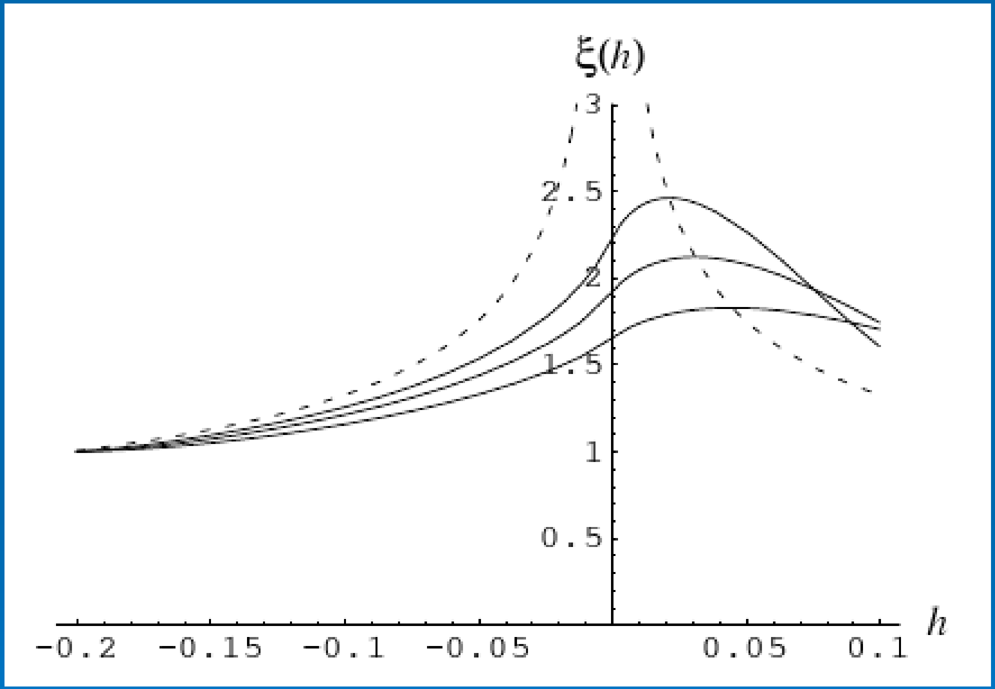
In the vicinity of the critical point the relaxation time for the order parameter diverges - no restoring force

$$\tau_{\mathrm{rel}}(T) \sim \frac{1}{\left|T - T_c\right|^{\nu}} \rightarrow \infty, \quad \nu \sim 2$$

(Landau&Lifshitz, vol. X, Physical kinetics)

Critical slowing down 2

B. Berdnikov, K. Rajagopal, Phys. Rec. D61 (2000)



Critical fluctuations have not enough time to build up. One can expect only a factor 2 enhancement in the correlation length (even for slow cooling rate, dT/dt=10 MeV/fm).

Dynamical model of chiral phase transition

Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}),$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \ \ <\sigma>_{\mathrm{vac}} = f_\pi \rightarrow H = f_\pi m_\pi^2$$

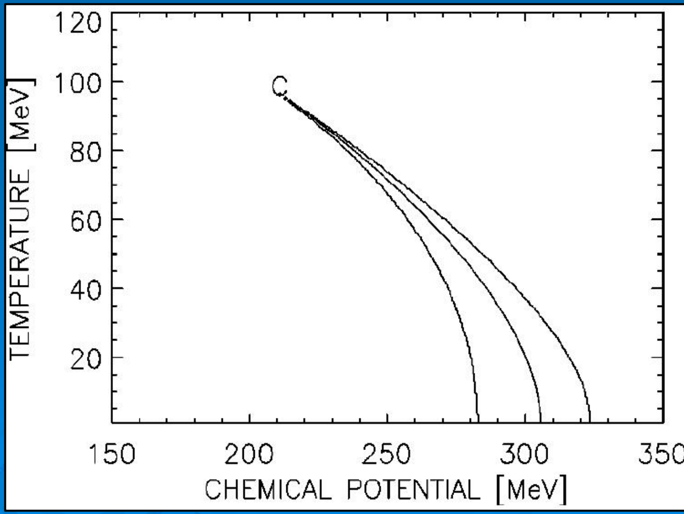
Thermodynamics of LσM on the mean-field level was studied in

Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

Effective thermodynamic potential:

$$U_{\mathrm{eff}}(\sigma; T, \mu) = U(\sigma, \boldsymbol{\pi}) + \Omega_q(m; T, \mu)$$

$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2)$$

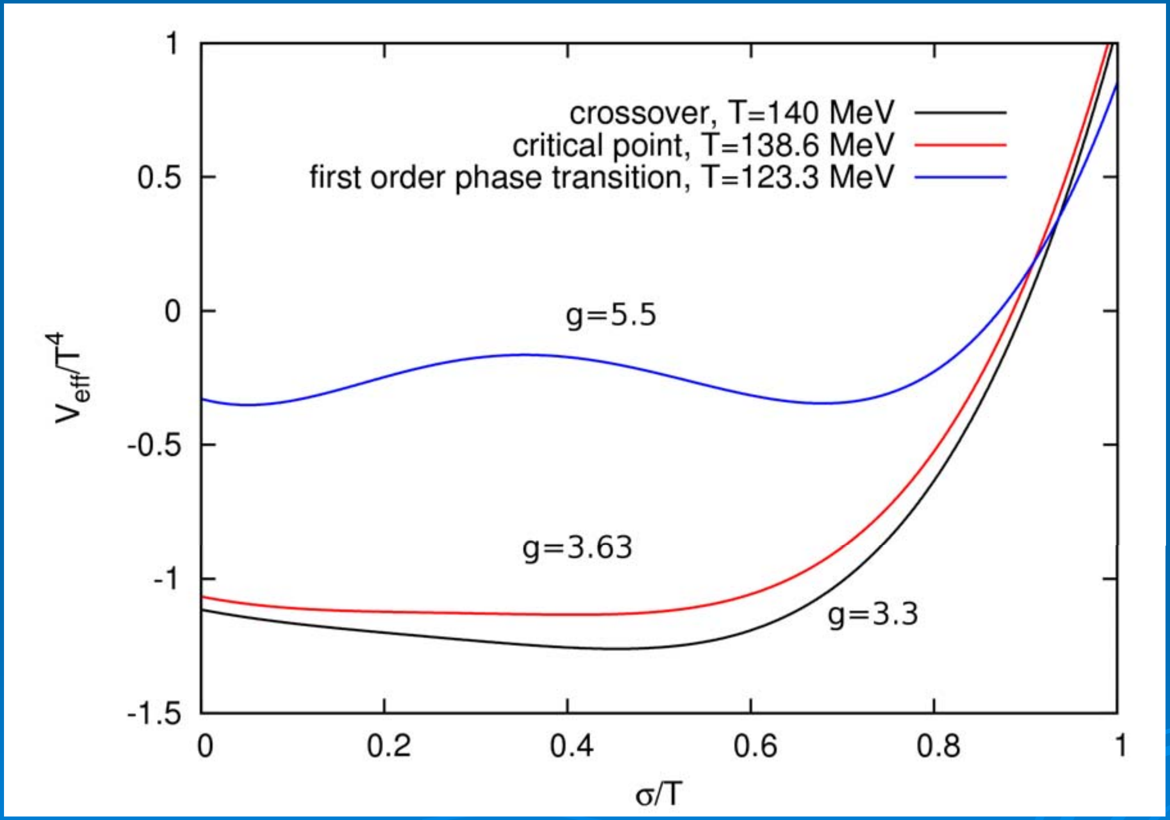


CO, 2nd and 1st order chiral transitions

are obtained in T-μ plane.

Effective thermodynamic potential

$$\Omega_q(m;T,\mu) = -\nu_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \ln\left[1+\exp\left(\frac{\mu-\sqrt{m^2+p^2}}{T}\right)\right] + (\mu\to-\mu) \right\},\ \ \nu=2N_f N_c$$

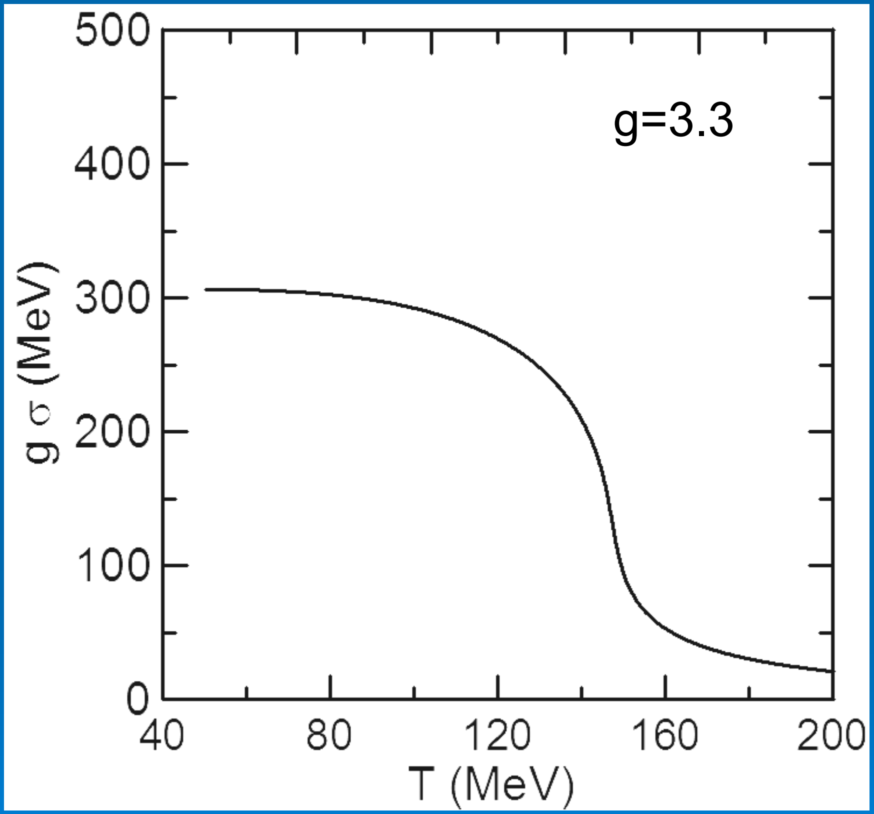


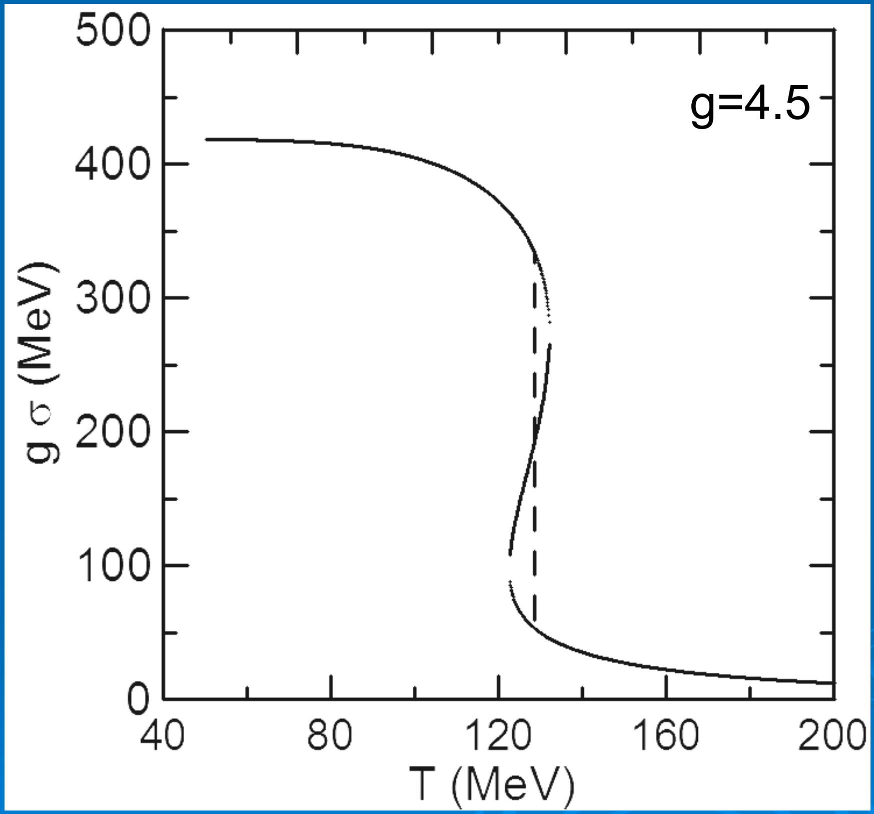
Here we consider μ=0 system but tune the order of the chiral phase transition by changing the coupling g.

Order parameter field vs T

crossover

1-st order





unstable states at 122 MeV<T<132 MeV

$\Rightarrow$ spinodal instability

Chiral fluid dynamics (CFD)

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system fluid+field

$$\partial_\nu (T_{\text{fluid}}{}^{\mu\nu} + T_{\text{field}}{}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}{}^{\mu\nu} = -\partial_\mu T_{\text{field}}{}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma})\partial^\nu\sigma = (g\rho_s + \eta\partial_t\sigma)\partial^\nu\sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu\partial^\mu\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma} + g < \bar{q}q > + \eta\partial_t\sigma = \xi$$

Langevin equation for the order parameter

$$< \xi(t,\vec{r}) >= 0, \quad < \xi(t,r)\xi(t',r') >= \frac{1}{V} m_\sigma \eta \delta(t-t')\delta(r-r')\coth\left(\frac{m_\sigma}{2T}\right)$$

Calculation of damping term

T.Biro and C. Greiner, PRL, 79. 3138 (1997)

M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC 84, 024912 (2011)

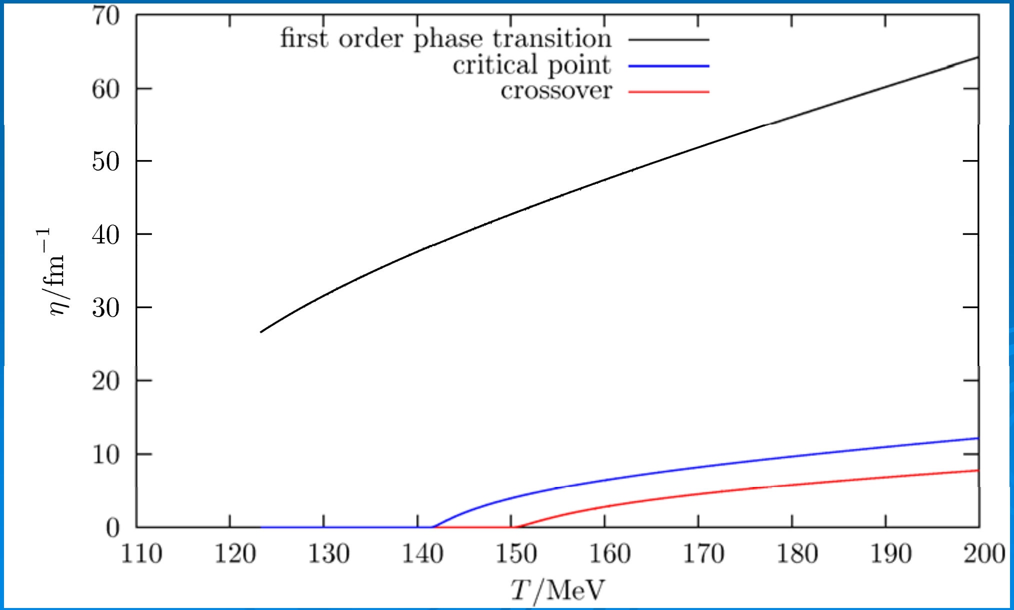
The damping is associated with the processes:

$$\sigma \to \bar{q}q, \ \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2}\left[1 - 2n_F\left(\frac{m_\sigma}{2}\right)\right]\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2}$$

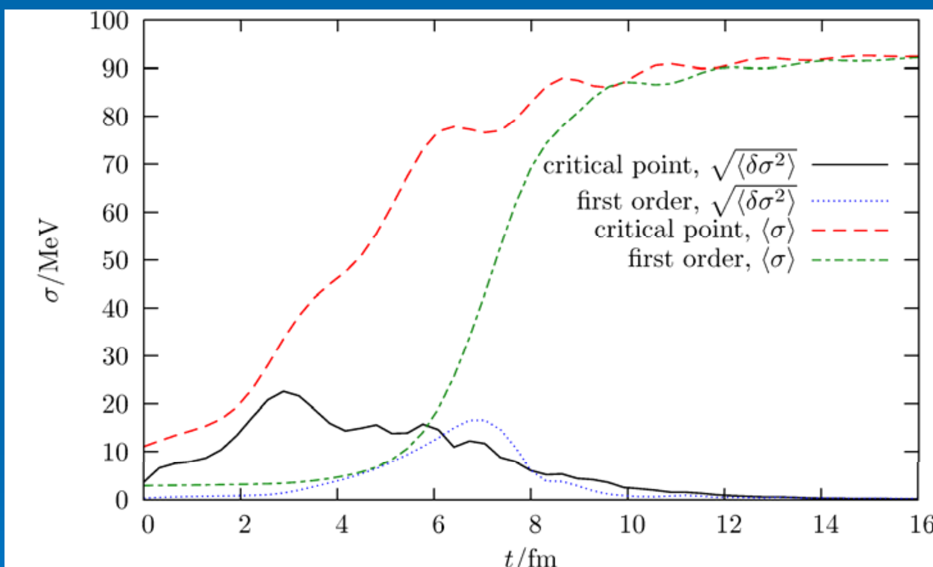
Around Tc the damping is due to the pion modes, η=2.2/fm

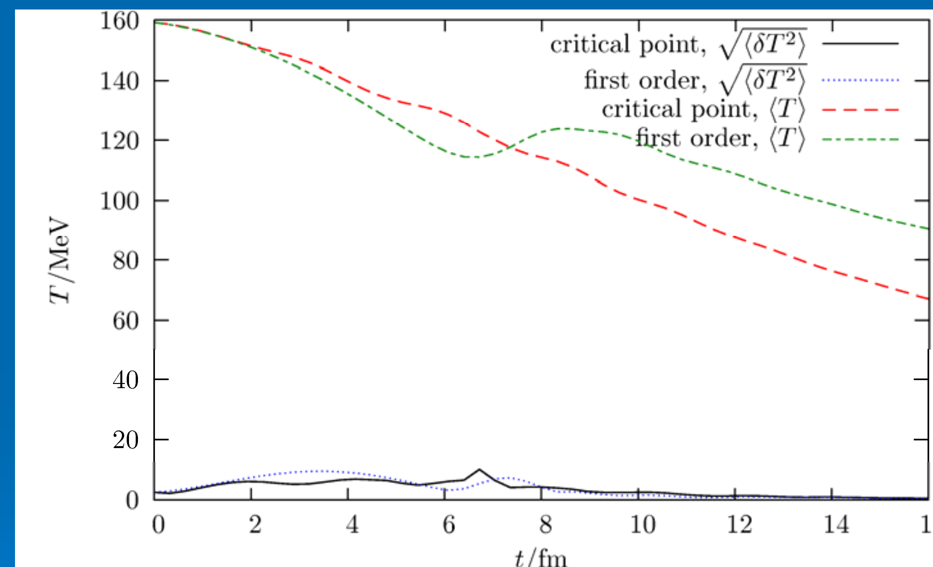


Dynamic simulations: Bjorken-like expansion

Initial state: cylinder of length L in z direction, with ellipsoidal cross section in x-y direction

$$\text{At } t=0:\ v(z)=\frac{2z}{L}0.2c,\ \ -\frac{L}{2}<z<\frac{L}{2};\ \ \mathrm{v}_{\mathrm{x}}=v_y=0;\ T=160\ MeV$$



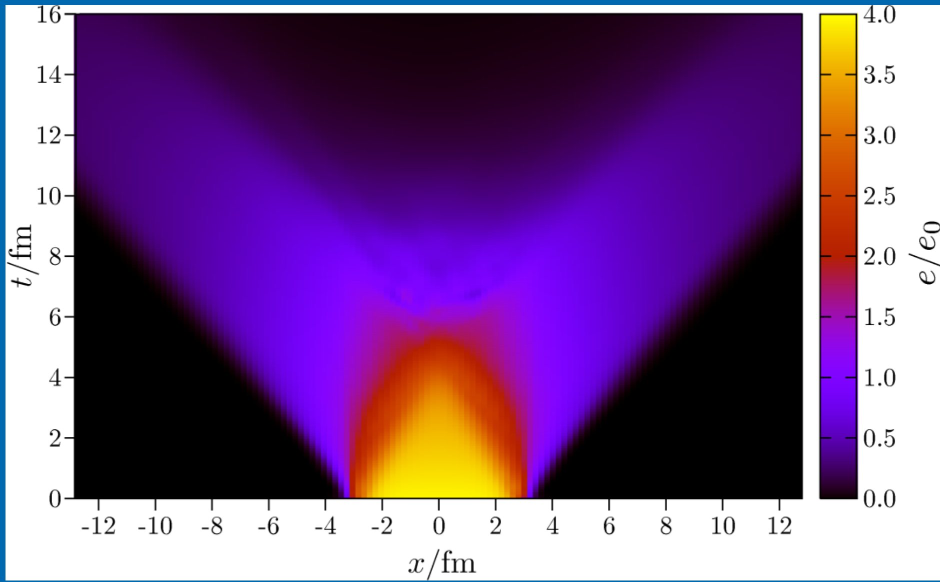


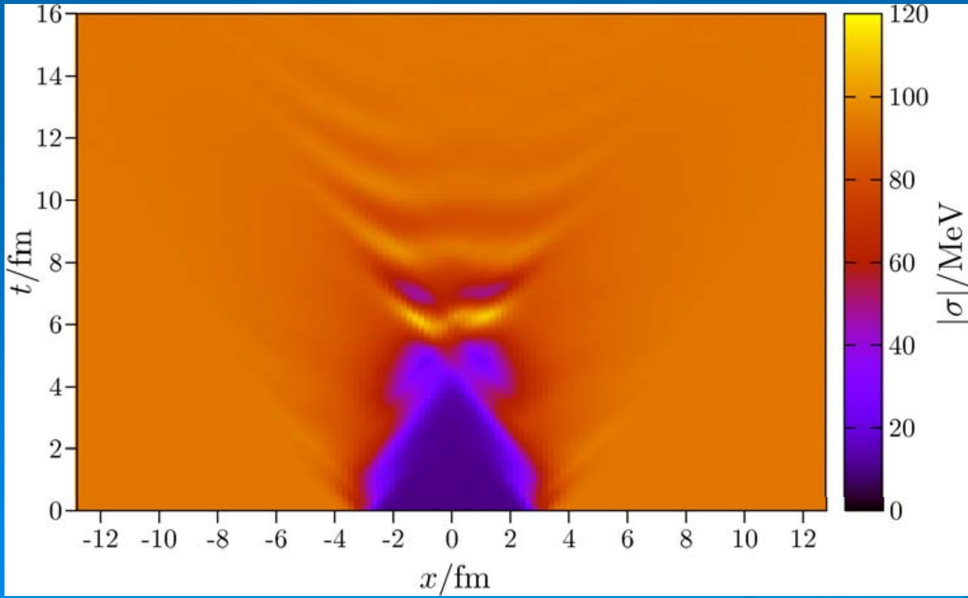
In the vicinity of the c.p. sigma shows pronounced oscillations since damping term vanishes $(m_\sigma < 2m_q)$

Supercooling and reheating effects are clearly seen in the 1-st order transition.

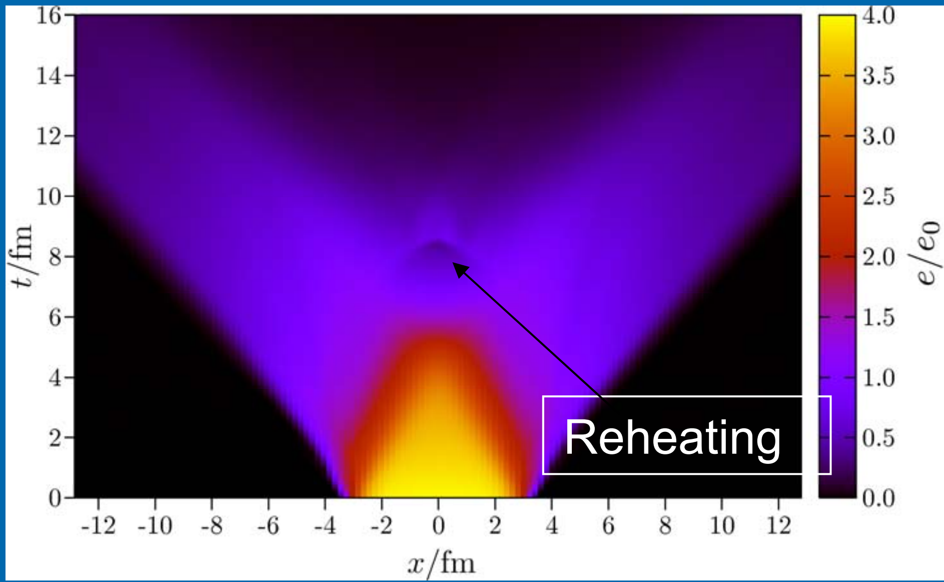
Dynamical evolution of chiral fluid

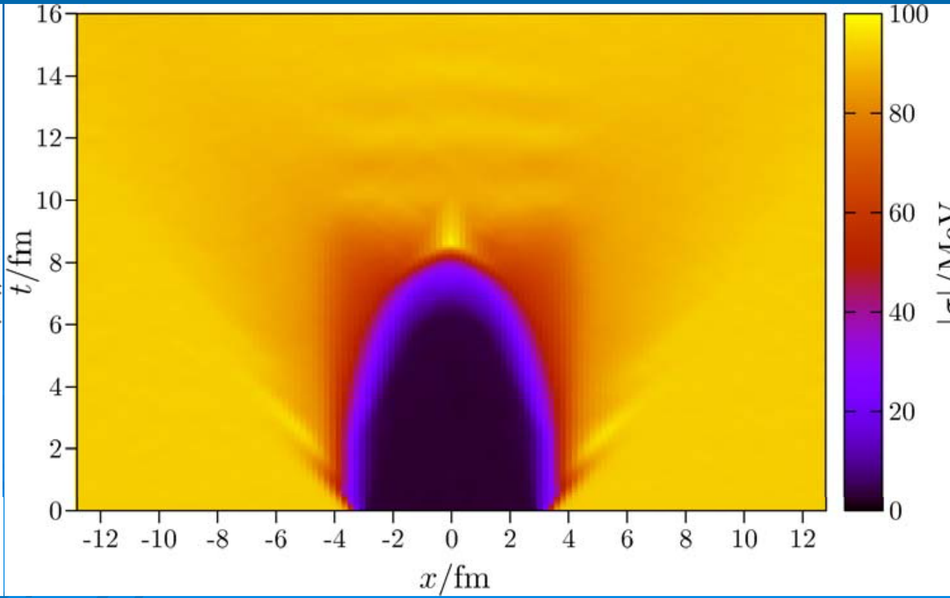
2-nd order transition (g=3.63)



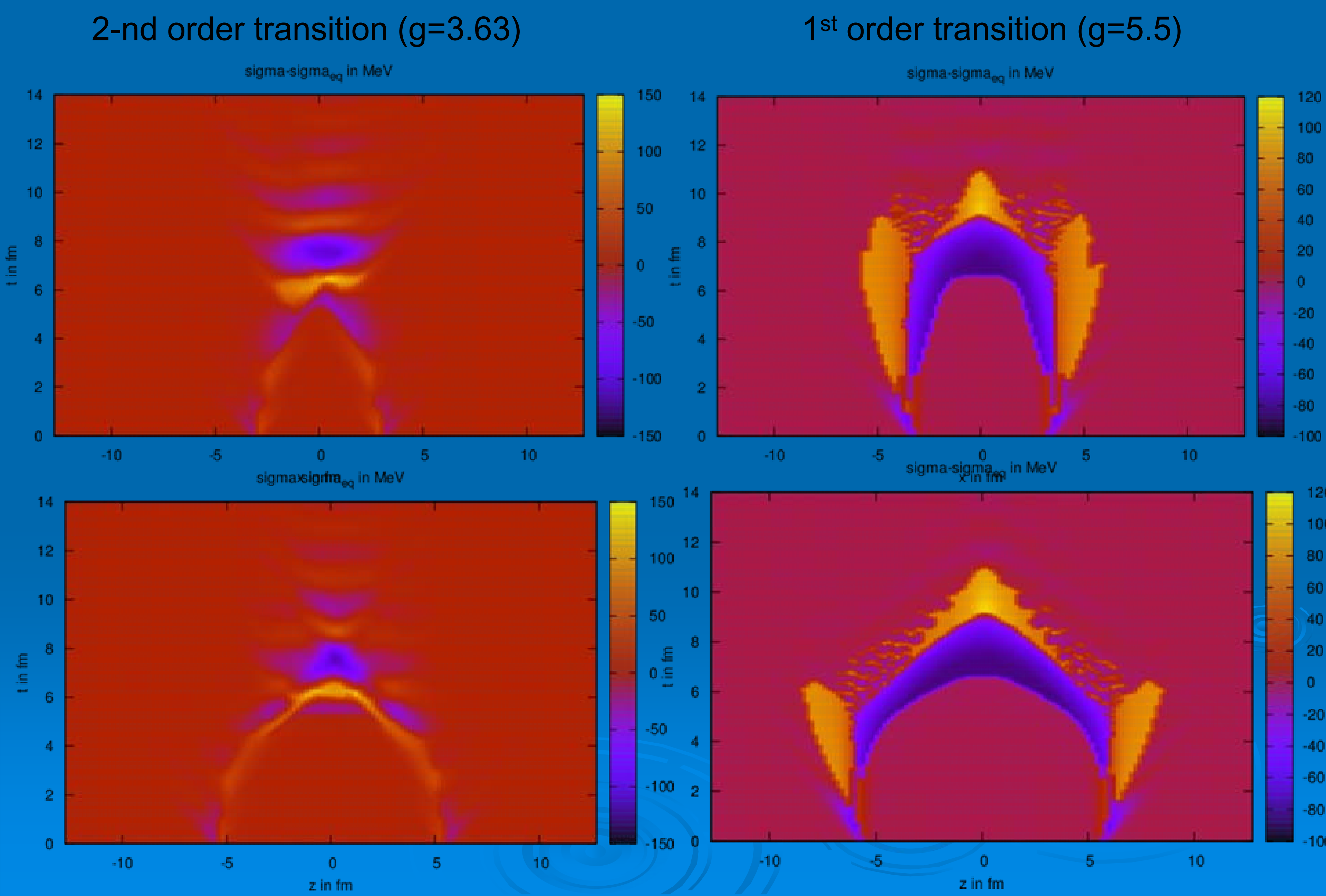


1st order transition (g=5.5)



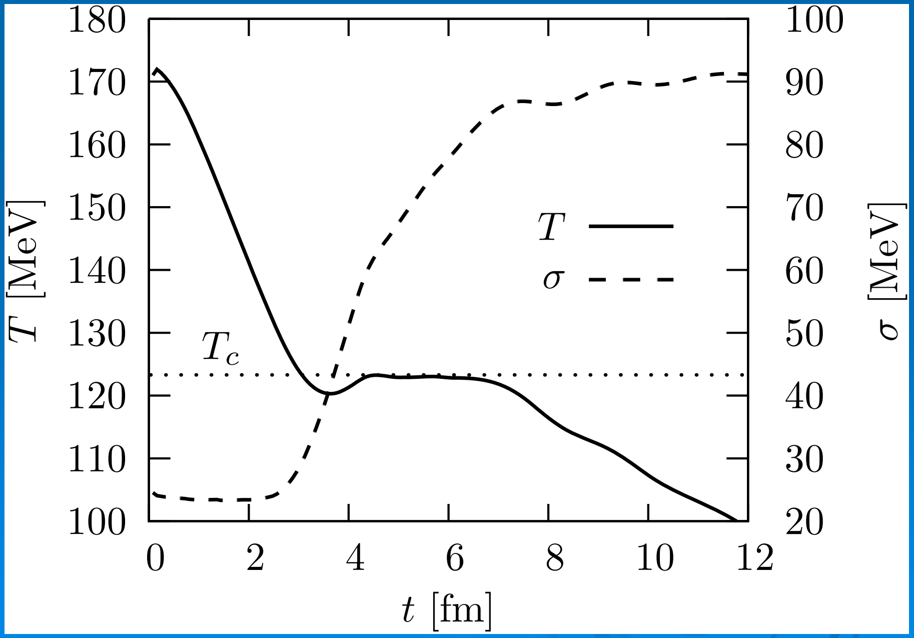


Dynamical evolution of sigma fluctuations

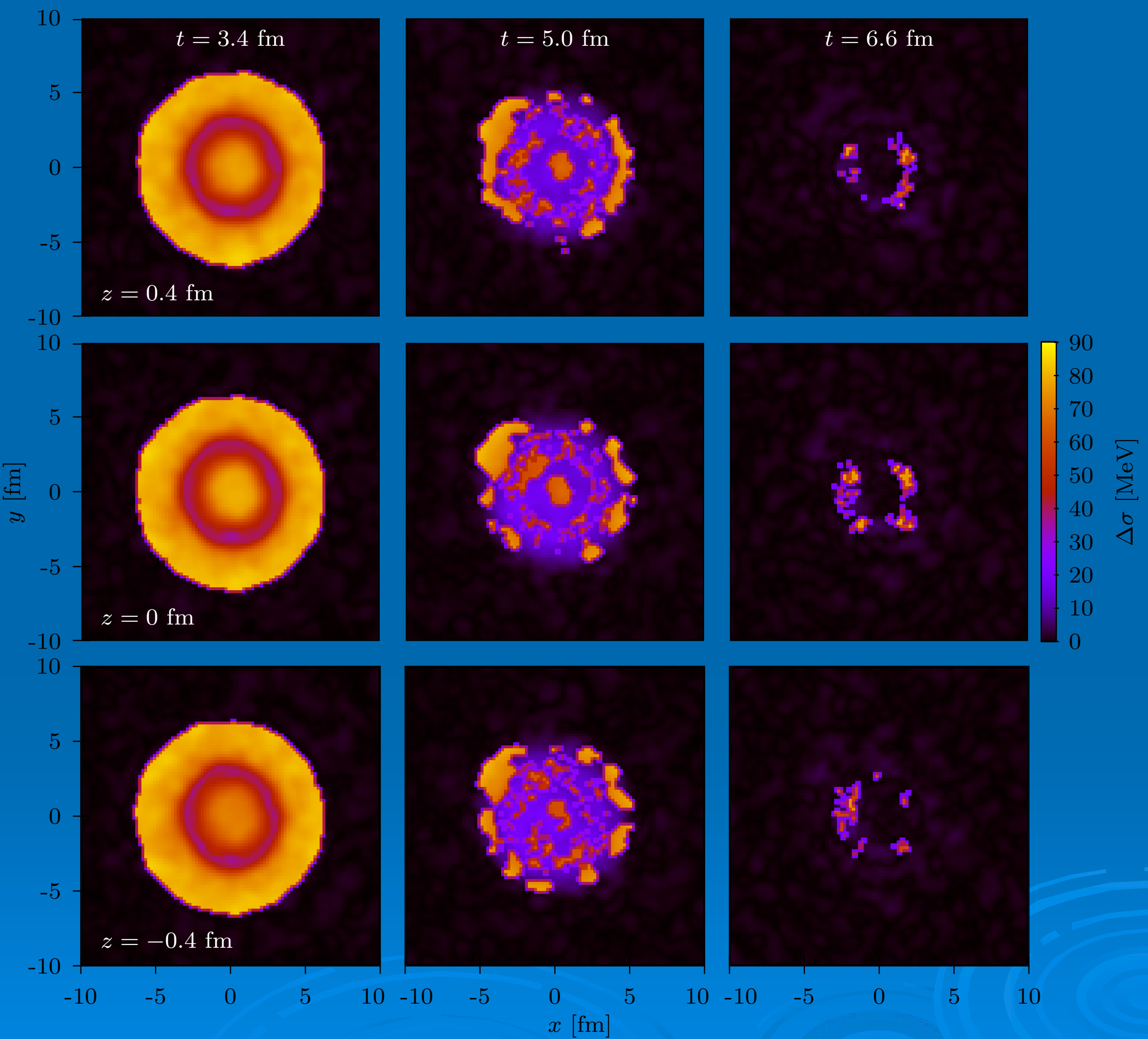


Latest developments

- Realistic initial conditions are obtained from UrQMD simulations;
- Correlations of noise variables in neighboring cells are introduced by averaging over V=1fm^3;
- Dynamical domain formation studied

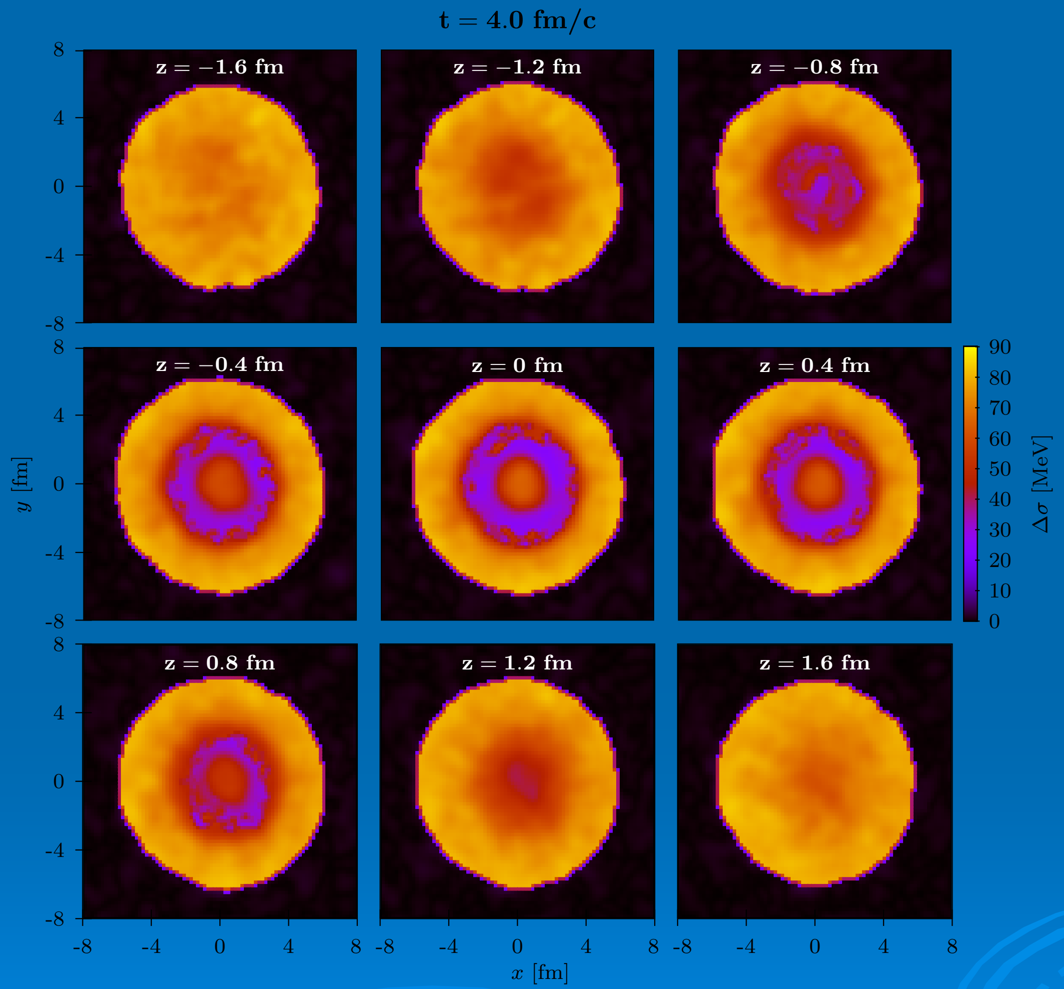


Tomographic visualization of domains



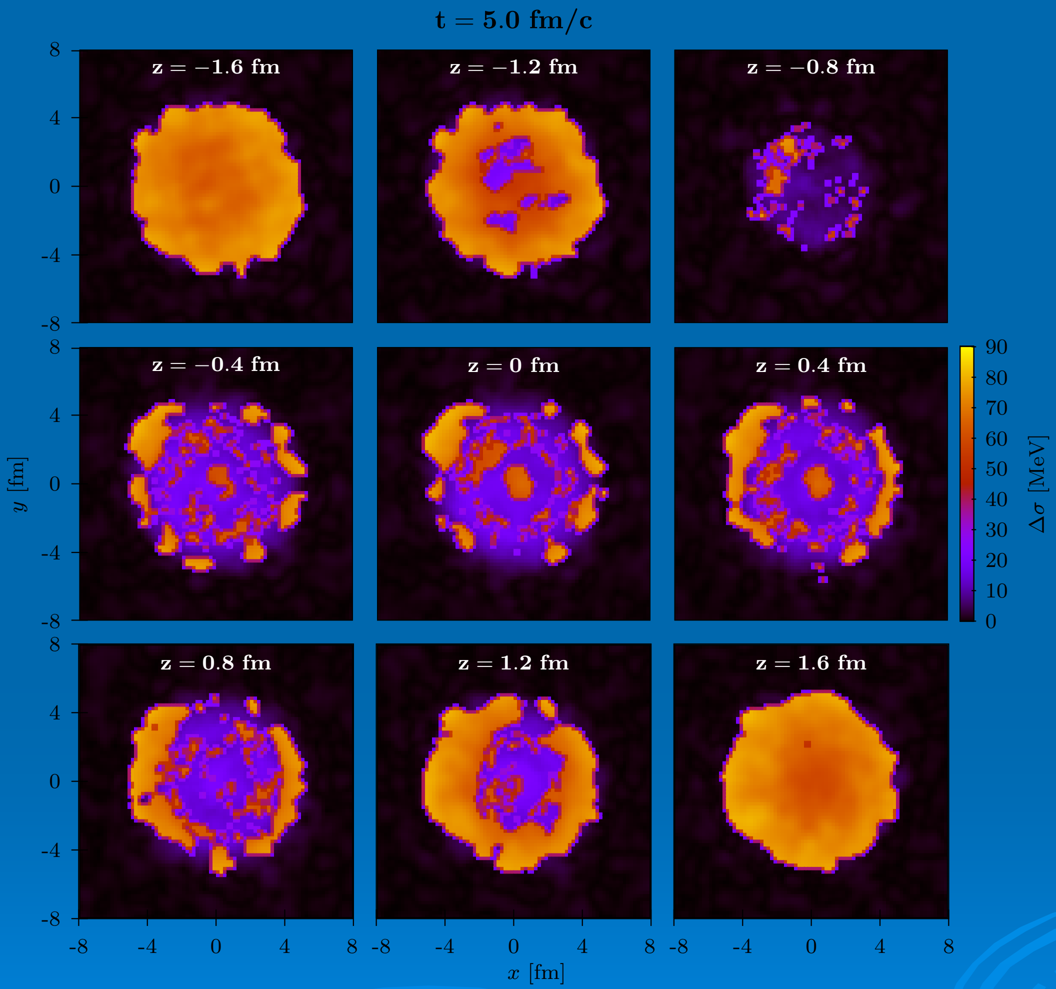
Domains of restored phase are embedded into a chirally-broken background

Tomography of the system at t=4 fm/c



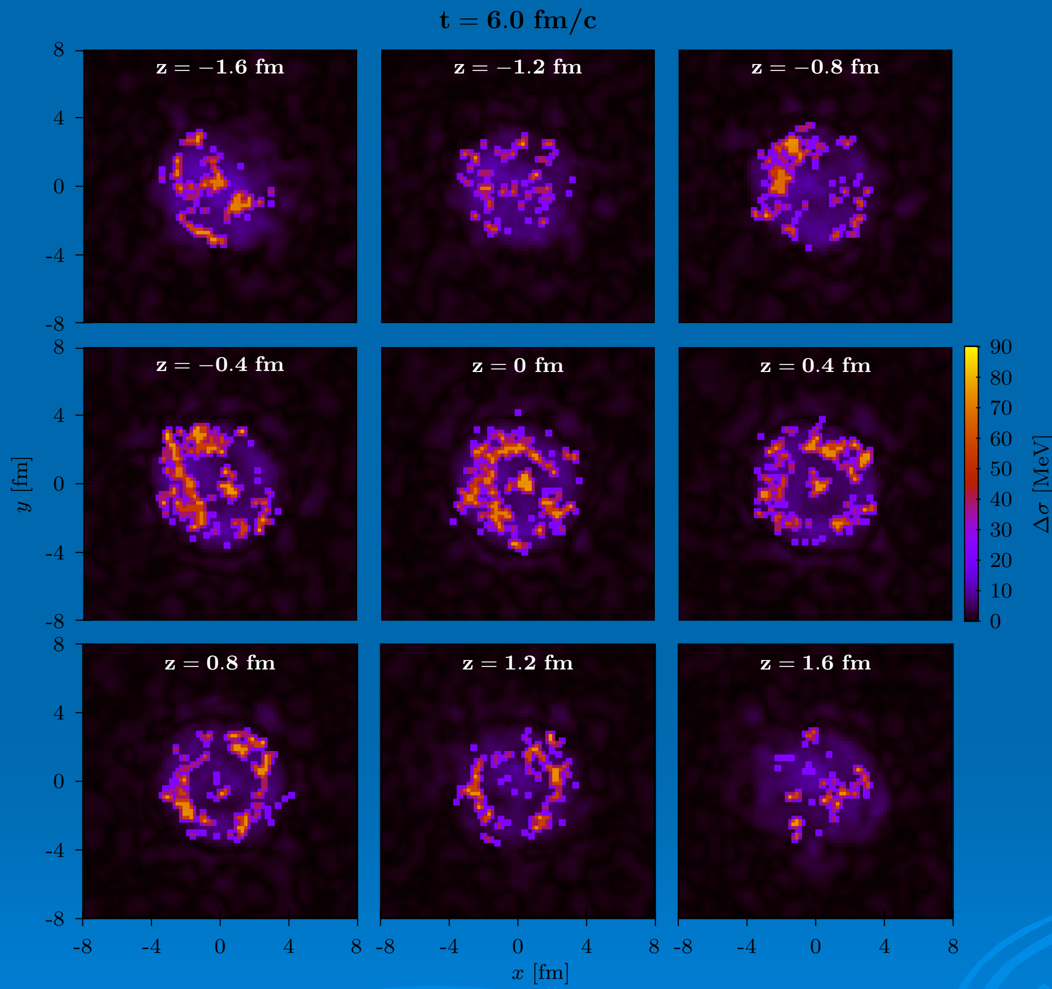
One can clearly see a big domain in the central region (r<0.8 fm)

Tomography of the system at t=5 fm/c



Smaller domains have been formed in the bigger region (r<1.2 fm)

Tomography of the system at t=6 fm/c

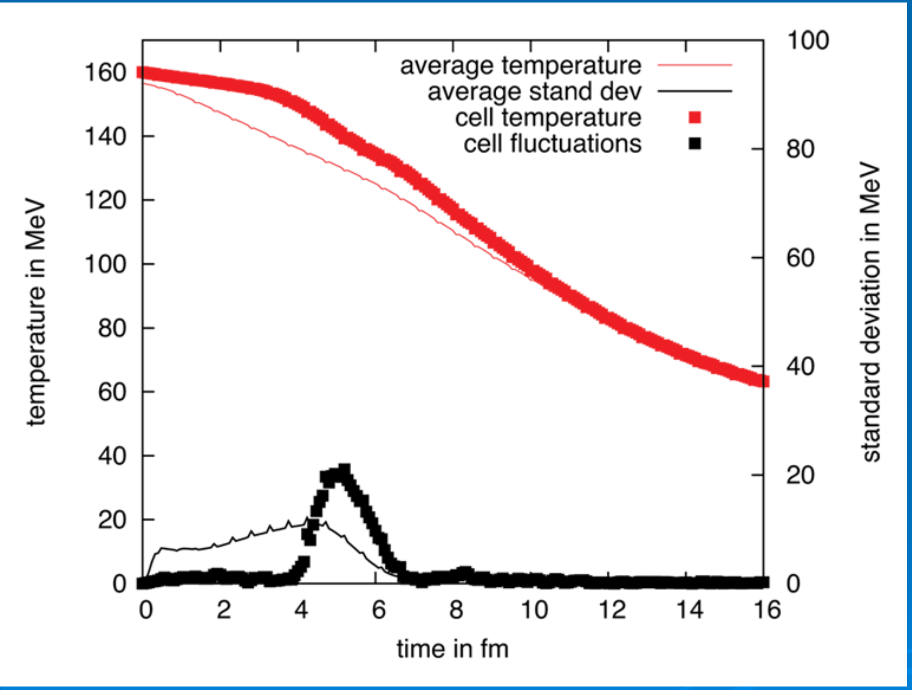


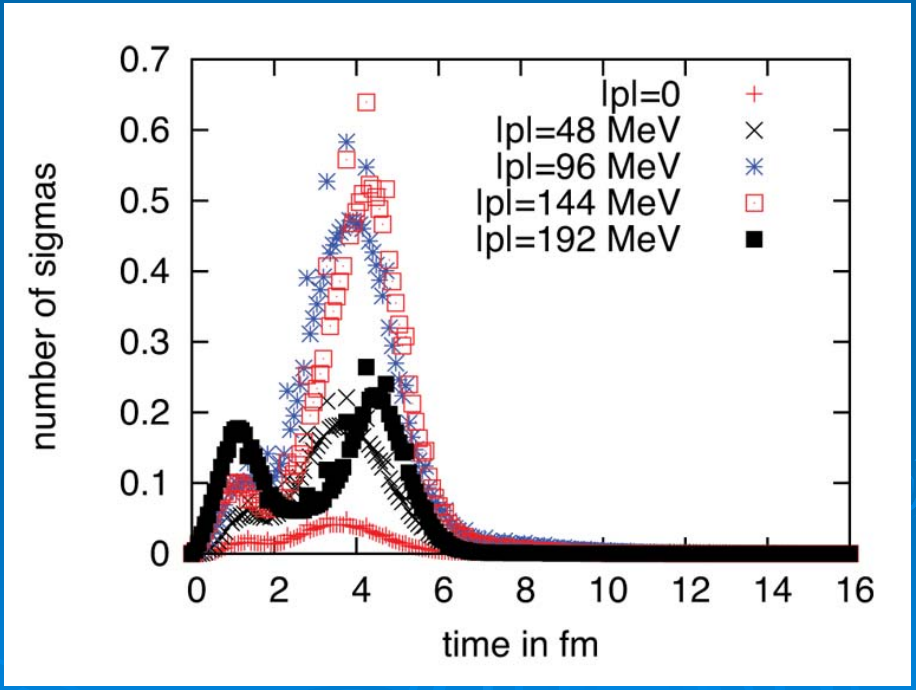
Domains (droplets) are spred over the large region (r<1.6 fm)

Strength of sigma fluctuations transition

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2], \ \omega_k = \sqrt{m_\sigma^2 + k^2}, \ \ m_\sigma^2 = \frac{\partial^2 U_{\text{eff}}}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\text{eq}}}$$

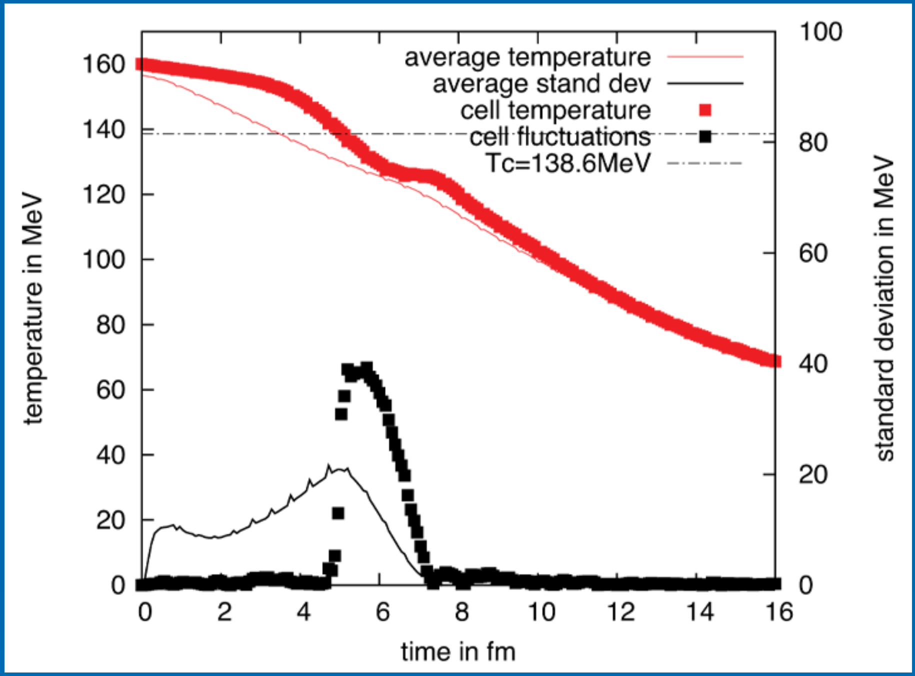
Crossover transition (g-3.3)

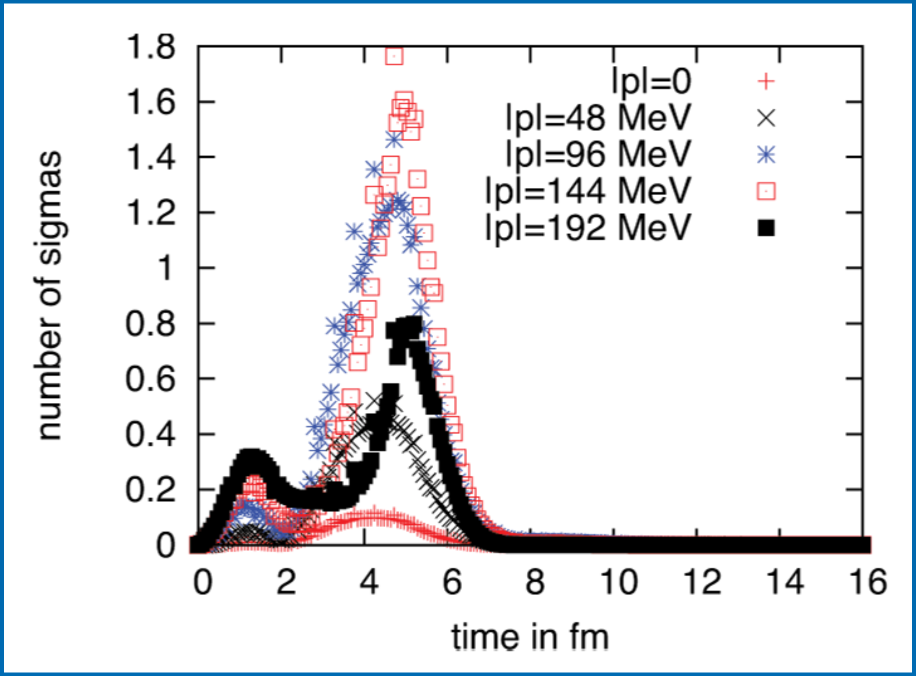




Fluctuations are rather weak, effective number of sigmas is small

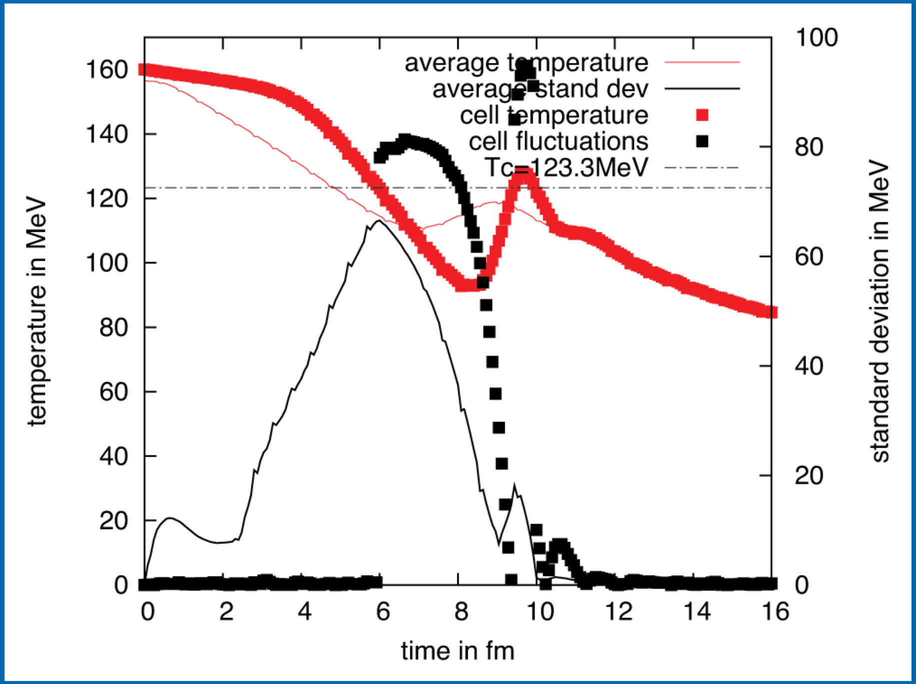
Second order transition with critical point (g=3.63)

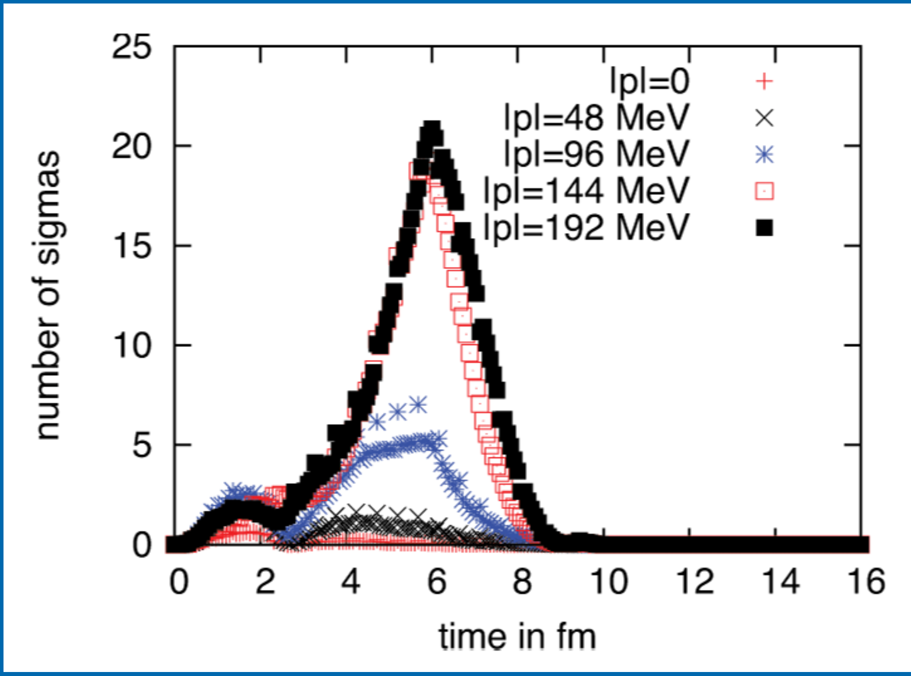




Fluctuations are stronger, but no traces of divergence are seen

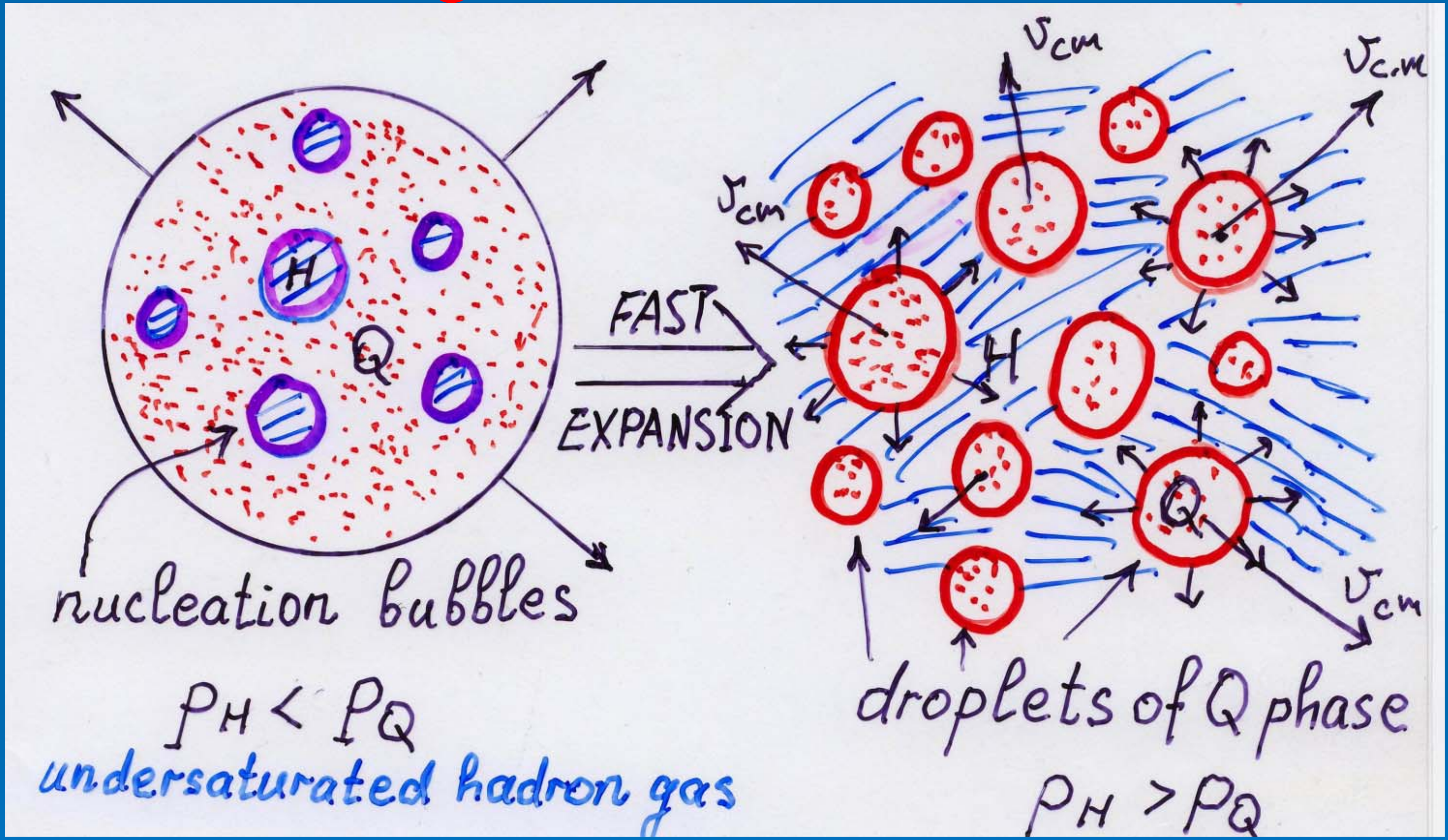
Strong first order transition (g=5.5)





- Strong supercooling and reheating effects are clearly seen.
- Sharp rise of fluctuations after 6 fm/c, when the barrier in thermodynamic potential disappears. Effective number of sigmas increases by two orders of magnitude!

Rapid expansion should lead to dynamical fragmentation of QGP



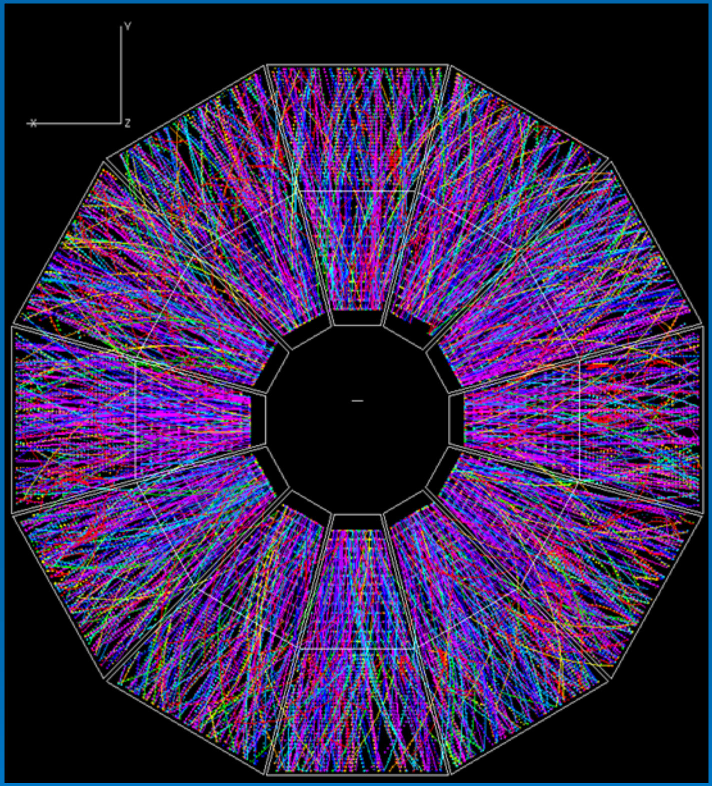
In the course of fast expansion the system enters spinodal instability when Q phase becomes unstable and splits into QGP droplets/hadron resonances

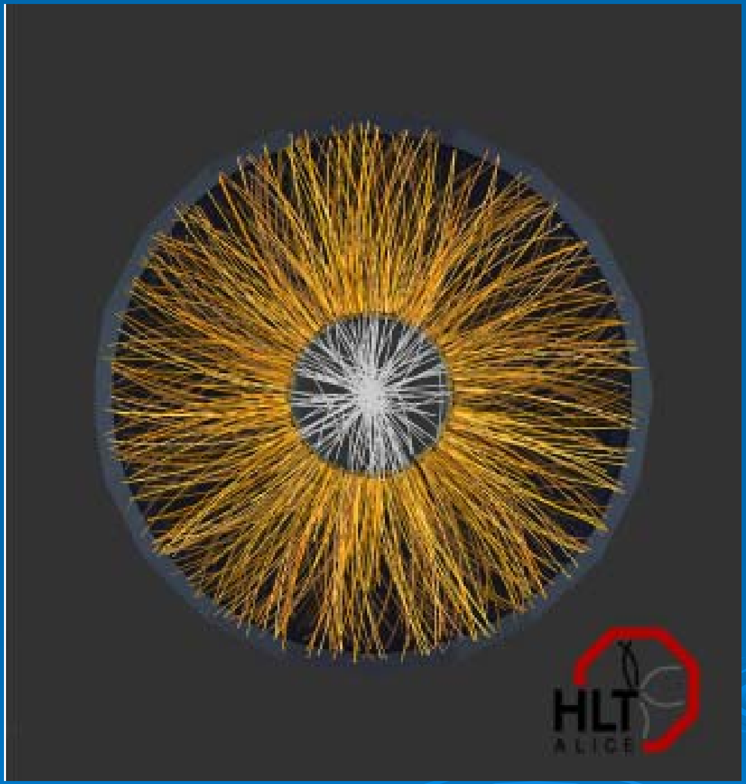
Csernai&Mishustin 1995, Mishustin 1999, Rafelski et al. 2000, Koch&Randrup, 2003

Extreme possibility - direct transition from quarks to hadrons without mixed phase

Ultrarelativistic A+A collisions

RHIC (STAR) Charged particles tracks LHC (ALICE)





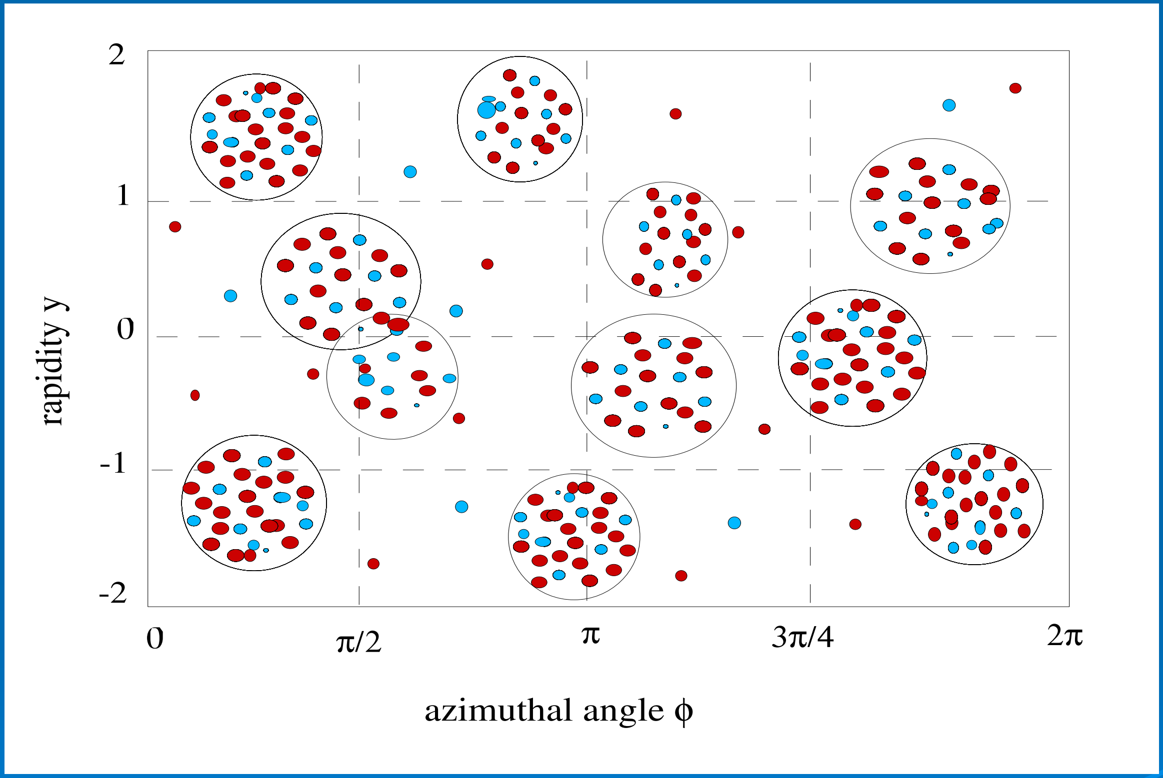
Au+Au, $\sqrt{s} = 130\,\mathrm{AGeV}$

Pb+Pb, $\sqrt{s} = 2.76\,\mathrm{ATeV}$

one of these tracks could be an antinucleus $\bar{d}, \overline{\alpha}, \ldots$

Experimental signal of droplets in the rapidity-azimuthal angle plane

Select hadrons in different p_t interwals



Look for event-by-event distributions of hadron multiplicities in momentum space associated with emission from QGP droplets. Such measurements should be done in the broad energy range!

Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium

- 2^{nd} order phase transitions (with CEP) are too weak to produce significant observable effects

- Non-equilibrium effects in a1^{st} order transition (spinodal decomposition, dynamical domain formation) may help to identify the phase transition

- If large QGP domains are produced in the 1-st order phase transition they will show up in large non-statistical multiplicity fluctuations in single events