Transport Theory at fixed $\eta/s(T)$: Collective flows & QGP composition

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This is my first baby
25th may 2012!

Outline

- Transport Theory at fixed η/s :
  - Motivations
  - Green-Kubo in a box vs Chapmann-Enskog

$$\eta \leftrightarrow \sigma(\theta), \rho, \mathcal{M}, \mathcal{T}, \ldots$$

- First application to HIC:
  - RHIC = LHC ?
  - $v_{2,4}$ sensitivity to $\eta/s(T)$
  - $v_{2,4}$ <-> η/s or microscopic details?

- Transport Theory with Mean Field:
  - Dynamics associated to Quasi-Particle Model:

    Chemical equilibration and quark dominance

Transport approach

$$\left\{p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^*\right]\partial^{p^*}_\mu\right\} f(x,p^*) = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3} + \ldots$$

Free streaming Field Interaction -> $\varepsilon \neq 3P$ Collisions -> $\eta \neq 0$

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

$$\sigma_{22} = \frac{1}{2s}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2)$$

C_{23} better not to show…

- Provide a measure of η/s other than viscous hydro
- Microscopic scale has some relevance?
  Can we know more about QGP?
- Can we link the effective $\mathcal{L}$ <-> transport dynamics in HIC
  Thermodynamics <-> phenomenology of HIC

$$\mathrm{L} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m\right)\Psi + \frac{G}{2}\left(\bar{\Psi}\Psi\right)^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + U(\Phi) + \ldots$$

Wigner Tranforms +
semiclassical approx.

Motivation for Transport approach

$$\left\{ p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^* \right] \partial^{p^*}_\mu \right\} f(x,p^*) = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3} + \dots$$

Free streaming

Field Interaction -> $\varepsilon \neq 3P$

Collisions -> $\eta \neq 0$

- valid also at intermediate p_T out of equilibrium: region of modified hadronization at RHIC
- valid also at high η/s –> LHC- $\eta/s(T)$, cross-over region
- Relevant at LHC due to large amount of minijet production
- Appropriate for heavy quark dynamics
- CGC p_T non-equilibrium phase (beyond the difference in ε_x):

A unified framework against a separate modelling with a wider range of validity in η, ζ, p_T + microscopic level (-> hadronization)

Simulate a constant shear viscosity

Relax. Time approx.

$$\frac{\eta}{s} = \frac{1}{15}\frac{\bar{p}}{\sigma_{tr} n} = \frac{4}{5}\frac{T}{\sigma_{tr} s}\,\text{cost.}$$

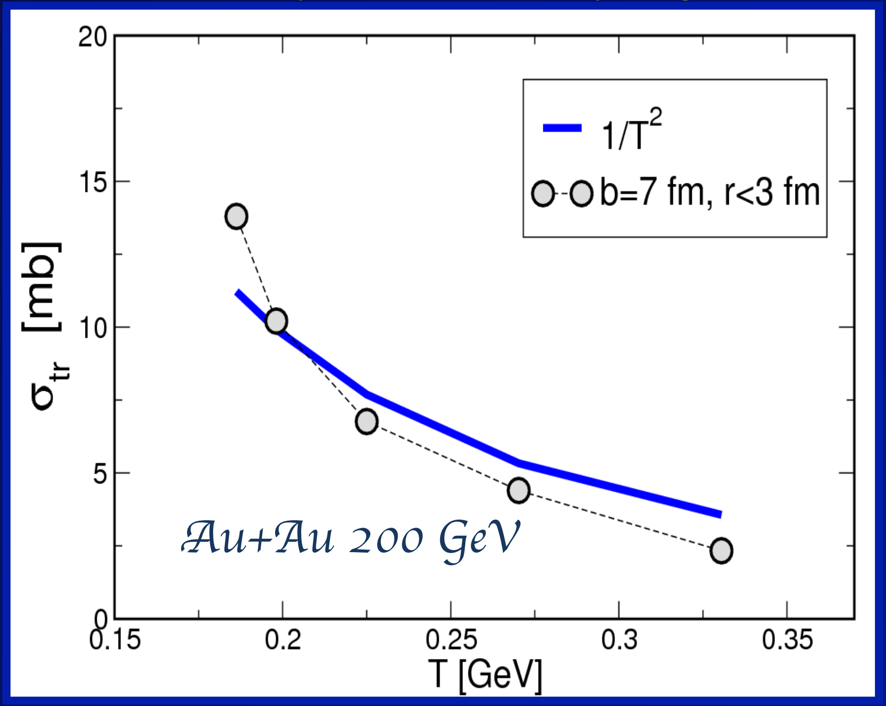
Space-Time dependent cross section evaluated locally

Cascade code

$$\sigma_{tr}(\rho(\vec{r}),T) = \sigma_{tr,\alpha} = \frac{1}{15}\frac{\bar{p}_\alpha}{n_\alpha}\frac{1}{\eta / s} \quad (*)$$

α=cell index in the r-space

Viscosity fixed varying σ



G. Ferini et al., PLB670 (09) V. Greco at al., PPNP 62 (09)

Do we really have the wanted shear viscosity with the relax. time approx.?

- Check the viscosity with the Green-Kubo correlator

Shear Viscosity in Box Calculation

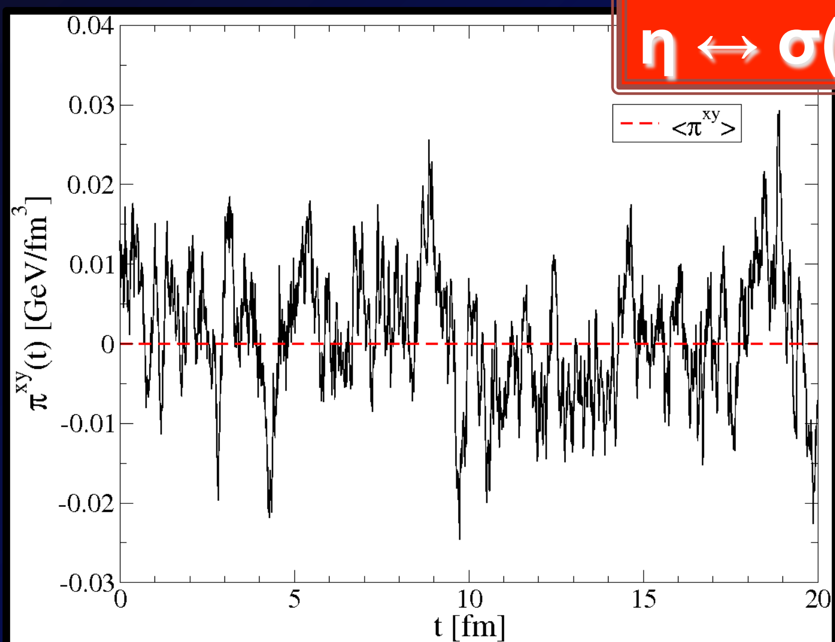
Green-Kubo correlator

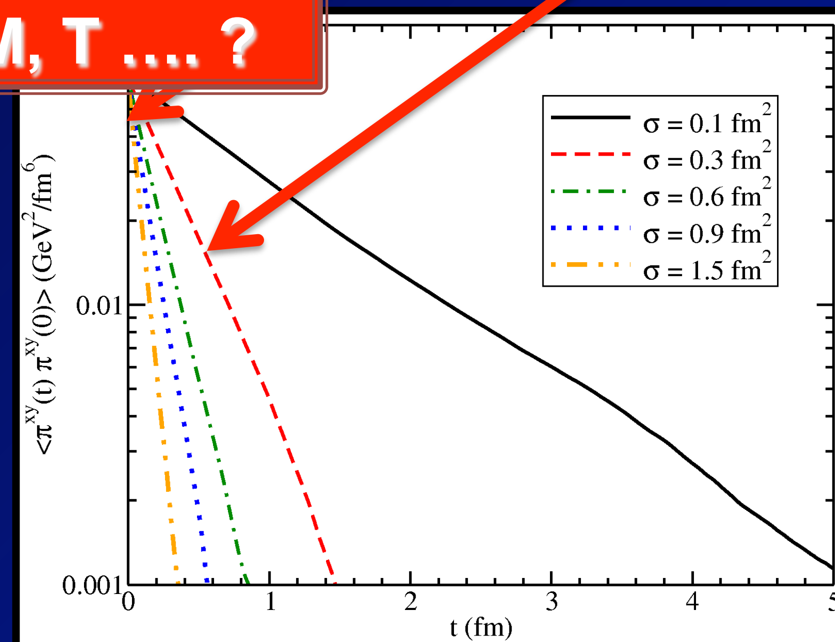
$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \langle \pi^{xy}(x,t)\,\pi^{xy}(0,t)\rangle$$

$$\langle \pi^{xy}(\vec{x},t)\,\pi^{xy}(\vec{0},t)\rangle = \langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle \cdot e^{-t/\tau}$$

$$\Rightarrow \quad \eta = \frac{V}{T}\langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle \cdot \tau$$

η ↔ σ(θ), ρ, M, T …. ?





C. Wesp et al., Phys. Rev. C 84, 054911 (2011).
J. Fuini III et al. J. Phys. G38, 015004 (2011).

Needed very careful tests of convergency vs. N_{test}, Δx_{cell}, # time steps !

Relaxation time approximation?

Kapusta, PRC(2010); Gavin NPA(1985); Redlich and Sasaki, PRC79, NPA832(2010)…

$$\eta \;=\; \frac{1}{15T}\int_0^{\infty}\frac{d^3p_a}{(2\pi)^3}\,\frac{|p_a|^4}{E_a^2}\,\frac{1}{w_a(E_a)}\,f_a^{eq}$$

$$w_a(I\tau_a^{-1} = w_a(E_a) = \rho\,\sigma_{tot}\,\langle v_{rel}\rangle \;\; \sigma_{tot}$$

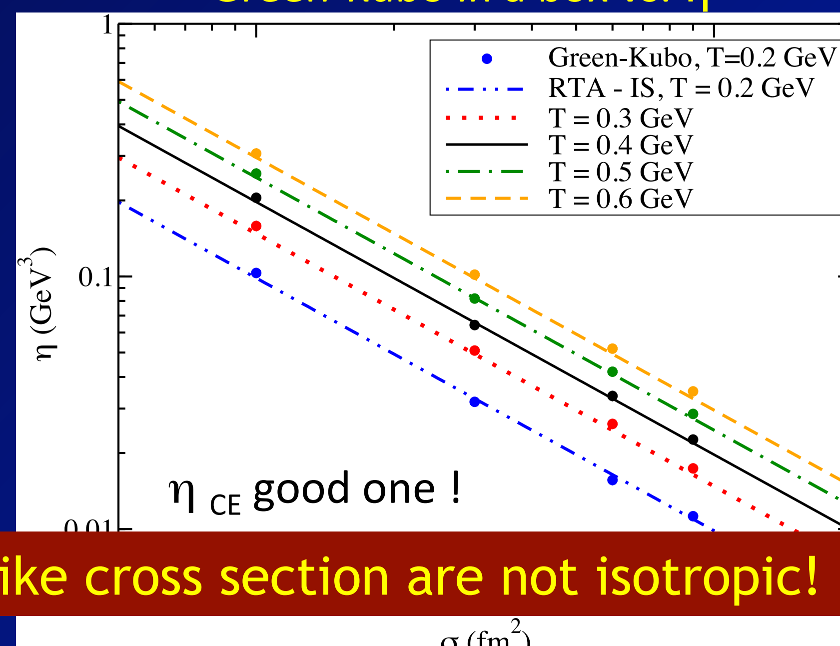
$$\eta_{relax} = 0.8\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Usual as relax. time approx. – Israel Stewart $\quad\sigma_{tr}=2/3\,\sigma_{tot}$ for isotropic cross section

$$\eta_{relax}^{IS} = 0.8\frac{T}{\sigma_{TR}\langle v_{rel}\rangle} = 1.2\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Molnar-Huovenin PRC(2009), G. Ferini PLB(2009), Khvorostukhin PRC (2010) …

Green-Kubo in a box vs. η^{IS}



Isotropic Cross section – massless particles

1st order Chapman-Enskog

$$\eta_{CE}^{I} = 1.2\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

16th order Chapman-Enskog

$$\eta_{CE}^{XVI} = 1.267\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Prakash et al., arxiv:1203.0281 [nucl-**th**]

pQCD-like cross section are not isotropic!

Chapmann-Enskog vs Green-Kubo- $\sigma(\theta)$

$$\eta^{IS}_{RTA} = 0.8\frac{T}{\sigma_{tr}\langle \mathrm{v}_{rel}\rangle} = 0.8\frac{T}{\sigma_{TOT}\langle f(a)\,\mathrm{v}_{rel}\rangle}$$

Employed also for non-isotropic cross section:
G.Ferini, PLB(2009); D. Molnar, JPG35(2008); V.Greco, PPNP(2009); Khvorostukhin PRC (2010)…

$$\sigma_{tr} = \int d\Omega_{cm}\,\sin^2\theta_{cm}\,\frac{d\sigma}{d\Omega_{cm}} = \sigma_{TOT}\,f(a) \le \frac{2}{3}\sigma_{TOT}$$

$$\frac{d\sigma}{d\Omega} = \frac{9\pi\alpha_s^2}{2}\frac{1}{\left(q^2(\theta)+m_D^2\right)^2}\left(1+\frac{m_D^2}{s}\right)$$

For standard pQCD-like cross section

$$f(a) = 4a(1+a)\left[(2a+1)\ln(1+a^{-1})-2\right] \quad , a = m_D^2/s$$

m_D regulates the angular dependence
$m_D \to \infty$ isotropic cross section

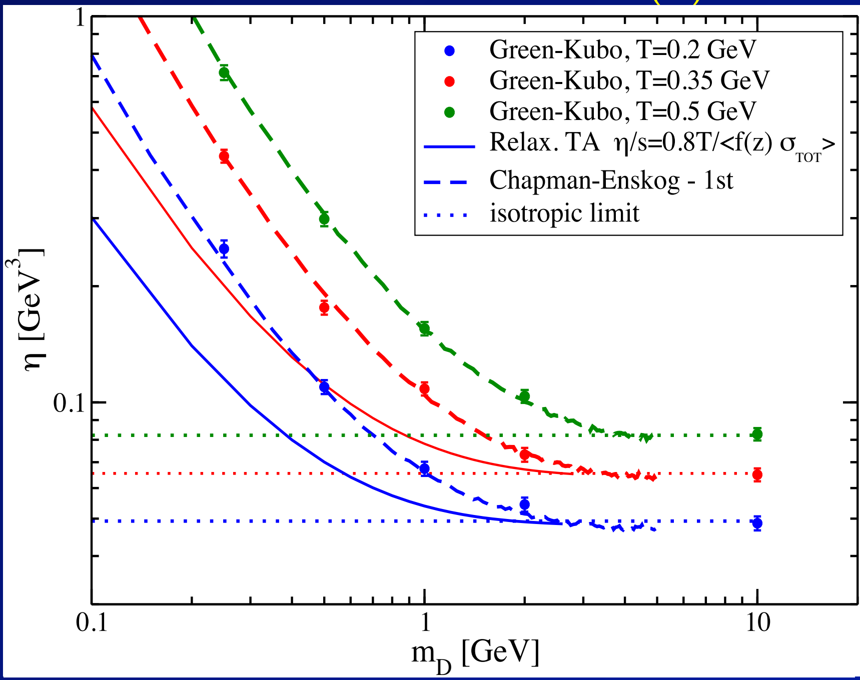
1^{st} Chapmann-Enskog - anisotropic σ

$$[\eta]^{1st}_{CE} = 10T\left[\frac{K_3(z)}{K_2(z)}\right]^2\frac{1}{c_{00}} = g(m_D,T)\,\frac{T}{\sigma_{TOT}}$$

$$c_{00} = 16\left[\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + (3z^2)^{-1}\omega_0^{(2)}\right]$$

$$\omega_i^{(2)} = \frac{z^3}{[K_2(z)]^2}\int_1^\infty dy\,(y^2-1)^3 y^i K_2(2zy)\underbrace{\int_0^\pi d\Omega\,\frac{d\sigma}{d\Omega}\,sen^2\theta}_{\sim\,\sigma_{tr}}$$

Green-Kubo in a box - $\sigma(\theta)$



- CE and RTA can differ by about a factor of 2
- Green-Kubo agrees with CE

Chapmann-Enskog vs Green Kubo:massive case

Massive case is relevant in quasiparticle models where $M_{q,g}(T)=g(T)T$
Hence we need it to extend the approach to Boltzmann-Vlasov transport

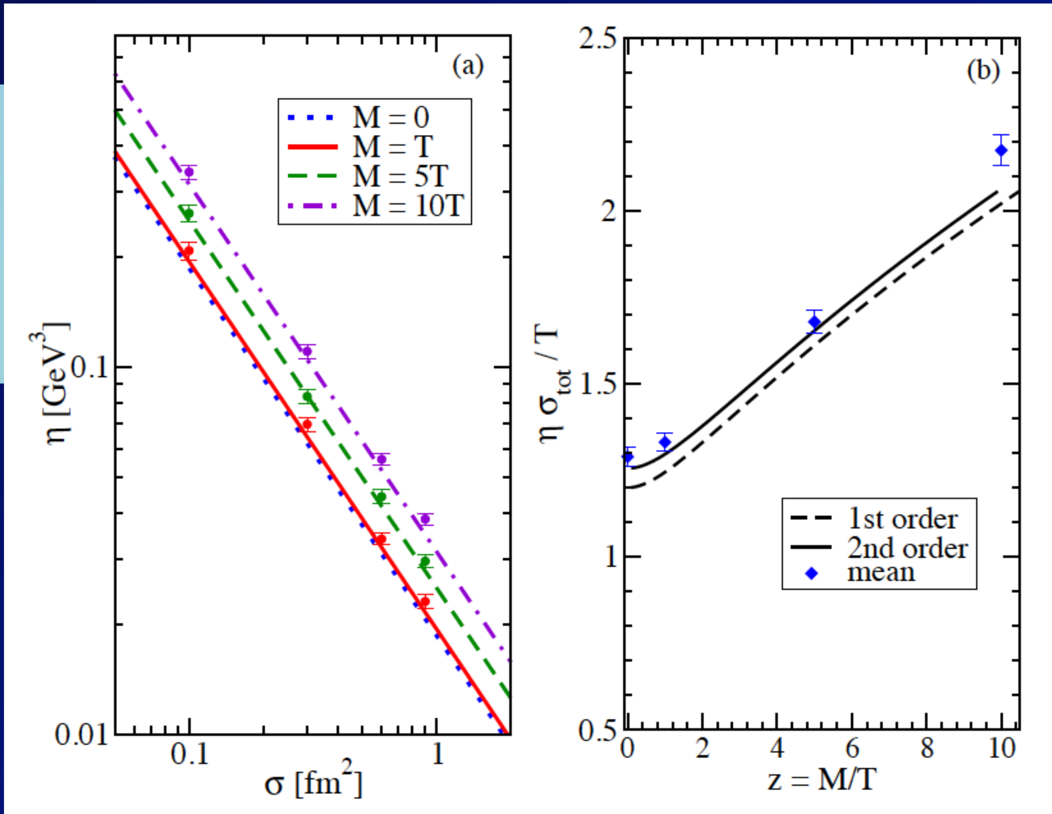
Again good agreement with CE 1st order for $\sigma(\theta)$=cost.

Isostropic σ – massive particles

$$[\eta]^{CE}_{1st} = f(z)\frac{T}{\sigma_{tot}} \qquad z=M/T$$

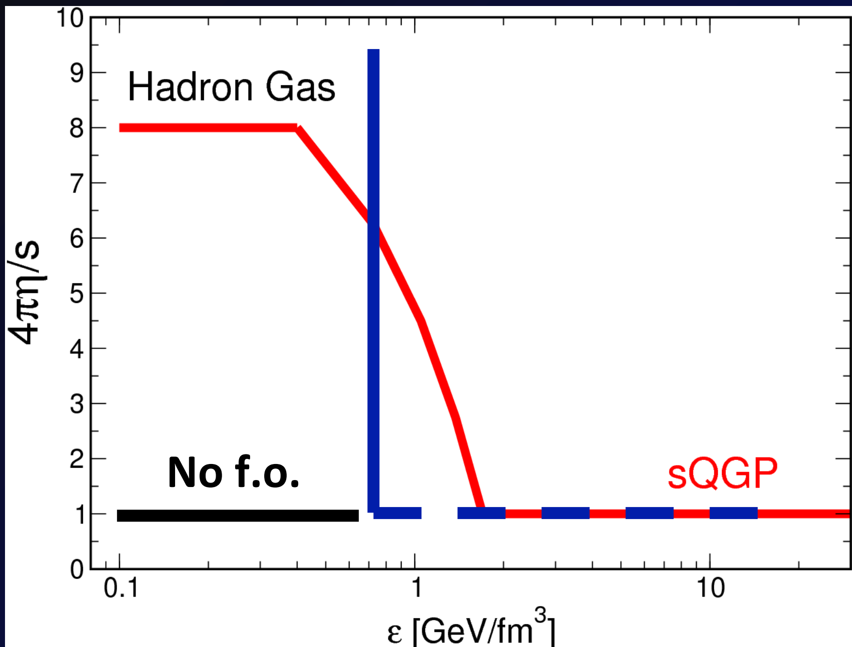
$$f(z) = \frac{15}{16}\frac{z^4 K_3^2(z)}{(15z^2+2)K_2(2z)+(3z^3+49z)K_3(2z)}$$

Still missing Chapmann-Enskog for massive & anisotropic cross section



- ✧ We know how to fix locally $\eta/s(T)$
- ✧ We have checked the Chapmann-Enskog approx. ≈ 3%
- ✧ We have check also the transport code

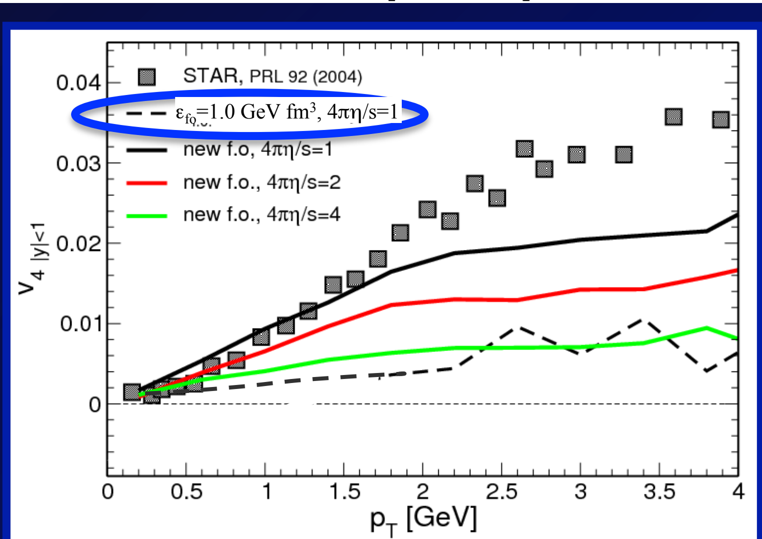
Terminology on freeze-out



a) collisions switched off for $\varepsilon < \varepsilon_c = 0.7$ GeV/fm^3

b) η/s increases in the cross-over region, faking the smooth transition in the cross-over region: small s -> natural f.o.

$$\sigma_{tr} = \frac{1}{15}\frac{\bar{p}}{n}\frac{1}{\eta/s}$$



A sudden f.o. tend to kill v_4!

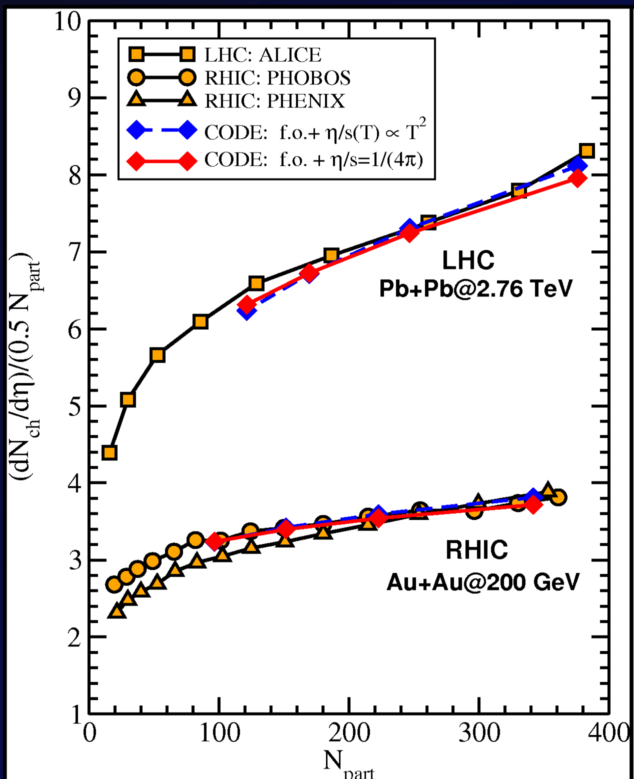
V. Greco et al., PPNP(2009)

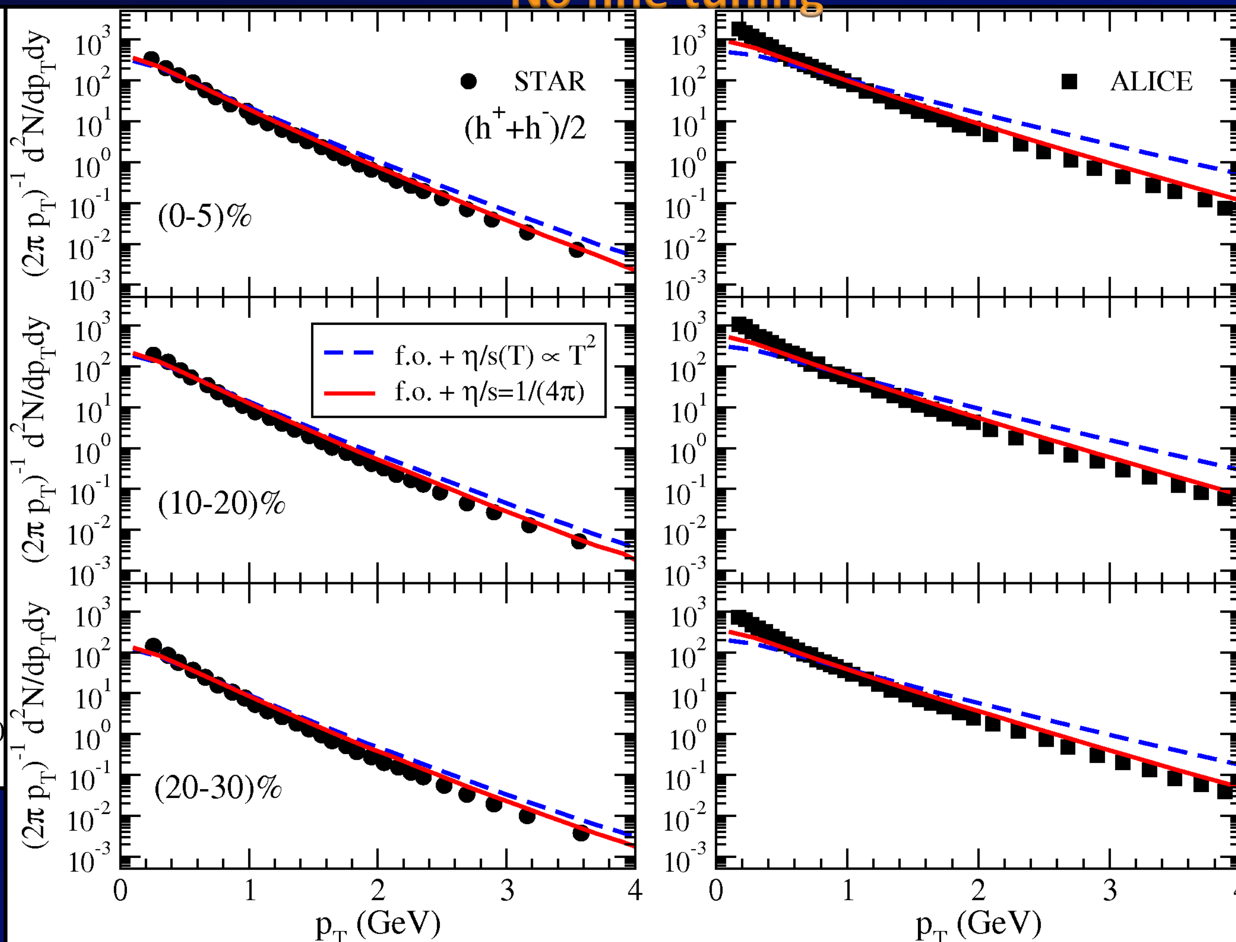
Multiplicity & Spectra

✧ r-space: standard Glauber condition

✧ p-space: Boltzmann-Juttner T_{max}=2(3) T_c [p_T<2 GeV]+ minijet [p_T>2GeV]

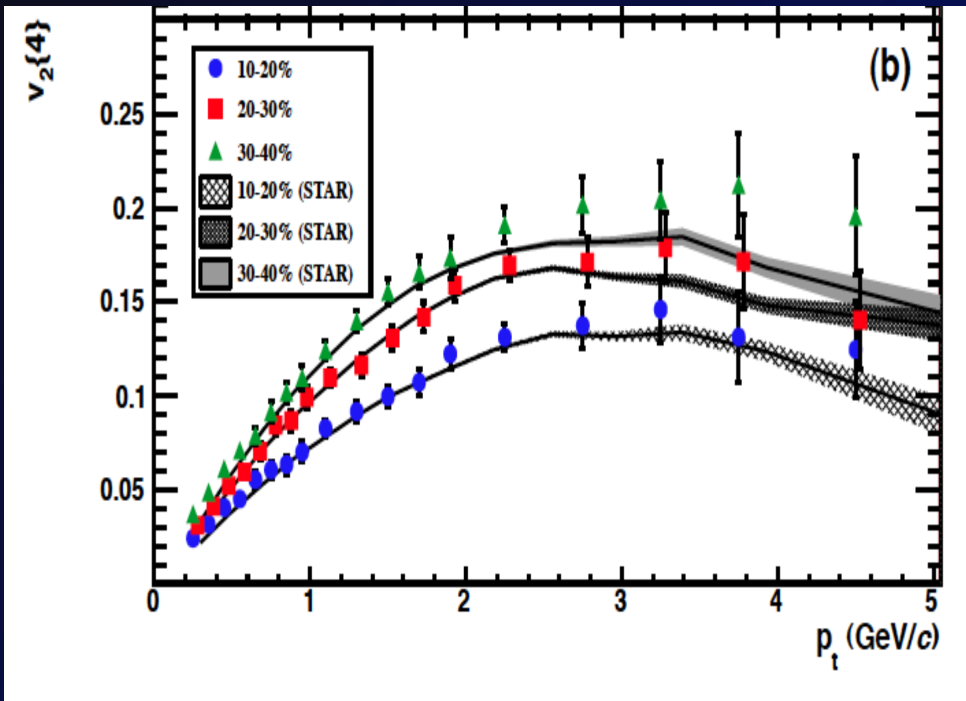
No fine tuning





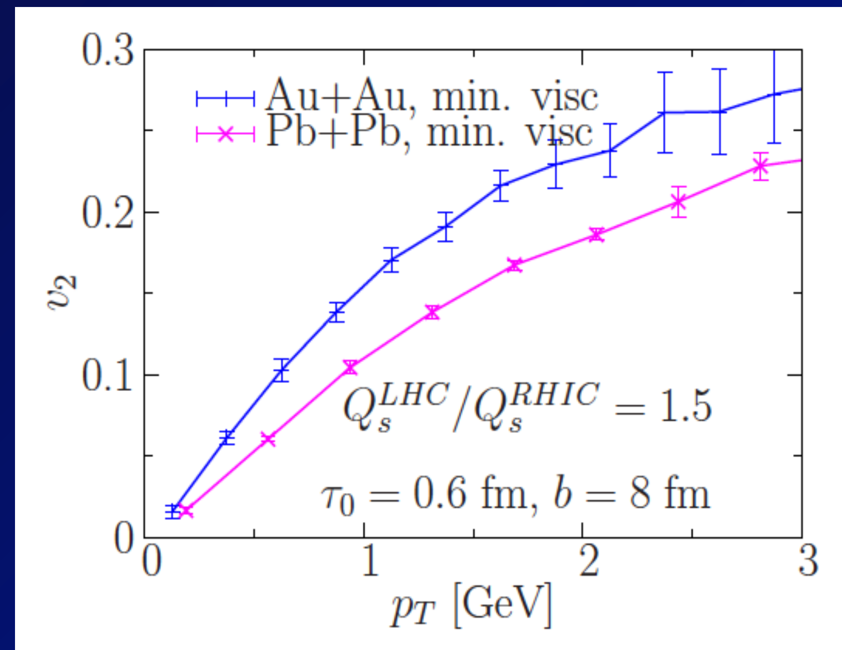
First Result on v_2: RHIC vs LHC

Going to LHC?



ALICE, PRL105 (2010)

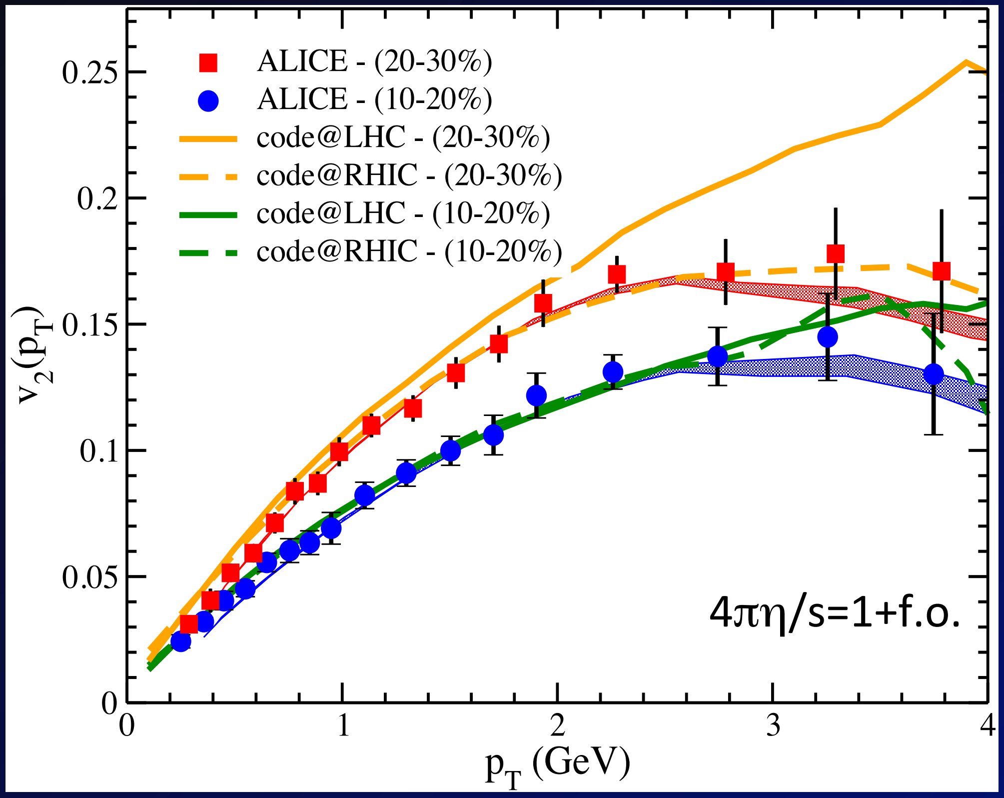
Cascade prediction



D. Molnar, LHC last call for prediction, JPG35

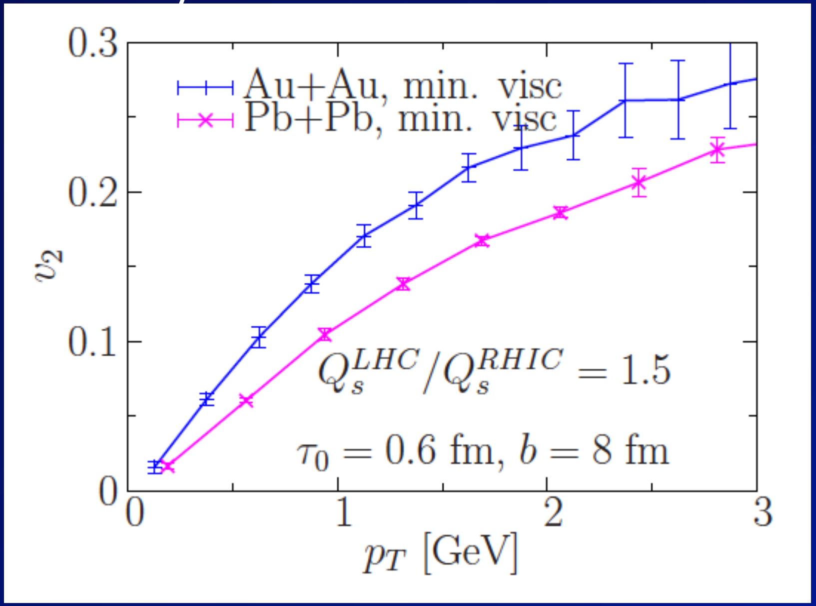
most pronounced for heavy particles. Models based on a parton cascade [19], including models that take into account quark recombination for particle production [20], predict a stronger decrease of the elliptic flow as function of transverse momentum compared to RHIC energies. Phenomenological extrapolations [21] and models

First application: v2 at RHIC & LHC



<u>At least 3 differences respect to MPC:</u>

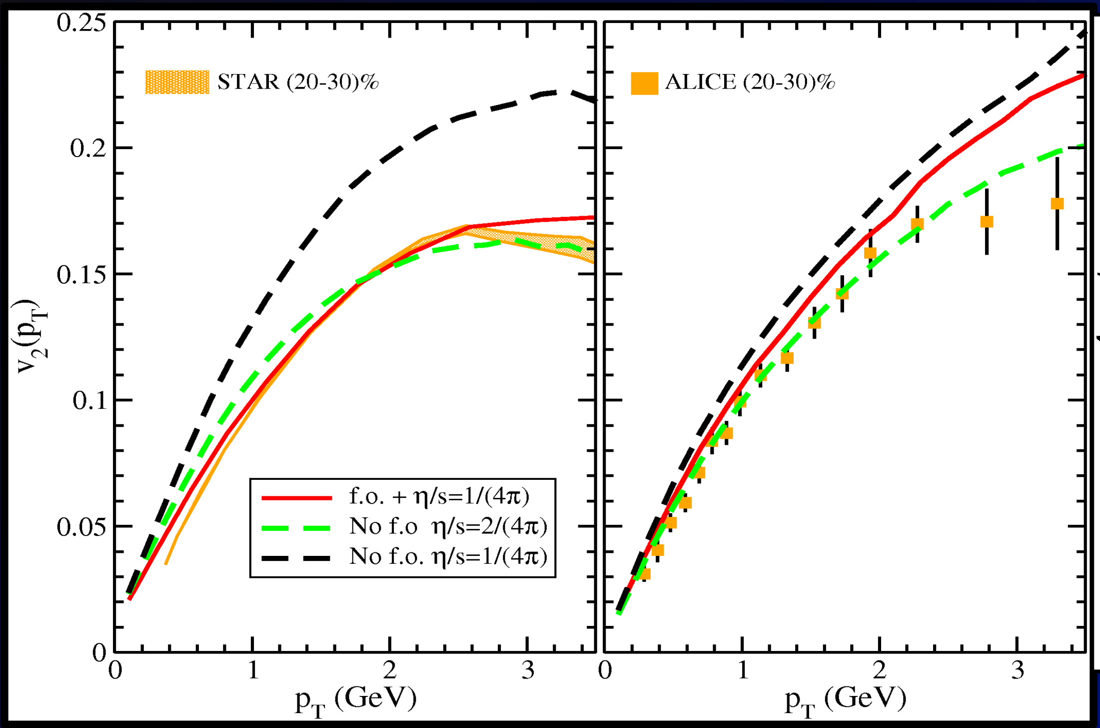
- Initial conditions *(non only minijets)*
- way to fix η/s *(not on average+ with CE)*
- f.o. dynamics included

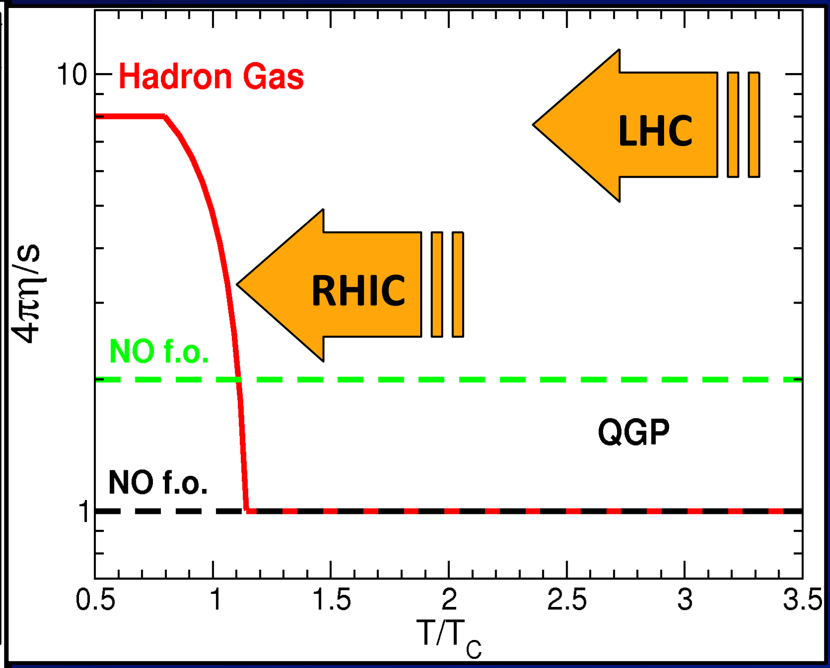


- Good agreement for 4πη/s=1 up to $p_T \approx 3$ GeV *(wider range than hydro)*
- <u>Same $v_2(p_T)$ at RHIC and LHC like in exp-data, up to p_T<2-3 GeV,</u>

  but for semi-central at LHC always over-predict larger v_2

First application: f.o. at RHIC & LHC



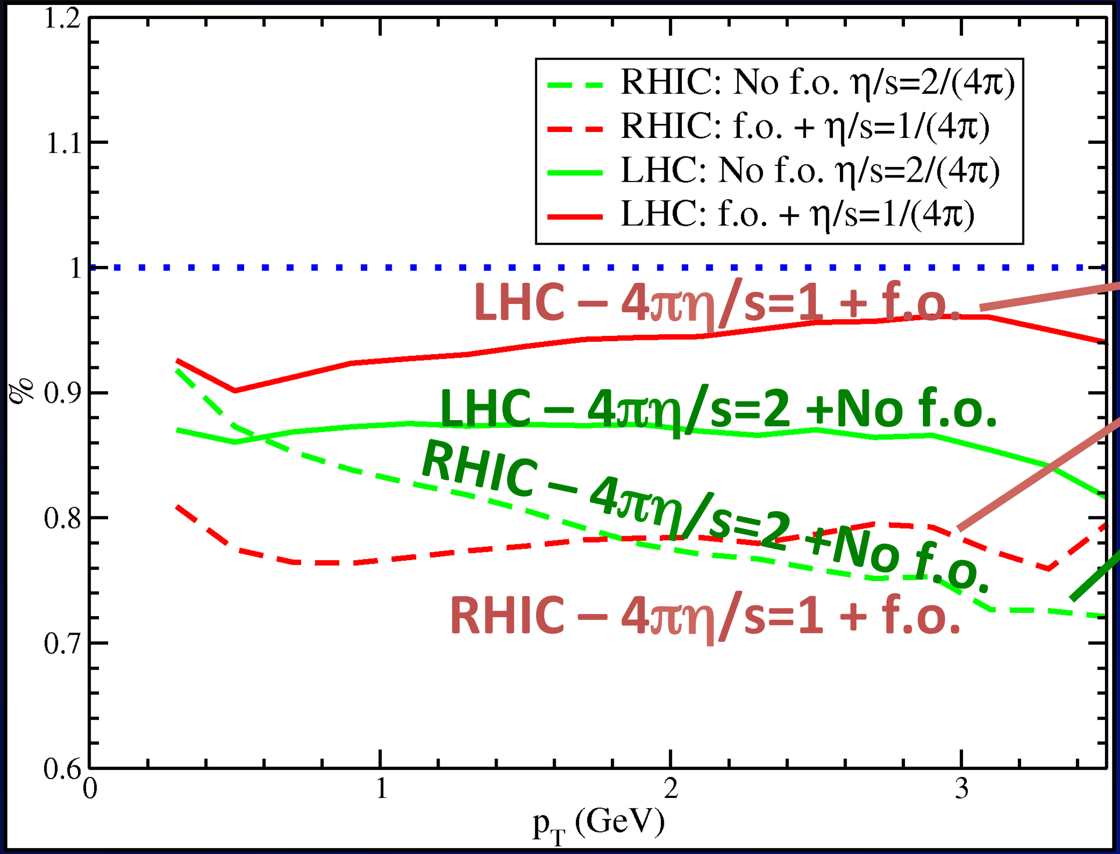


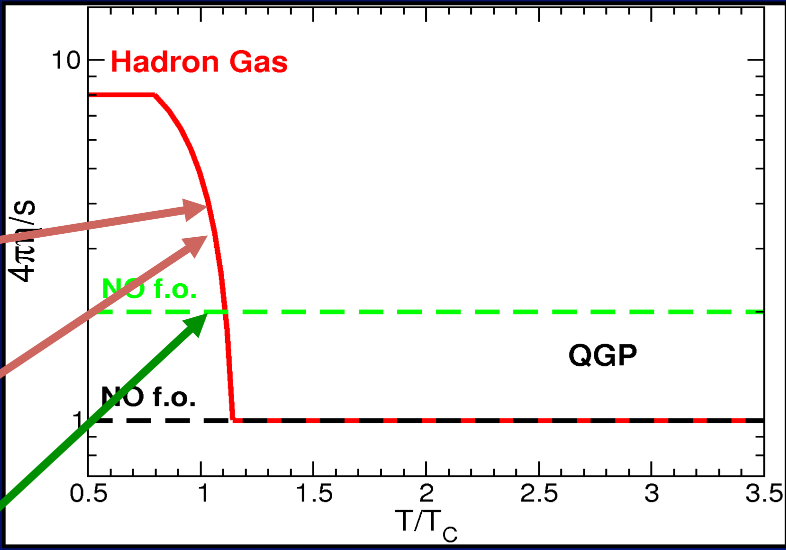
- RHIC: η/s increase in the cross-over region equivalent to double η/s in the QGP
- LHC: almost insensitivity to cross-over (≈ 5%) : v_2 from pure QGP,

  but at LHC less sensitivity to T-dependence of η/s? *see later*

Effect of η/s of the hadronic phase at LHC

Suppression of v_2 respect the ideal **4πη/s=1**



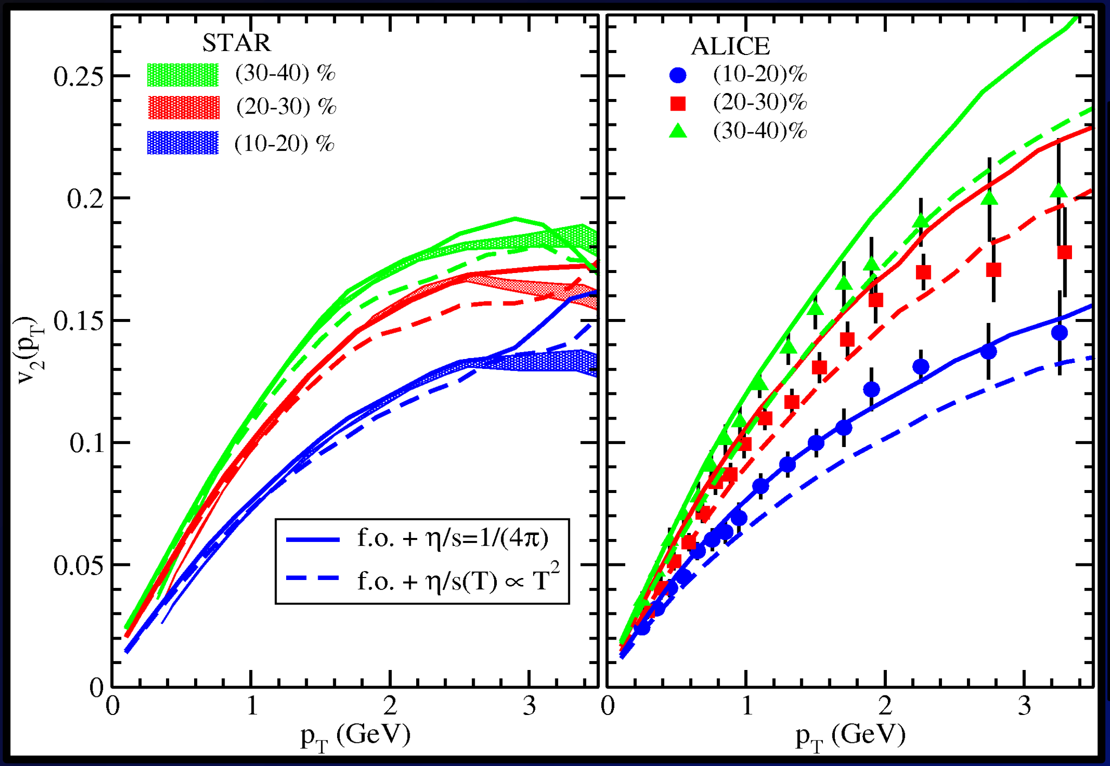


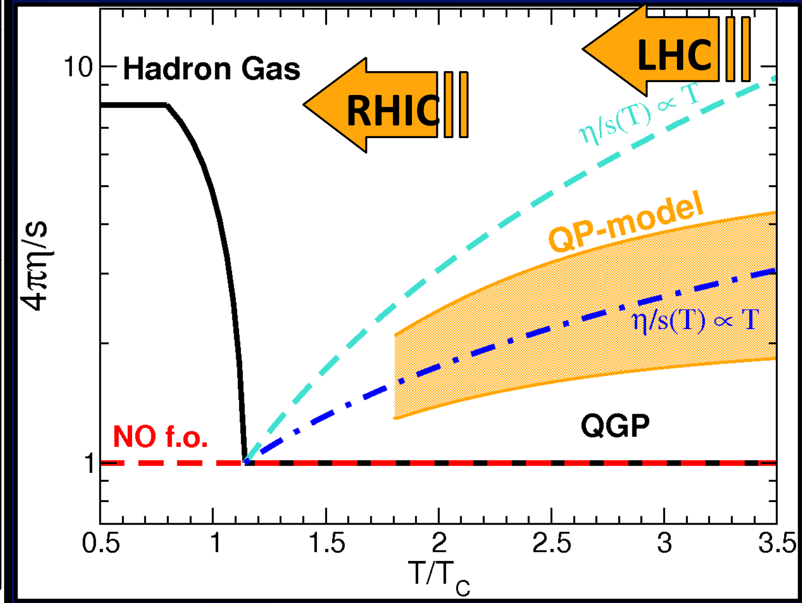
At LHC the contamination of cross-over & hadronic phase becomes negligible

Longer lifetime of QGP -> v_2 completely developed in the QGP phase

S. Plumari and VG, EPIC@LHC, AIP1422(2012)- arXiV:1110.4138 [hep-ph]

Sensitivity to Temperature dependent η/s(T)





Plumari et al., arxiv:1103.5611[hep-ph].

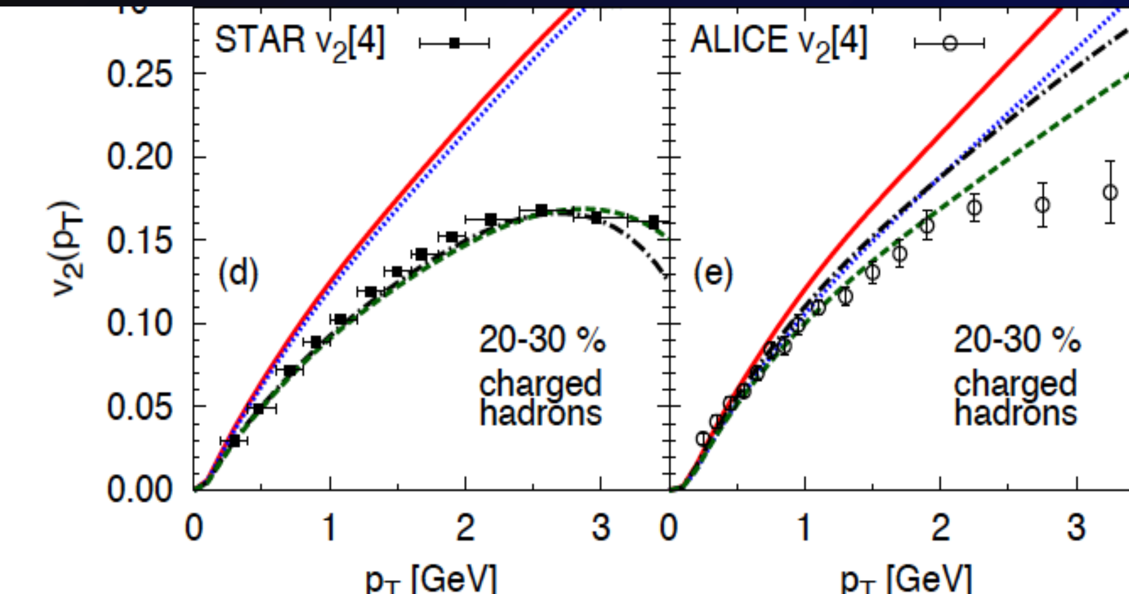
RHIC:

- The v_2 is nearly insensitive to the value of η/s(T) in the QGP phase

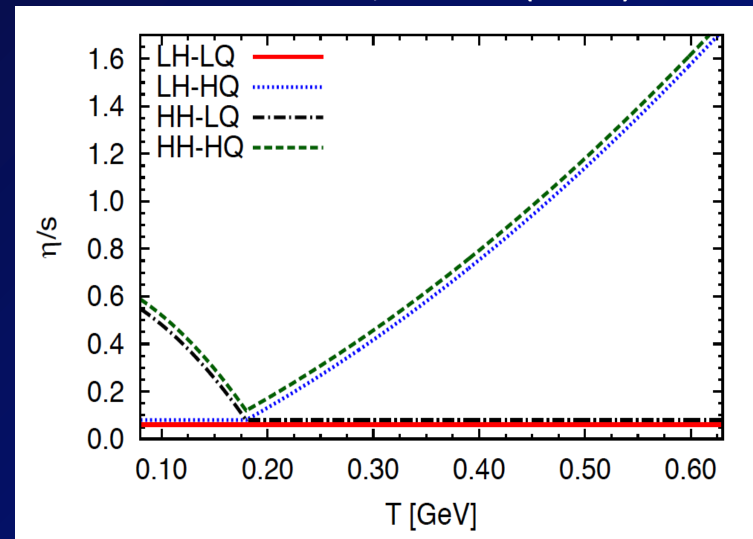
LHC:

- The v_2 more sensitive to the QGP phase η/s(T)
- [[$\eta/s \sim T^2$ cannot account for the v_2 decrease for p_T > 2.5 GeV.]]

Sensitivity η /s(T) in Hydro – Niemi et al.



Niemi et al., PRL106(2011)



$$T^{\mu\nu}_{eq} + \delta T^{\mu\nu} \Leftarrow f_{eq} + \delta f$$

An Asantz (Grad)

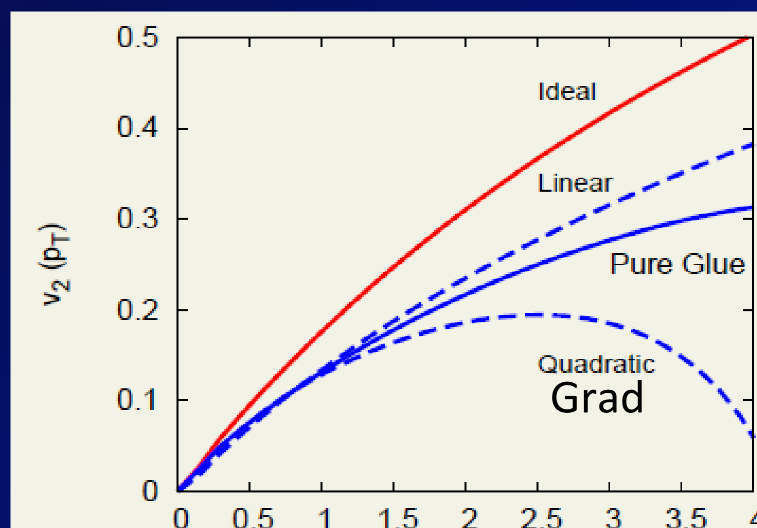
$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P} \frac{p_\mu p_\nu}{T^2} f_{eq}$$

$$\approx \frac{\eta}{3s} \frac{p_T^2}{\tau T^2} f_{eq}$$

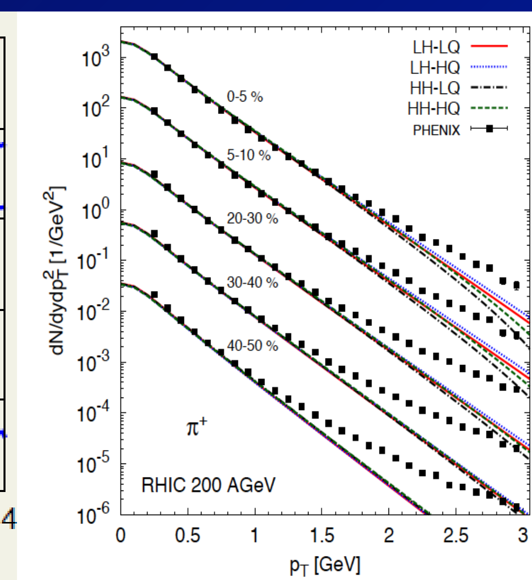
R. Lacey et al.,PRC82

Hydro up to p_T~3 GeV

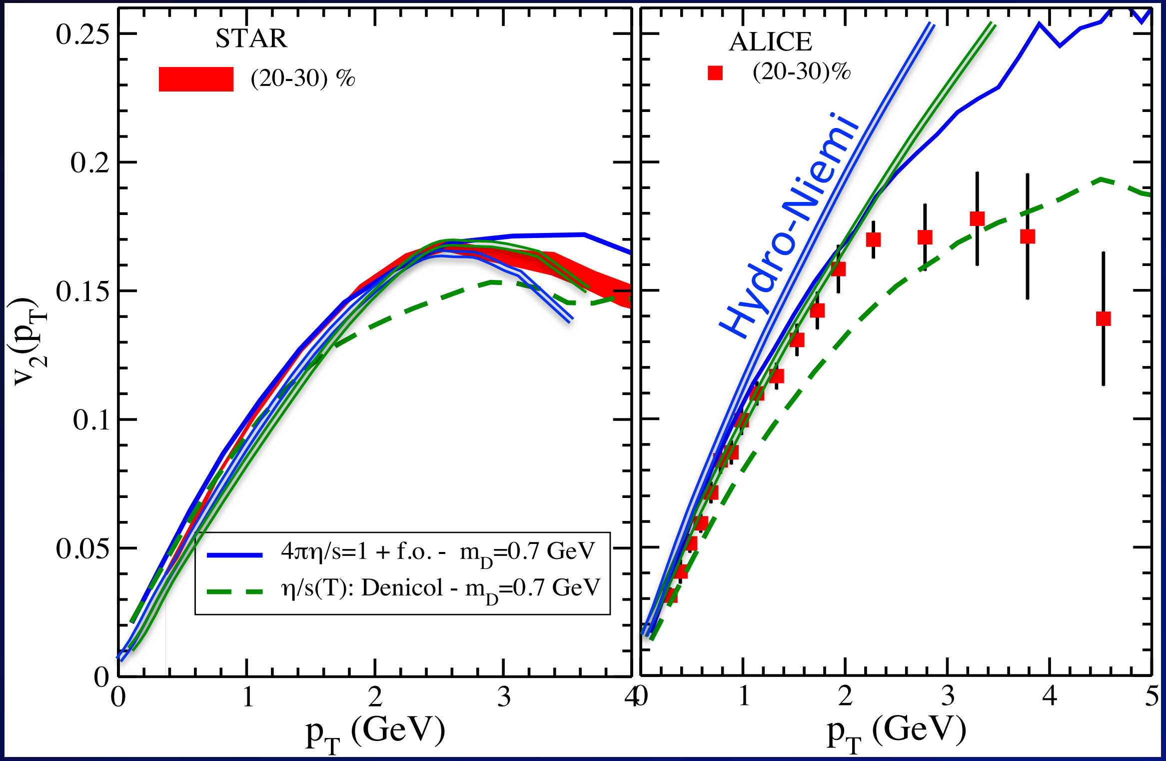
There is no one to one correspondence!



D. Molnar, HQ2010



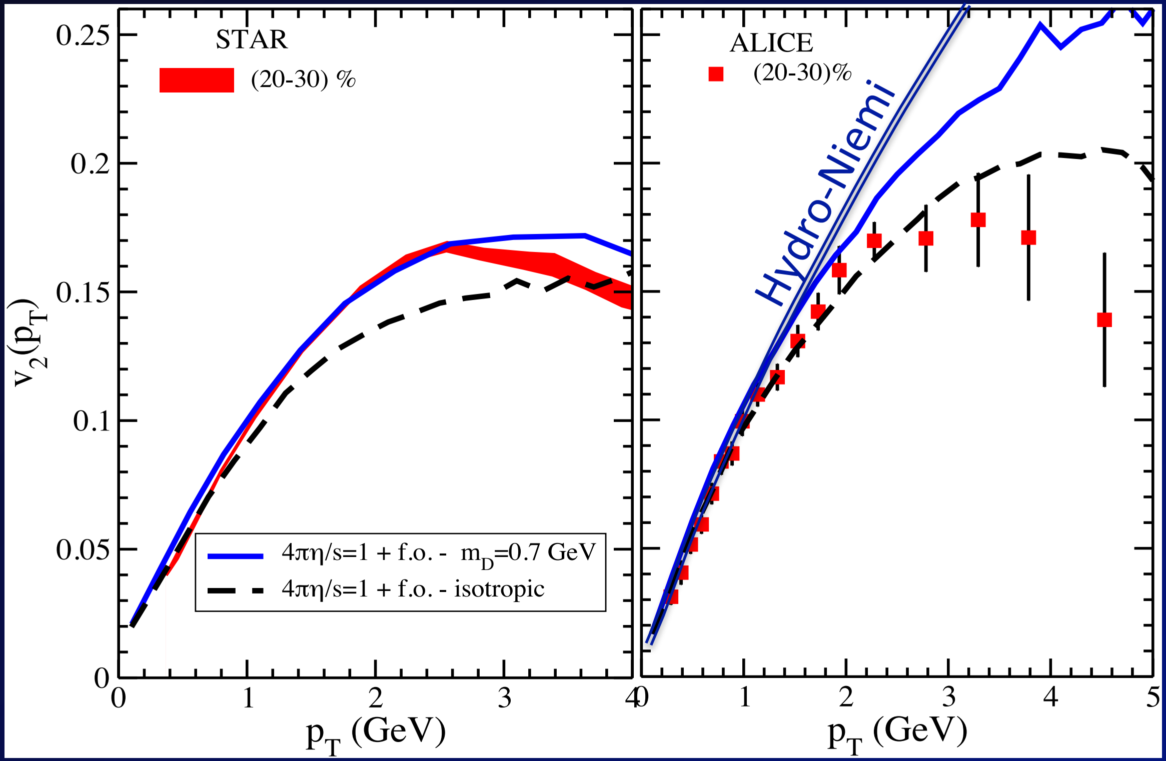
Sensitivity in transport using same η /s(T)



- ✓ Larger sensitivity at LHC on η/s (T) at LHC
- ✓ Effect larger respect to viscous hydro, but this depends also on δf

Does it depends also on some detail of the cross section?

Relevance of microscopic scale: $\sigma(\theta)$



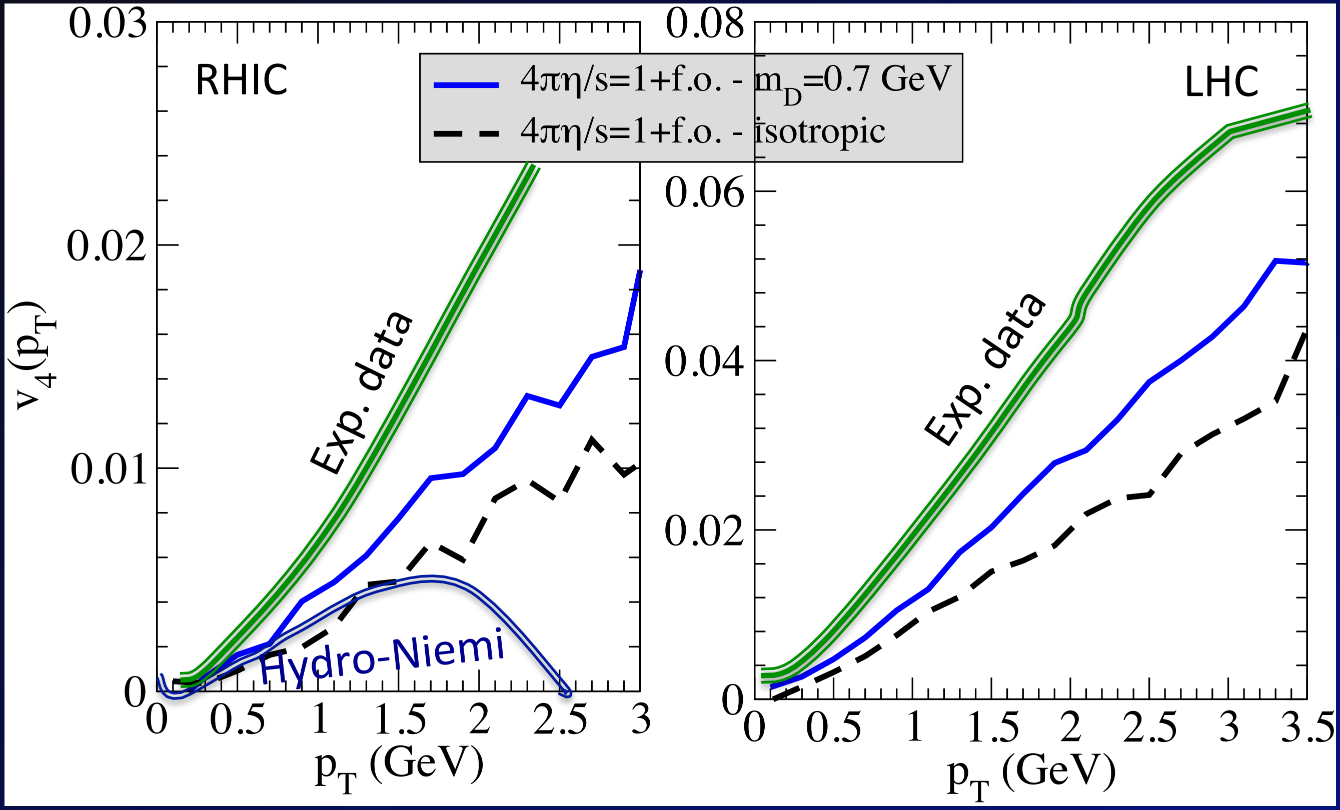
Out of pure hydro language!

- Microscopic details of the cross section matter at $p_T > 1.5$ GeV
- Fixed the η/s of the fluid isotropic cross section leads to smaller v_2
- An appropriate $m_D(T) = 3/2\ g(T)T$ at LHC?

-> damps v_2 (p_T) at LHC for $p_T > 2$ GeV ?

v_4 - Relevance of microscopic scale: $\sigma(\theta)$

Initial space fluctuation are discarded -> no possible an absolute comparison



- ✧ V_4 even more affected by microscopic details
- ✧ V_4 enhancement from RHIC to LHC correctly predicted
- ✧ Hydro seems to have more problems, again an effect of δf?

Quasiparticle model:

- Study quark-gluon chemical equilibrium
- Imply a transport evolution with EoS-lQCD

$$\begin{cases} \left\{ p^{*\mu}\partial_\mu + m^*(x)\partial^\mu m^*(x)\,\partial_\mu^{p^*} \right\} f(x,p^*) = C[f] \\ \text{Field Interaction -> } \varepsilon \neq 3P \\ \dfrac{\partial B}{\partial m_i} + d_i \displaystyle\int \frac{d^3p}{(2\pi)^3} \frac{m_i(x)}{E_i(x)} f(x,p) = 0 \end{cases}$$

Using a simple QP-model

U.Heinz and P. Levai, PRC (1998)

$$P(T) = \sum_{i=g,q,\bar{q}} \frac{D_i}{(2\pi)^3} \int_0^\infty d^3k \frac{k^2}{3\omega_i(k)} f_i(k) - B(T)$$

$$\varepsilon(T) = \sum_{i=g,q,\bar{q}} \frac{d_i}{(2\pi)^3} \int_0^\infty d^3k\ \omega_i(k)\ f_i(k) + B(T) + \widetilde{W}_B(T)$$

$W_B=0$ guarantees Thermodynamicaly consistency

$$B(T) = B(T^*) - \sum_{i=g,q,\bar{q}} \frac{D_i}{(2\pi)^3} \int_{T^*}^T dT' M_i(T') \frac{dM_i(T')}{dT'} \int_0^\infty \frac{d^3k}{\omega_i} f_i(k)$$

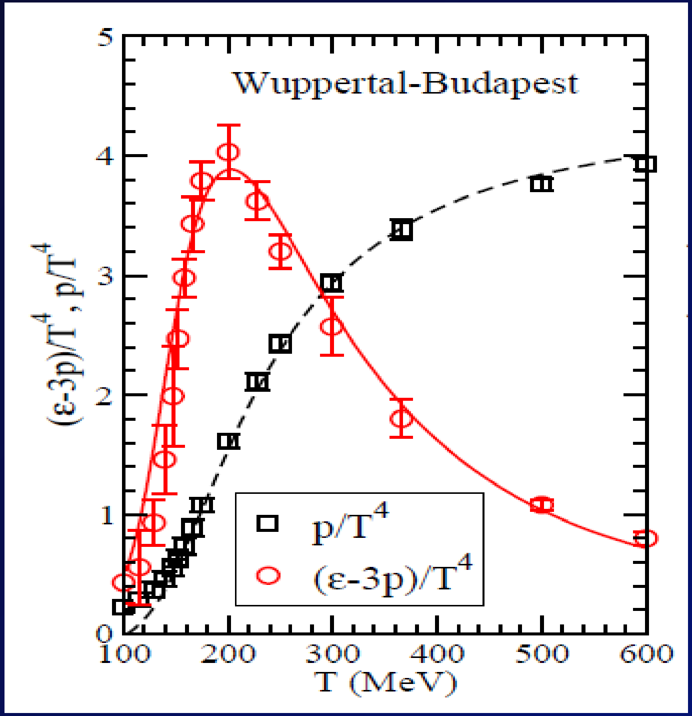
Plumari, Alberico, Greco, Ratti, PRD(2011)

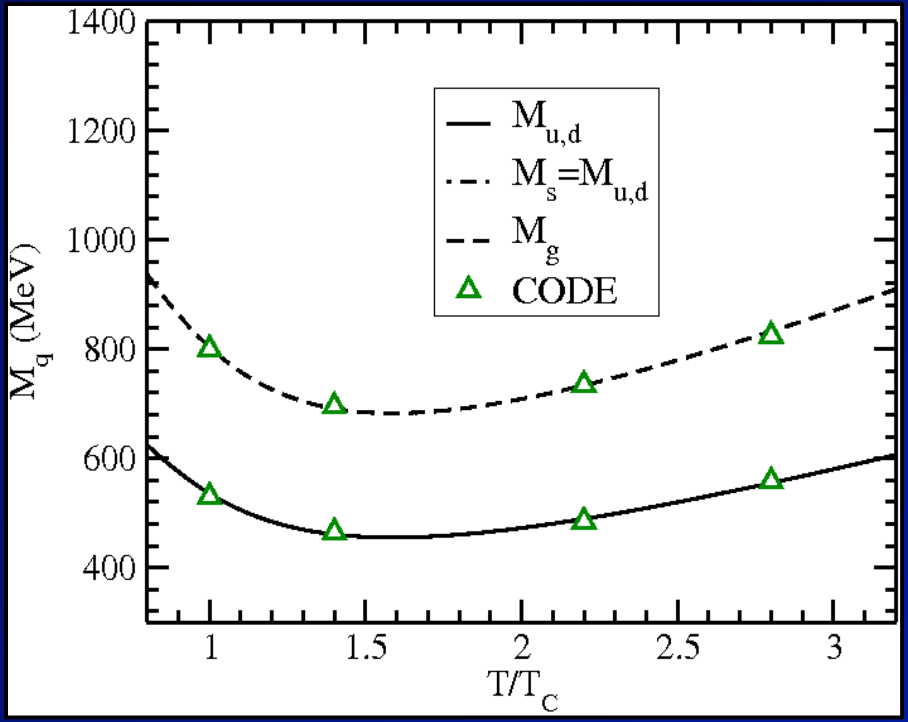
See Plumari TALK

$$\omega^2 = k^2 + M^2(T)$$

$M_g(T)=3/2\ g(T)T$

$M_g(T)$ from a fit to ε from lQCD -> good reproduction of P, e-3P, c_s



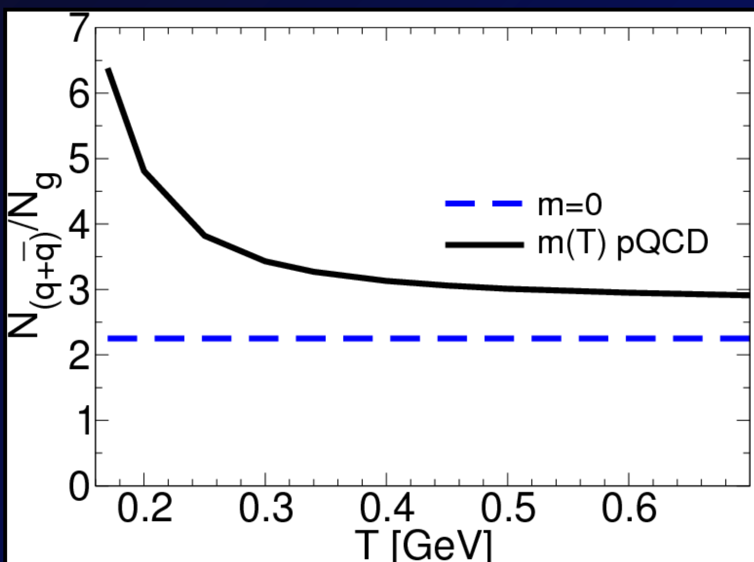


QP-model: implications for chemical composition

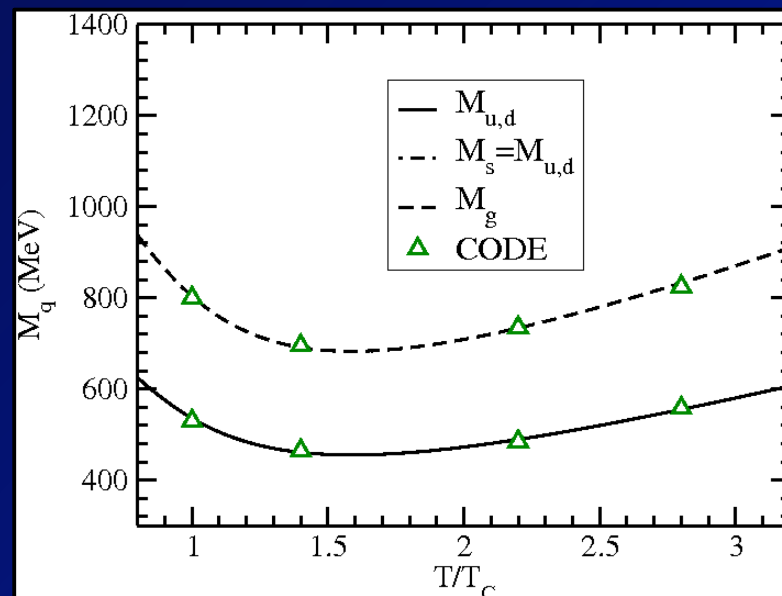
Passed several numerical test on the box. We reproduce the lQCD EoS .

$$\begin{cases} p^{\mu}\partial_{\mu}f(x,p) + m_i(x)\partial_{\mu}m_i(x)\partial_p^{\mu}f(x,p) = C_{22}[f] \\ \dfrac{\partial B}{\partial m_i} + d_i\displaystyle\int\frac{d^3p}{(2\pi)^3}\frac{m_i(x)}{E_i(x)}f(x,p) = 0 \end{cases}$$

, i=g,u,d,s



$$\frac{N_{q+\bar{q}}}{N_g} = \frac{d_{q+\bar{q}}}{d_g}\frac{m_q^2(T)\,K_2(m_q/T)}{m_g^2(T)\,K_2(m_g/T)}$$



Using the QP-model: q<-> g conversion

Inelastic cross section with massive parton gg->qq

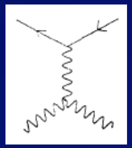
$$|M_t|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2m_q^2(m_q^2 + t) - m_g^2 s - 4m_q^2 m_g^2}{(t - \mu_q^2)^2}$$

$$|M_u|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2m_q^2(m_q^2 + u) - m_g^2 s - 4m_q^2 m_g^2}{(u - \mu_q^2)^2}$$

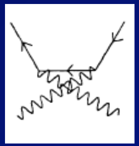
$$|M_s|^2 = \alpha_s^2 \pi^2 12 \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 3m_g^2 s + 2m_q^2 m_g^2}{(s - \mu_g^2)^2}$$

→

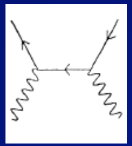
$$|M_t|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{t}{u}$$

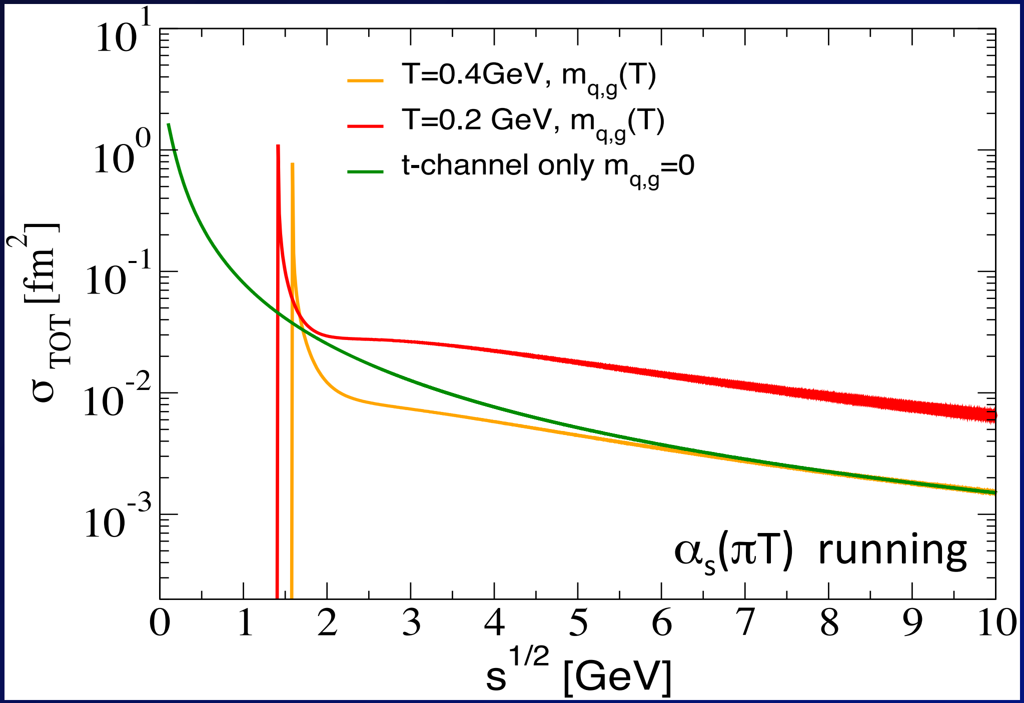


$$|M_u|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{u}{t}$$

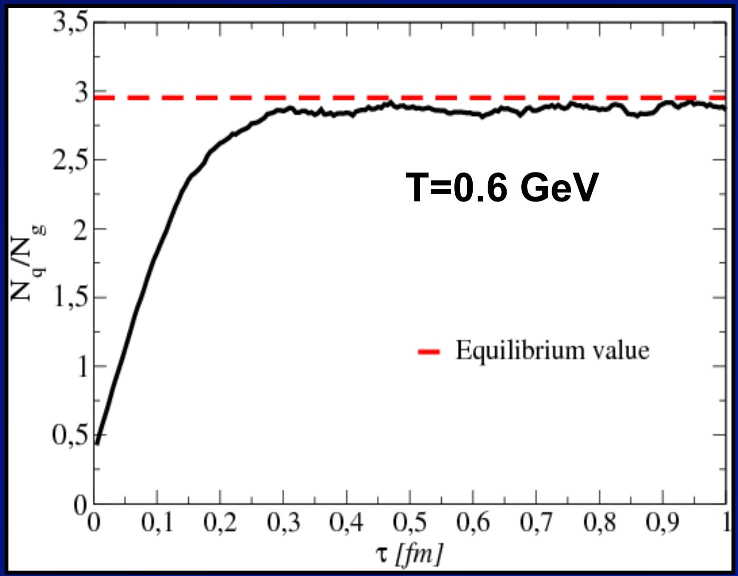


$$|M_s|^2 = \alpha_s^2 \pi^2 12 \frac{u \cdot t}{s^2}$$



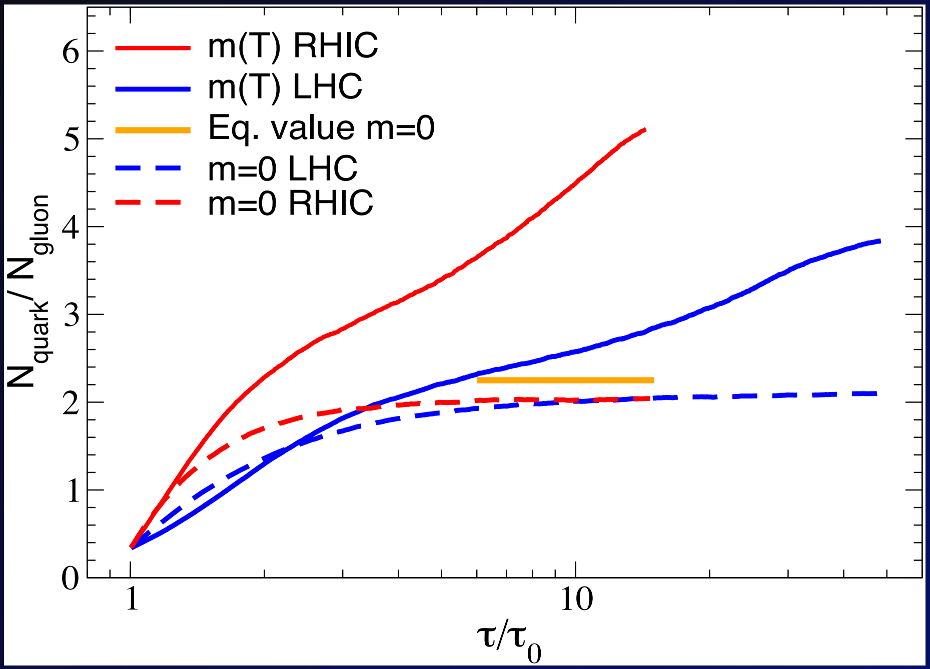


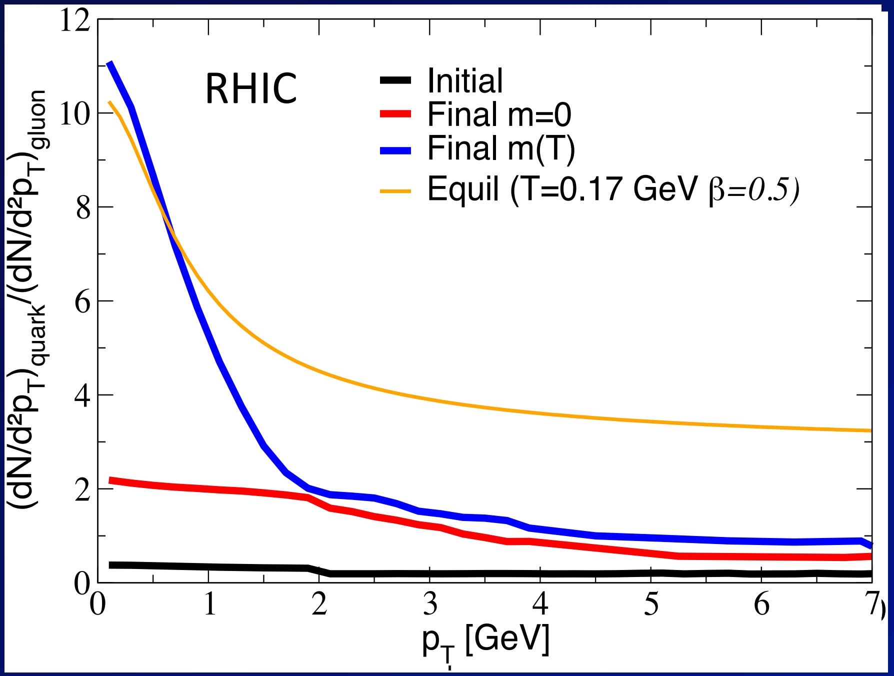
Evolution in a Box



QP-model:QGP composition in HIC

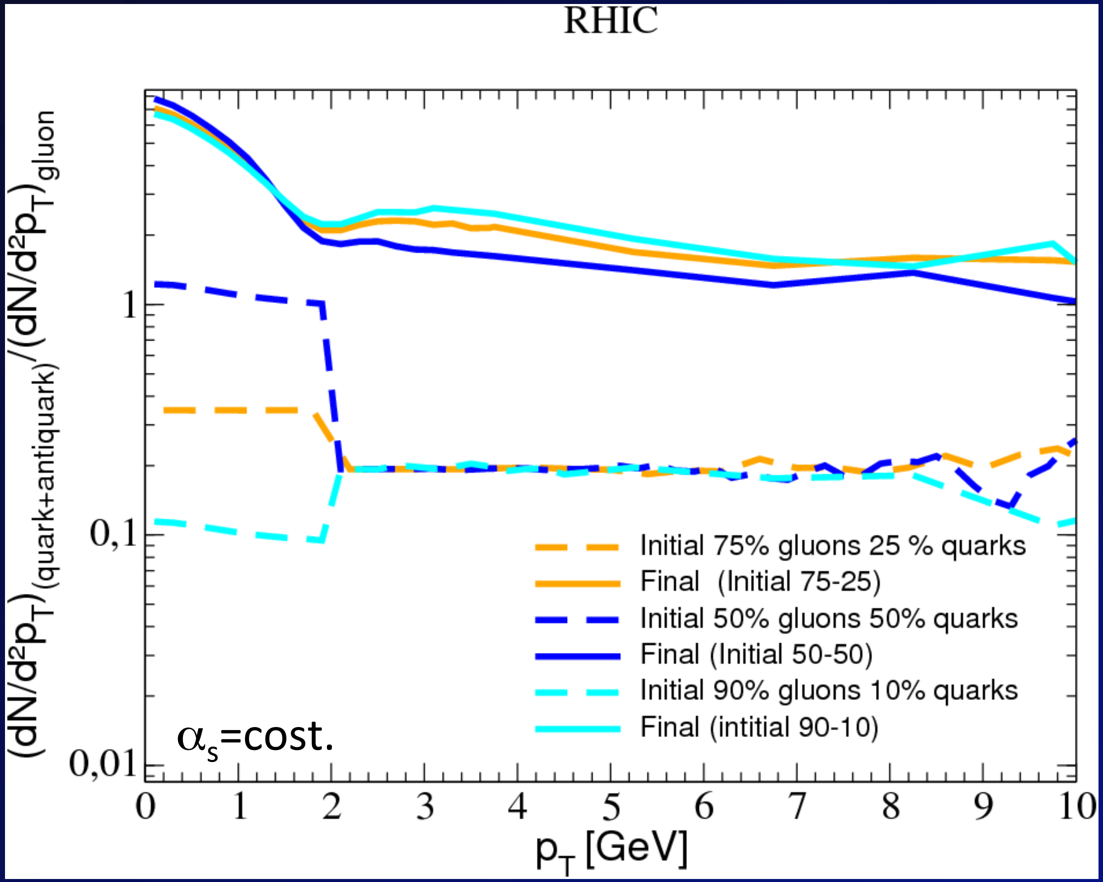
F. Scardina et al., arXiv:1202.2262 [nucl-th].





- Quark dominance clode to T_c : ~ 85% of total partons composed of q + anti-q starting from a 80% gluon matter
- The plasma reaches the equilibrium value at low p_T but at high p_T the N_q/N_g is still significantly modified
- Massive quarks close to T_c -> coalescence
- Should one rivisit jet quenching, quarkonia suppression…

Impact of different q/g initial ratio



Summary

Developing a Boltzmann-Vlasov like partonic transport:

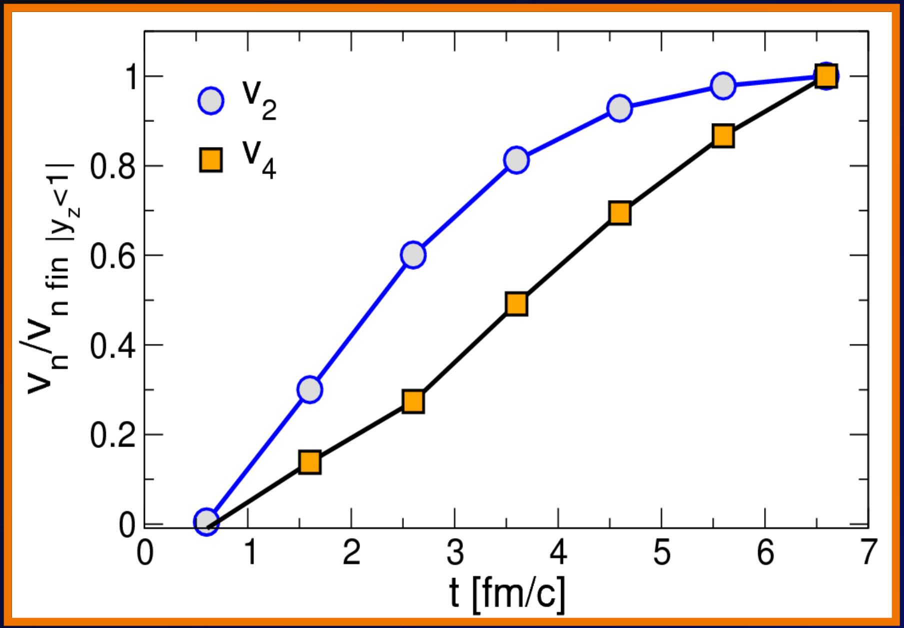
- Chapmann-Enskog agree with Green-Kubo (3%)
  - Allow to work with transport at fixed $\eta/s(T)$
- Agreement with data for $4\pi\eta/s \approx 1$
  similarly to hydro (but in a wider p_T-range):
  - LHC more sensitive to $\eta/s(T)$ from pure QGP
  - $p_T < 2$ GeV same $v_2(p_T)$ at RHIC and LHC
  - $p_T > 2$ GeV microscopic $\sigma(\theta)$ matters
  - v_4 more sensitive to both $\eta/s(T)$ and $\sigma(\theta)$
- **Quasi-particle transport:**
  - **Massive quark plasma close to T_c (-> coalescence)**

Outlook

- Include initial state fluctuations - >v_2, v_3, v_4,v_5
- Include hadronization

V_4

Cascade simulation

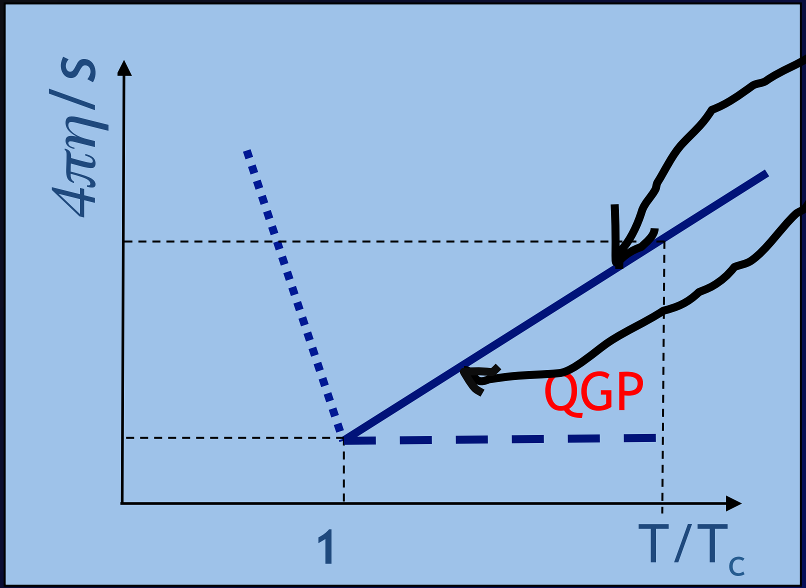


VG et al., PPNP(2009), HQ2010

- V_4 develops later - > more affected by η/s at T ~ T_c
- V_4 more sensitive probe of $\eta/s(T)$

Initial space fluctuation are discarded -> no possible an absolute comparison

Effect of η/s(T) on v_4

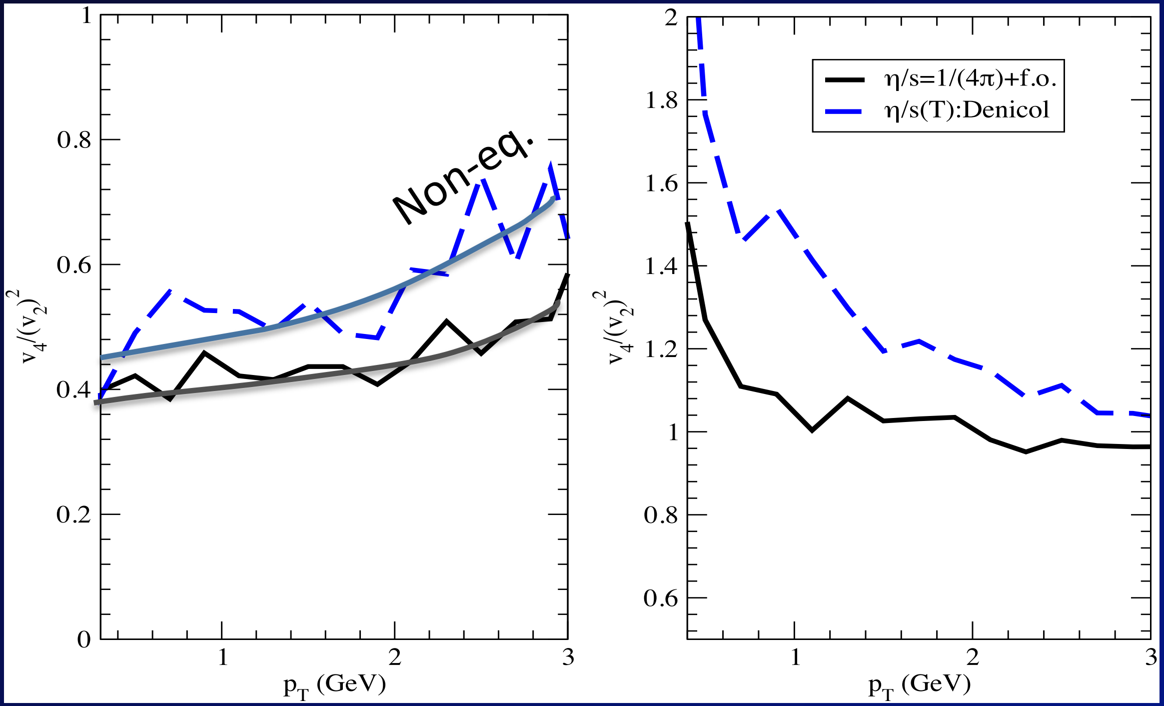


- V_2 develops earlier at higher η/s
- V_4 develops later at lower η/s

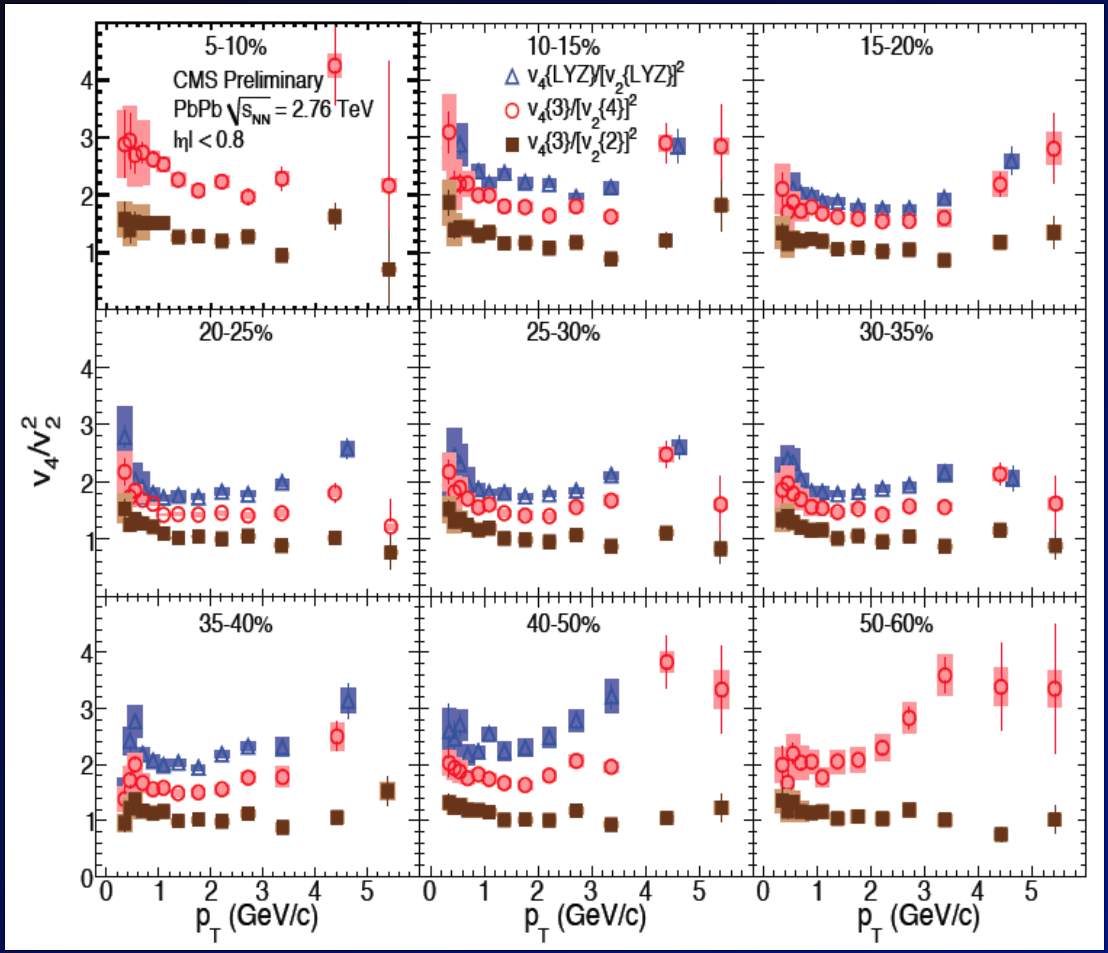
-> $v_4/(v_2)^2$ increase stronger η/s(T)

✧ $v_4/(v_2)^2$ increase for T-dep η/s ≈ 25-30%

✧ Increase from RHIC to LHC (observed exp. from 0.8 to 1.6)

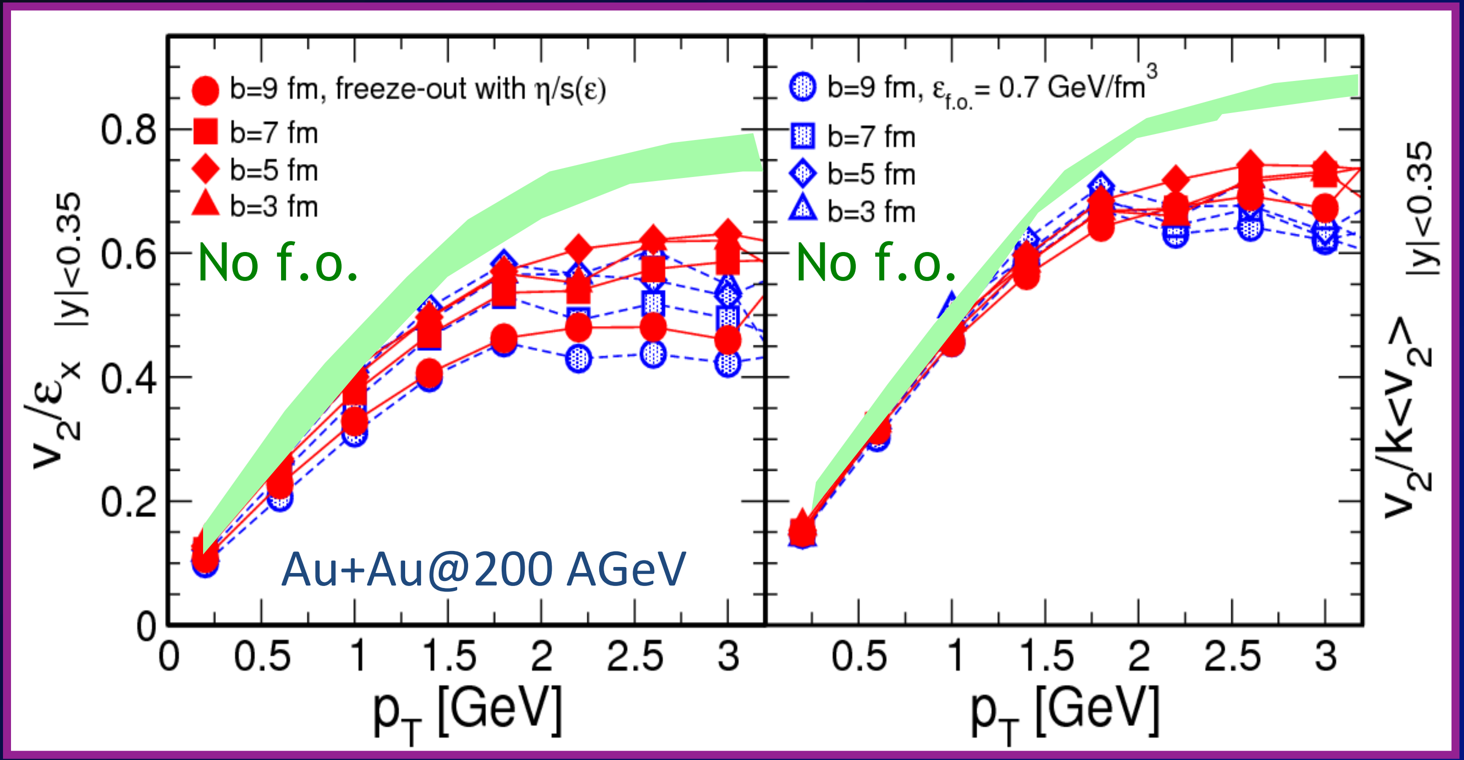


Lack of an increase at high p_T due to the increasing $v_2(p_T)$ that overshoot exp. data



5-10%
CMS Preliminary
PbPb $\sqrt{s_{NN}}$ = 2.76 TeV
$|\eta| < 0.8$
10-15%
$v_4\{LYZ\}/[v_2\{LYZ\}]^2$
$v_4\{3\}/[v_2\{4\}]^2$
$v_4\{3\}/[v_2\{2\}]^2$
15-20%
20-25%
25-30%
30-35%
35-40%
40-50%
50-60%
v_4/v_2^2
p_T (GeV/c)

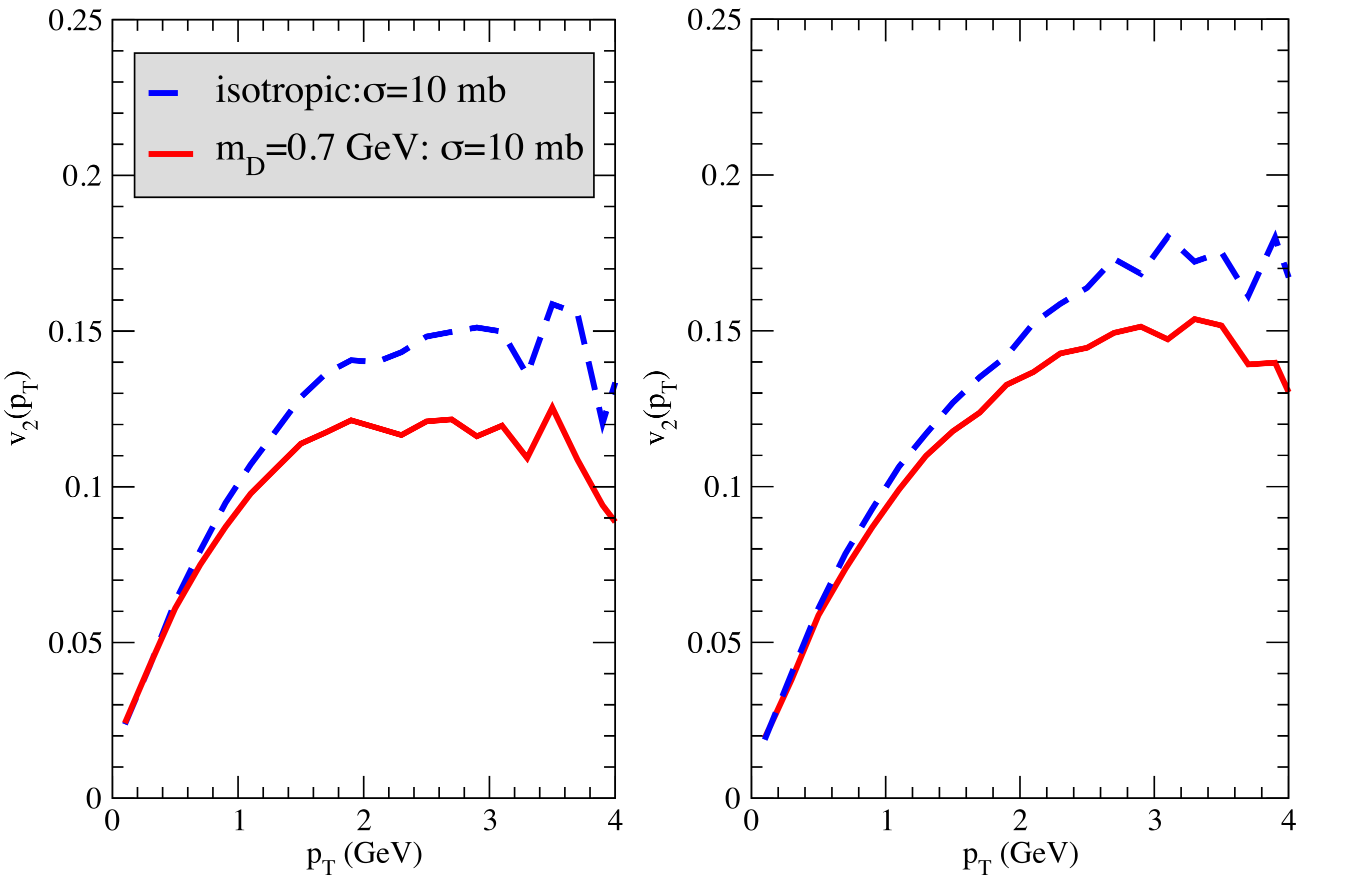
Results with both freeze-out and no freeze-out



v_2/ε scaling broken $\qquad$ $v_2/\langle v_2\rangle$ scaling kept!

Cascade at finite η/s + freeze-out :

- v_2/ε broken in a way similar to STAR data
- Agreement with PHENIX and STAR scaling of $v_2/\langle v_2\rangle$
- Freeze-out + η/s lowers the $v_2(p_T)$ at higher p_T …



isotropic:σ=10 mb
m_D=0.7 GeV: σ=10 mb
v_2(p_T)
p_T (GeV)
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4