Gluon plasma in a bottom-up AdS/QCD approach: Thermodynamics and transport coefficients

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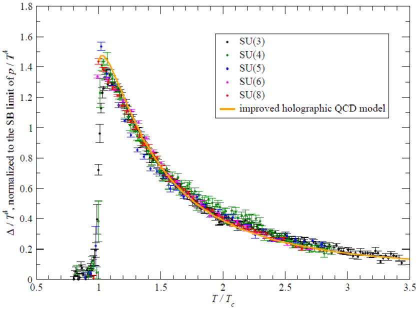
SU(N) Yang Mills Thermodynamics



$\frac{e-3p}{T^4} / \frac{p_{SB}}{T^4}$ from [Panero Phys. Rev. Lett. 103 (2009) 232001]
IHQCD: [Kiritsis et al. Lect. Notes Phys. 828 (2011) 79] etc.

AdS/QCD

Mild N-dependence ⇒ use gauge/gravity duality
Maldacena conjecture (1998), “low energy form“:

- 5D classical gravity on AdS_5 ⇔ strongly coupled large N 4D conformal field theory
- isometry group on AdS_5 = conformal group of 4D gauge theory

⇒ bottom-up AdS/QCD:

- Einstein gravity on AdS_5 coupled to fields
- 5th coordinate in AdS_5: gauge theory energy scale
- idea: deform AdS_5 ⇒ break conformal invariance

Shear and bulk viscosities - AdS vs. QPM

Philosophy as in:
[Bluhm, Kaempfer, Redlich Phys. Rev. C84 (2011) 025201]:
Equation of state $\Rightarrow \eta$, ζ via

- effective kinetic theory
- extra parameter: relaxation time τ
- ad hoc assumption: $\tau_\eta = \tau_\zeta$

Hope: no extra parameter in AdS/QCD

Anti-deSitter space

Vacuum solution of 5D Einstein equations ($\kappa_5 \equiv 1$):

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \Lambda g_{\mu\nu}$$

with $\Lambda < 0$ (unique up to conformal transformations)

- maximally symmetric, homogeneous, isotropic
- AdS scale: $\frac{1}{L^2} = -\frac{\Lambda}{12}$
- black hole: horizon $z = z_h$ defined by $f(z_h) = 0$

AdS + black hole: $f(z) = 1 - \frac{z^4}{z_h^4}$:

$$ds^2 = \frac{L^2}{z^2} \left(-f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right)$$

The setting

5D-action:

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3} \partial^\mu \phi \partial_\mu \phi + V(\phi) \right)$$

Ansatz - deformed AdS_5:

$$ds^2 = \frac{L^2}{z^2} e^{2A_s(z) - \frac{4}{3}\phi(z)} \left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right)$$

Einstein equations and scalar field EoM:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

$$D^\mu T_{\mu\nu}(\phi) = 0$$

Boundary conditions, solutions

Boundary conditions:

1. UV $z \to 0$: asymptotically AdS space $\Leftrightarrow$ asymptotic freedom of SU(3) YM
2. IR $z = z_h$: $A_s(z_h)$, $\phi(z_h)$, $V(\phi)$ regular, $f(z_h) = 0$

3 independent equations for 4 unknown functions:
$A_s(z)$, $f(z)$, $\phi(z)$, $V(\phi)$

- Kiritsis et al., Gubser et al.: fix $V(\phi)$
- Li, Huang et al., we: fix $A_s(z) \Rightarrow \phi(z)$, $f(z)$ easily obtained, $V(z)$ not needed

Black hole thermodynamics

Hawking temperature:

$$T(z_h) = \frac{\mid f'(z) \mid}{4\pi} \Big|_{z=z_h}$$

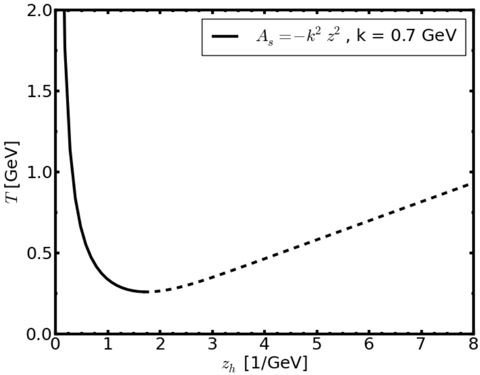
Bekenstein-Hawking entropy density:

$$s(z_h) = \frac{L^3}{4\,G_5} \left(\frac{e^{A_s - \frac{2}{3}\phi}}{z} \right)^3 \Big|_{z=z_h}$$

Equation of state:

$$s(T) = s(z_h(T))$$

The transition temperature



Heat capacity $C = T\frac{ds}{dT}$:

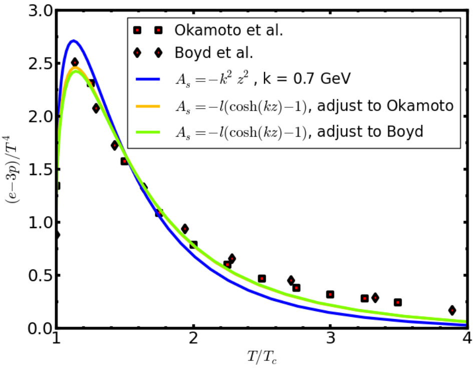
- $0 < z_h < z_{min}$: $C > 0$ stable
- $z_h > z_{min}$: $C < 0$ unstable

Assume:

$$T_c = T(z_{min})$$

use $C > 0$-branch

Results



- Fit $A_s(z)$ to LQCD [Boyd et al. Nucl. Phys. B469 (1996) 419] [Okamoto et al. Phys. Rev. D60 (1999) 094510]
- Parameters: k, l, $\frac{L^3}{G_5}$

Kubo formulae

Shear η and bulk ζ viscosities from 4D Kubo formulae:

$$G_\eta(\omega) = -i \int dt d^3x e^{i\omega t} \theta(t) \langle [T_{12}(t, \vec{x}), T_{12}(0,0)] \rangle_T$$

$$G_\zeta(\omega) = -i \frac{1}{4} \int dt d^3x e^{i\omega t} \theta(t) \left\langle \left[T_i^i(t, \vec{x}), T_k^k(0,0) \right] \right\rangle_T$$

$$\{\eta, \zeta\} \propto - \lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} G_{\eta,\zeta}(\omega)$$

Holographic Kubo formulae

AdS/CFT dictionary - Minkowski correlator:
[Gubser et al. JHEP 0808 (2008) 085]

$$-\operatorname{Im} G_{\eta,\zeta}(\omega) = \frac{\mathcal{F}_{\eta,\zeta}(\omega)}{16\pi G_5}$$

1. Perturb metric $g_{\mu\nu} \to g_{\mu\nu} + H_{\mu\nu}$
2. $\Rightarrow$ linearized EoM for $H_{\mu\nu}$
3. $\Rightarrow$ 5D-correlator $\mathcal{F}_{\eta,\zeta}(H_{\mu\nu}, \omega)$
4. solution $H_{\mu\nu} \Rightarrow \zeta, \eta$

1. Perturbation of the metric

Change gauge:

$$A_s(z) \to A(\phi), B(\phi)$$

Perturbations:

$$\begin{aligned} ds_\eta^2 &= e^{2A(\phi)}(-f(\phi)dt^2 + d\vec{x}^2 + \lambda H_{12}dx_1dx_2) + e^{2B(\phi)}\frac{d\phi^2}{f(\phi)} \\ ds_\zeta^2 &= e^{2A(\phi)}(-f(\phi)dt^2(1+\frac{\lambda}{2}H_{00})^2 + d\vec{x}^2(1+\frac{\lambda}{2}H_{11})^2) + \\ &\quad + e^{2B(\phi)}\frac{d\phi^2}{f(\phi)}(1+\frac{\lambda}{2}H_{44})^2 \end{aligned}$$

- η- and ζ- perturbations decouple
- ζ: SO(3)-invariance $H_{11} = H_{22} = H_{33}$

2. Linearized equations of motion

$$\eta : \qquad H''_{12} + \left(4A' - B' + \frac{f'}{f}\right) H'_{12} + \frac{e^{2(B-A)}}{f^2}\omega^2 H_{12} = 0$$

$$\zeta : \qquad H''_{11} + k_1(\phi)H'_{11} + k_2(\phi,\omega)H_{11} = 0$$

- $H'_{ij} \equiv \frac{dH_{ij}}{d\phi}$
- $H_{ij}(t,\phi) = e^{-i\omega t}H_{ij}(\phi)$
- $H_{11} \Rightarrow H_{00}$ and H_{44}
- Now: $H_{ij} \to h_{ij}$ complex

3. 5D Correlators

[Gubser et al. JHEP 0808 (2008) 085]

$$\mathcal{F}_\eta(\omega) = -i\frac{fe^{4A-B}}{2}(h_{12}^* h_{12}' - h_{12}^{*\prime} h_{12})$$

$$\mathcal{F}_\zeta(\omega) = -i\frac{fe^{4A-B}}{3A'^2}(h_{11}^* h_{11}' - h_{11}^{*\prime} h_{11})$$

At the horizon: $\mathcal{F}_{\eta,\zeta} \propto \omega \mid h_{ij}(\phi_h) \mid^2$

Shear viscosity

$$\mathcal{F}_\eta(\omega) = \omega e^{3A(\phi_h)} \mid h_{12}(\phi_h) \mid^2$$

Boundary conditions (AdS/CFT dictionary): $h_{12}(\phi_h) = 1$

$$\eta = \frac{e^{3A(\phi_h)}}{16\pi G_5}$$

Reminder:

$$s = \frac{e^{3A(\phi_h)}}{4 G_5}$$

$\Rightarrow$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

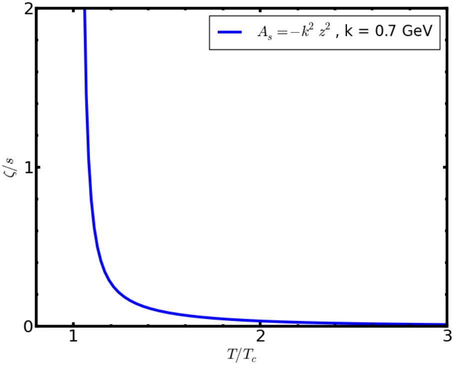
[Kovtun, Son, Starinets Phys. Rev. Lett. 94 (2005), pp. 111601]

Bulk viscosity

$$\mathcal{F}_\zeta(\omega) = \omega e^{3A(\phi_h)} \frac{2 \mid h_{11}(\phi_h) \mid^2}{3A'(\phi_h)^2}$$

$$\frac{\zeta}{s} = \frac{2 \mid h_{11}(\phi_h) \mid^2}{27A'(\phi_h)^2}$$

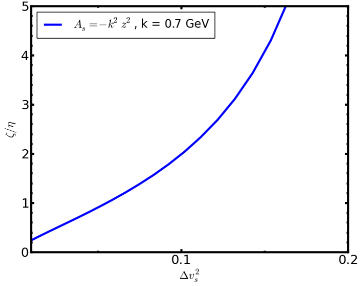
Bulk viscosity



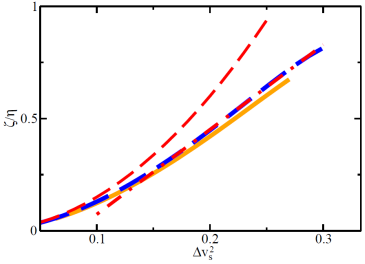
- sharp rise at $\approx 1.04\, T_c$
- numerics indicate peak/divergence at $\approx 1.02\, T_c$ (further checks needed)
- Kiritsis et al., Gubser et al.: mild rise $\frac{\zeta}{s}(T_c) \approx 0.06$
- Kharzeev et al. [JHEP 0809 (2008), pp. 093] sharp rise $\frac{\zeta}{s}(T_c) \approx 1$

Comparison with QPM

$$\Delta v_s^2 = \frac{1}{3} - v_s^2$$



[Bluhm, Kaempfer, Redlich AIP Conf.Proc. 1441 (2012) 877]



Outlook

Work in progress:

- better fits to EoS
- $\Rightarrow$ bulk viscosity
- understand approach of Gubser and Kiritsis

Extensions:

- finite ω: spectral function
- quarks
- finite chemical potential