Trace Anomaly, Chiral Symmetry Breaking and Baryons at High Density

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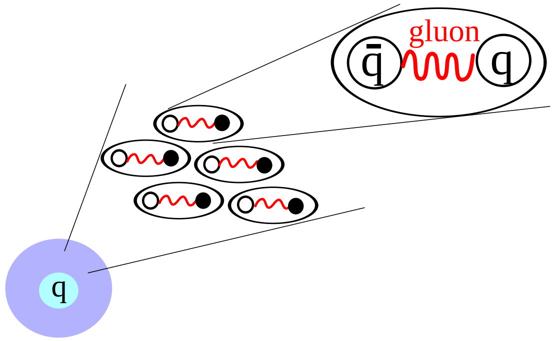
Outline

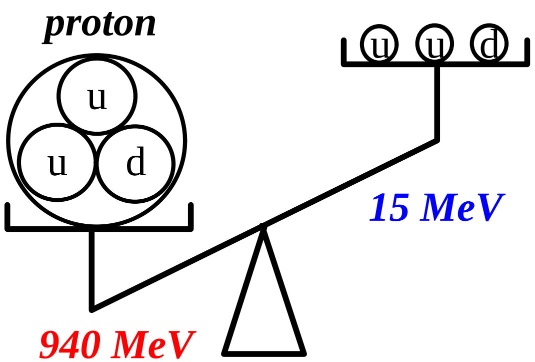
1. The problem of hadron mass
2. How to embed nucleon and scalar fields?
3. Dilaton limit: consequences and implications
4. Polyakov loops and pure gluo-thermodynamics

I. Main Objectives

Origin of hadron masses?

- spontaneous chiral symmetry breaking $\cdots$ dynamics of strong int., $\Lambda_{\rm QCD}$





- scale symmetry breaking $(x^\mu \to e^\tau x^\mu)$ $\cdots$ emergence of a scale in QCD

$$\partial_\mu J^\mu = T^\mu_\mu = -\left(\frac{11}{24}N_c - \frac{1}{12}N_f\right)\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu} + \sum_f m_f \bar{q}_f q_f$$

- chiral SB and trace anomaly closely related $\to$ hadron masses

$$m_H = \mathcal{F}\,(\text{CSB}\,,\text{non-CSB})$$

- baryons near CS restoration? $\cdots$ dynamical origin of nucleon mass?
  - standard assignment: DχSB generates entire masses. $m_N \overset{\sigma\to 0}{\to} 0$
  - mirror assignment: DχSB generates mass difference of parity doublers.

$$m_{N_\pm} = \frac{1}{2}\left[\sqrt{c_1\sigma^2 + 4m_0^2} \mp c_2\sigma\right] \overset{\sigma\to 0}{\to} m_0 \neq 0$$

[Detar-Kunihiro (89)]

Role of scalar bosons in nuclear matter

- how to have empirical saturation in Walecka model? [e.g., Serot-Walecka (97)]

$$\mathcal{L} = \bar{N}\left(i\partial\!\!\!/ - g_V \omega\!\!\!/ - M + g_S \phi\right) N + \mathcal{L}_{\text{kin+mass}}(\omega) + \mathcal{L}_{\text{kin+mass}}(\phi) - \kappa\phi^3 - \lambda\phi^4$$

  – chiral symmetry: the signs and magnitudes of interactions

$$\kappa = \left(m_\sigma^2 - m_\pi^2\right)/2\,, \quad \lambda = \left(m_\sigma^2 - m_\pi^2\right)/8f_\pi$$

  – LσM yields no stable ground state! [Kerman-Miller (74)] thus $\phi \neq \sigma$!

- MF studies of nuclear matter and finite nuclei [Heide-Rudaz-Ellis (92-93), Mishustin-Bondorf-Rho (93), Furnstahl-Tang-Serot (95), Papazoglou et al. (97-99), ...]

NM ground state requires an additional scalar other than genuine chiral partner of pion.

- **low density:**
  Walecka's ϕ: a mixture of quarkonia, tetraquarks and glueballs

- **higher density towards chiral restoration:**
  scalar meson gets lighter $\Rightarrow$ O(4) multiplet with pions $(\vec{\pi}, s)$

How does Walecka's scalar transmute into the 4th component of O(4) vector?

- **gluon condensate vs. light quark mass**
  - low-energy theorem ($q = u, d, s$) [Novikov-Shifman-Vainstein-Zakharov (81)]

$$\frac{d}{dm_q}\frac{\alpha_s}{\pi}\langle G^2\rangle = \frac{-24\langle\bar{q}q\rangle}{\frac{11}{3}N_c - \frac{2}{3}N_f}$$

  - **decomposition ("PCDC" hypothesis)** [Miransky-Gsynin (89), Lee-Rho (09)]

$$\langle G^2\rangle = \underbrace{\langle G^2\rangle_{\rm soft}}_{\chi\text{SB}\,,\, f_\pi^2} + \underbrace{\langle G^2\rangle_{\rm hard}}_{\text{SSB}}\,,$$

$$F(T,\mu;m_q,g) \to \langle\mathcal{O}\rangle(T,\mu;m_q,g)$$

  - from Lattice EoS: gluon *decondensation* at finite T [Miller (07)]

$$\langle G^2\rangle_{T_{\rm chiral}} \simeq \frac{1}{2}\langle G^2\rangle_{T=0} \quad\Rightarrow\quad \text{melting } \langle G^2\rangle_{\rm soft}$$

  - soft and hard dilatons

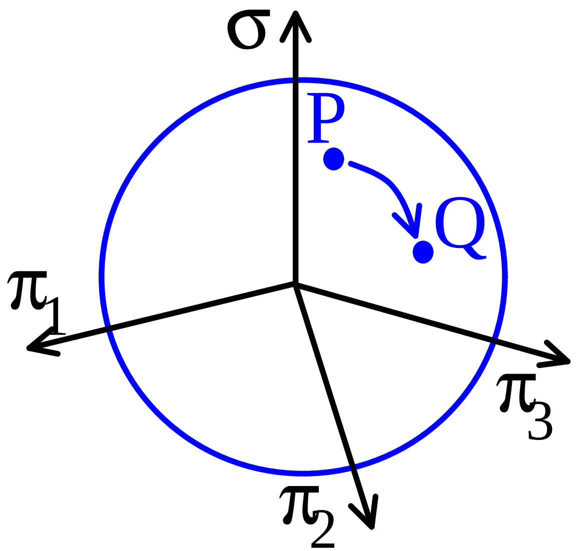
$$\chi = \chi_s + \chi_h\,, \quad V_{s,h} = \frac{1}{4}B\left(\frac{\chi_{s,h}}{F_{\chi_{s,h}}}\right)^4\left[\ln\left(\frac{\chi_{s,h}}{F_{\chi_{s,h}}}\right)^4 - 1\right]$$

  - **role of hard dilaton** $\Rightarrow$ origin of m_0

II. From Low Density to High Density

Role of scalar mesons: nonlinear vs linear

- combine chiral symmetry breaking and trace anomaly in a single theory
- non-linear chiral Lagrangian, chiral perturbation theory:
  a minimal theory for NG bosons, reliable in low density
- **from linear to non-linear basis, or the other way around**



$P \longrightarrow Q$: chiral transformation

$(\pi_1, \pi_2, \pi_3, \sigma) \longrightarrow (\theta_1, \theta_2, \theta_3; f_\pi)$

$$\Phi = \sigma + i\vec{\tau}\cdot\vec{\pi} \qquad f_\pi = \sqrt{\sigma^2 + \vec{\pi}^2}$$
$$= (\sigma_0 + \tilde{\sigma})U\,, \quad U = e^{-i\vec{\tau}\cdot\vec{\pi}/f_\pi}$$
$$\Rightarrow \mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]$$

changeover of effective theories:
from NLSM (low T, ρ) to LSM (around χSR)
what is/are constraint(s) from symmetries?

- **transmutation of a scalar from NLSM to LSM** [Beane-van Kolck (94)]

1. non-linear chiral Lagrangian plus χ_s: $U = \xi^2 = e^{2i\pi/F_\pi}$, $\sqrt{\kappa} = F_\pi / F_{\chi_s}$

$$U \to LUR^\dagger , \quad \xi \to v\xi R^\dagger = L\xi v^\dagger , \quad \psi \to v\psi , \quad \chi_s \to \chi_s$$

2. linearization: $\Sigma = \sqrt{\kappa} U \chi_s = s + i\vec{\tau}\cdot\vec{\pi}$ & $B = \frac{1}{2}[(\xi + \xi^\dagger) - \gamma_5(\xi - \xi^\dagger)]\psi$

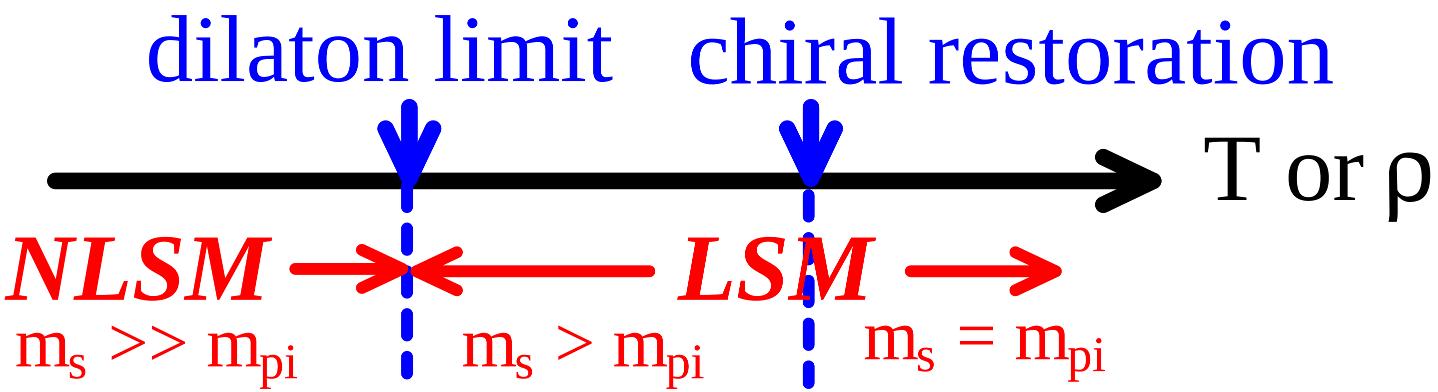
$$\Sigma \to L\Sigma R^\dagger , \quad B_L \to LB_L , \quad B_R \to RB_R$$

3. a LSM $\mathcal{L}(s, \vec{\pi}, B)$ emerges when $\kappa \to 1$ & $g_A \to 1$ (dilaton limit).

$$\mathcal{L}_{\rm sing} = (1 - \kappa)\mathcal{F}(1/\mathrm{tr}[\Sigma\Sigma^\dagger]) + (1 - g_A)\mathcal{G}(1/\mathrm{tr}[\Sigma\Sigma^\dagger]) \to 0$$

– higher dim. ops. suppressed by scale invariance

– emergent LSM renormalizable

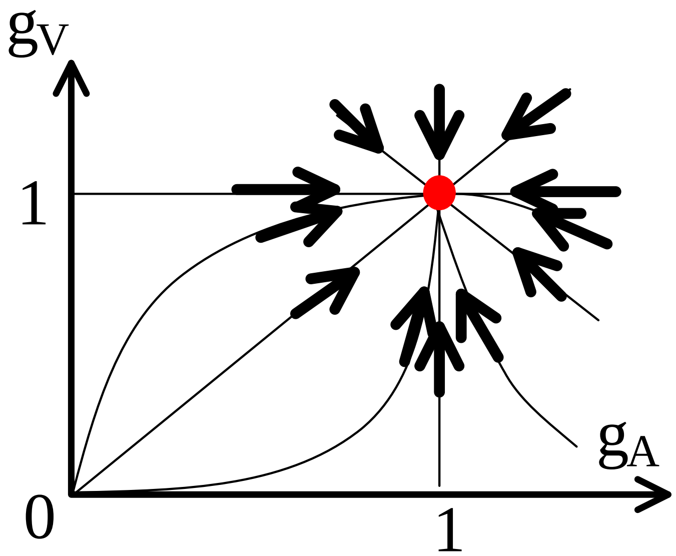


- **introduce vector mesons:** $(N\,,\pi\,,\rho\,,\omega\,,\chi)$ [CS-Lee-Paeng-Rho (2011)]

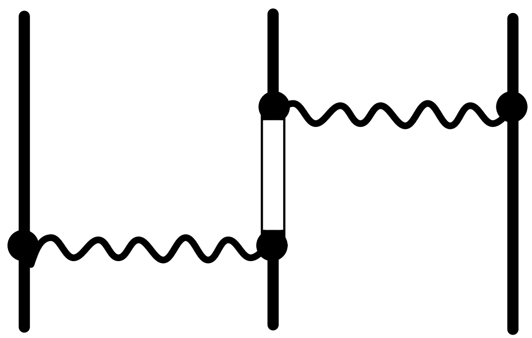
  hidden local symmetry (HLS): $U=\xi^2\rightarrow\xi_L^\dagger\xi_R$ [Bando et al. (85)]

  – dilaton limit: $\kappa=1$ and $g_A=g_V$ (common to standard and mirror)

  – $g_A=g_V=1$ as IR fixed point of RGEs
  $\Rightarrow$ DL unaffected by quantum loops!
  [Paeng-Lee-Rho-CS (2011)]

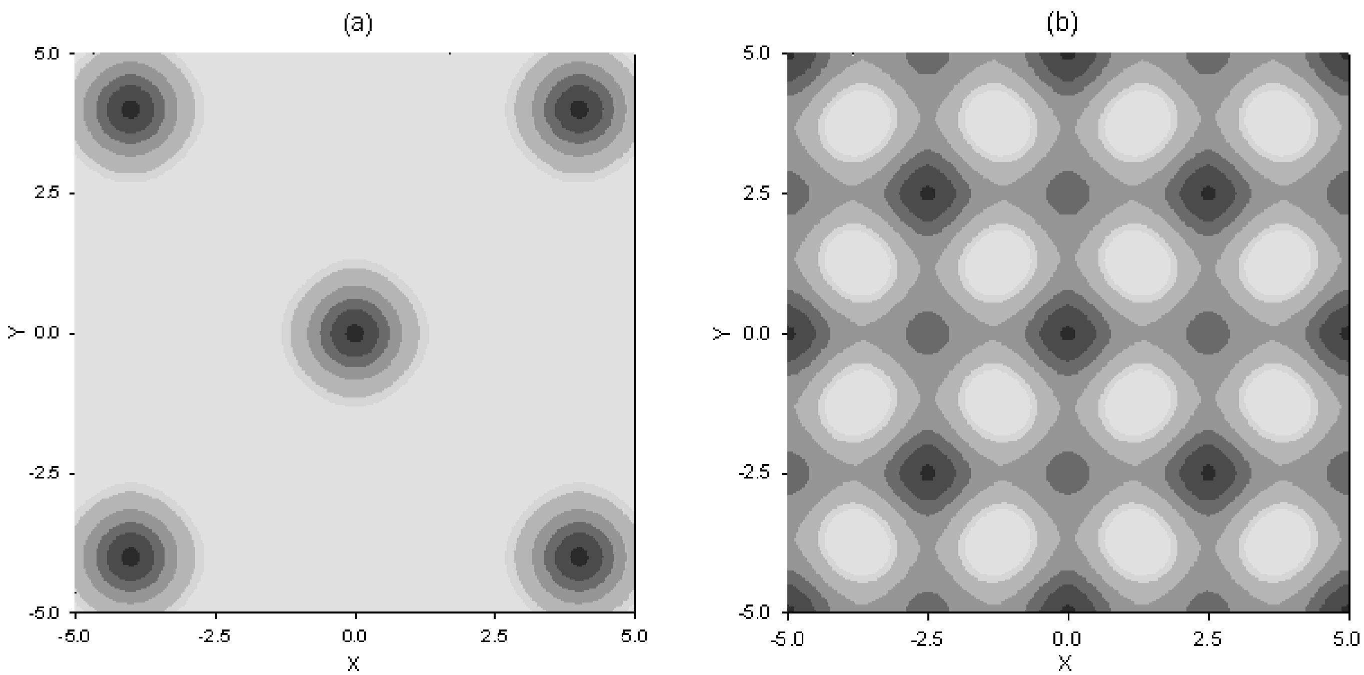


  – consequence: VN repulsion suppressed $g_{VN}=g\,(1-g_V)\rightarrow 0$
  - $*$ **softer EoSs at high density**
  - $*$ **suppression of n-body repulsion via vector-meson exchanges**
  - $*$ **short-range repulsion suppressed?**
    higher states $V', V''\cdots$: KK modes in hQCD



Ground state of a skyrmion matter [Park et al. (2002)]

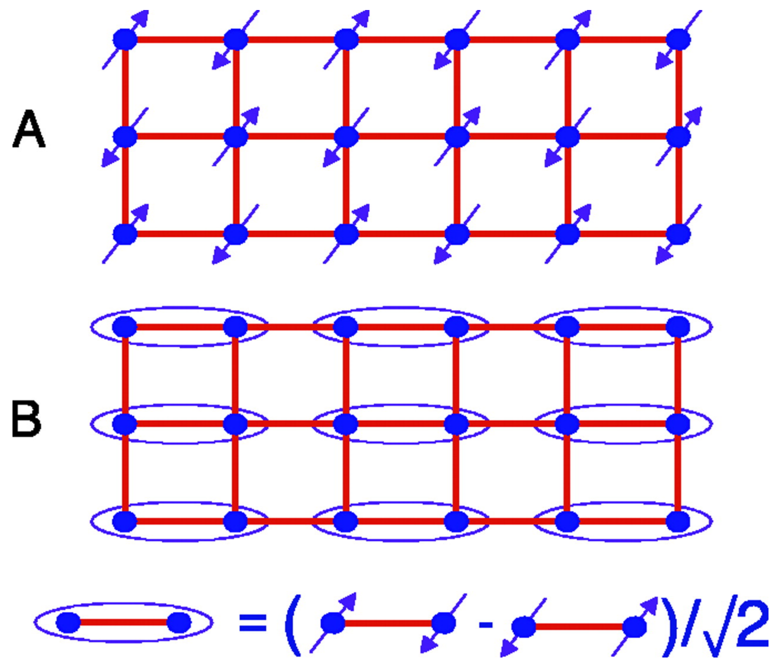
- baryons as solitons generated from pions: skyrmions
- simulate dense matter: put skyrmions on a crystal lattice and squeeze them
  $\Rightarrow$ half-skyrmions appear at $\rho_{1/2} > \rho_0$, each carries a half baryon charge



what is it in continuum?

lessons from condensed matter physics [Senthil et al., Science 303, 1490 (2004)]

- Néel magnet (A: broken spin rotation) and valence bond solid (VBS) paramagnet (B: broken lattice rotation)
- topology captured by CP^1 model ($\hat{n} = z^\dagger \vec{\sigma} z$: skyrmion $\longrightarrow$ half-skyrmions)
- Berry phase (VBS), *emergent* gauge symmetry: U(1)

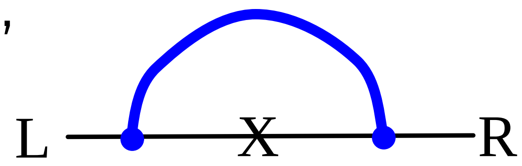


similar gauge structures expected in dense QCD!

- integrating out "fast" modes $\Rightarrow$ induced gauge fields [Shapere-Wilczek (89)]
- HLS: $(F_V^2/F_\pi^2, g_V - g_A) = (1, 0) \Rightarrow L$-$R$ mixing only via gauge boson ex.

  $L \times R$ "restored", $\mathcal{L} \sim (D_\mu \xi_L)^2 + (D_\mu \xi_R)^2$,

  whereas



III. Pure Yang-Mills Thermodynamics and Polyakov Loops

"Confinement" in PNJL/PQM models???

- NJL/QM under a constant background A_0 [Meisinger-Ogilvie (96), Fukushima (03)]

$$\mathcal{L}_{\rm kin} = \bar{q}\,(i\partial\!\!\!/ - A\!\!\!/_0)\,q$$

$$\Rightarrow\ \Omega_q = d_q\,T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\Phi e^{-E/T}+3\Phi e^{-2E/T}+e^{-3E/T}\right]$$

$\langle\Phi\rangle \simeq 0$ at low T: 1- and 2-quark states thermodynamically irrelevant
$\Rightarrow$ mimicking confinement

- **NOTE!**
  - no confinement: only quarks existing at any T
  - no baryons: $3\sqrt{p^2+M_q^2}$ vs. $\sqrt{p^2+(3M_q)^2}$

- color confinement: pure SU(3) YM theory
cf. pure gauge sector of PNJL/PQM $\cdots$ $\Omega_g = T^4\,\mathcal{U}(\Phi;T)$
$\Rightarrow$ no dynamical fields! where are gluons?

How does thermodynamics potential look like?
Gluon thermodynamics in low-T phase?

$$Z = \int \mathcal{D}A_\mu \mathcal{D}C\mathcal{D}\bar{C}\, \exp\left[i\int d^4x\mathcal{L}\right], \quad \mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

1. employ background field method. [Gross-Pisarski-Yaffe (81)]

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$

2. collect terms quadratic in quantum fields.

$$\mathcal{L}^{(2)} = -\frac{1}{2}\check{A}^a_\alpha\left[\delta_{ab}g^{\alpha\beta}\partial^2 - f_{abc}\left(\partial^\beta\bar{A}^{\alpha,c} + 2g^{\alpha\beta}\bar{A}^c_\mu\partial^\mu\right)\right.$$
$$\left. + f_{ac\bar{c}}f_{cb\bar{d}}g^{\alpha\beta}\bar{A}^{\bar{c}}_\mu\bar{A}^{\mu,\bar{d}} + 2f_{abc}\bar{A}^{\alpha\beta,c}\right]\check{A}_\beta{}^b$$

3. consider a constant uniform background $\bar{A}_0$.

$$\bar{A}^a_\mu = \bar{A}^a_0\delta_{\mu 0}, \quad \bar{A}_0 = \bar{A}^3_0T^3 + \bar{A}^8_0T^8$$

4. calculate propagator inverse and diagonalize it.

5. from Minkowski to Euclidean space: carry out Matsubara summation.

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

- Polyakov loop matrix in adjoint representation (8x8 matrix)

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

rank of SU(3) group $= 2 \;\Rightarrow$ elements expressed in 2 variables

- thermodynamic potential (gluon part) [CS-Redlich (2012)]

$$\Omega_g = 2T\int\frac{d^3p}{(2\pi)^3}\text{tr}\ln\left(1-\hat{L}_A\,e^{-|\vec{p}|/T}\right)$$

traced Polyakov loops $\Phi = \text{tr}\hat{L}_F/N_c\,,\ \bar{\Phi} = \text{tr}\hat{L}_F^\dagger/N_c$ (gauge invariant)

full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\text{Haar}}$

$$\begin{aligned}
\Omega_g &= 2T\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\sum_{n=1}^{7}C_n\,e^{-n|\vec{p}|/T}+e^{-8|\vec{p}|/T}\right)\,,\\
\Omega_{\text{Haar}} &= -a_0T\ln\left[1-6\bar{\Phi}\Phi+4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\bar{\Phi}\Phi\right)^2\right]\,,\\
C_1 &= C_7 = 1-9\bar{\Phi}\Phi\,,\quad C_2 = C_6 = 1-27\bar{\Phi}\Phi+27\left(\bar{\Phi}^3+\Phi^3\right)\,,\\
C_3 &= C_5 = -2+27\bar{\Phi}\Phi-81\left(\bar{\Phi}\Phi\right)^2\,,\\
C_4 &= 2\left[-1+9\bar{\Phi}\Phi-27\left(\bar{\Phi}^3+\Phi^3\right)+81\left(\bar{\Phi}\Phi\right)^2\right]
\end{aligned}$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

High and low temperature phases

- high temperature limit: $\Phi \to 1 \Rightarrow$ non-int. gluon gas

$$\Omega_g(\Phi = \bar{\Phi} = 1) = 16T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-|\vec{p}|/T}\right)$$

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\text{Haar}} = 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right) > 0$$

wrong sign! $\Rightarrow$ **Gluons are NOT correct variables below** T_c**!**

cf. PNJL/PQM: approx. $\Omega_g \sim a(T)\bar{\Phi}\Phi \cdots$ **unjustifiable near** T_c

- **unchanged by quarks** $(T < T_c, \Phi \sim 0)$

$$\Omega_{g+q} \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-E_g/T}\right) - 4N_f T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-3E_q/T}\right)$$

$$\sim \frac{T^2}{\pi^2}\left[M_g^2 K_2\left(\frac{M_g}{T}\right) - \frac{2N_f}{3} K_2\left(\frac{3M_q}{T}\right)\right] > 0$$

with effective masses: $M_g \equiv M_{\text{gluball}}/2\,, \quad M_q \equiv M_{\text{nucleon}}/3$

- applications $\Rightarrow$ talk by Pok Man Lo

Summary

- **an effective chiral Lagrangian with scale invariance**
  dilaton limit = IR fixed point $\Rightarrow$ *intrinsic* medium effects in couplings
- **role of Polyakov loops in quasi-particle approaches**
  Polyakov loops = group character $\Rightarrow$ gluons forbidden below T_c (MF!)

- **at which T or ρ does dilaton limit set in?**
  constraint from Danielewicz et al. (02), $1.97M_\odot$ neutron star
- **mixed scalar modes: quarkonium, tetraquarks, glueballs**
- **reliable estimate of m_0 in dense matter, thermodynamics**
  in-medium tensor forces, symmetry energy

- **analysis of RG flows, a-theorem**
- **higher KK modes**
- **half-skyrmion phase, its EFT in continuum and emergent gauge symmetry**