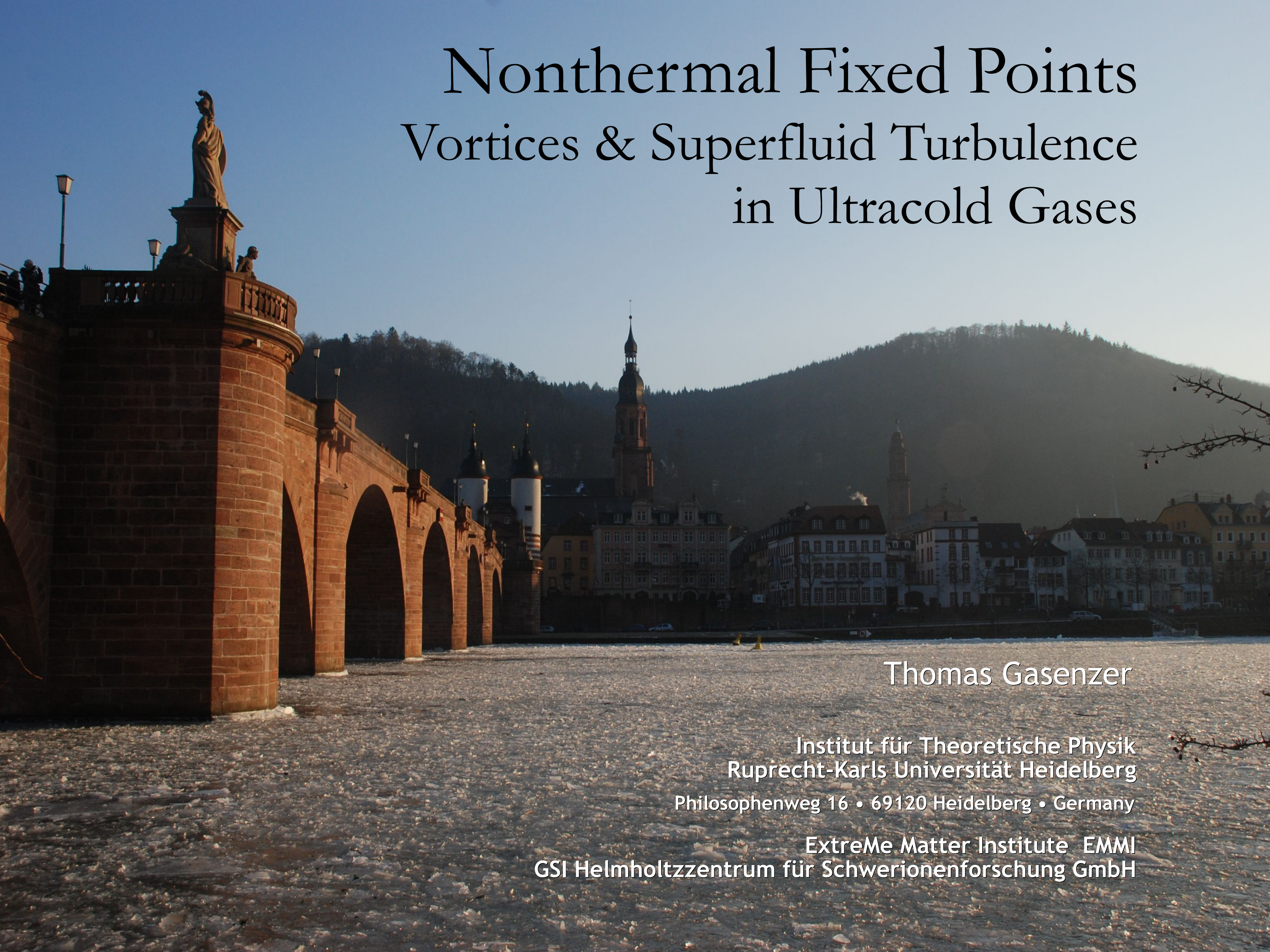

Nonthermal Fixed Points
Vortices & Superfluid Turbulence in Ultracold Gases

Thomas Gasenzer

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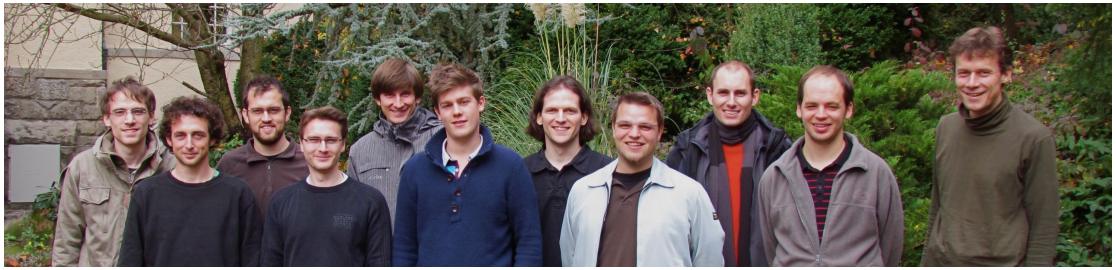
ExtreMe Matter Institute EMMI
GSI Helmholtzzentrum für Schwerionenforschung GmbH

Thanks & credits to...



...my work group in Heidelberg:

Sebastian Bock
Sebastian Erne
Martin Gärttner
Roman Hennig
Markus Karl
Steven Mathey
Boris Nowak
Nikolai Philipp
Jan Schole
Dénes Sexty
Martin Trappe
Pascal Weckesser

...my former students:

Jan Zill (→ Queensland), Maximilian Schmidt (→ Jülich), Cédric Bodet (→ NEC), Alexander Branschädel (→ KIT Karlsruhe), Stefan Keßler (→ U Erlangen), Matthias Kronenwett (→ R. Berger), Christian Scheppach (→ Cambridge, UK), Philipp Struck (→ Konstanz), Kristan Temme (→ Vienna)

€€€...

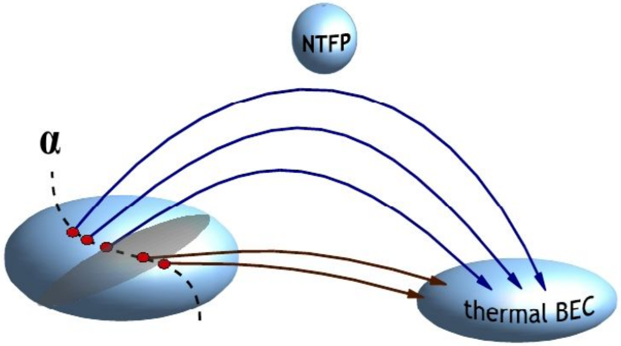
Alexander von Humboldt Stiftung / Foundation
Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft
Ruprecht-Karls-Universität Heidelberg
LGFG BaWue
DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service
Center for Quantum Dynamics
EMMI
Helmholtz Gemeinschaft

Equilibration

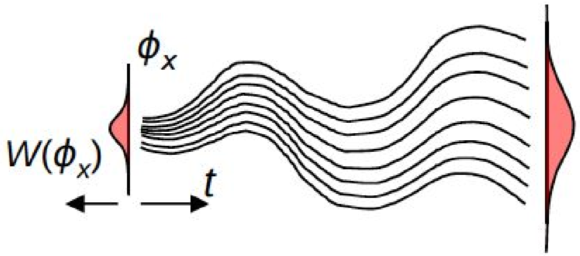
Initial state:
Far from equilibrium

Transient state,
e.g. Turbulence
Non-thermal fixed point

Final state:
Thermal equilibrium



Semi-classical Simulations



Classical field equation for ϕ(x,t):

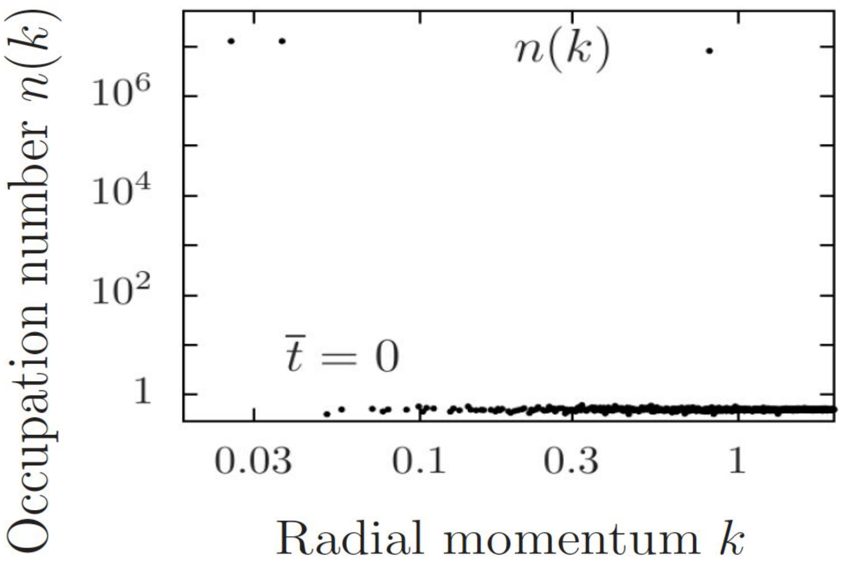
$$i\partial_t \phi(\mathbf{x},t) = \left[-\frac{\nabla^2}{2m} + g|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t)$$

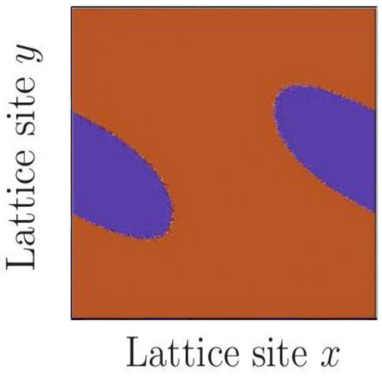
Observables: e. g. Momentum distribution

$$n(k) = \int \mathrm{d}^{d-1}\Omega_k \, \langle \phi^*(\mathbf{k})\phi(\mathbf{k})\rangle_{\mathrm{ensemble}}$$

2+1 D: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)





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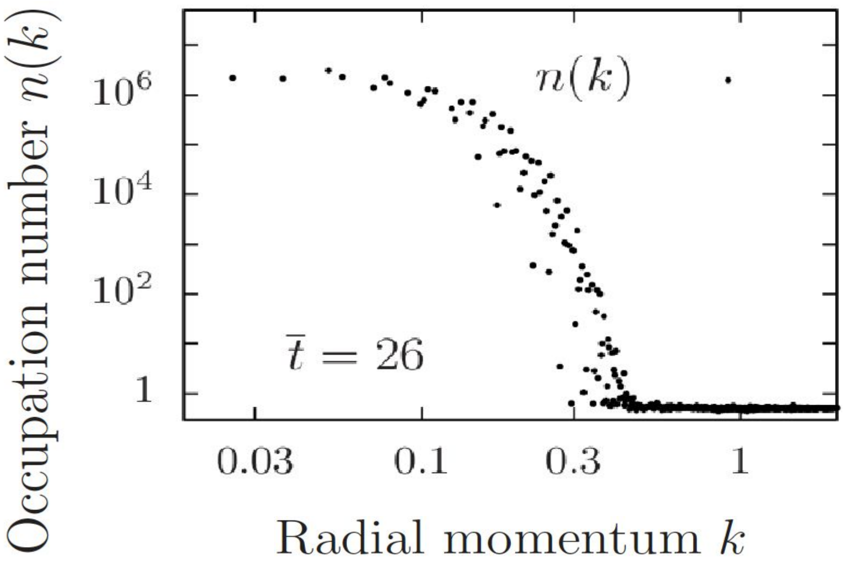

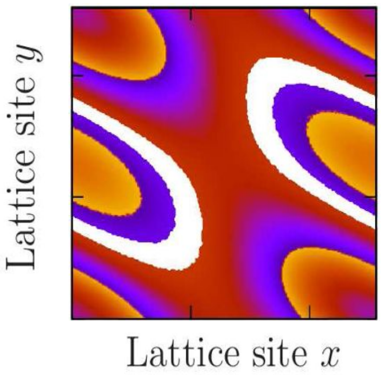

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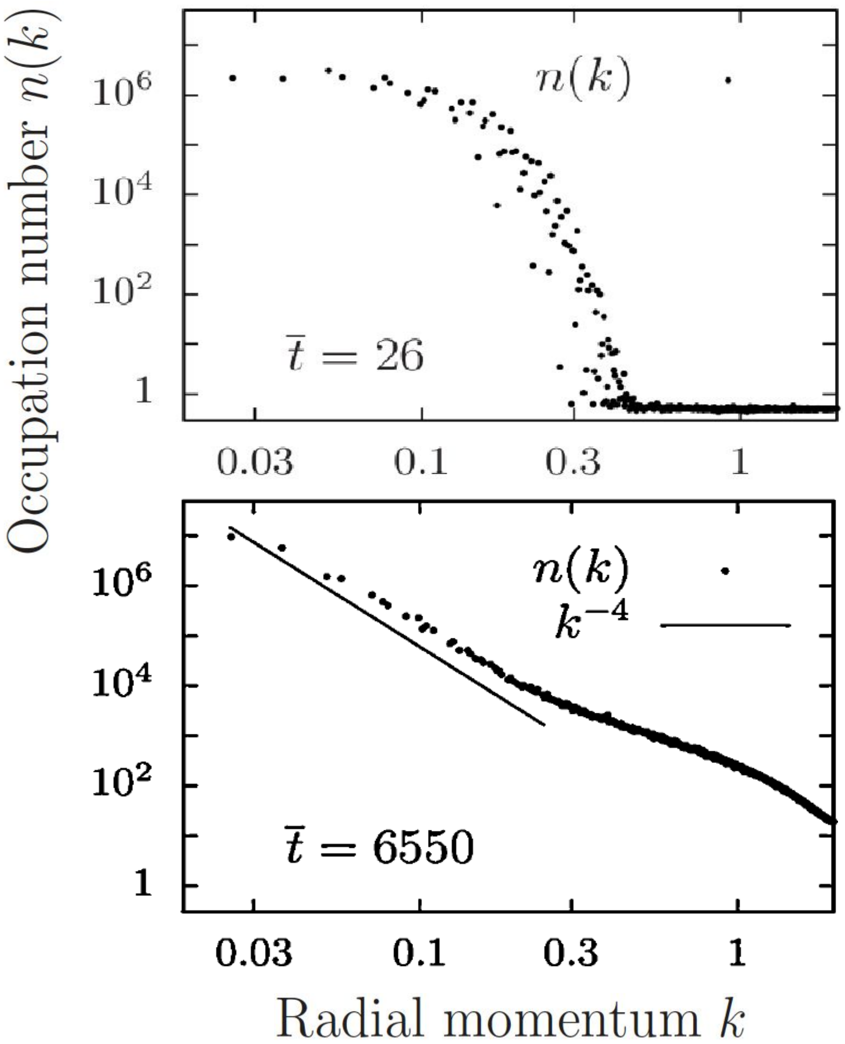

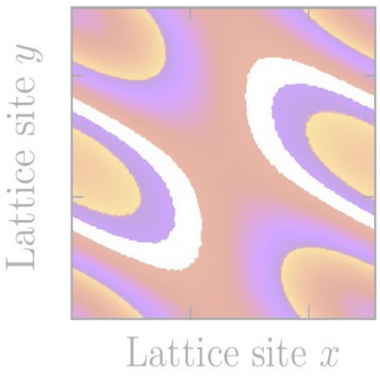

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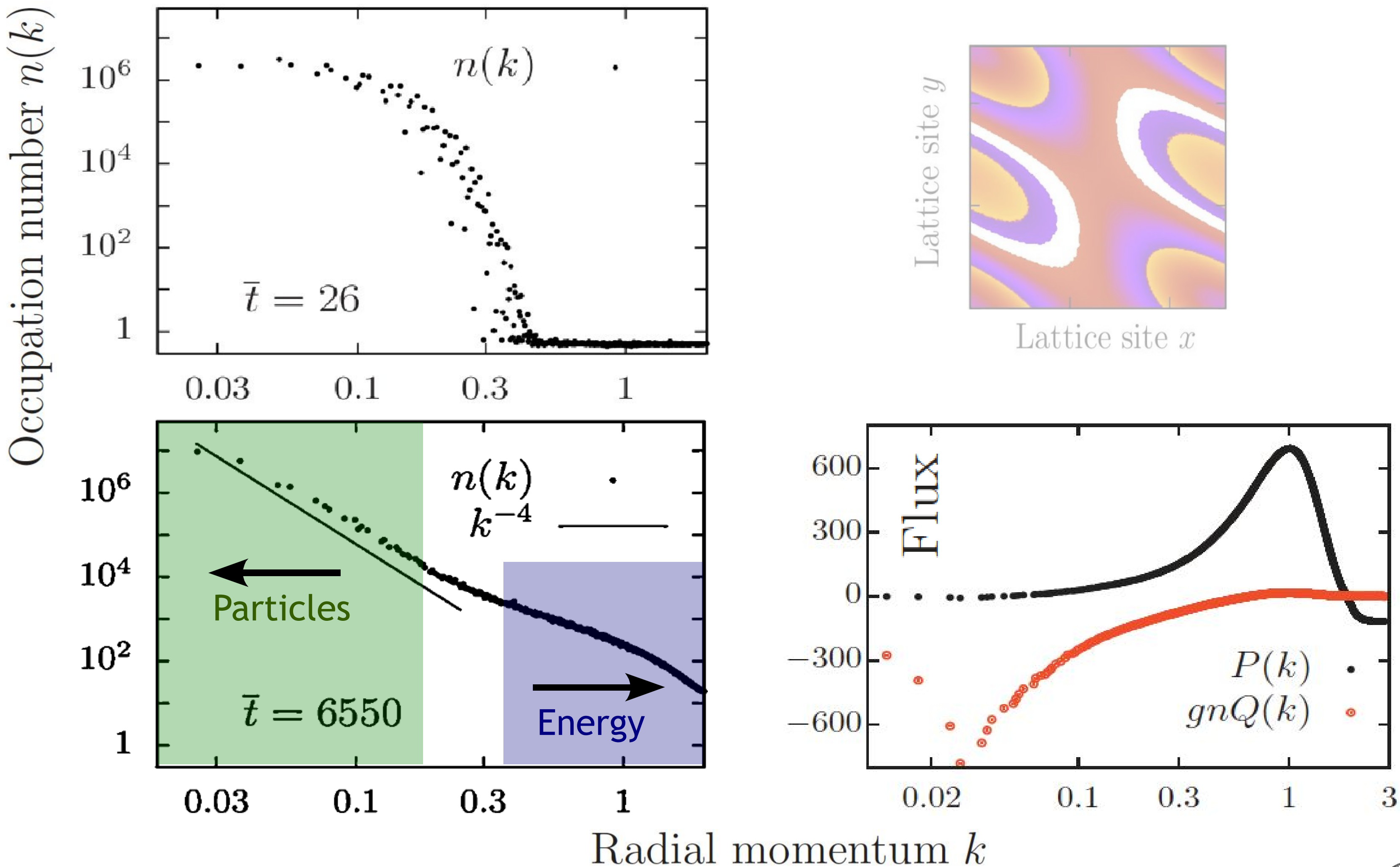

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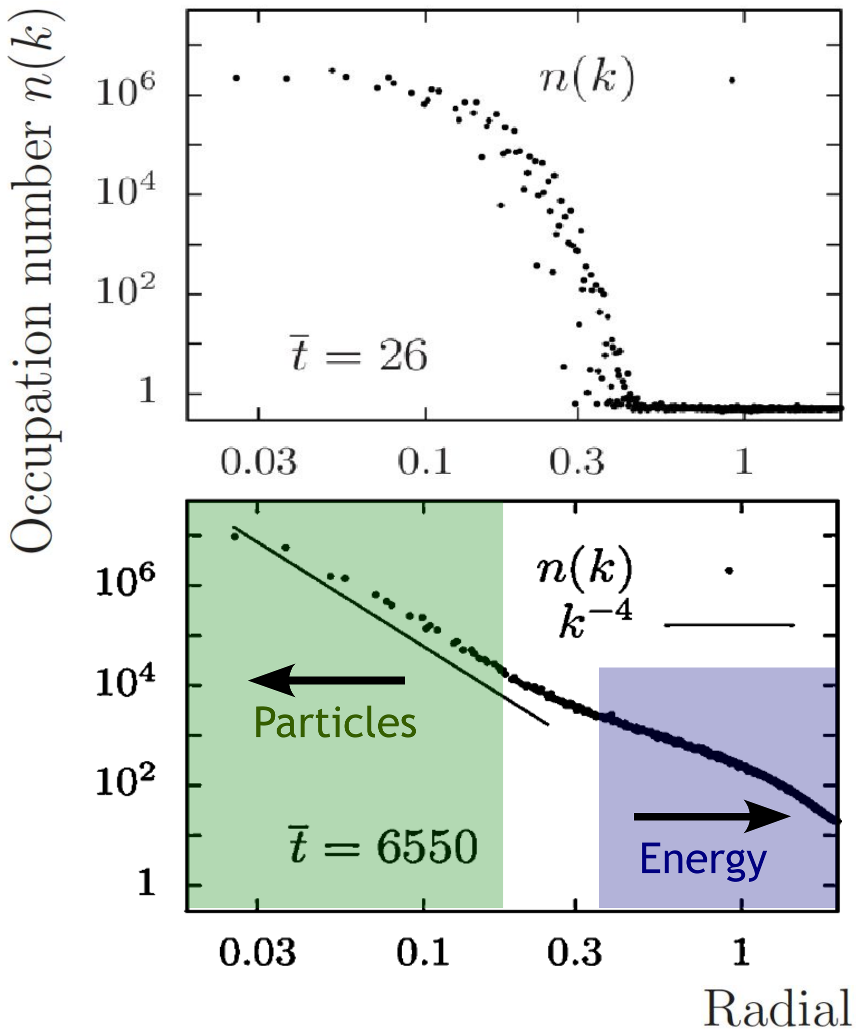
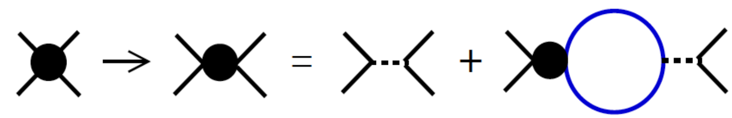
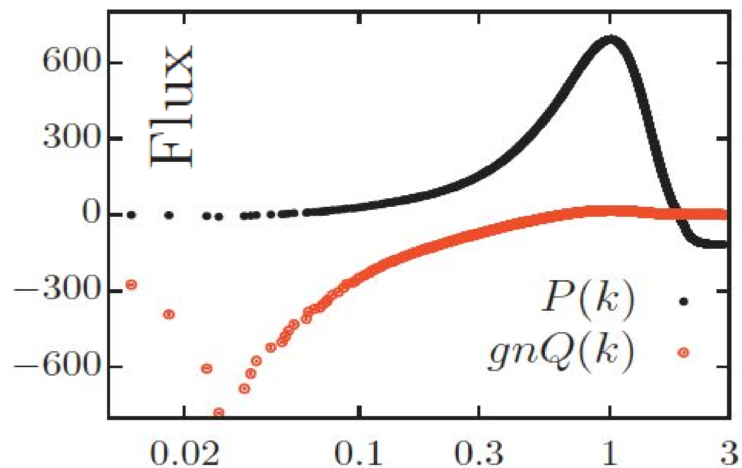
2+1 D: Quench dynamics





J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603,
J. Berges, G. Hoffmeister, NPB **813** (09) 383,
C. Scheppach, J. Berges, TG PRA **81** (10) 033611,



2+1 D: Quench dynamics

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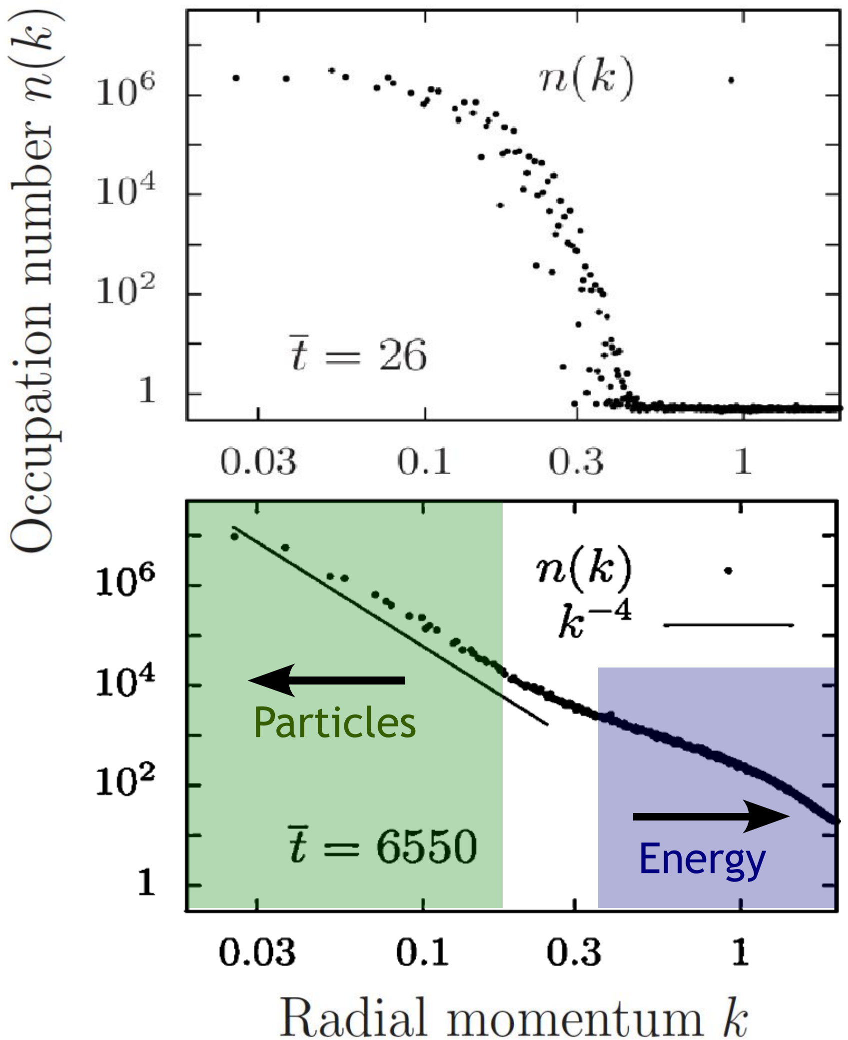
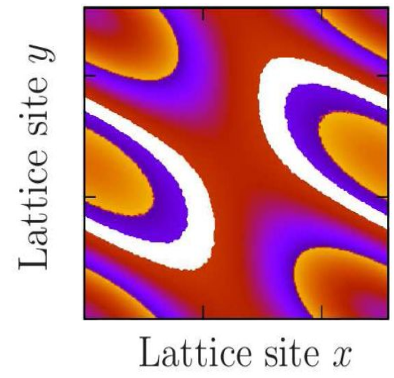
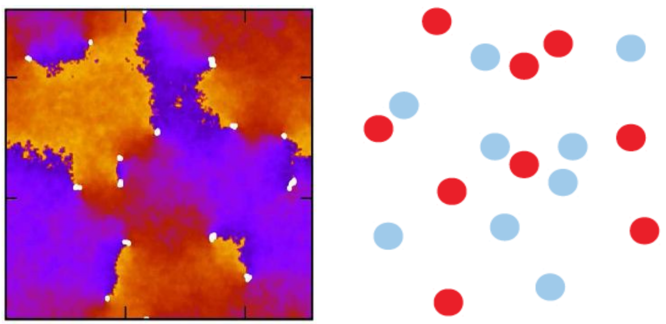


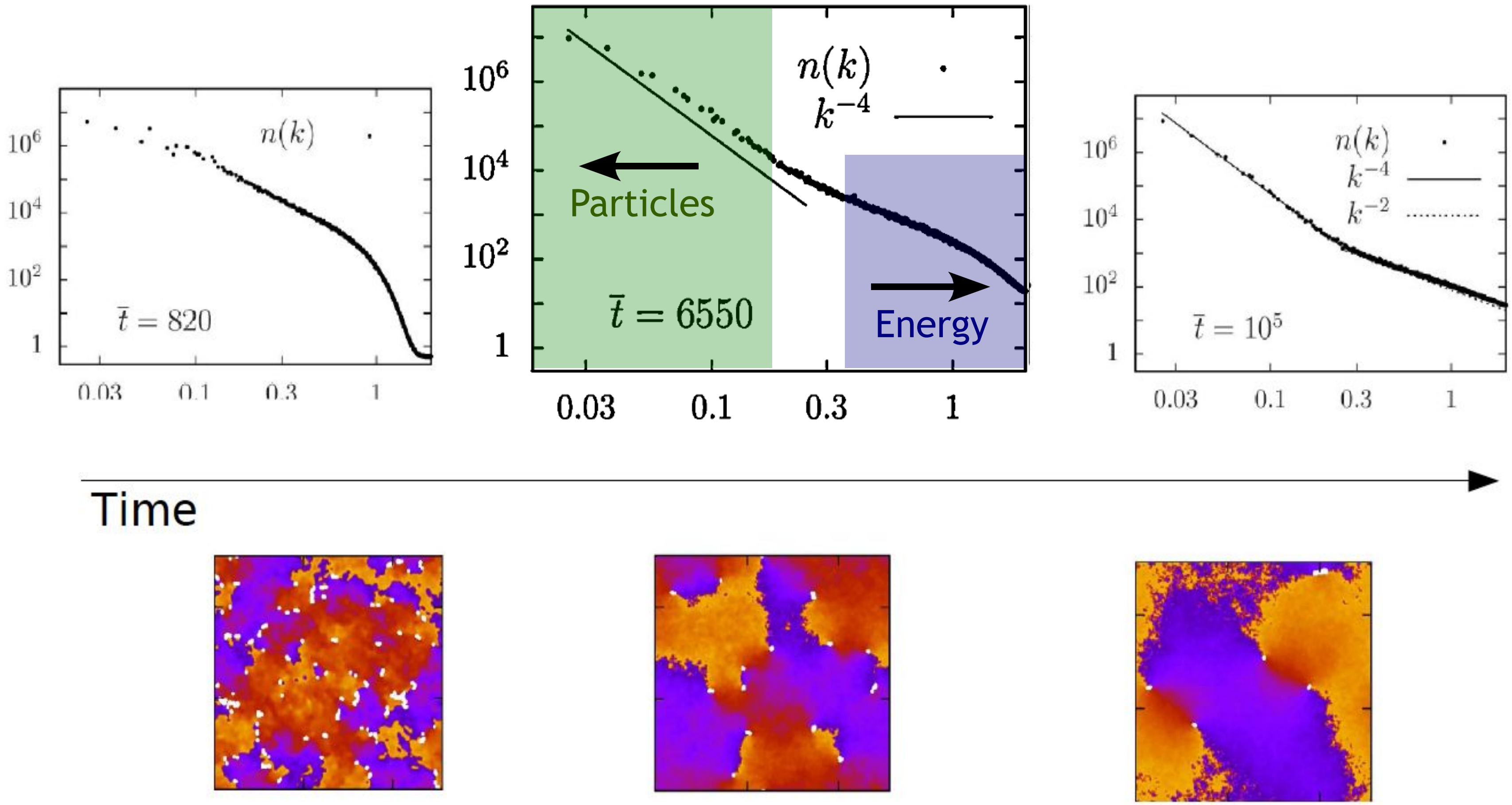




$n(k) \sim k^{-4}$

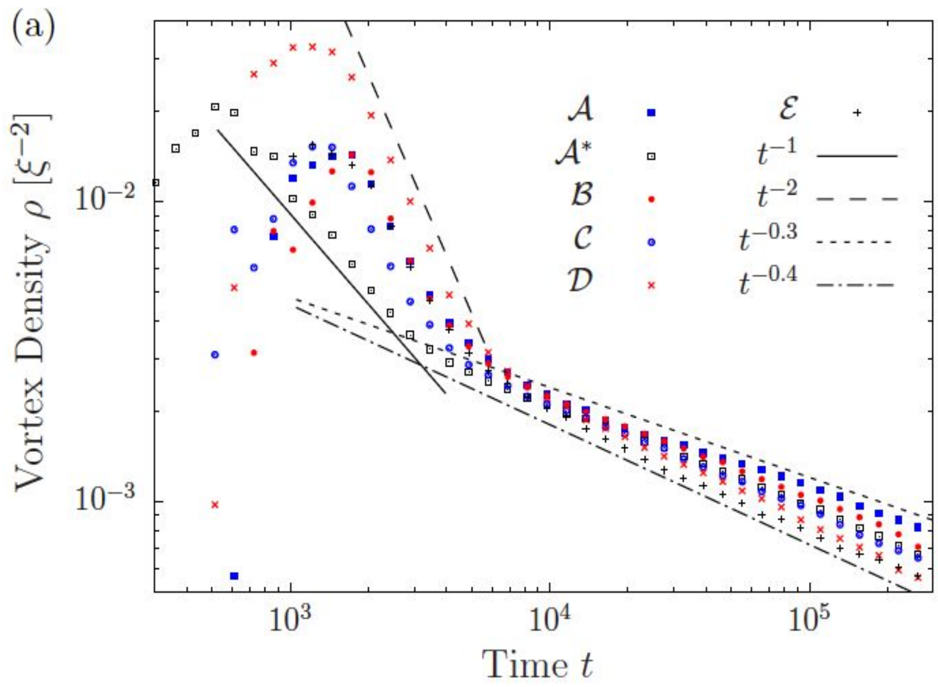
$\Leftrightarrow E(k) \sim k^{-1}$

2+1 D: Phase ordering dynamics



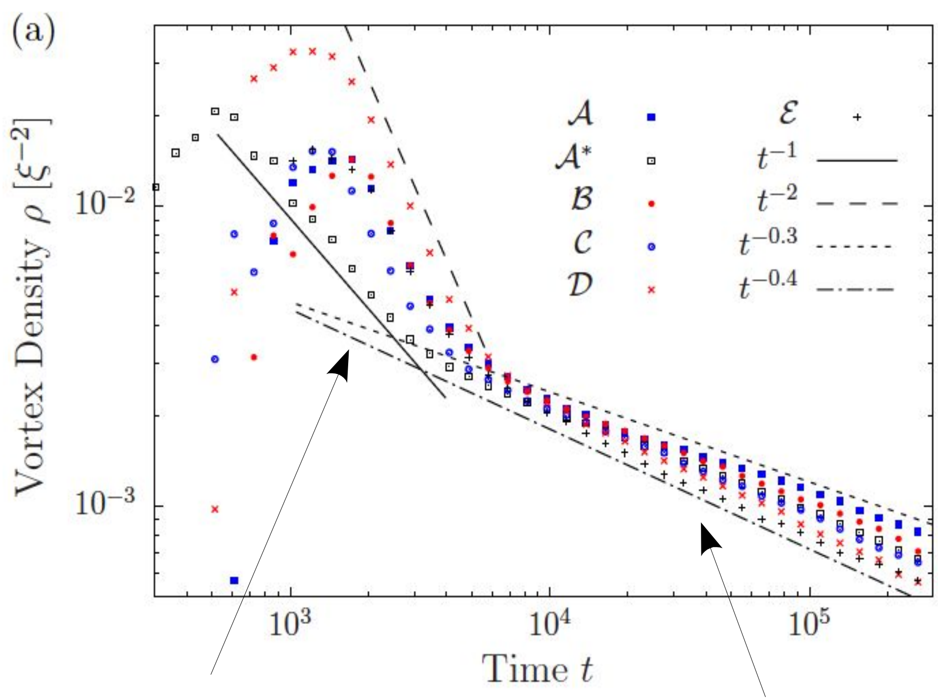
B. Nowak, D. Sexty, TG, PRB **84**(R) (11); B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)

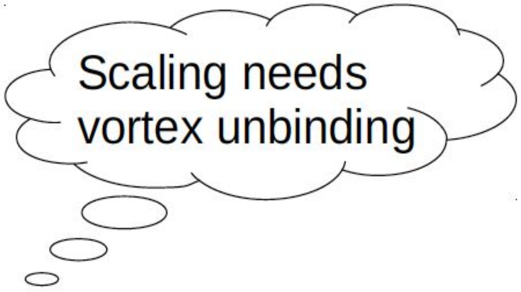
2+1 D: Phase ordering dynamics

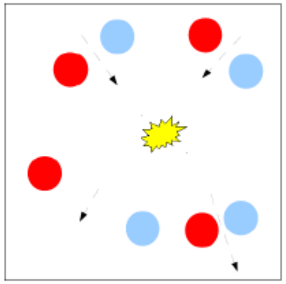


J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

2+1 D: Phase ordering dynamics





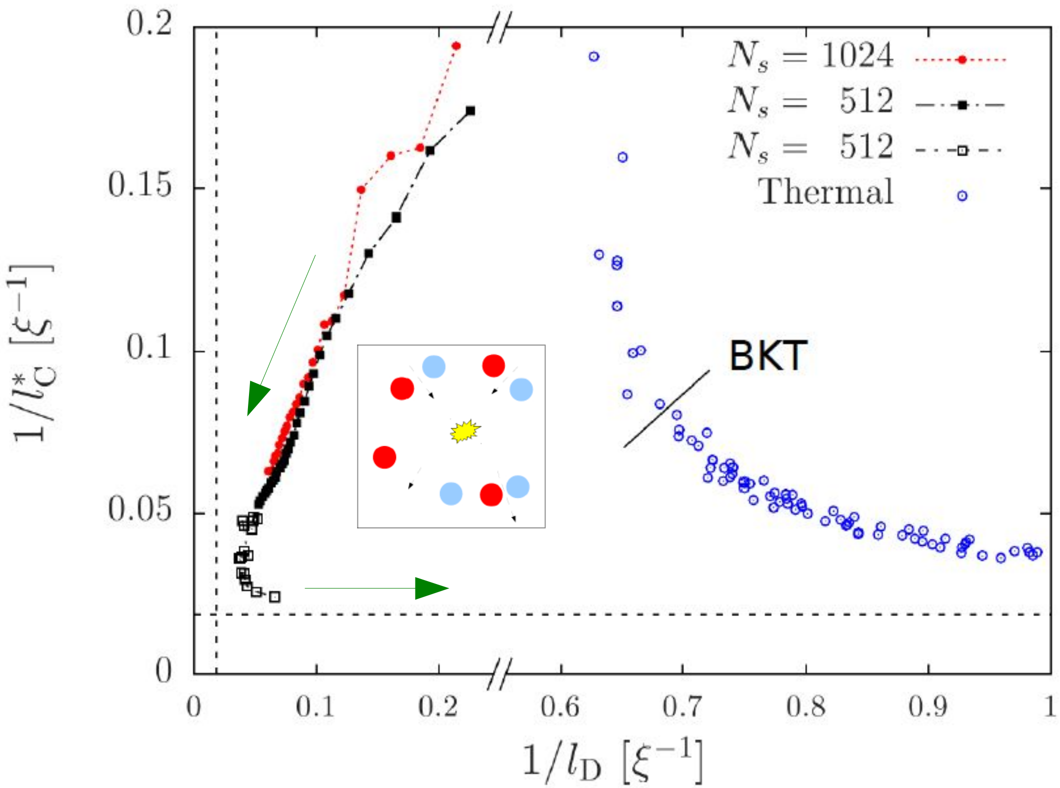


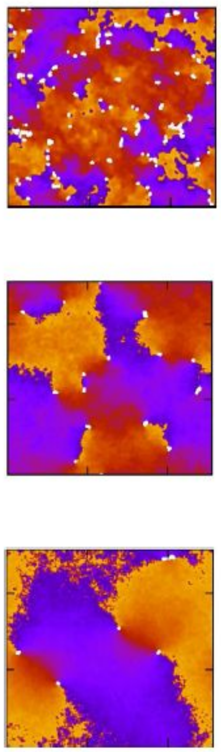
Non-universal decay law
(depends on initial vortex distribution)
Kinetic gas theory for dipoles

Universal decay regime
Strongly correlated dilute vortex gas
Scaling $n(k) \sim k^{-4}$

J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

Approach of the NTFP



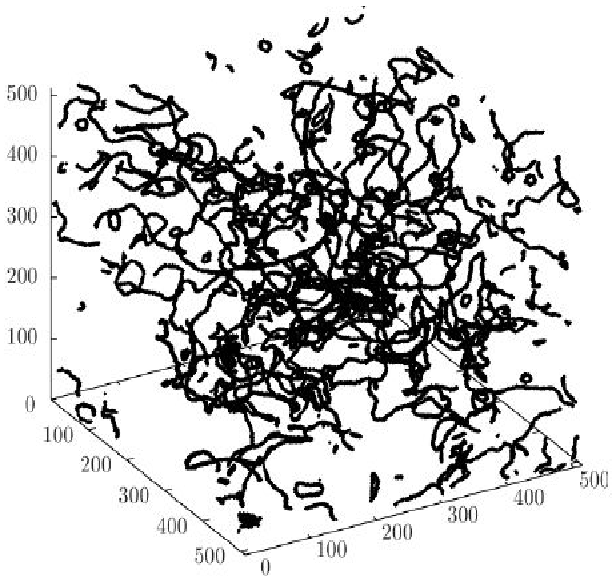


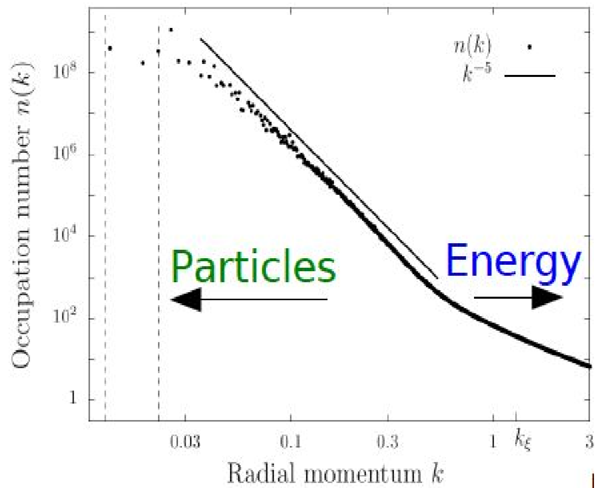
l_C^* = Phase coherence length

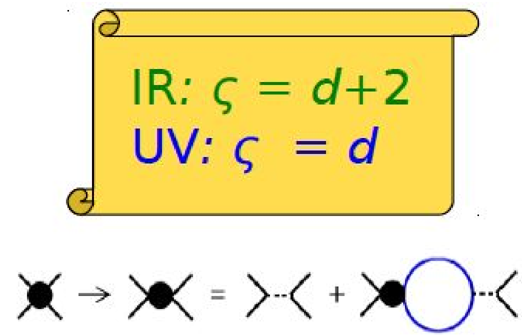
l_D = Vortex-Antivortex pair distance

J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

3D Nonthermal Fixed Point







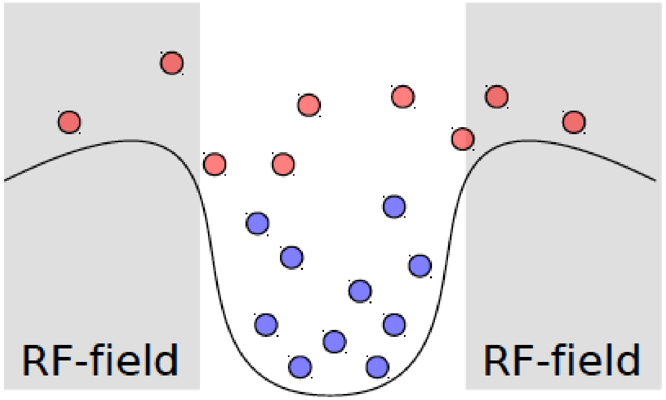
Vortices ← Spectrum → QFT

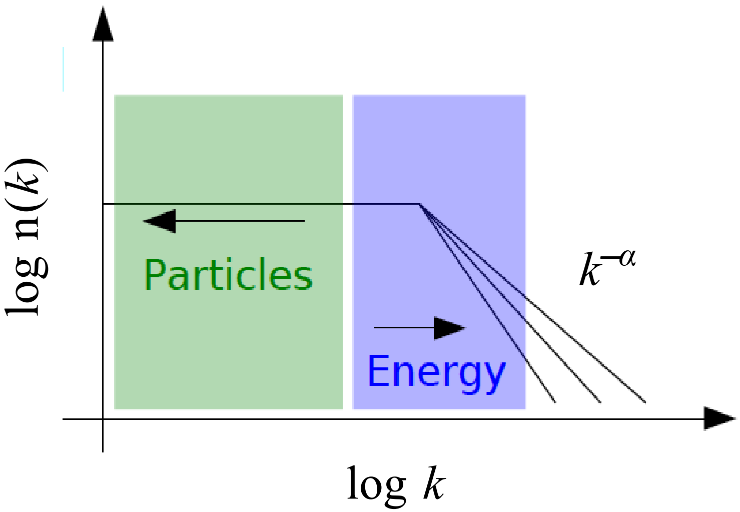
Quantum Turbulence

Strong & Weak Wave Turbulence

B. Nowak, D. Sexty, TG, PRB **84**(R) (11); B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)

3D: Bose Condensation





Experiments

Hänsch, Esslinger (02)
Esslinger (07)
Hadzibabic (12)

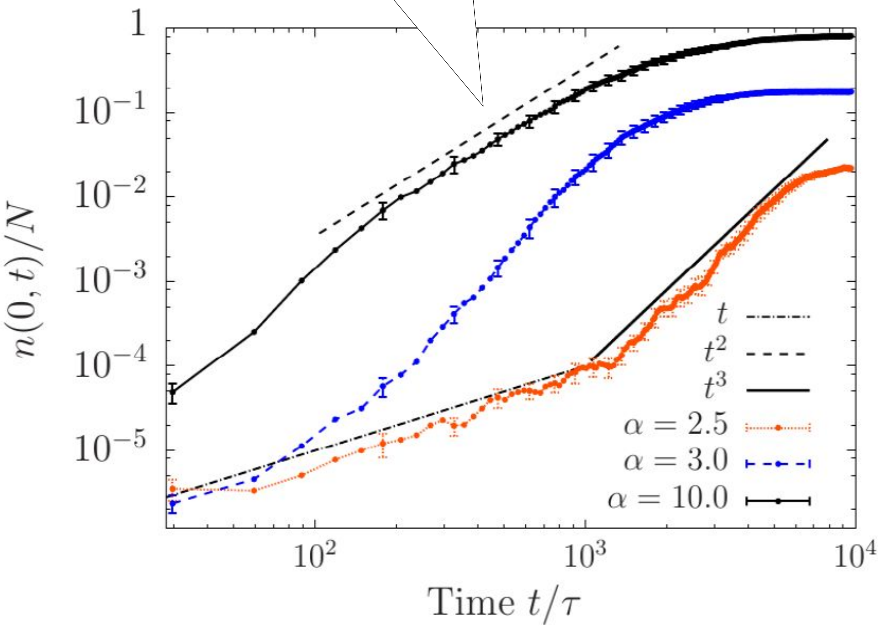
Condensation Dynamics

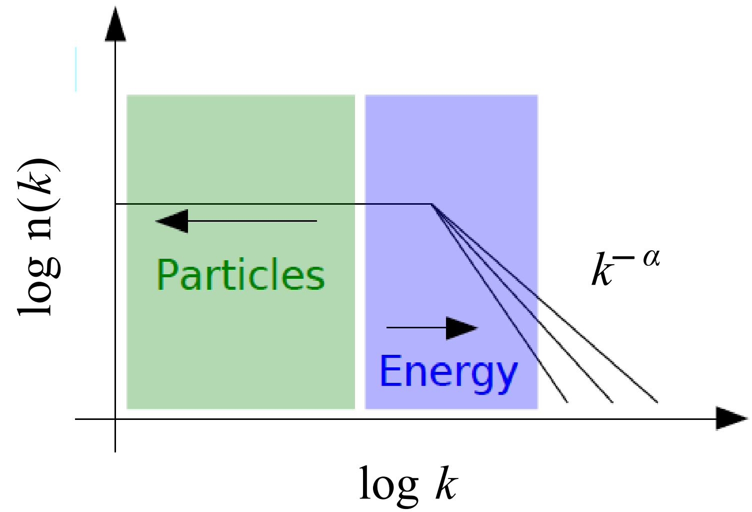
Levich, Yakhot (70s);
Snoke, Wolfe (89);
Kagan, Svistunov, Shlyapnikov (91-94);
Damle, Sachdev (96)
Semikoz, Tkachev (95)
Berloff, Svistunov (02)
Anderson, Davis (08)
Blaizot, McLerran (12)
Berges, Sexty (12)

B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

3D: Bose Condensation

Slowed condensation dynamics

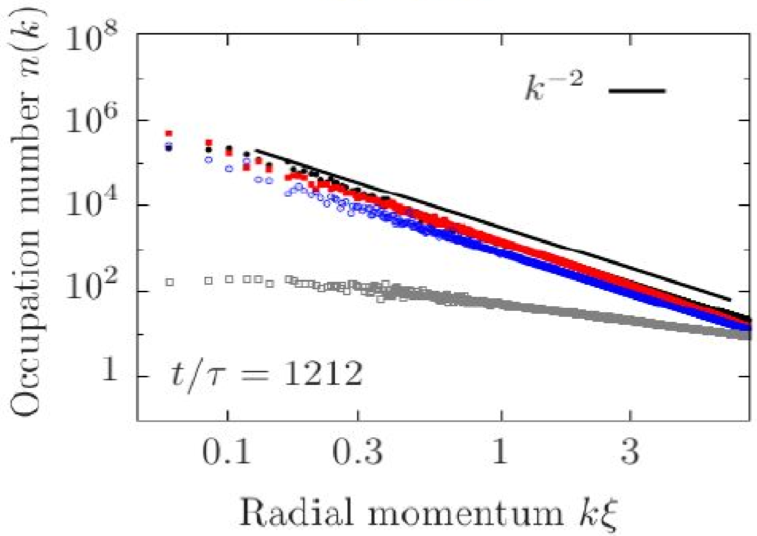




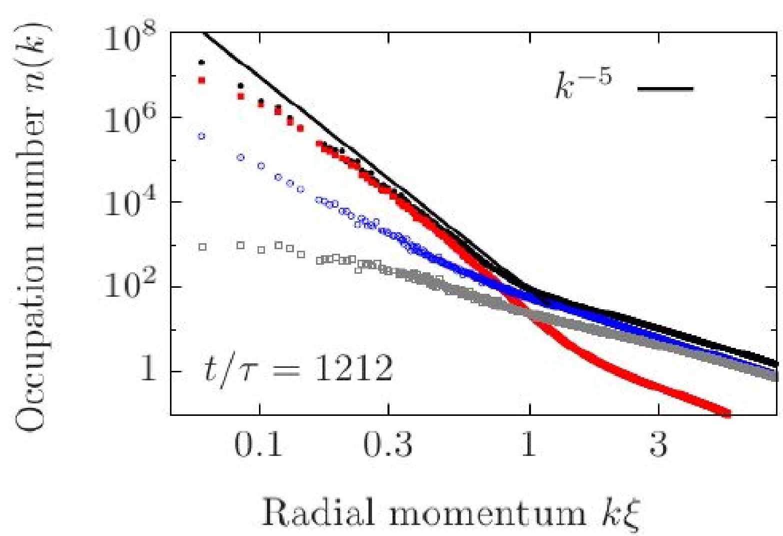
B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

3D: Bose Condensation

α=2.5

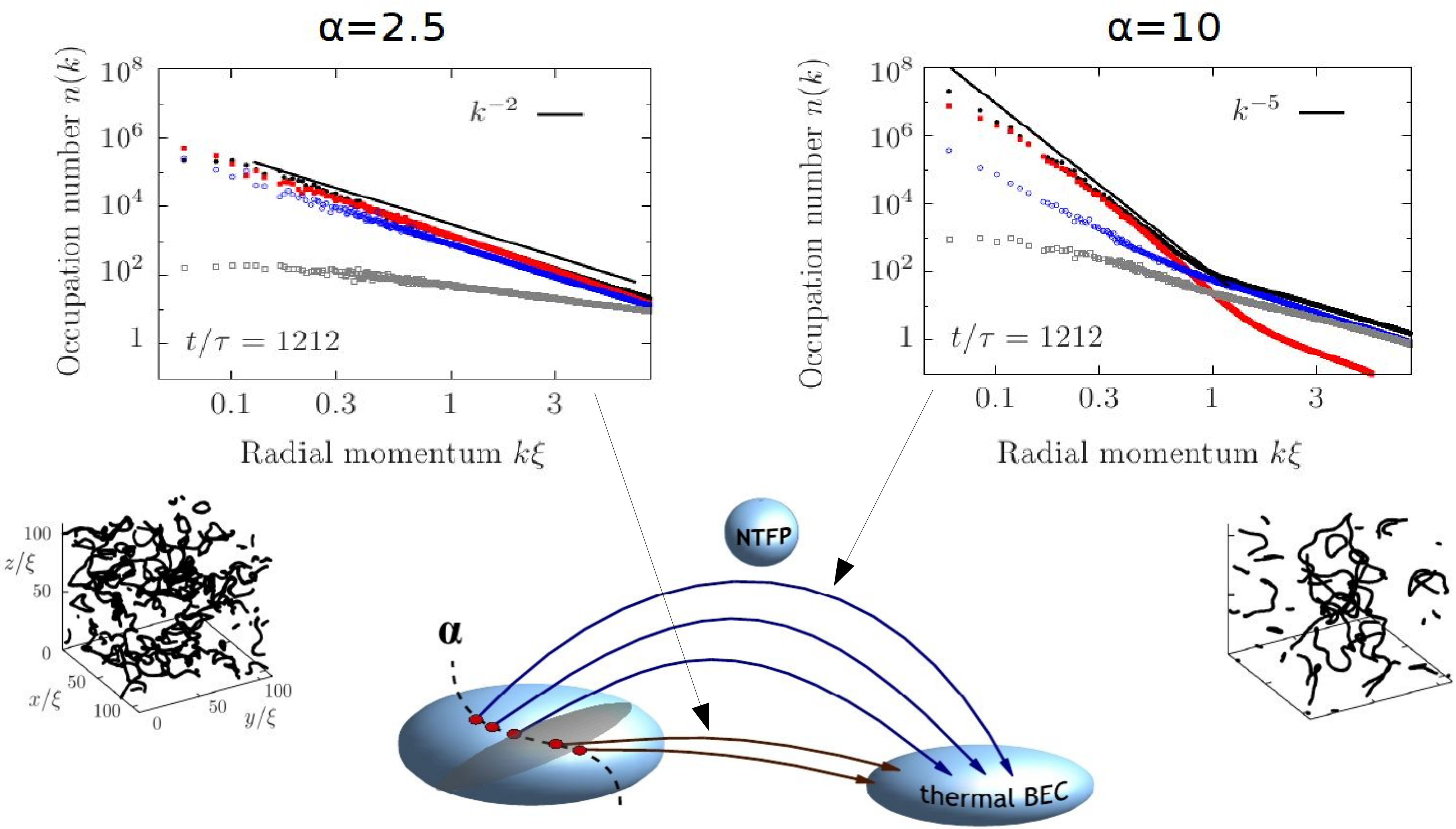


α=10



B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

3D: Bose Condensation

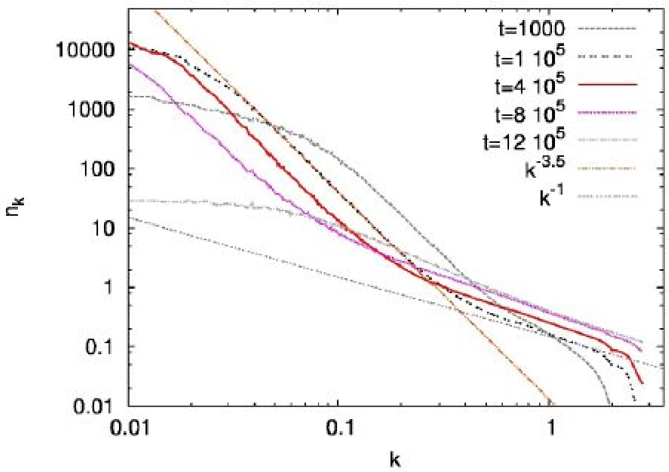


B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

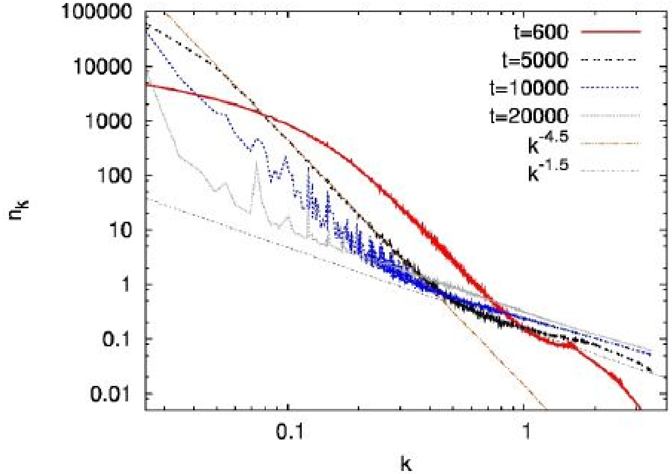
Relativistic simulations

Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

d=2



d=3



reheating after inflation

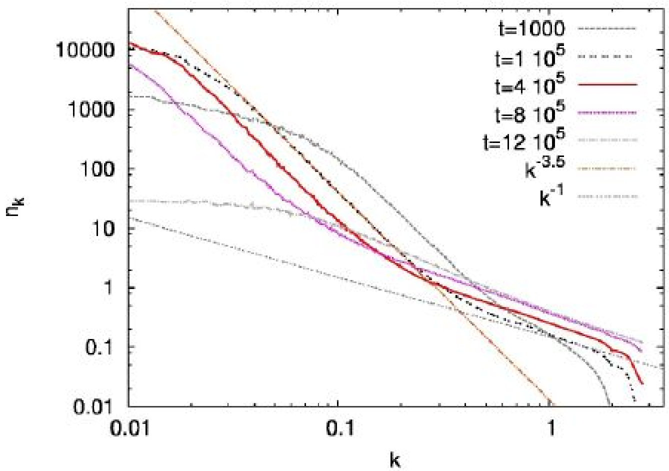
S. Khlebnikov, I. Tkachev, PRL (96)
R. Micha, I. Tkachev, PRD (04)
J. Berges, A. Rothkopf, J. Schmidt, PRL (08)
J. Berges, D. Sexty, PRD (11)

D. Sexty, B. Nowak, TG, PLB **710** (12)

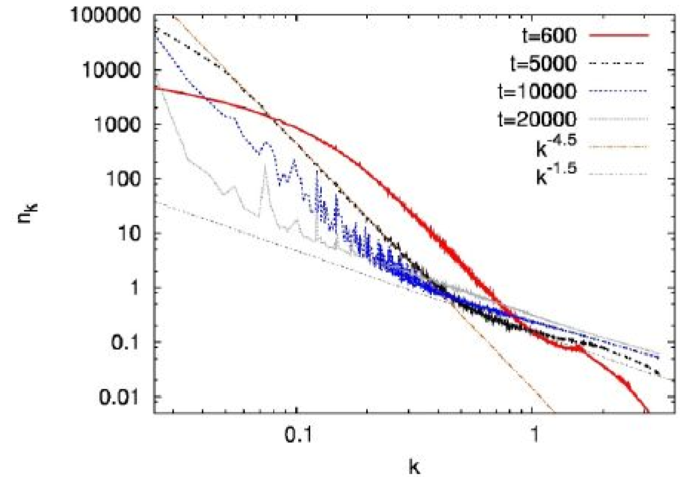
Relativistic simulations

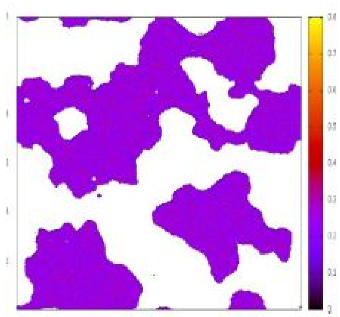
Classical field equation: $$\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$$

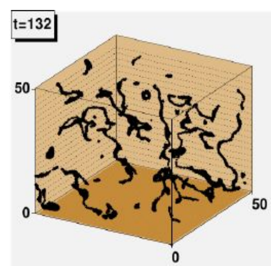
d=2



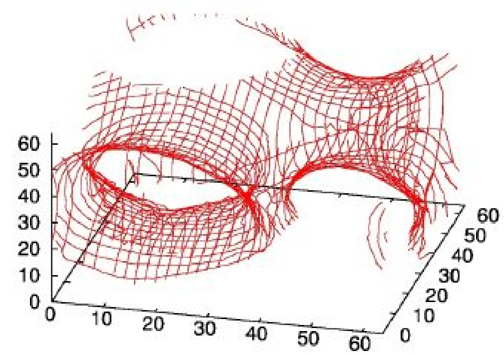
d=3







Tkachev (98)

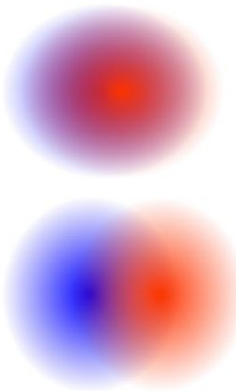


D. Sexty, B. Nowak, TG, PLB **710** (12)

2-component BEC

miscible
$g_{12} < g$

immiscible
$g_{12} > g$

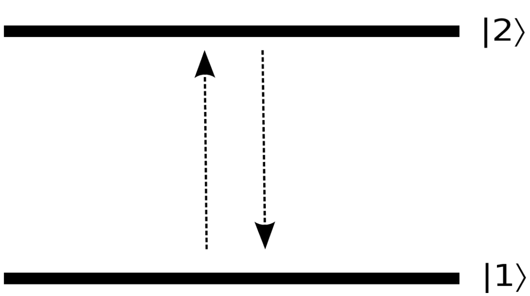


Bose gas with internal two-level structure

non-linear interactions:

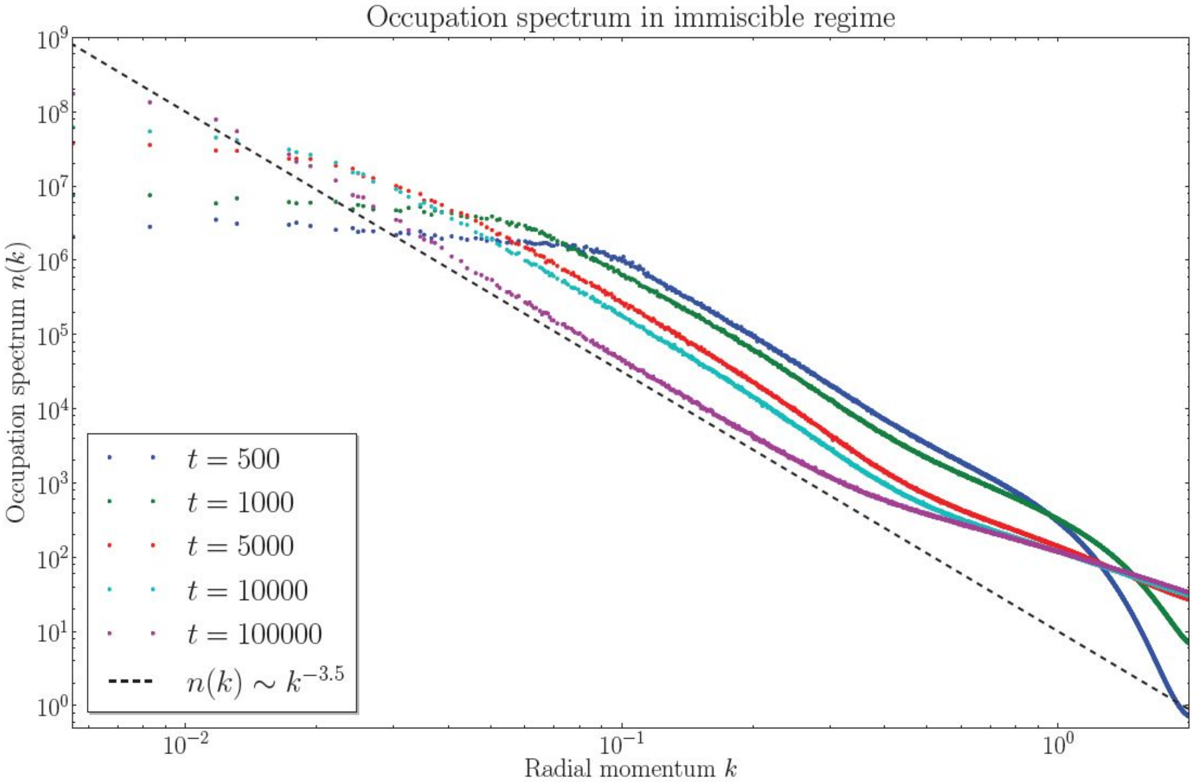
$$\hat{H}_{int} \sim \frac{g}{2}(\hat{n}_1^2 + \hat{n}_2^2) \quad \begin{matrix} |1\rangle \longleftrightarrow |1\rangle \\ |2\rangle \longleftrightarrow |2\rangle \end{matrix}$$

$$+ \; g_{12}\hat{n}_1\hat{n}_2 \quad |1\rangle \longleftrightarrow |2\rangle$$



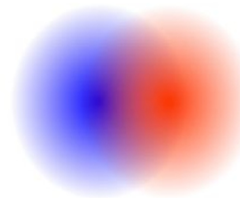
M. Karl, B. Nowak, TG, unpublished (12)

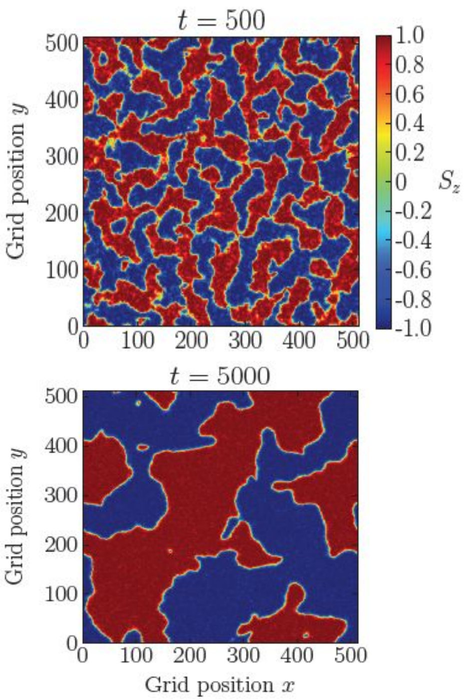
2-component BEC



miscible
$g_{12} < g$

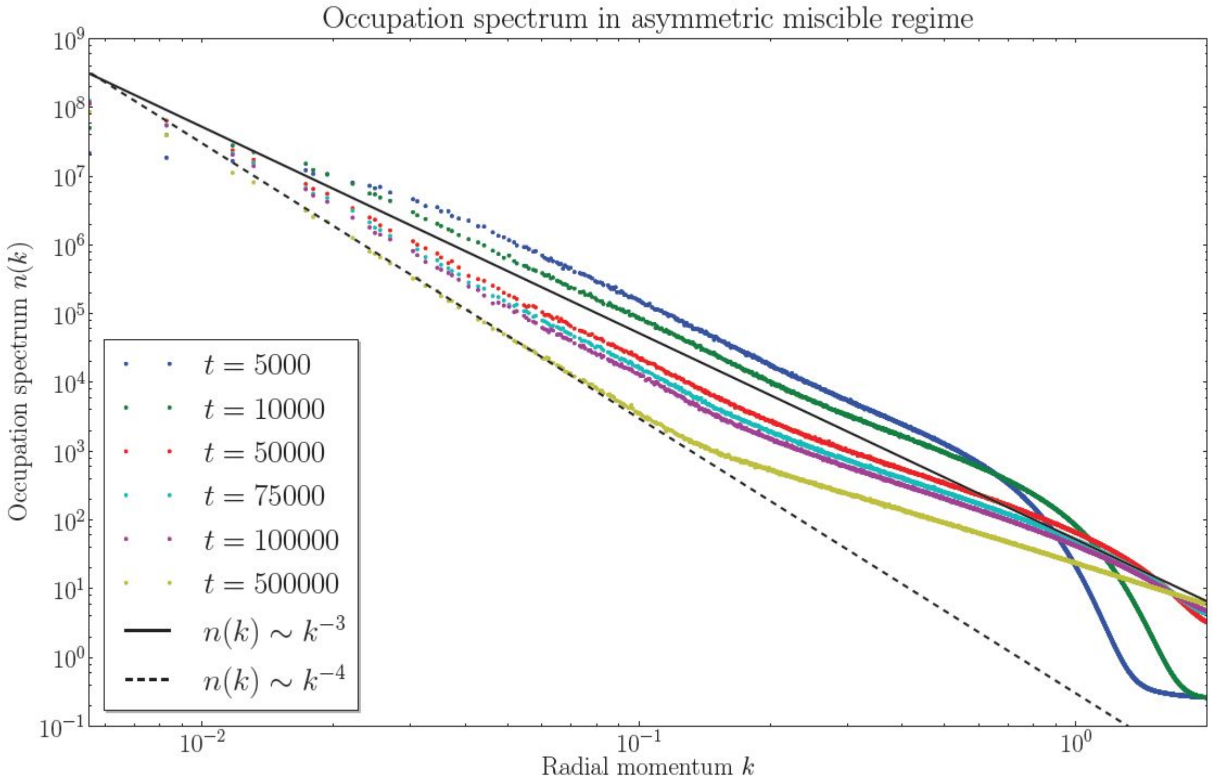
immiscible
$g_{12} > g$





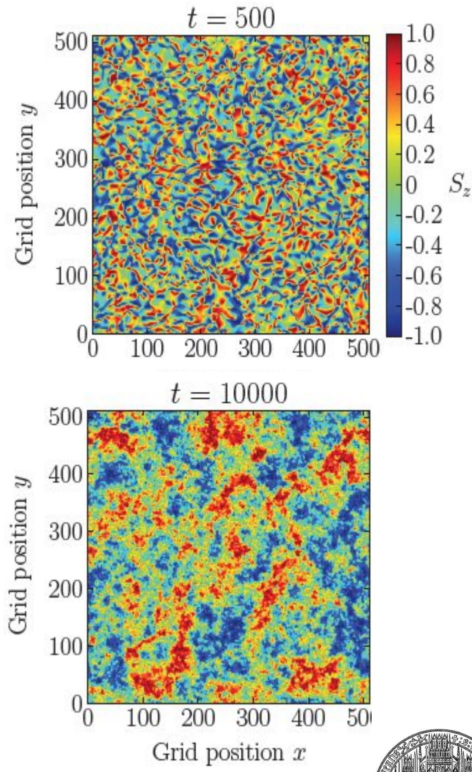
M. Karl, B. Nowak, TG, unpublished (12)

2-component BEC



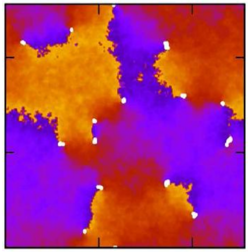
miscible
$g_{12} < g$

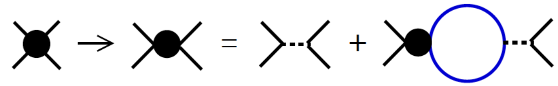
immiscible
$g_{12} > g$



M. Karl, B. Nowak, TG, unpublished (12)

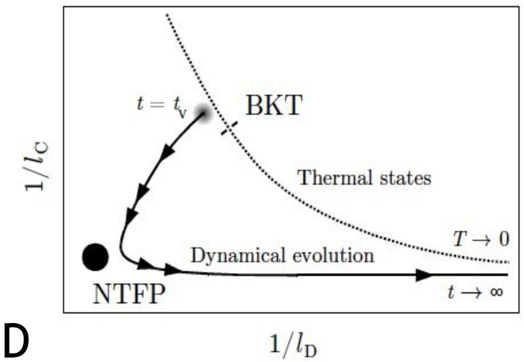
Summary



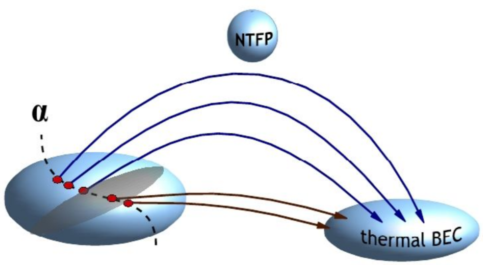


Non-Thermal Fixed Points

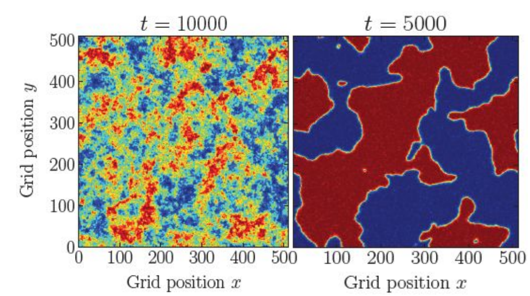
Superfluid Turbulence in 2D



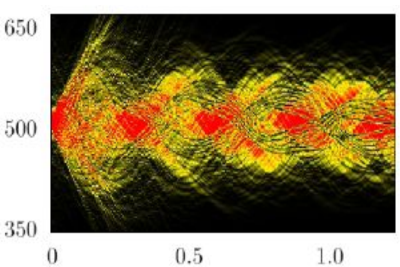
Approach of the NTFP in 2D



Dynamics of BE condensation

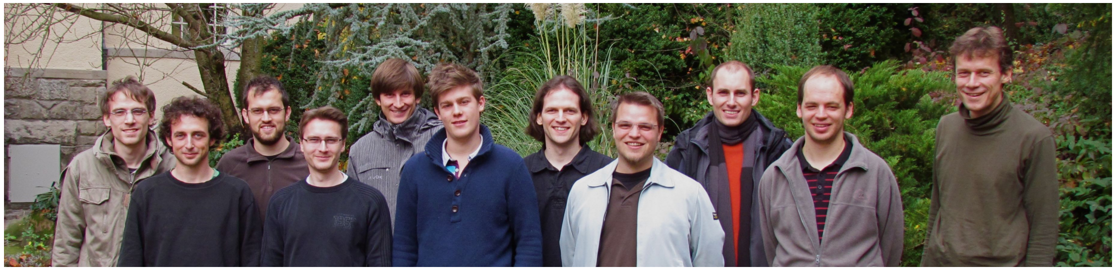


Charge separation/Pattern formation



1D Soliton Gas as an NTFP

Thanks & credits to...



...my work group in Heidelberg:

Sebastian Bock
Sebastian Erne
Martin Gärttner
Roman Hennig
Markus Karl
Steven Mathey
Boris Nowak
Nikolai Philipp
Jan Schole
Dénes Sexty
Martin Trappe
Pascal Weckesser
Jan Zill

...my former students:

Maximilian Schmidt (→ Jülich), Cédric Bodet (→ NEC), Alexander Branschädel (→ KIT Karlsruhe), Stefan Keßler (→ U Erlangen), Matthias Kronenwett (→ R. Berger), **Christian Scheppach** (→ Cambridge, UK), Philipp Struck (→ Konstanz), Kristan Temme (→ Vienna)

€€€...

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DFG Heisenberg-Programm Deutsche Forschungsgemeinschaft

Ruprecht-Karls-Universität Heidelberg

LGFG BaWue

DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service

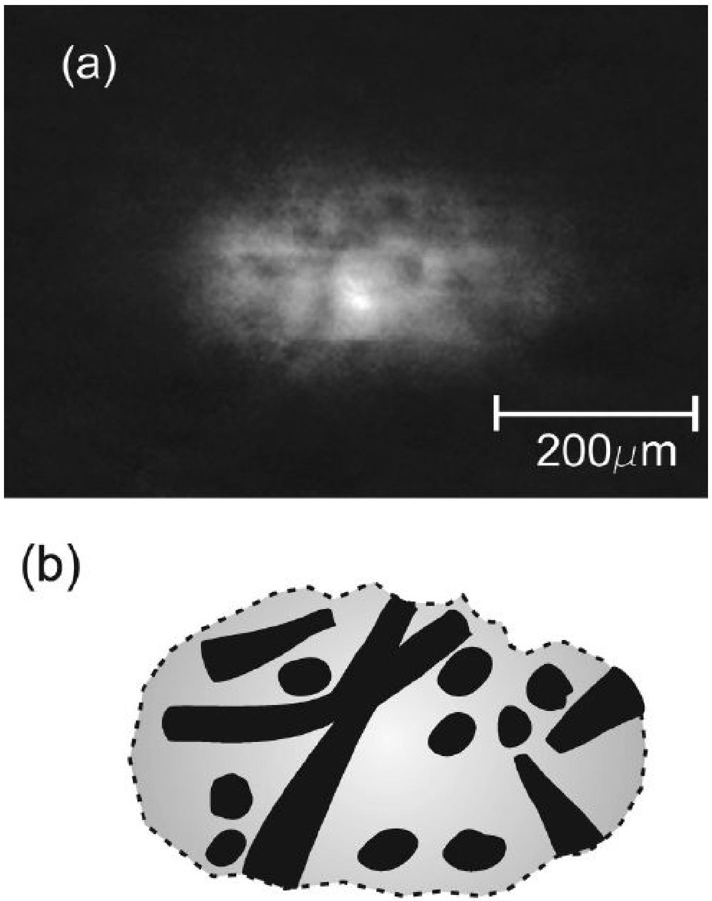
Center for Quantum Dynamics

EMMI

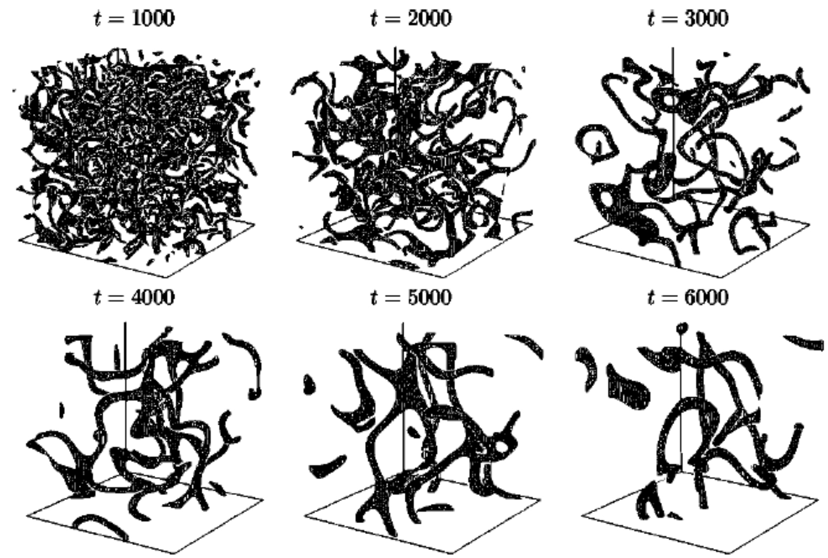
Helmholtz Gemeinschaft

Superfluid Turbulence

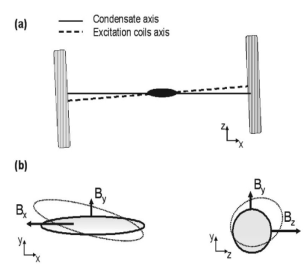
Vortex tangles in BEC, Superfluid Turbulence



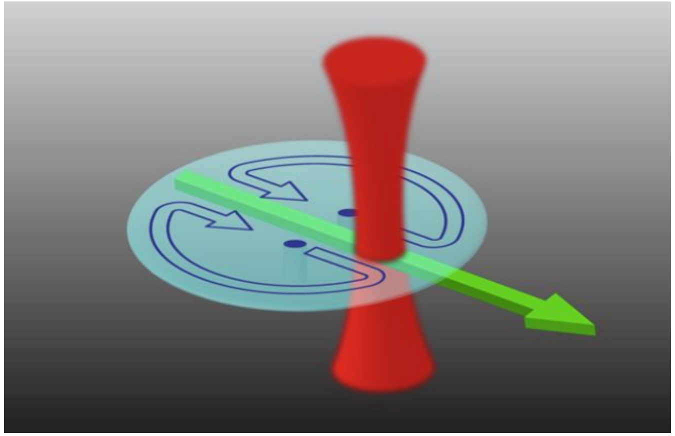
[E.A.L. Henn et al. PRL **103** (09)]

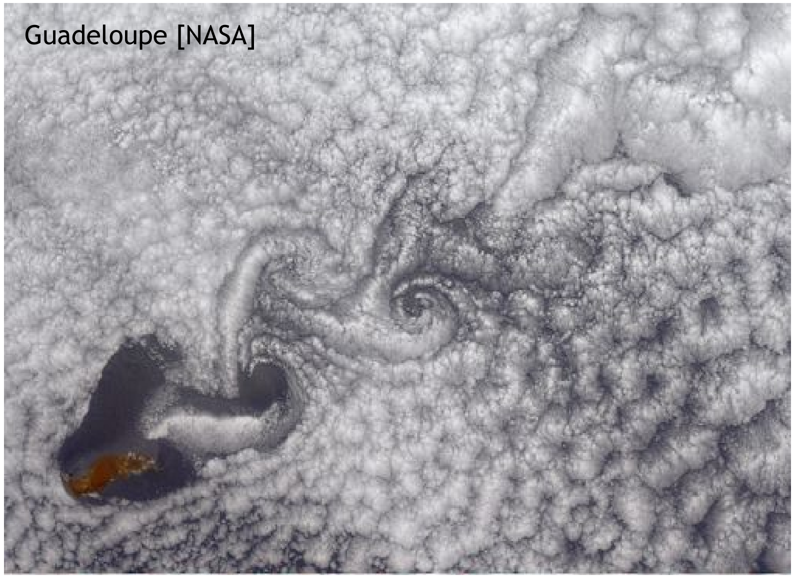


[N. Berloff & B. Svistunov, PRA (02)]

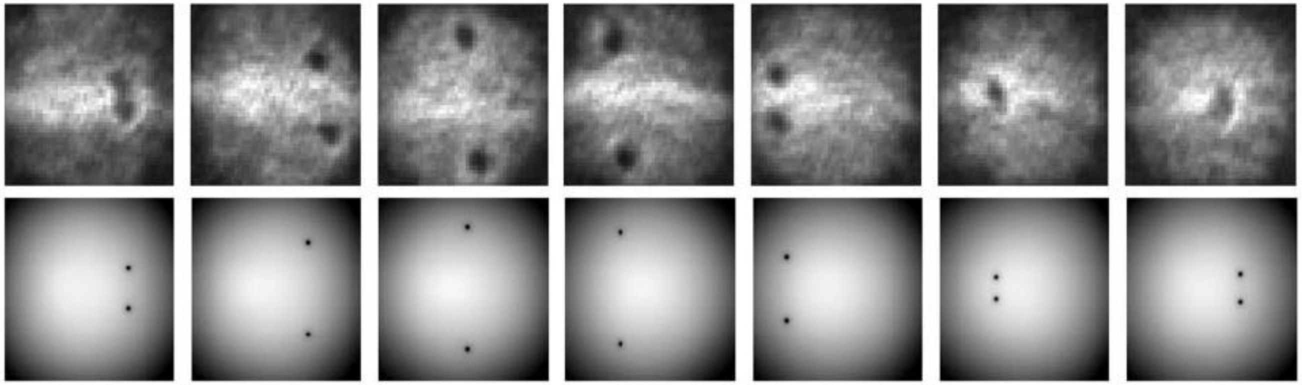


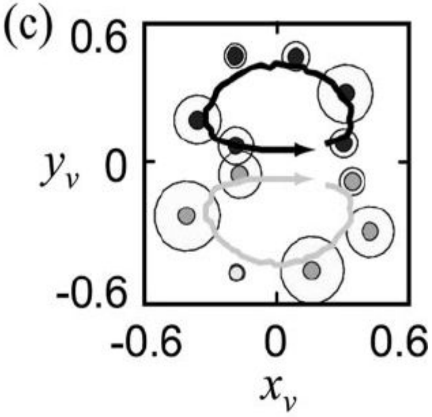
Vortex pairs





Tucson [AZ]





[T.W. Neely et al. PRL **104** (10)] (Anderson group Tucson)

Vortices in a Na condensate

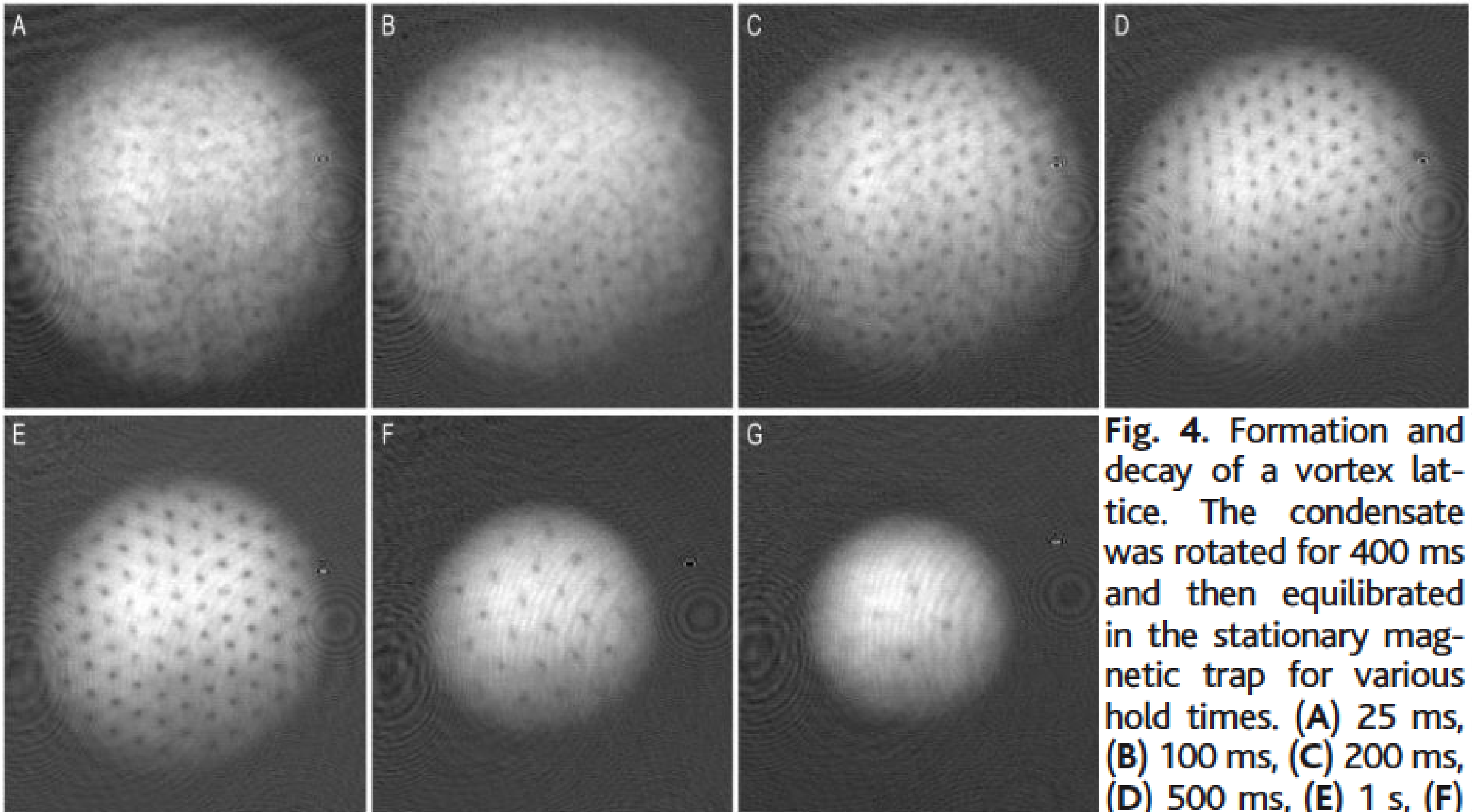
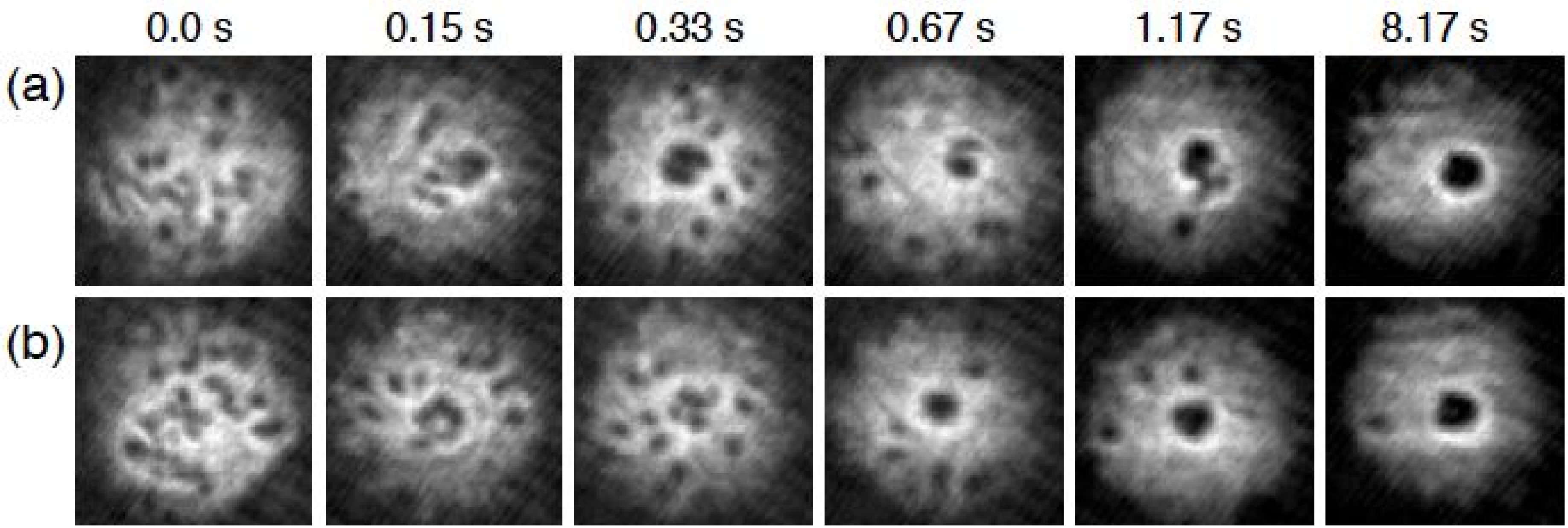


Fig. 4. Formation and decay of a vortex lattice. The condensate was rotated for 400 ms and then equilibrated in the stationary magnetic trap for various hold times. (A) 25 ms, (B) 100 ms, (C) 200 ms, (D) 500 ms, (E) 1 s, (F) 5 s, (G) 10 s,

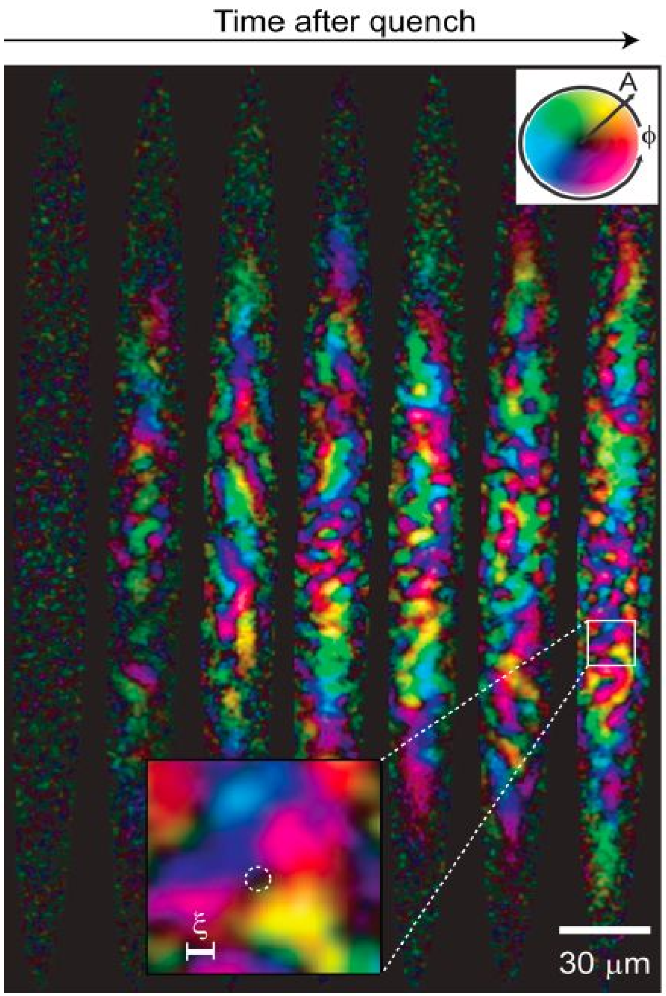
J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle
20 APRIL 2001 VOL 292 SCIENCE

Vortex pairs in experiment



T.W. Neely et al. 1204.1102 [cond-mat.quant-gas] (Anderson group, Tucson)

Nonthermal Fixed Points



[L.E. Sadler et al., Nature 443 (06), after A. Polkovnikov et al., RMP (11)]

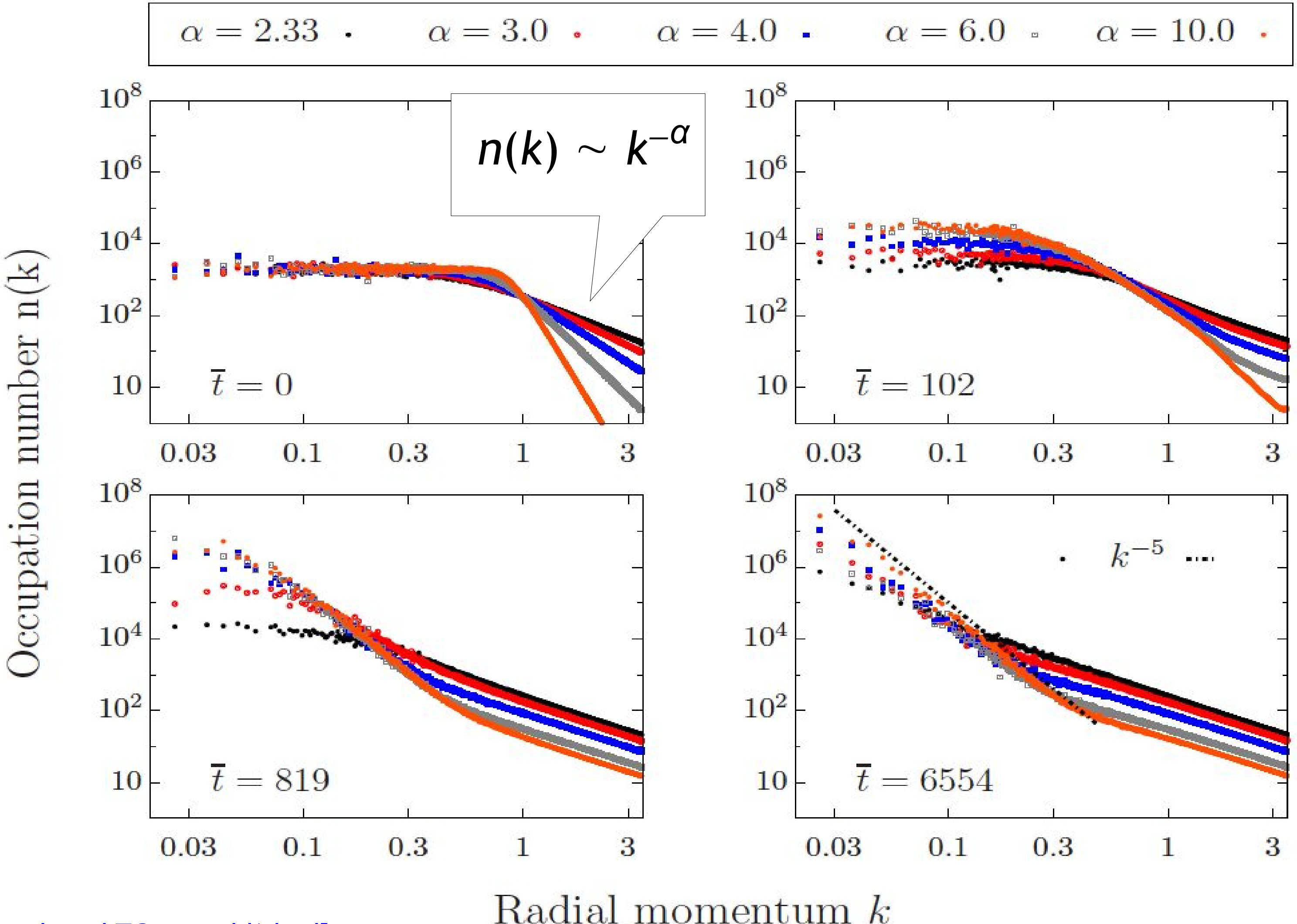
Domain & defect formation in Spinor BEC

(Stamper-Kurn group, Berkeley)

Supplementary slides

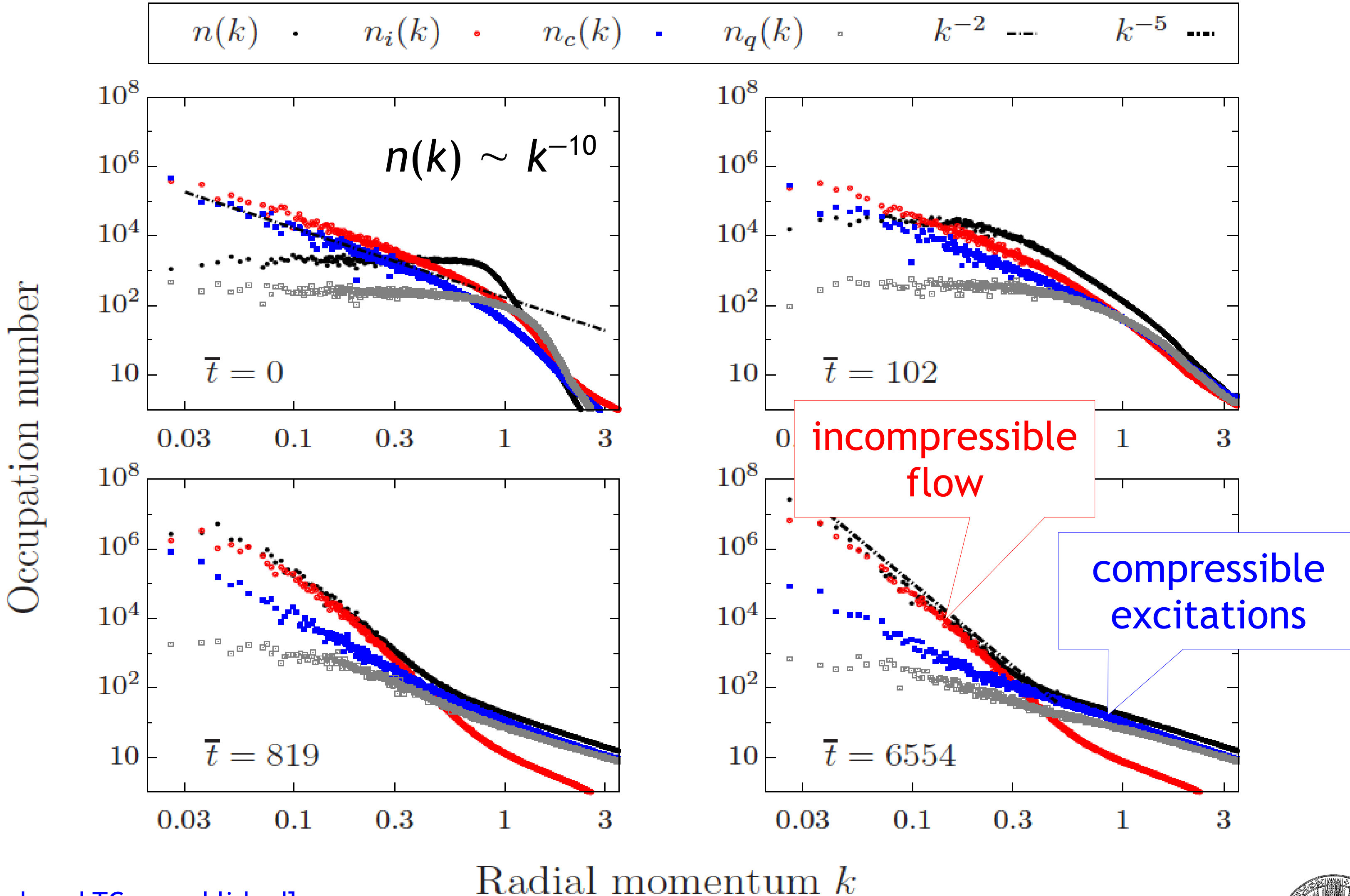
Bose-Einstein Condensation

Hydrodynamic vs. kinetic Condensation



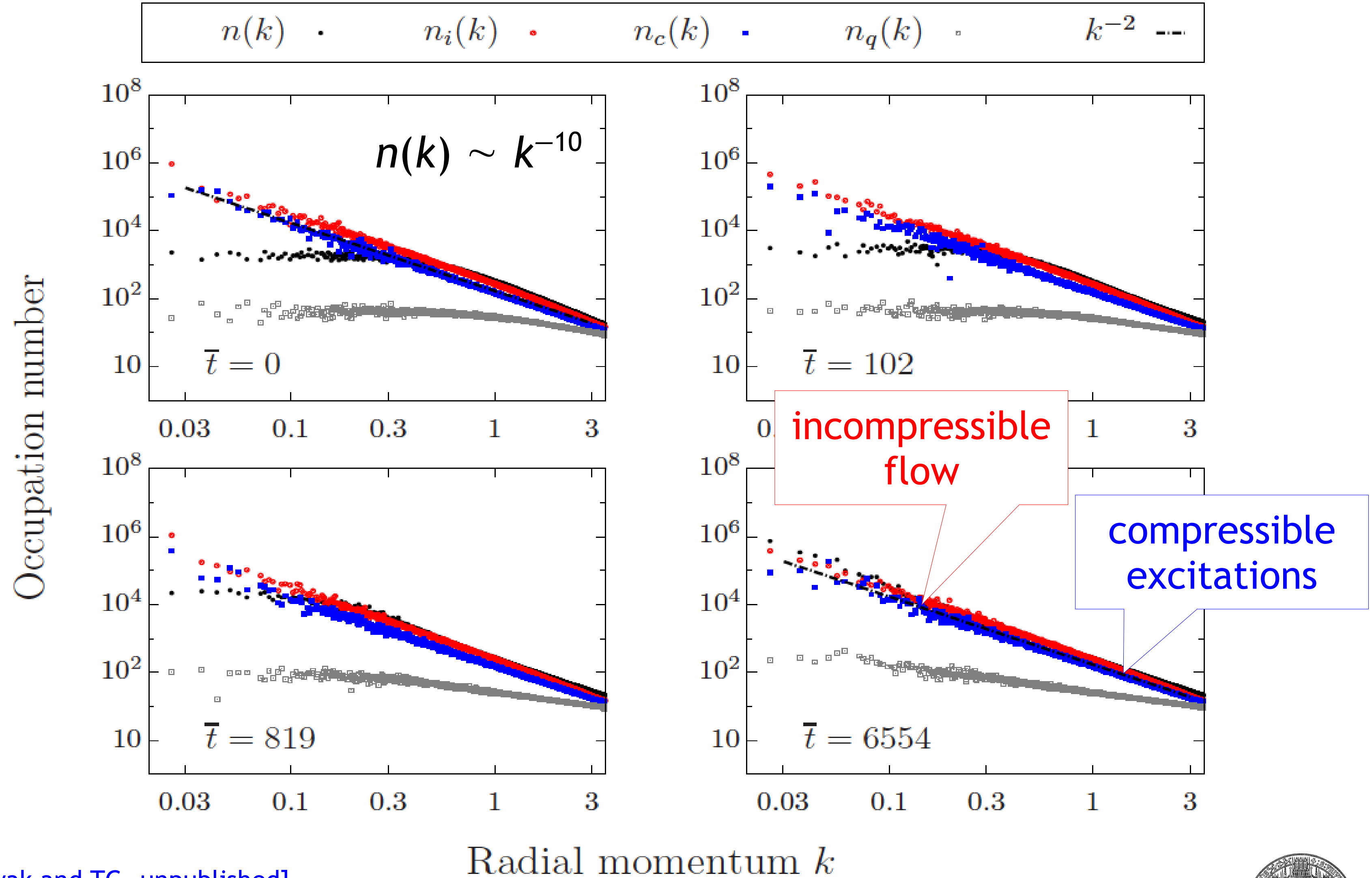
[B. Nowak and TG, unpublished]

Hydrodynamic vs. kinetic Condensation



[B. Nowak and TG, unpublished]

Hydrodynamic vs. kinetic Condensation



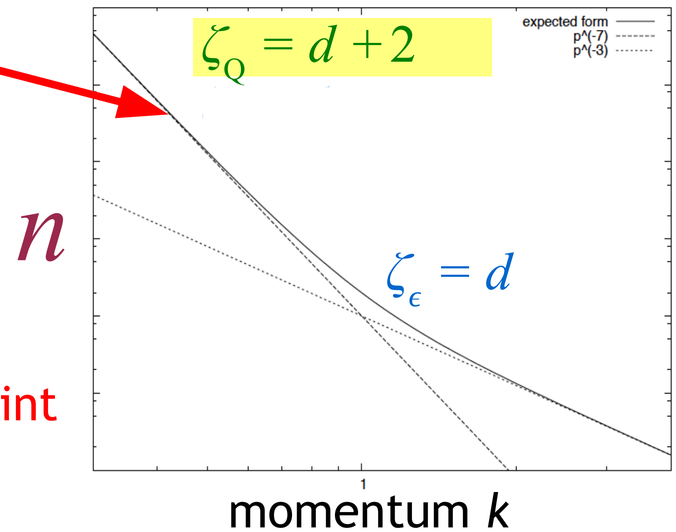
[B. Nowak and TG, unpublished]

Point vortex model

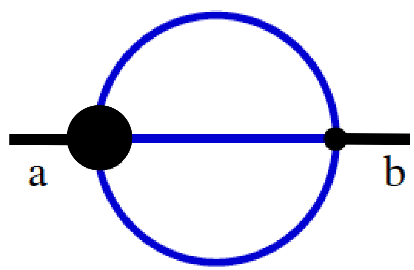
Bose gas in d spatial dimensions $n \sim k^{-\zeta}$

New exponent
beyond
Quantum Boltzmann!

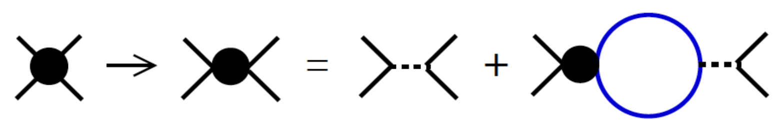
@ Nonthermal Fixed Point



$\Sigma_{ab}(x,y) =$



Vertex bubble resummation:
(2PI to NLO in $1/N$)



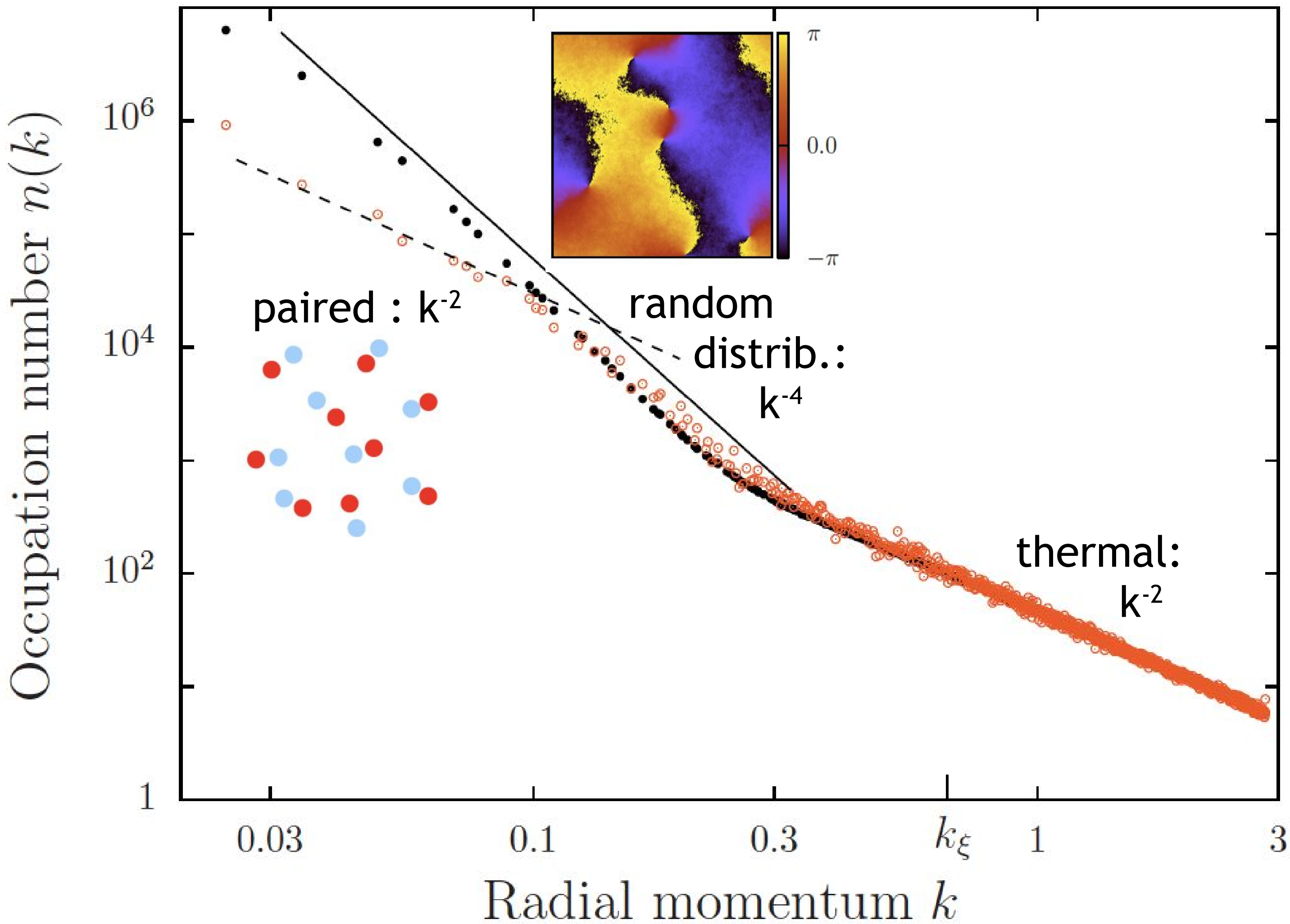
J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603, J. Berges, G. Hoffmeister, NPB **813** (09) 383
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Movie 2: Vortex “gas” & Spectrum

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$

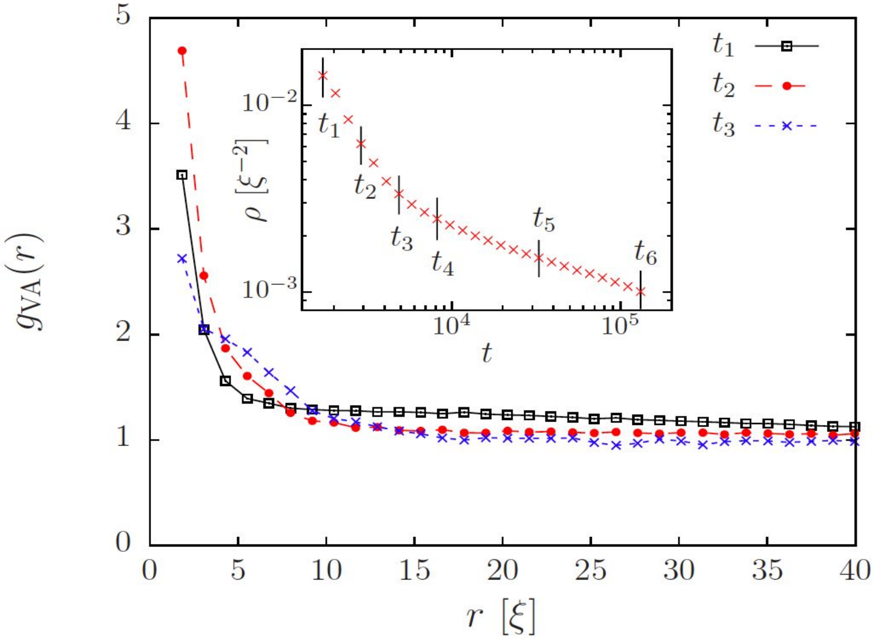
http://www.thphys.uni-heidelberg.de/~smp/gasenzer/videos/boseqt.html

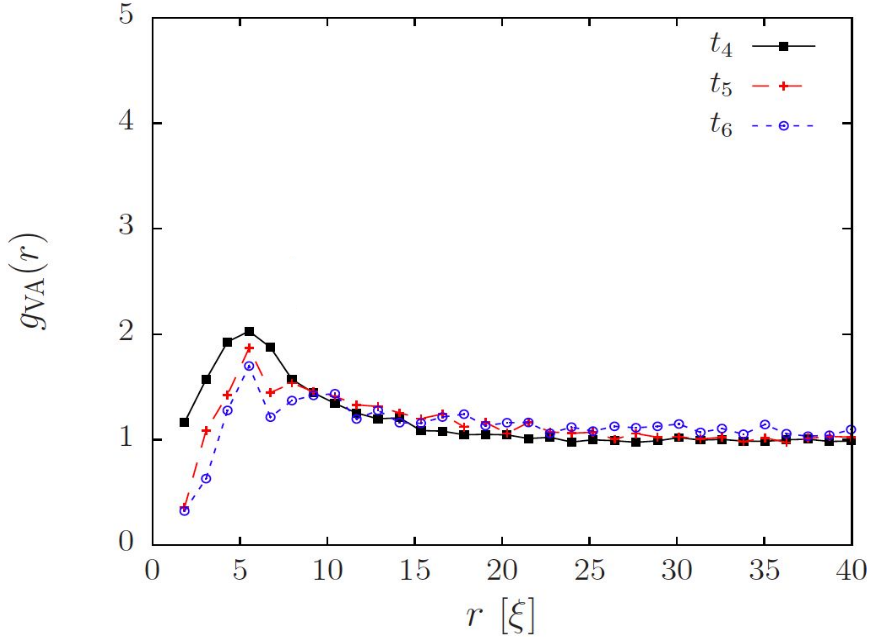
Scaling & Vortex correlations



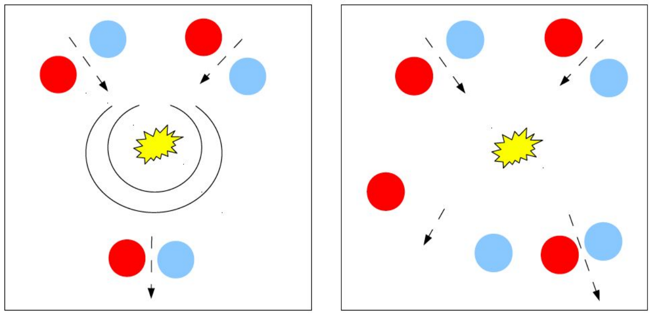
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127, PRA, to appear (12)

Dynamical vortex unpairing



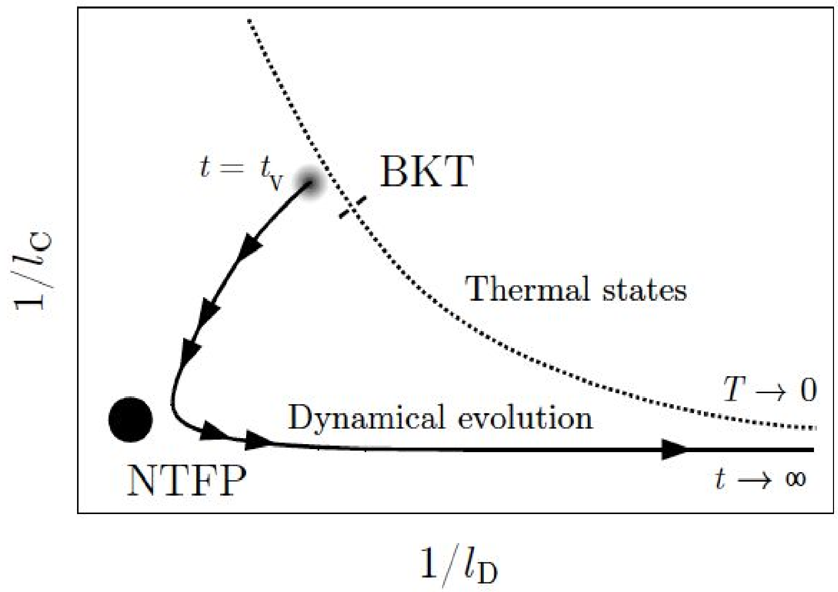


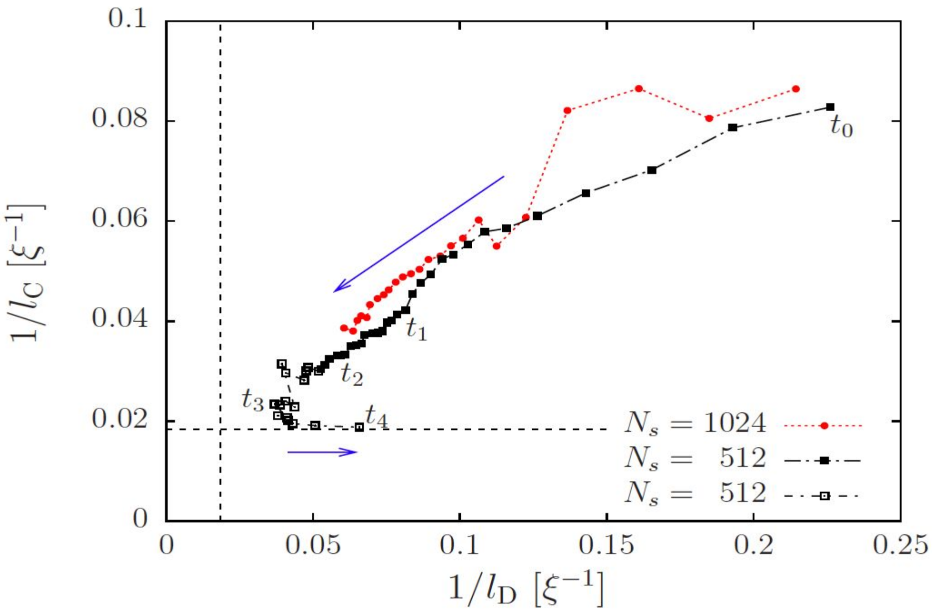
$$g_{\mathrm{VA}}(\mathbf{x},\mathbf{x}',t) = \frac{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rangle\langle \rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}$$



J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

Nonthermal fixed point in 2D

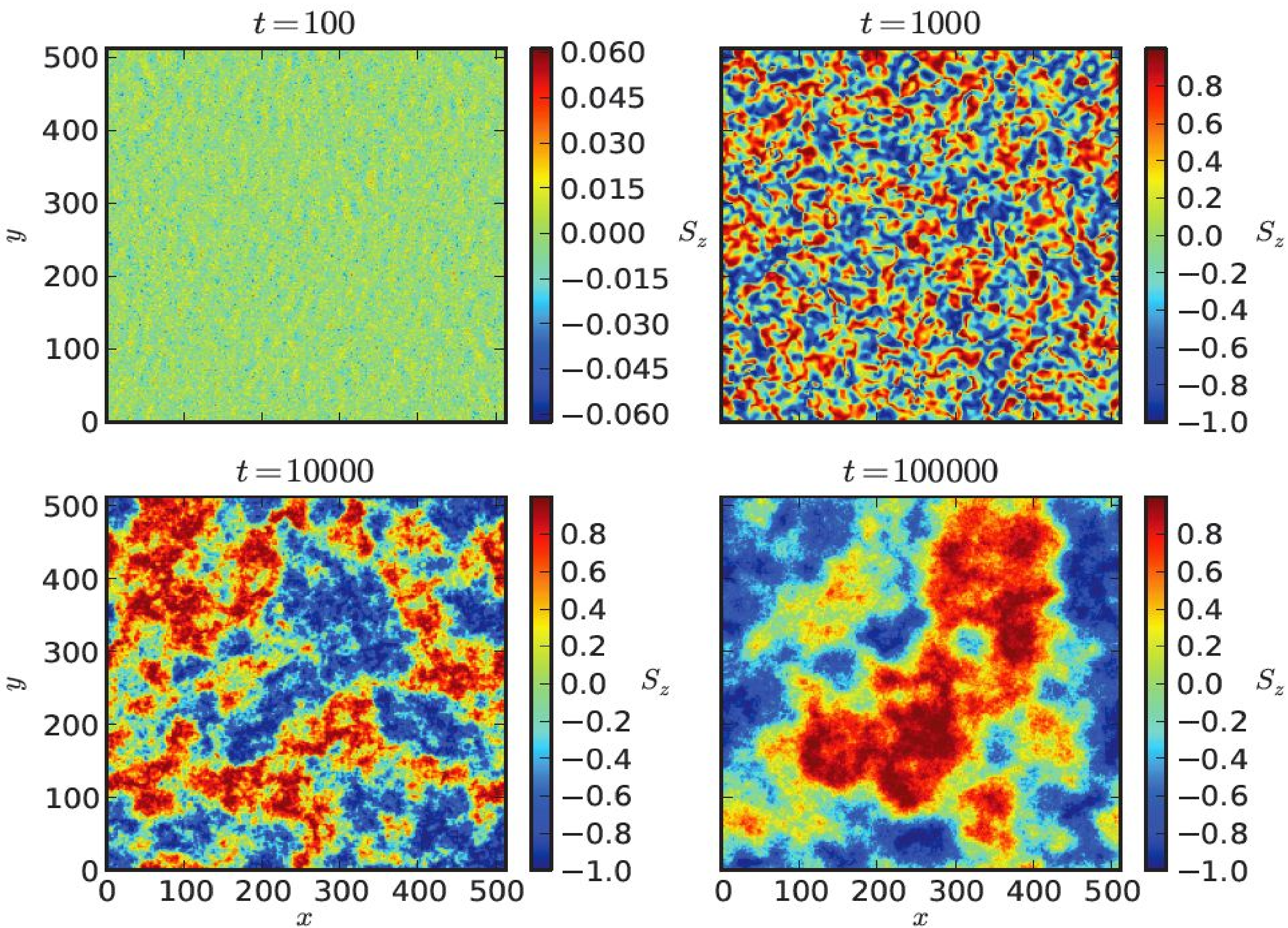




J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

Perturbative RG for dyn. near BKT: Mathey & Polkovnikov, PRA **80**, 041601R (09), **81**, 033605 (10)
See also: Jelic & Cugliandolo, J. Stat. Mech. P02032 (11)

Domain formation in spin systems

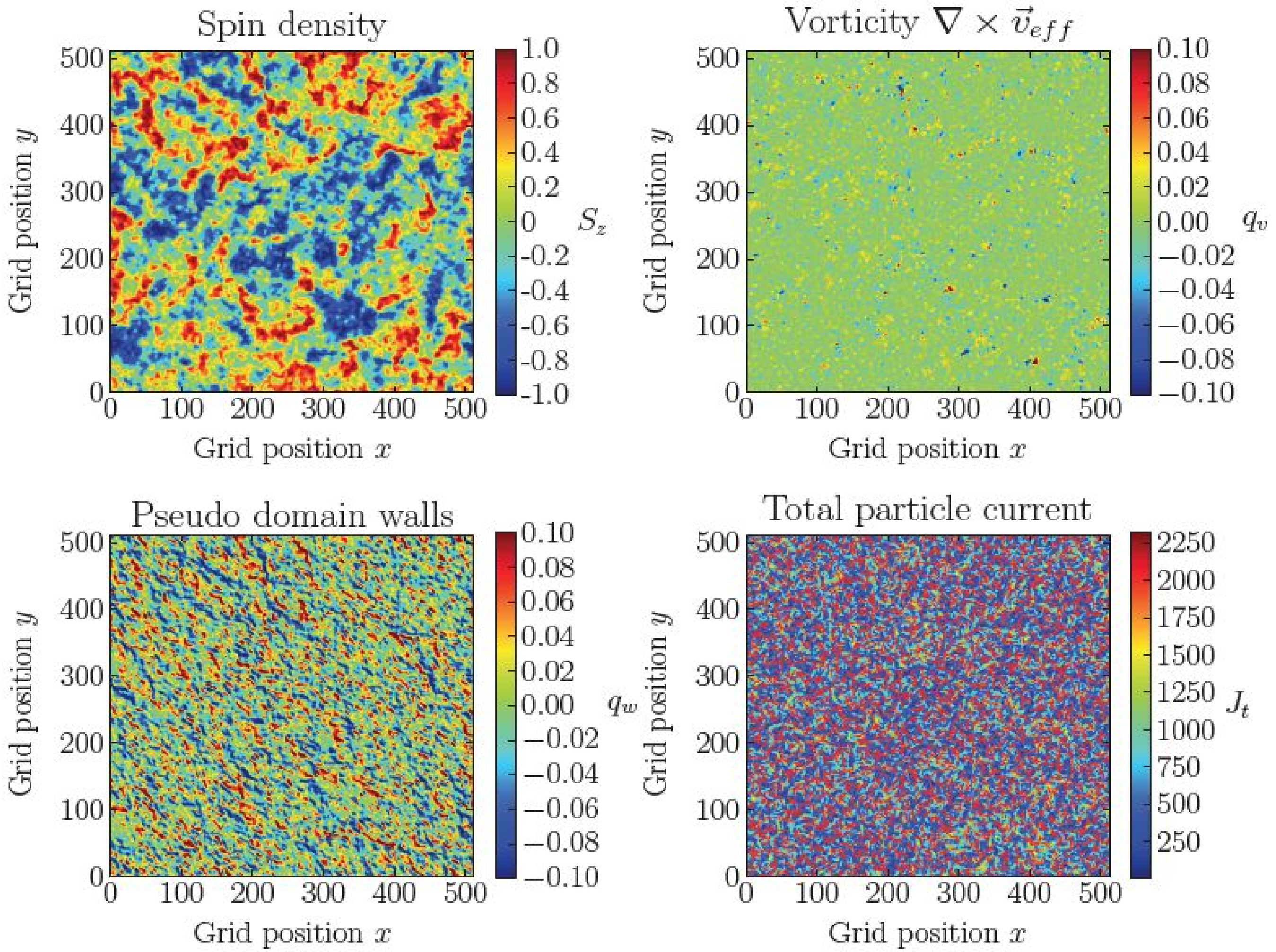


[M. Karl, B. Nowak, and TG, unpublished]

2-component BEC

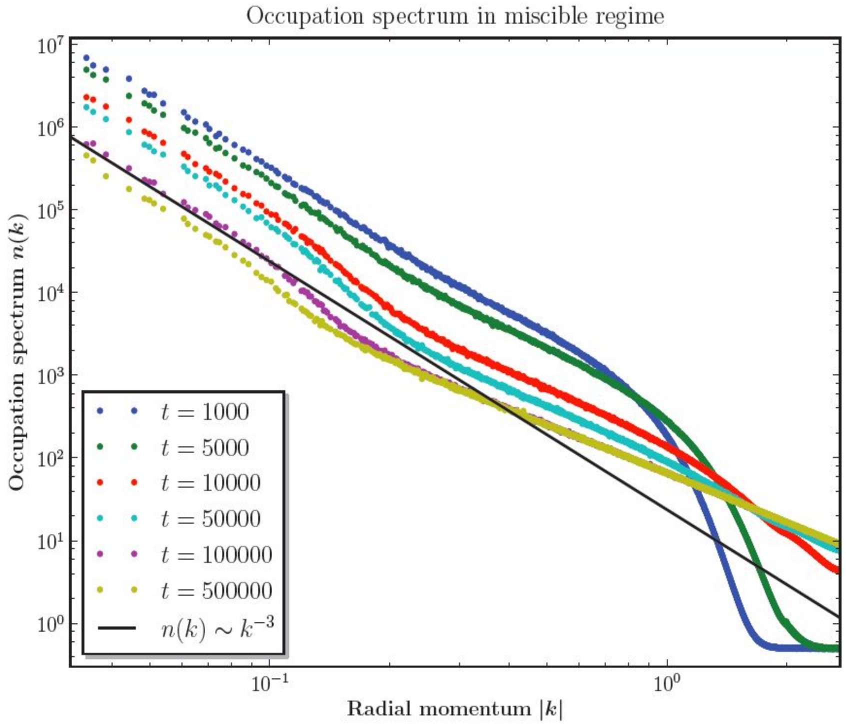
miscible

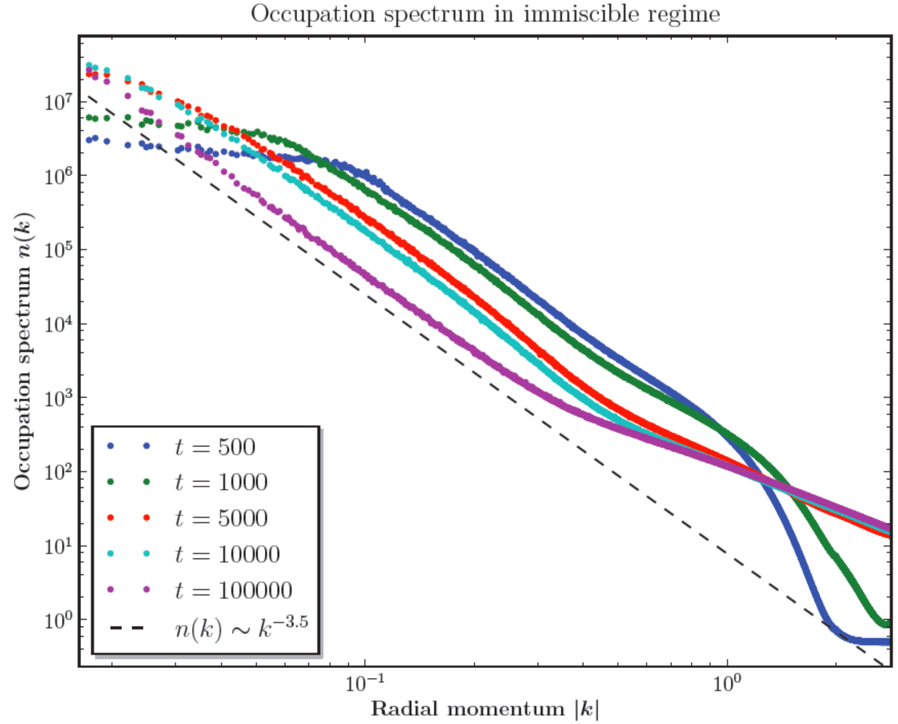
$g_{12} < g$



M. Karl, B. Nowak, TG, unpublished (12)

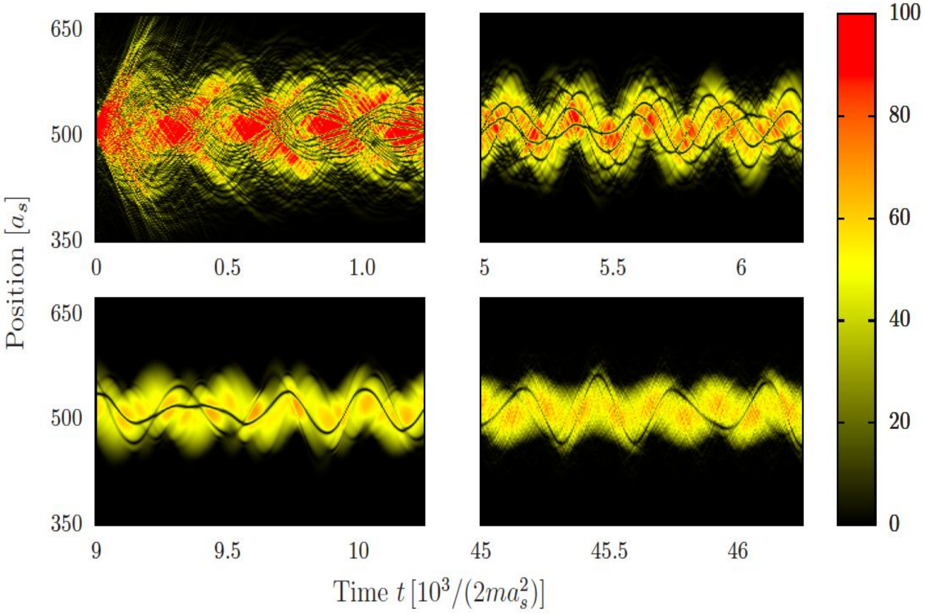
Domain formation in spin systems

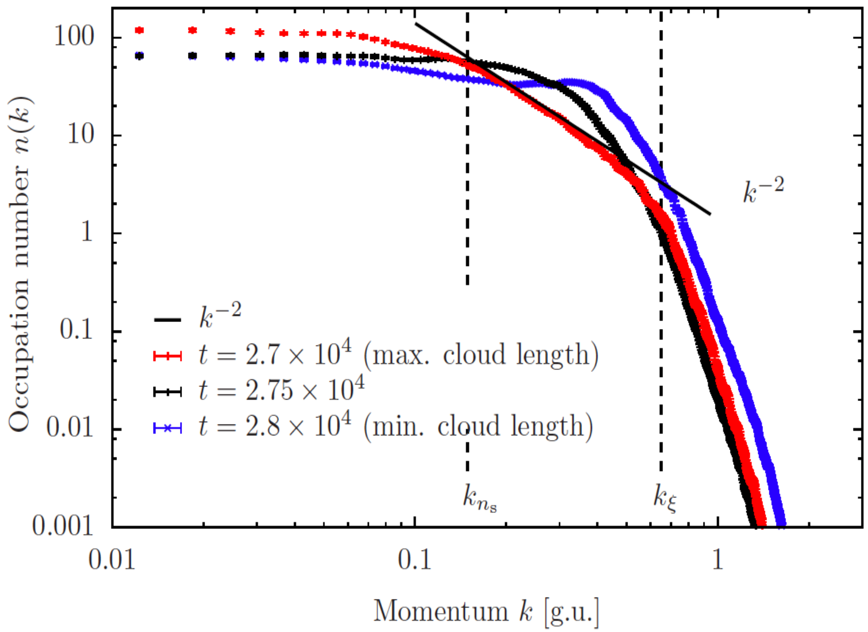


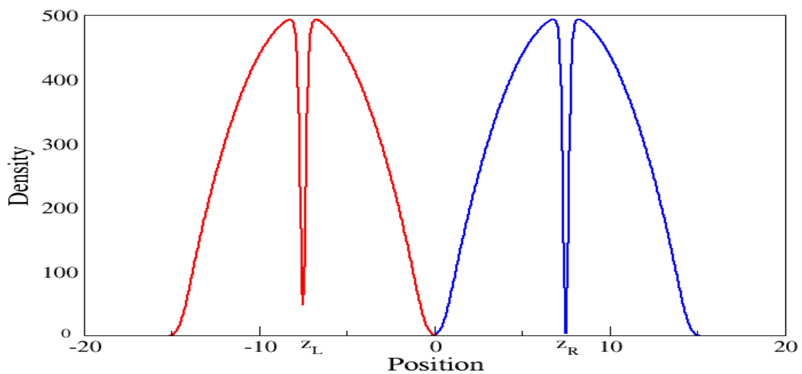


[M. Karl, B. Nowak, and T. Gasenzer, unpublished]

Solitons in 1 spatial dimension







M. Schmidt, S. Erne, B. Nowak, D. Sexty, and TG, arXiv:1203.3651 [cond-mat.quant-gas]

Relativistic scalar field

Non-linear Klein-Gordon equation

O(2) symmetry

$$(\partial_t^2 - \partial_x^2)\varphi(x,t) + \lambda\varphi^3(x,t) = 0$$

Initial condition: Highly occupied zero mode, Unoccupied modes with k>0

(video)

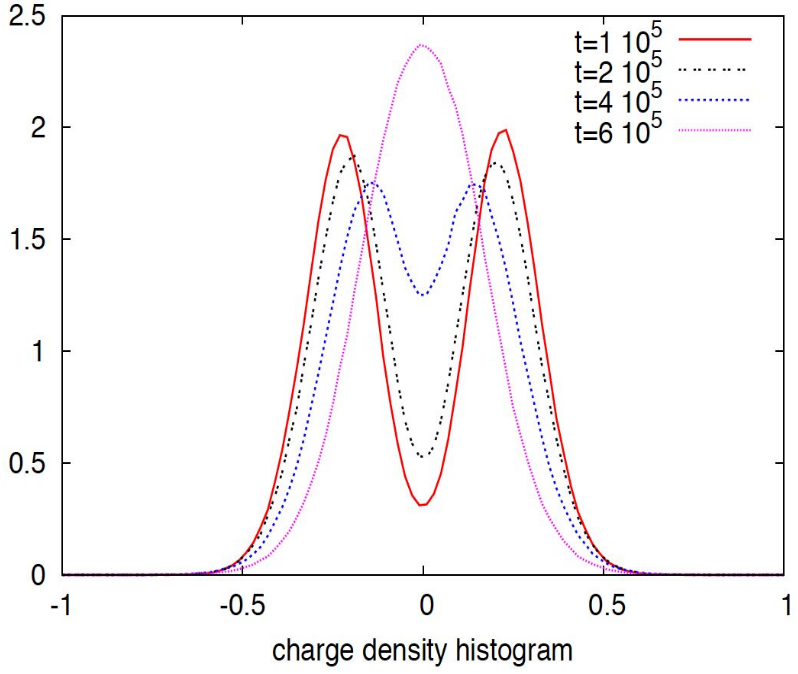
See also: http://www.thphys.uni-heidelberg.de/~sexty/videos

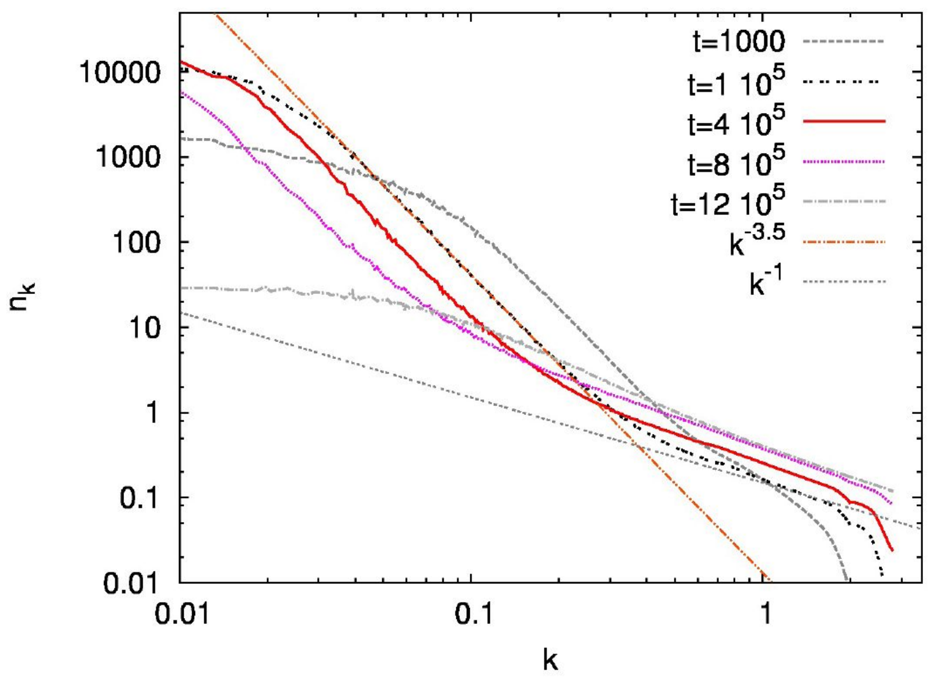
TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph]

Strong Turbulence = Charge Separation

Charge density distribution vs. power spectrum

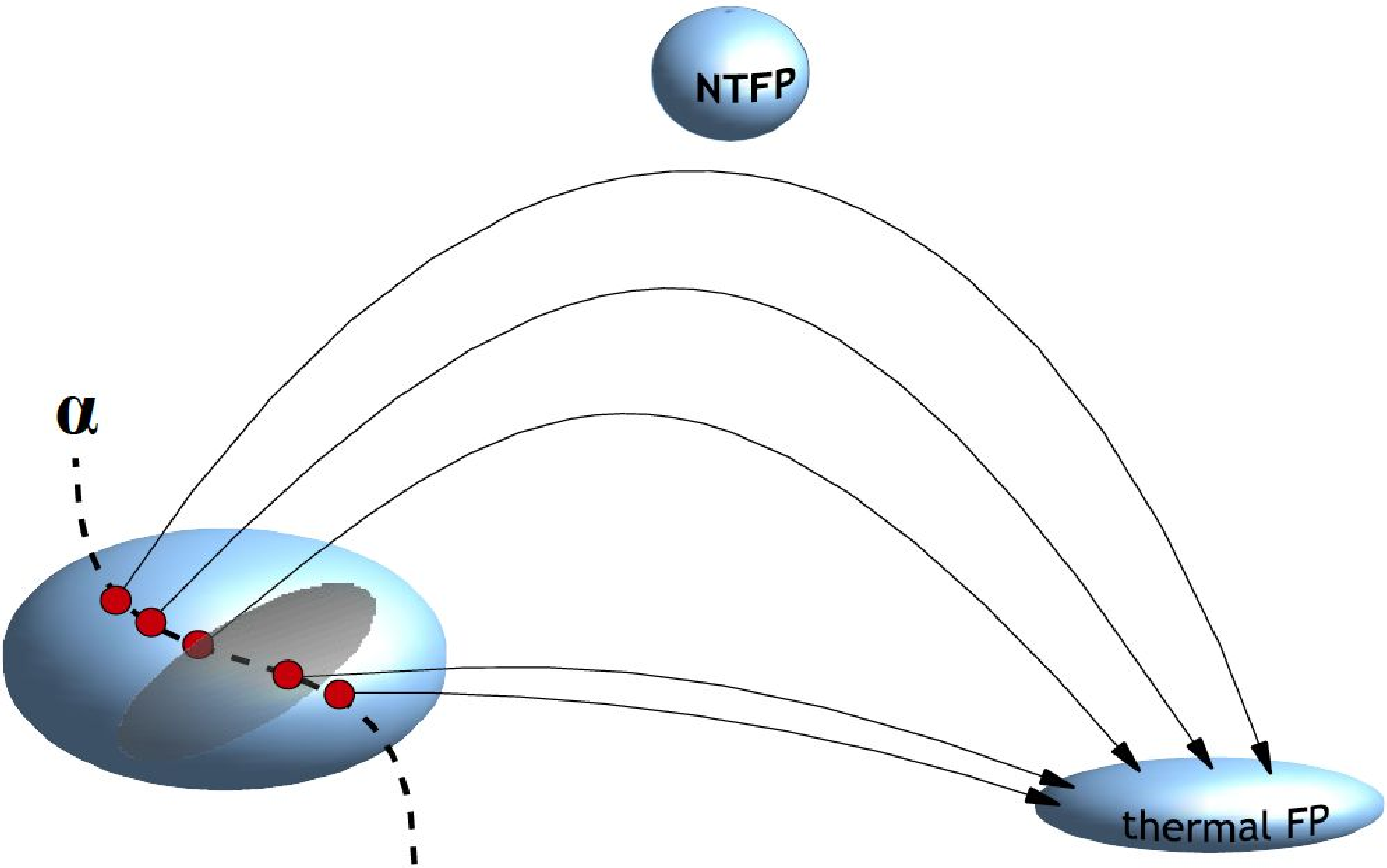
(d = 2, N = 2)





TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph], PLB, to appear

Hydrodynamic vs. adiabatic Condensation



[B. Nowak and TG, unpublished]

Non-thermal Fixed Point

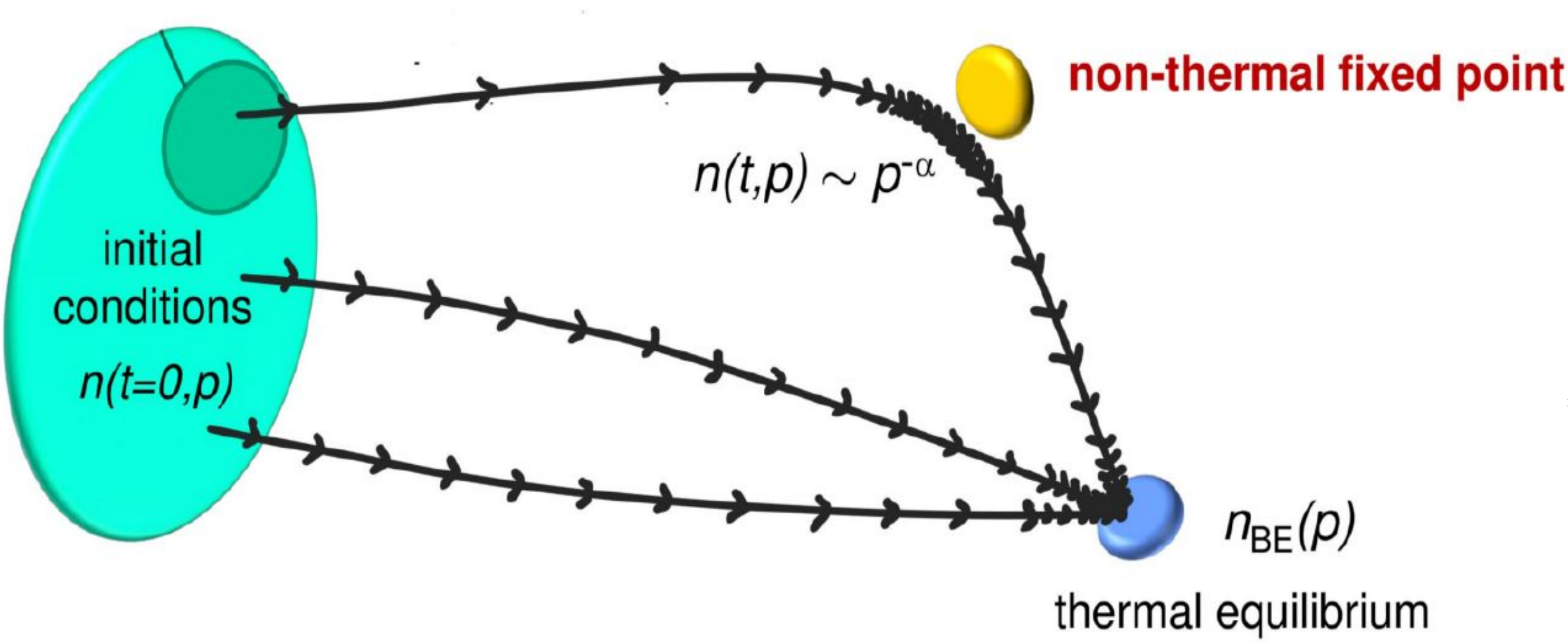
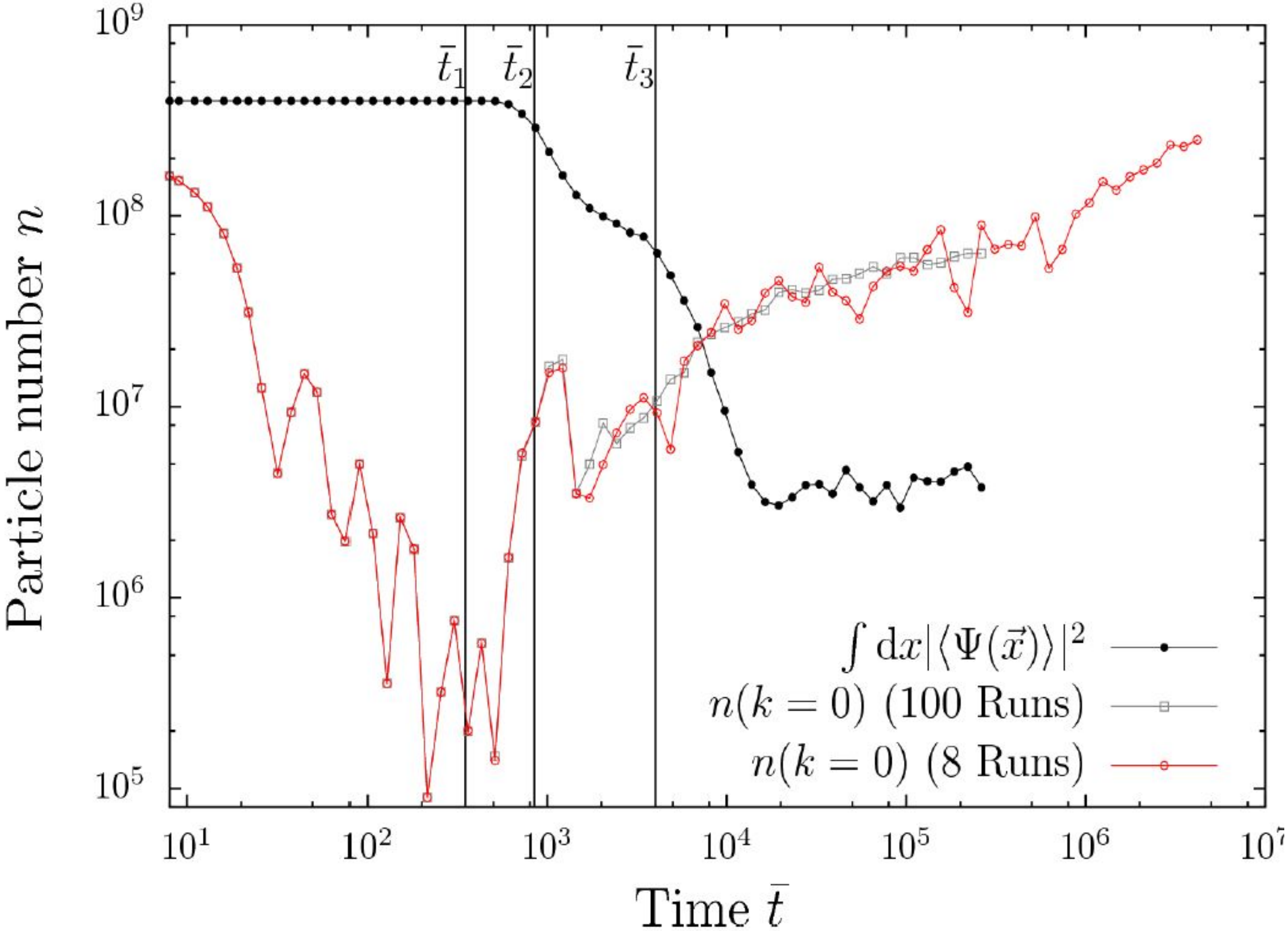


Fig. courtesy Jürgen Berges

J. Berges, A. Rothkopf, J. Schmidt, PRL **101,** 041603 (2008)
J. Berges and G. Hoffmeister, NPB **813**, 383 (2009)
C. Scheppach, J. Berges, TG PRA **81,** 033611 (2010)
B. Nowak, D. Sexty, TG, PRB **84**, 020506(R) (2011)
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127

Bose-Einstein condensation

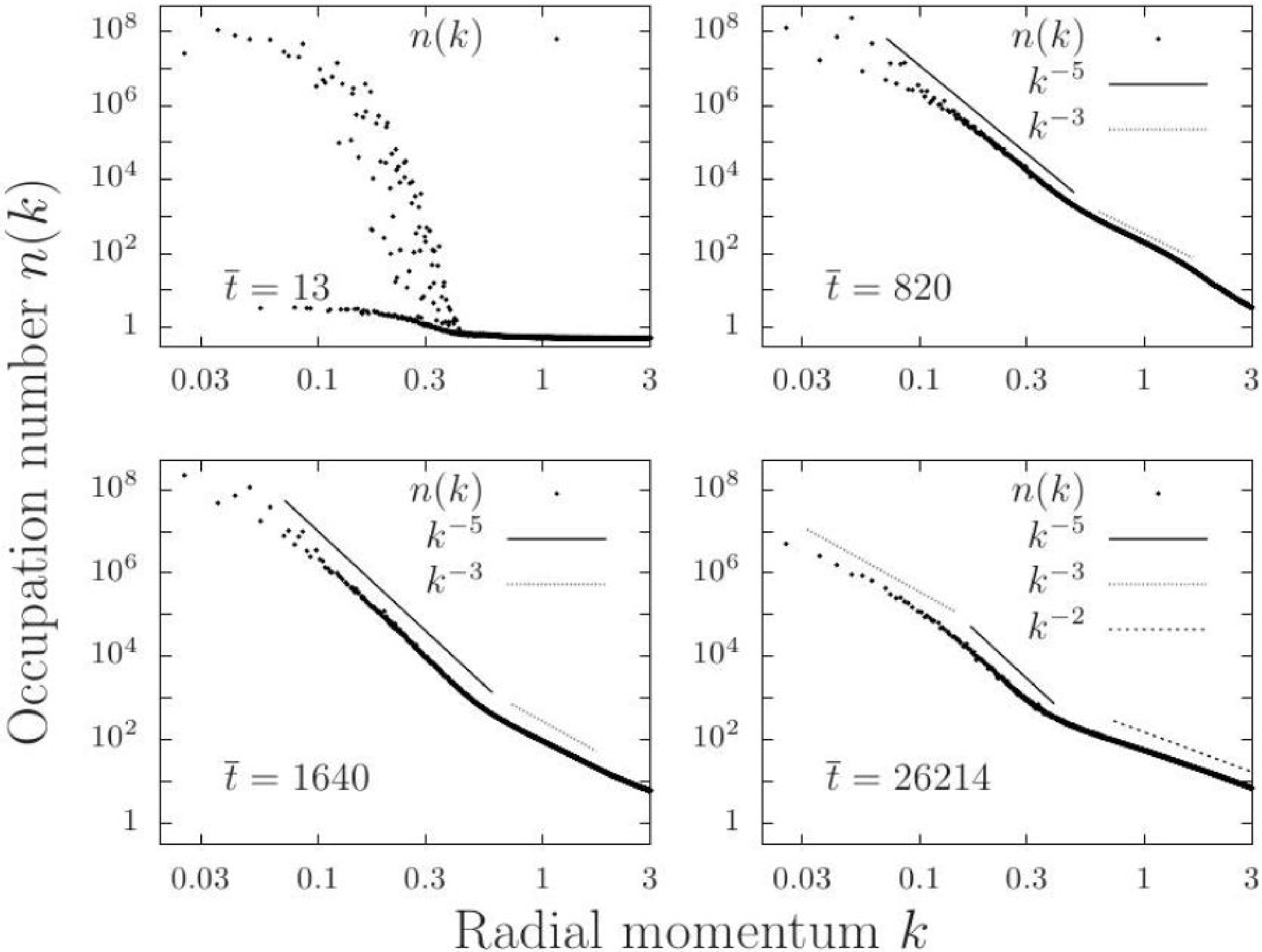


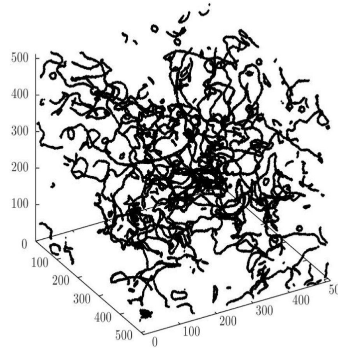
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

For 3D see also N. Berloff & B. Svistunov, PRA 66 (02)

B. Nowak, D. Sexty, TG, PRB **84**: 020506(R), 2011

3+1 D simulations





Acoustic Turbulence

Decomposition of Energy

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

$$\mathbf{u}(\boldsymbol{\rho}, t) = \nabla \varphi(\boldsymbol{\rho}, t)$$

$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

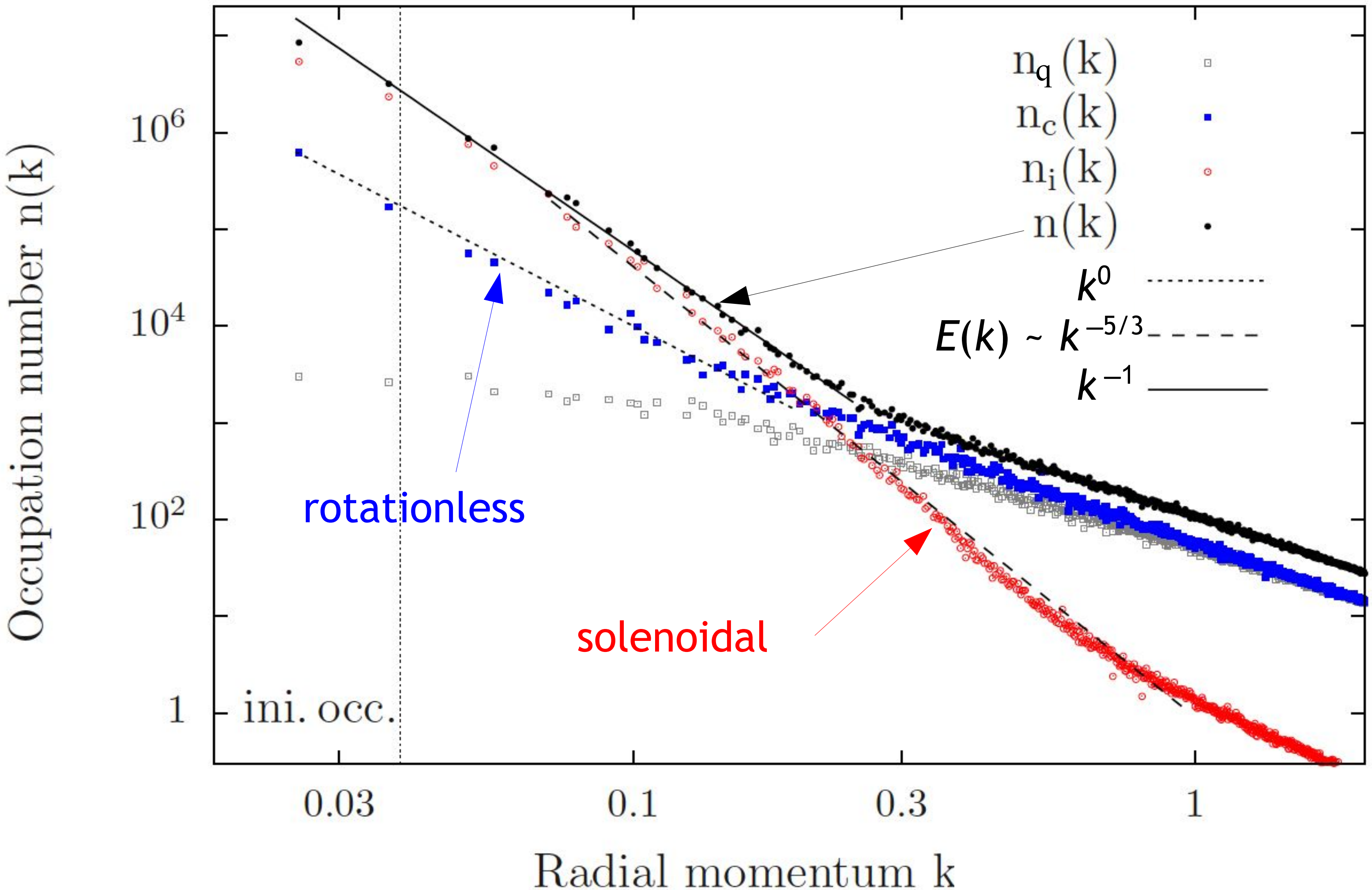
$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} \quad = E^i_{kin} + E^c_{kin}$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

$$E_q = \frac{1}{2} \int (\nabla \sqrt{n})^2 d\boldsymbol{\rho}$$

Simulations in 2+1 D

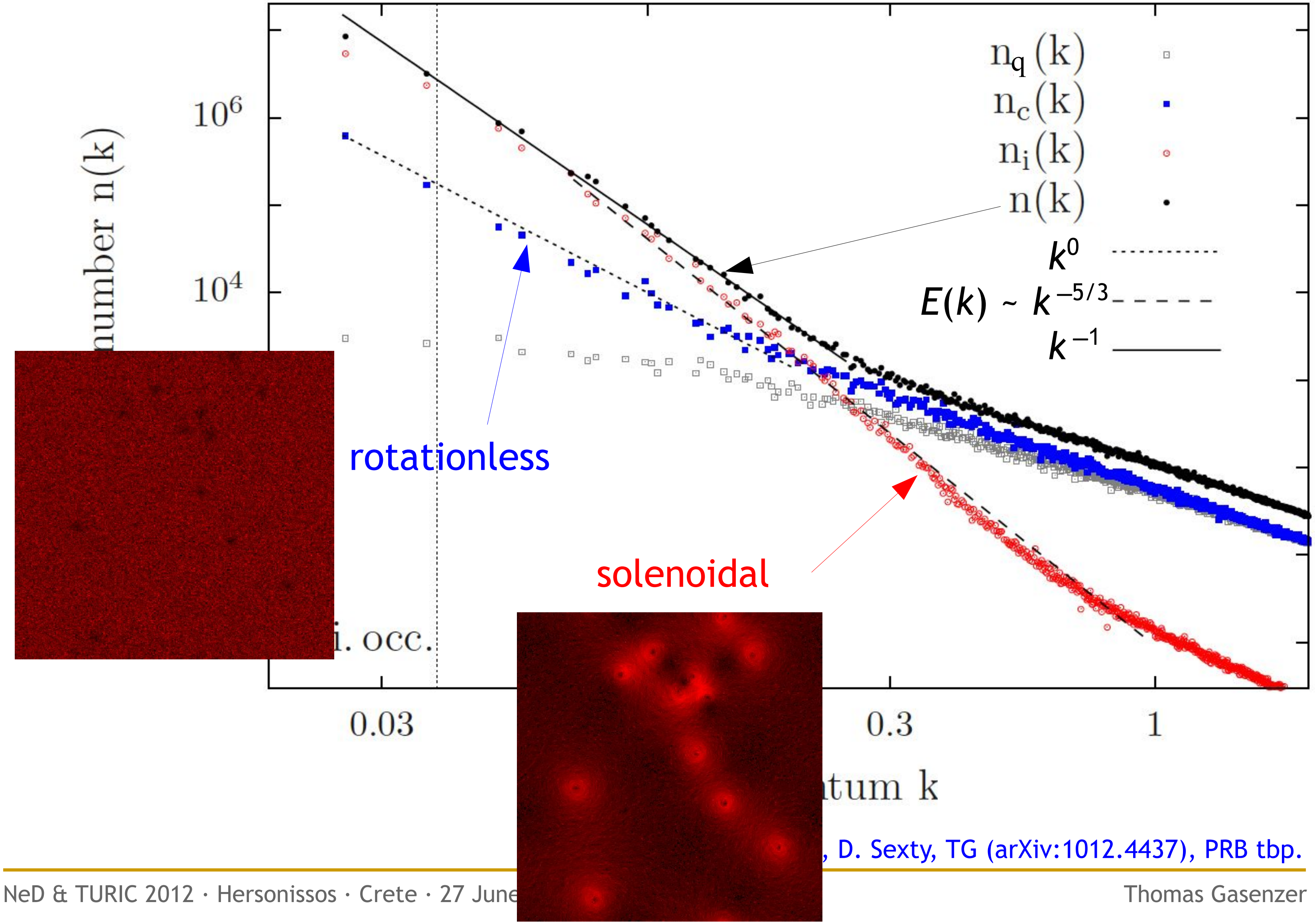
$$E(\mathrm{k}) = \omega(\mathrm{k})\mathrm{k}^{d-1}\mathrm{n}(\mathrm{k})$$



B. Nowak, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

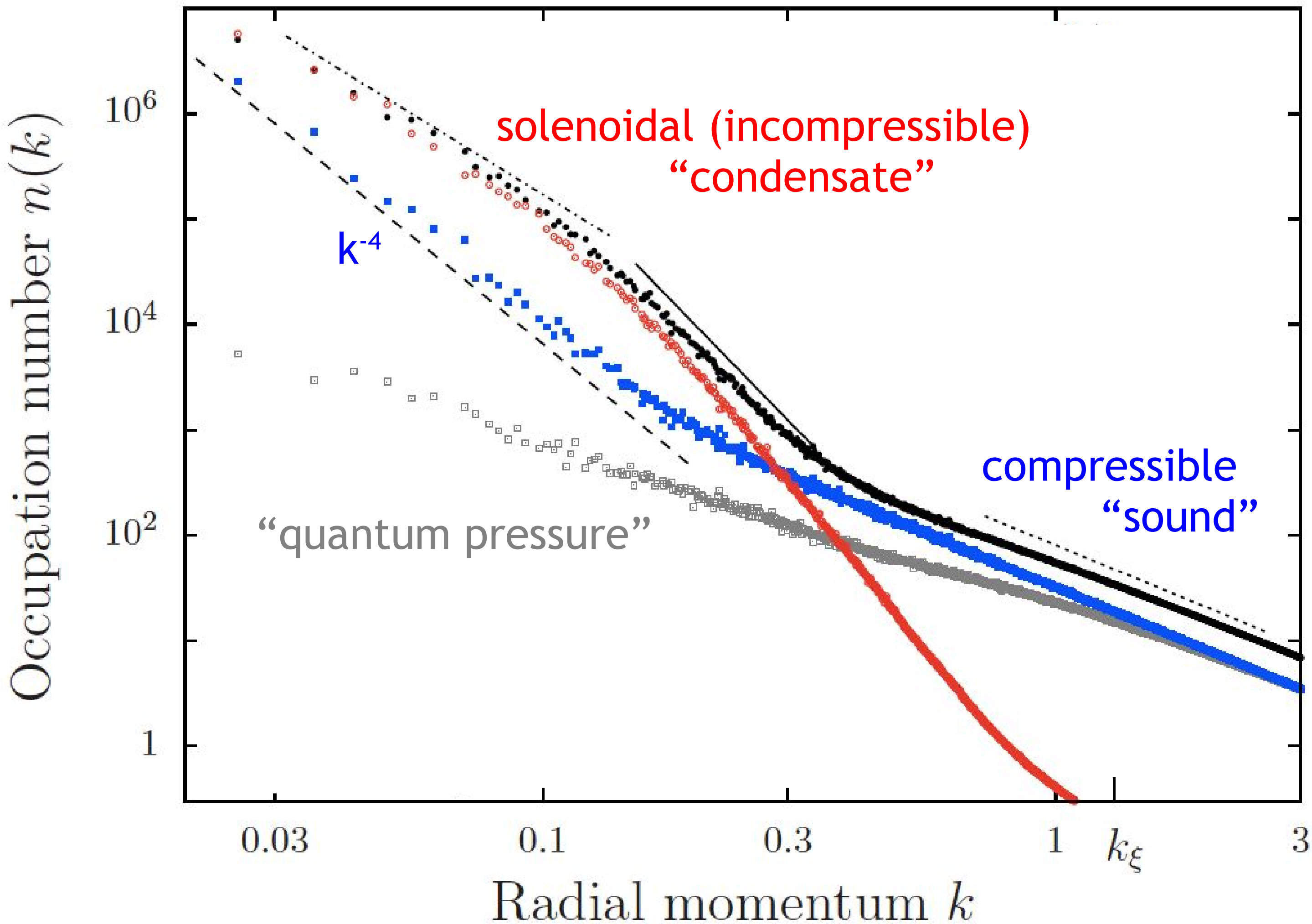
Simulations in 2+1 D

$$E(k) = \omega(k)k^{d-1}n(k)$$

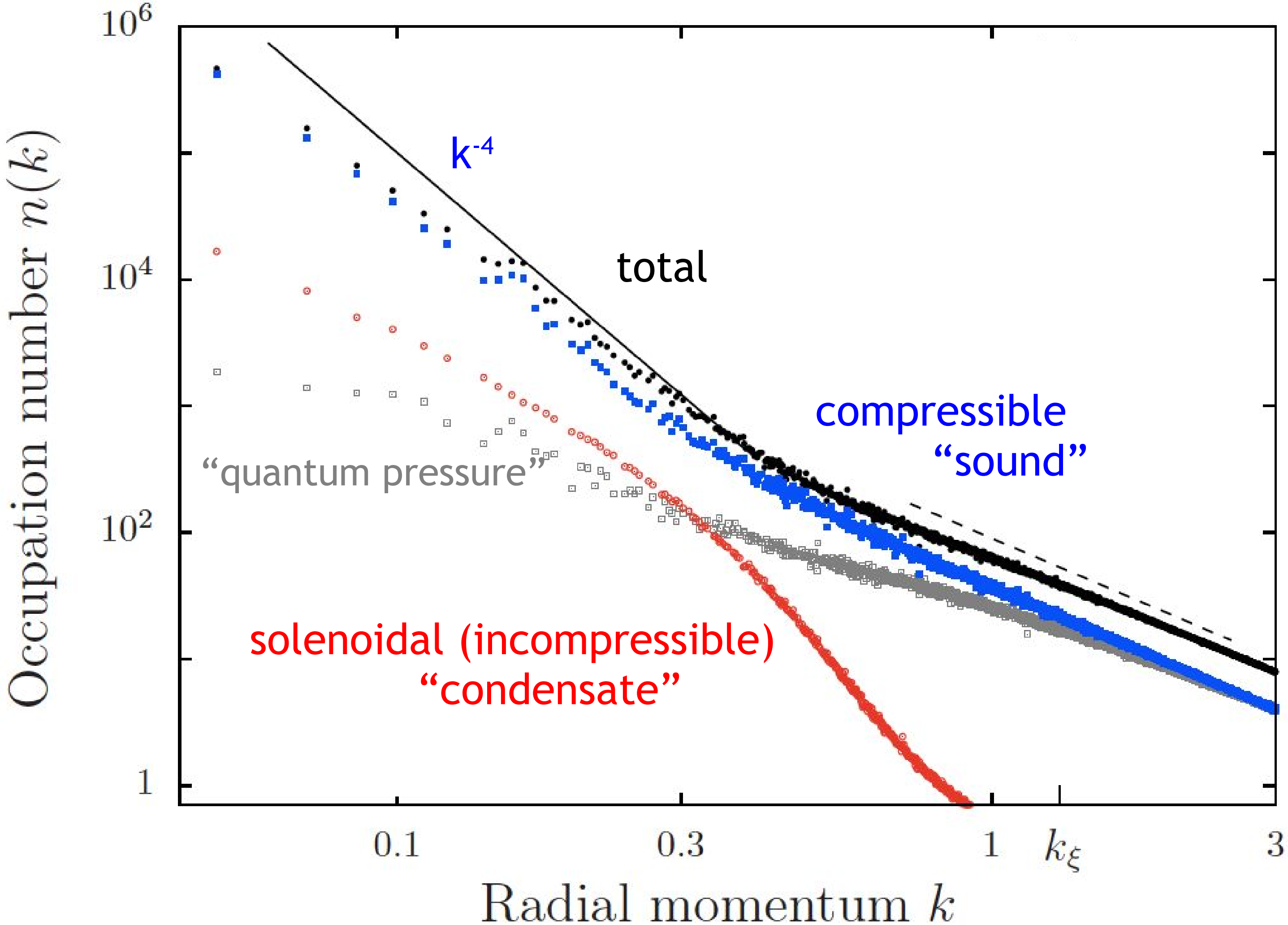


, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

Decomposition of flow

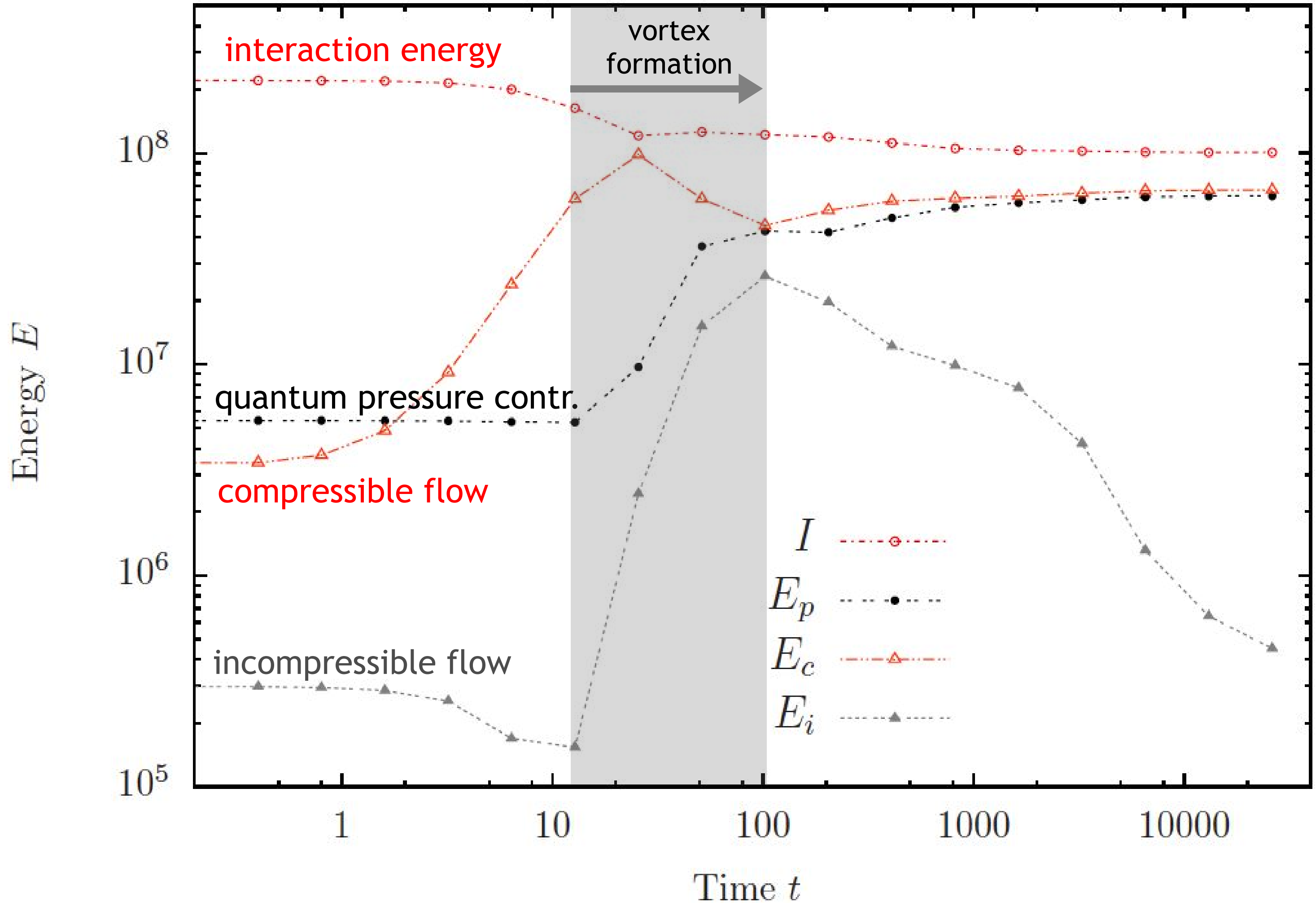


Acoustic turbulence



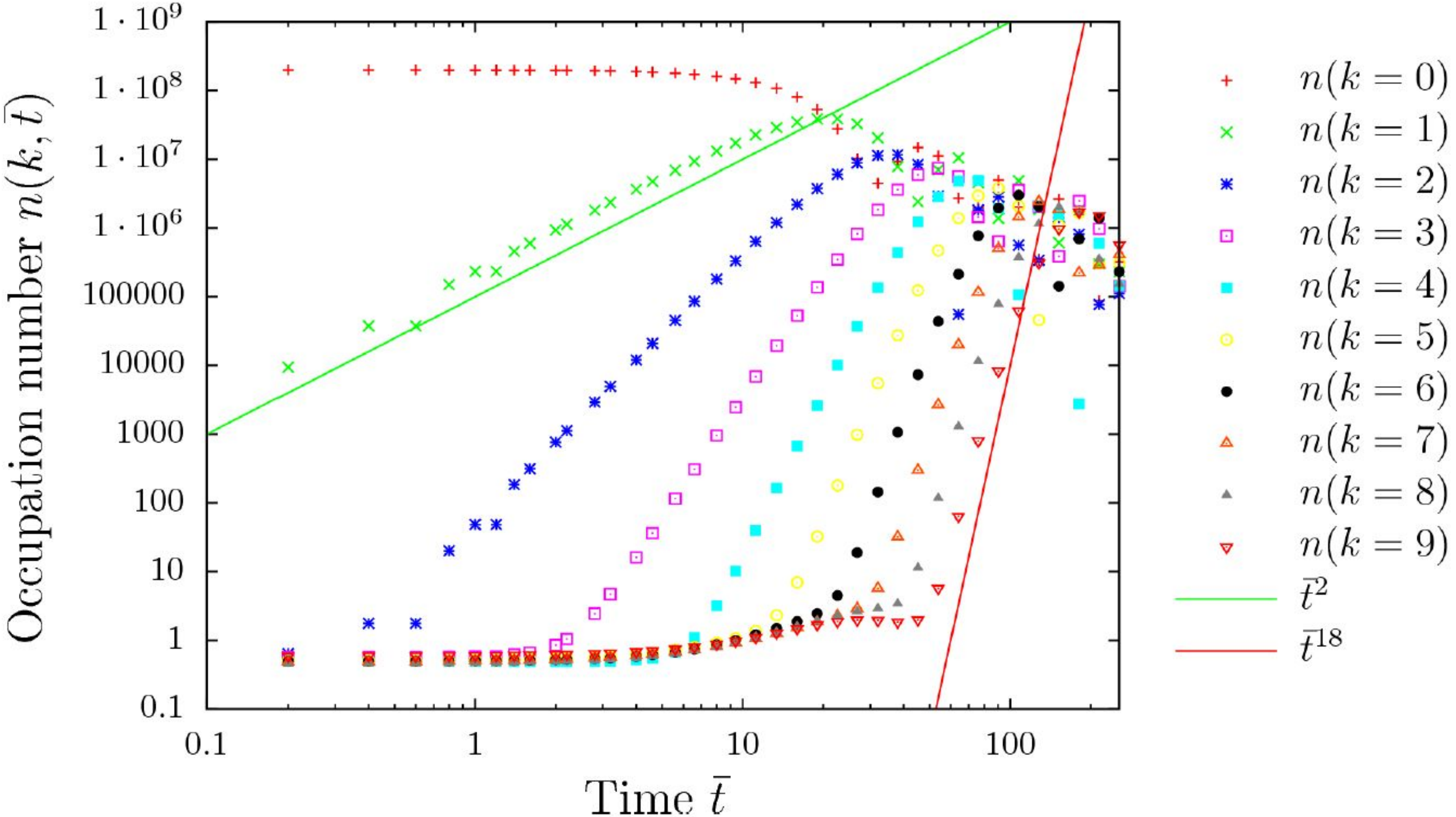
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Time evolution of Energy Components (3+1 D)



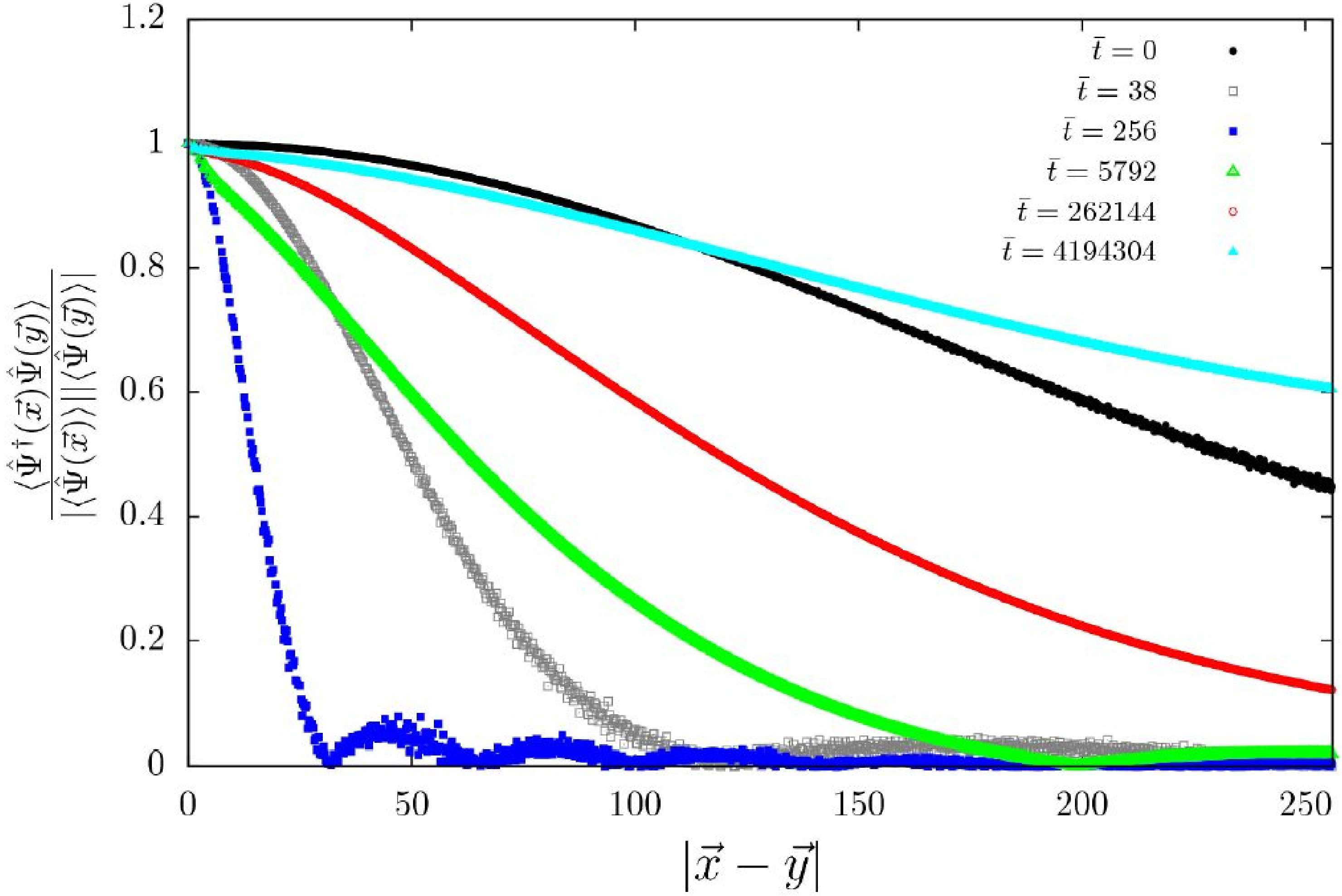
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Mode Occupations



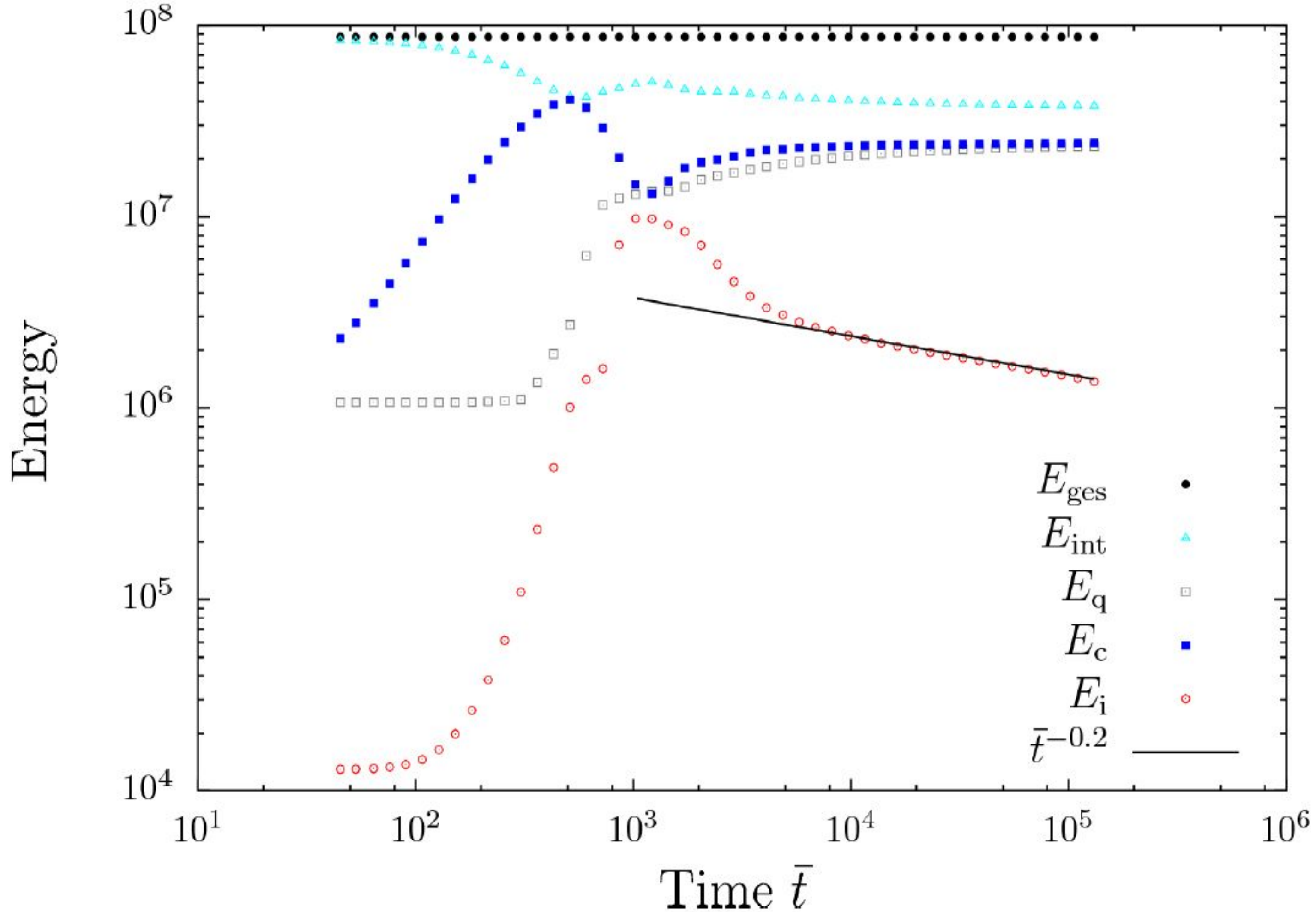
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

1st-order Coherence



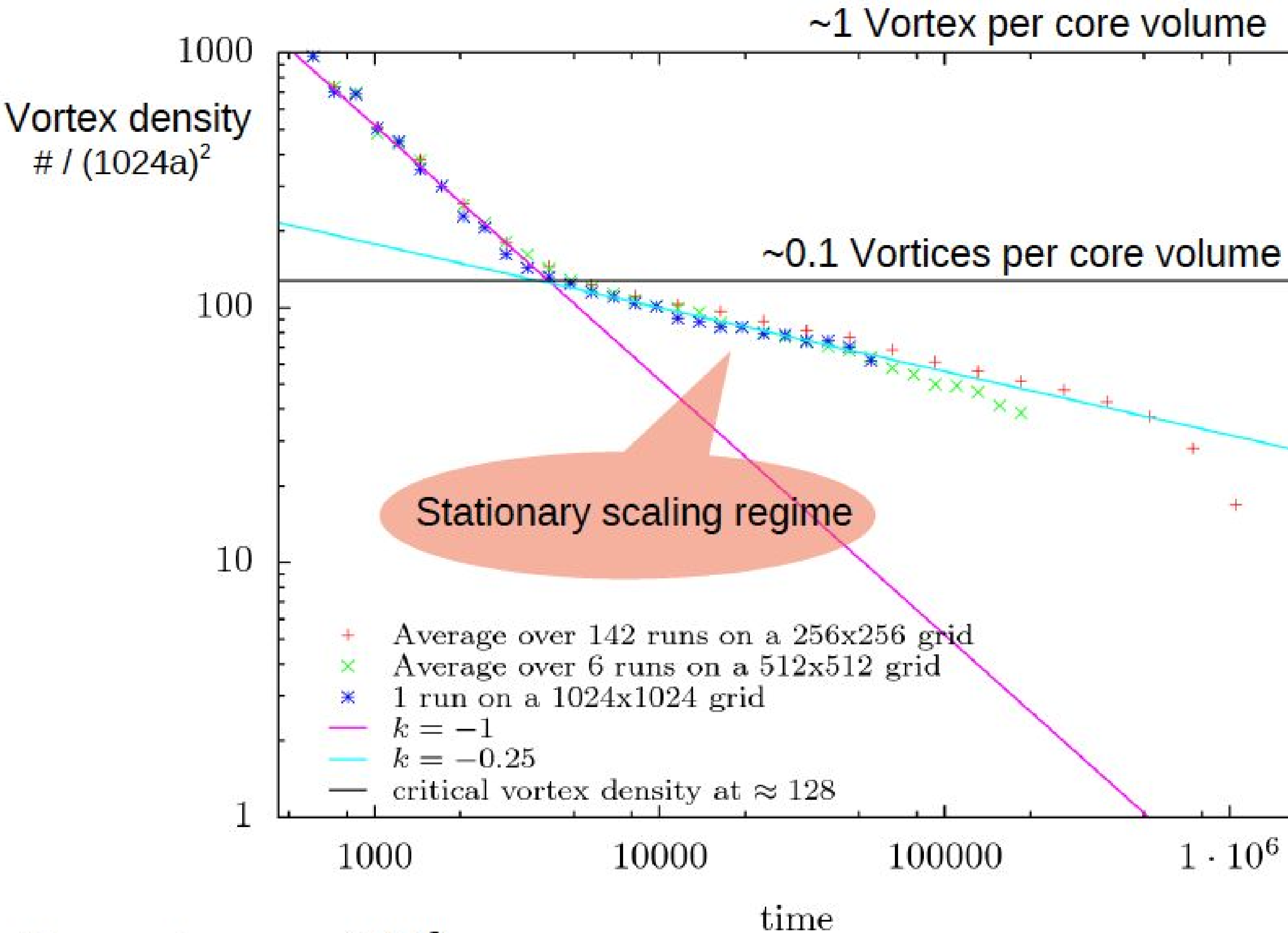
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Time-Evolution of Energy-components (2+1 D)



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Time evolution of vortex density



Core volume ~ $\pi(3\xi)^2$

J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Enstrophy in classical turbulence

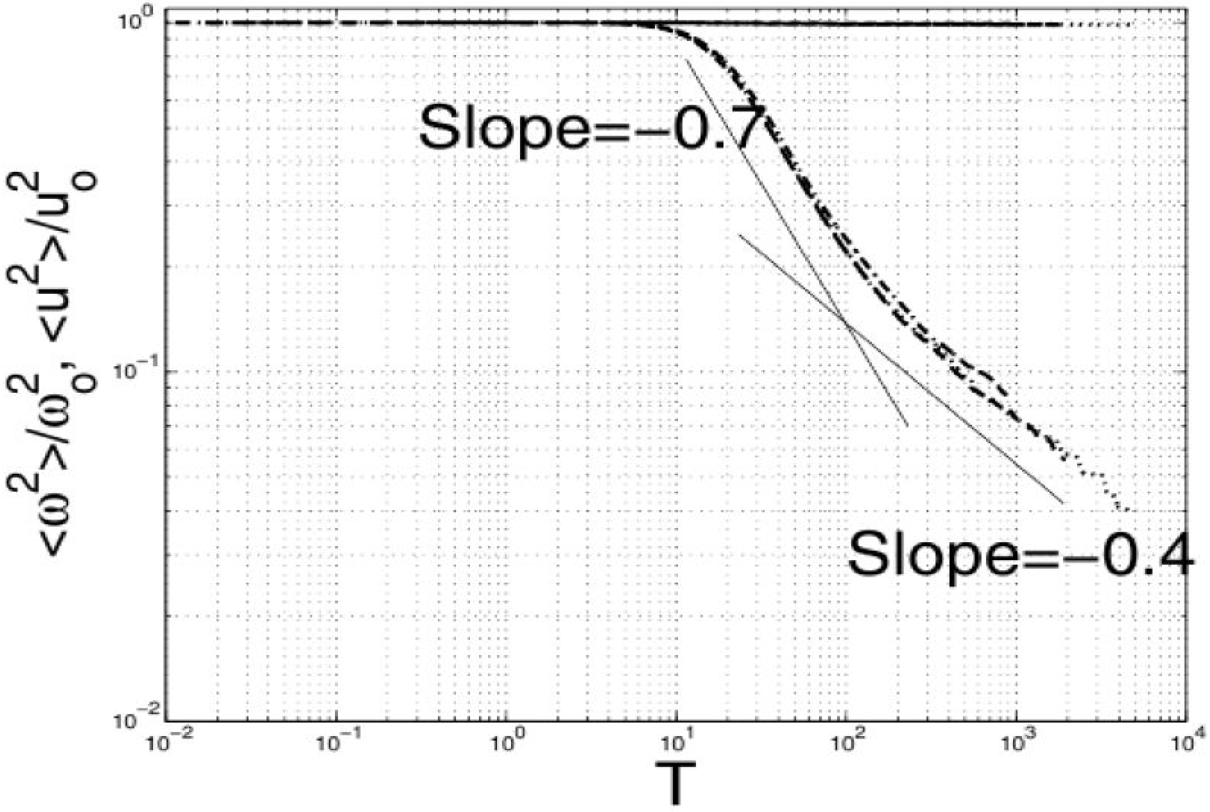
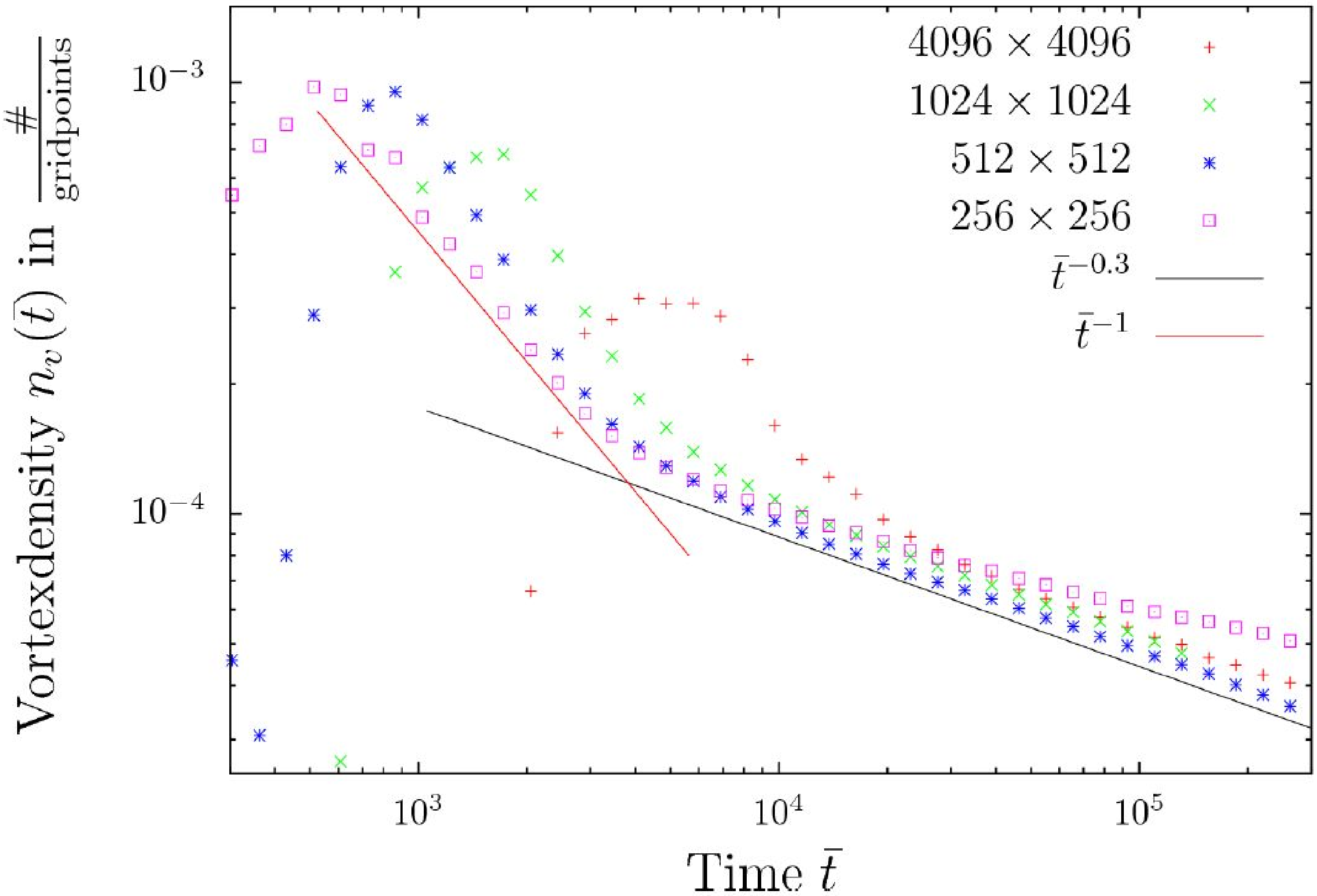


FIG. 1. Time evolution of energy (upper horizontal curve) and enstrophy. Resolution: dotted line (512^3); dashed line (1024^3); dash-dotted line (2048^3).

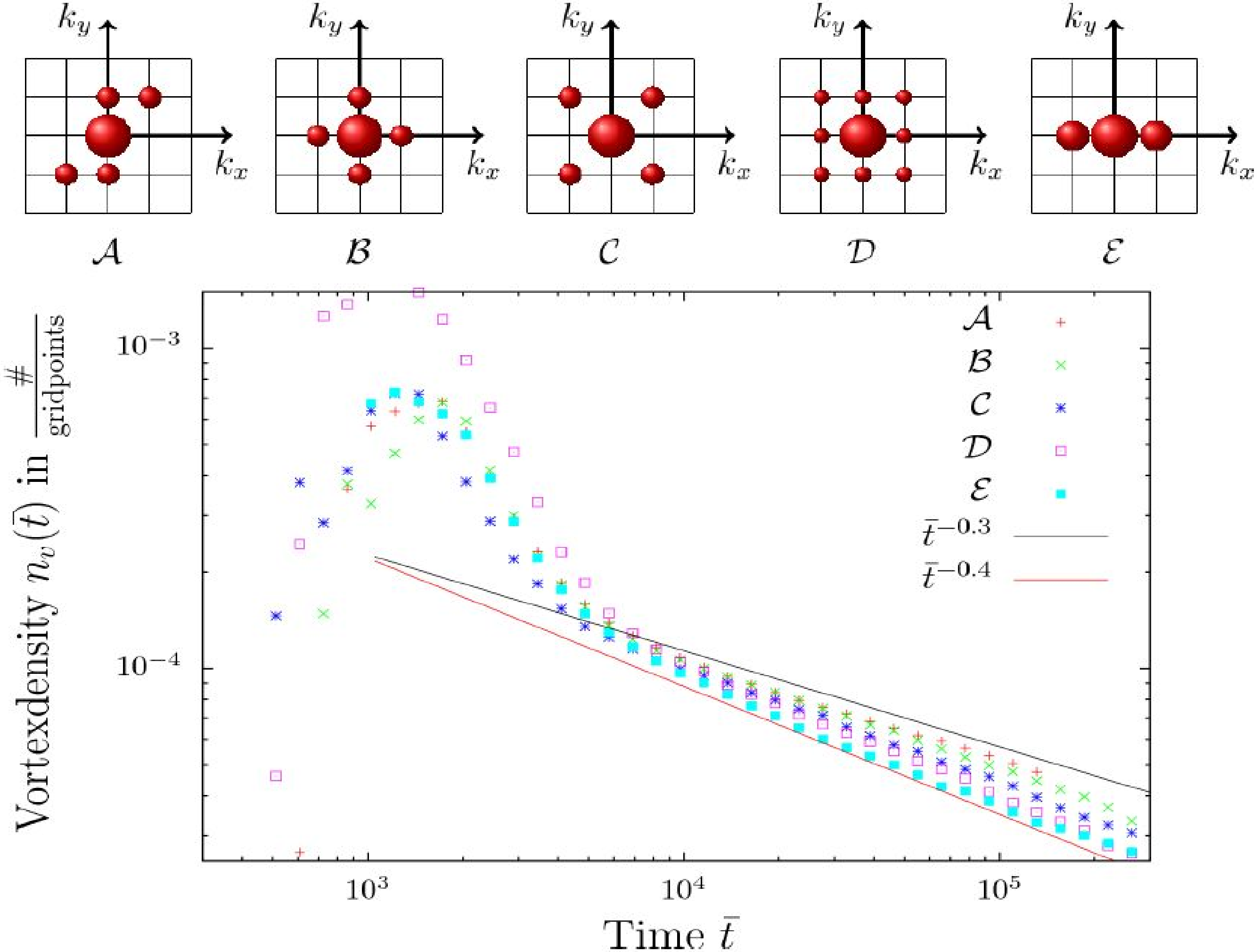
V. Yakhot, J. Wanderer, PRL 93:154502

Vortex-Density Decay in 2d



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Vortex-Density Decay in 2d



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Kinetic Theory

One of the power laws can be explained by a kinetic theory:

$$\partial_t n_v(t) = -\frac{n_{\text{dip}}}{\tau_{\text{ann}}}$$

$$n_{\text{dip}} \sim n_v \qquad \sigma \sim d$$

$$\tau_{\text{ann}} = \tau_{\text{coll}}\,\alpha \qquad \bar{v} = \frac{1}{d}$$

$$\tau_{\text{coll}} = \frac{l}{\bar{v}} \qquad d = \frac{1}{\sqrt{n_v}}$$

$$l \sim \frac{1}{n_v \sigma}$$

d : average pair distance

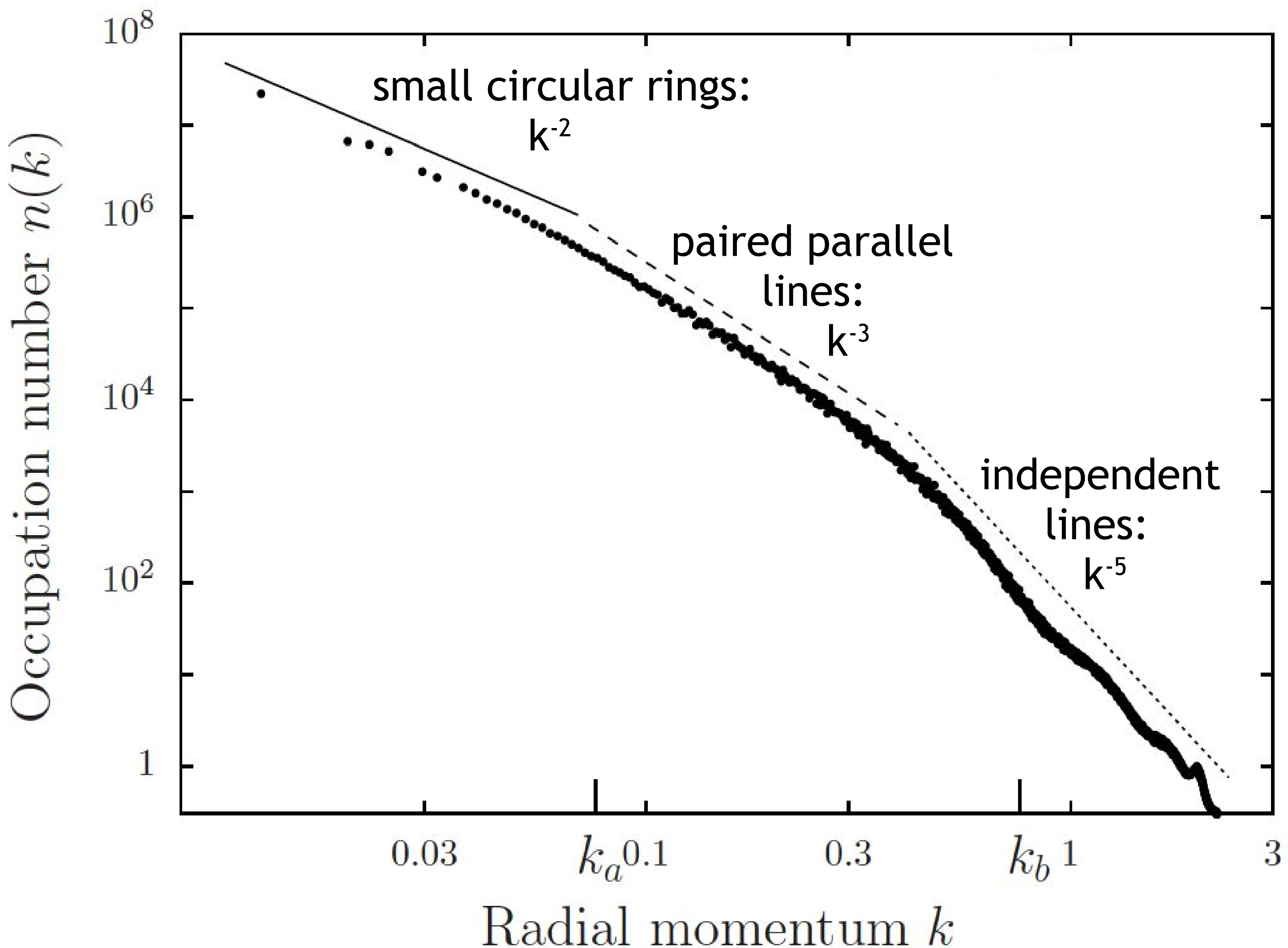
$\bar{v}$: average pair velocity

l : mean free path

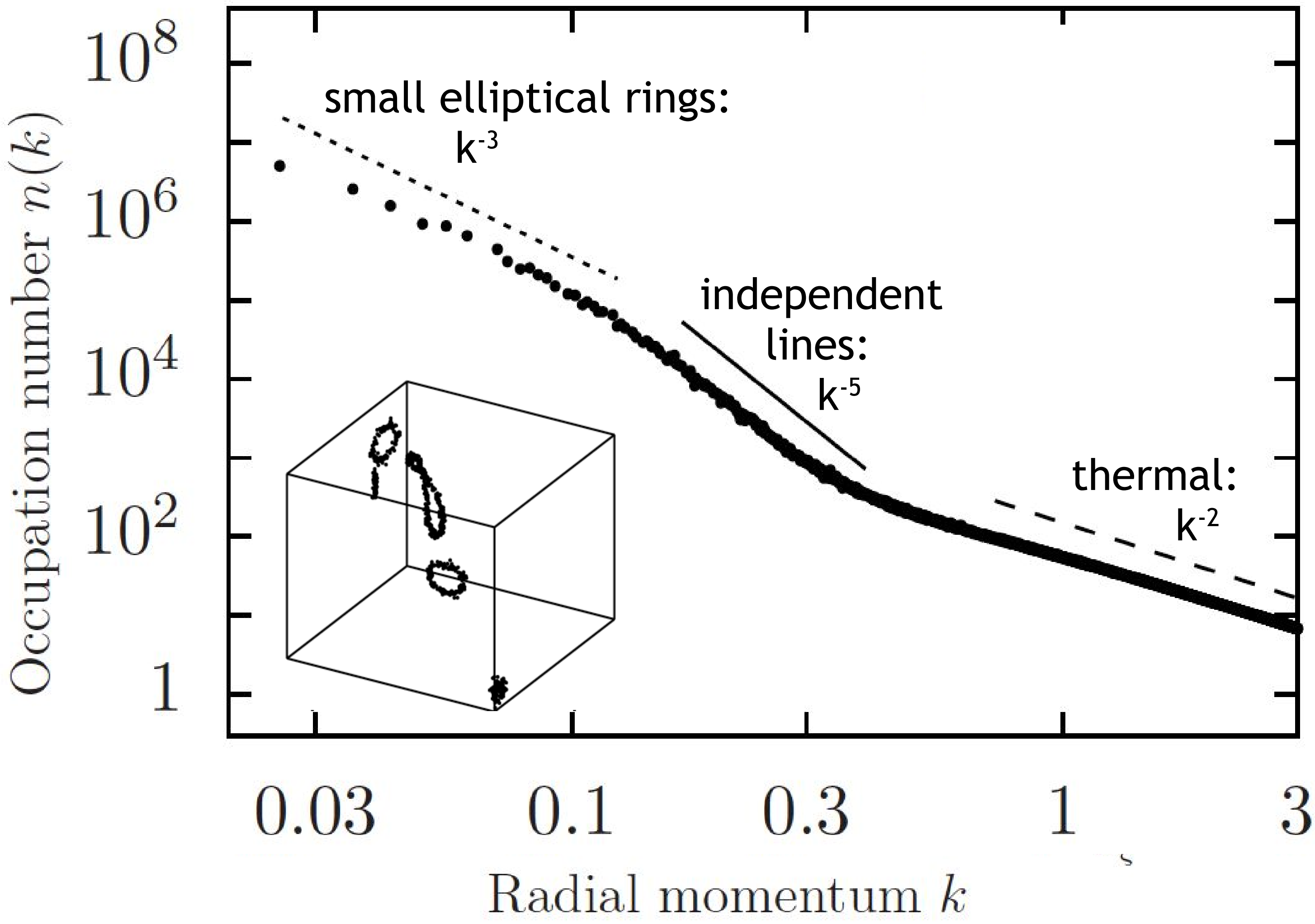
$$\Rightarrow \quad \partial_t n_v(t) \sim -n_v^2 \quad \Rightarrow \quad n_v(t) \sim t^{-1}$$

This result is valid under the assumption that the vortices are moving in pairs and that the pairs are homogeneously distributed.

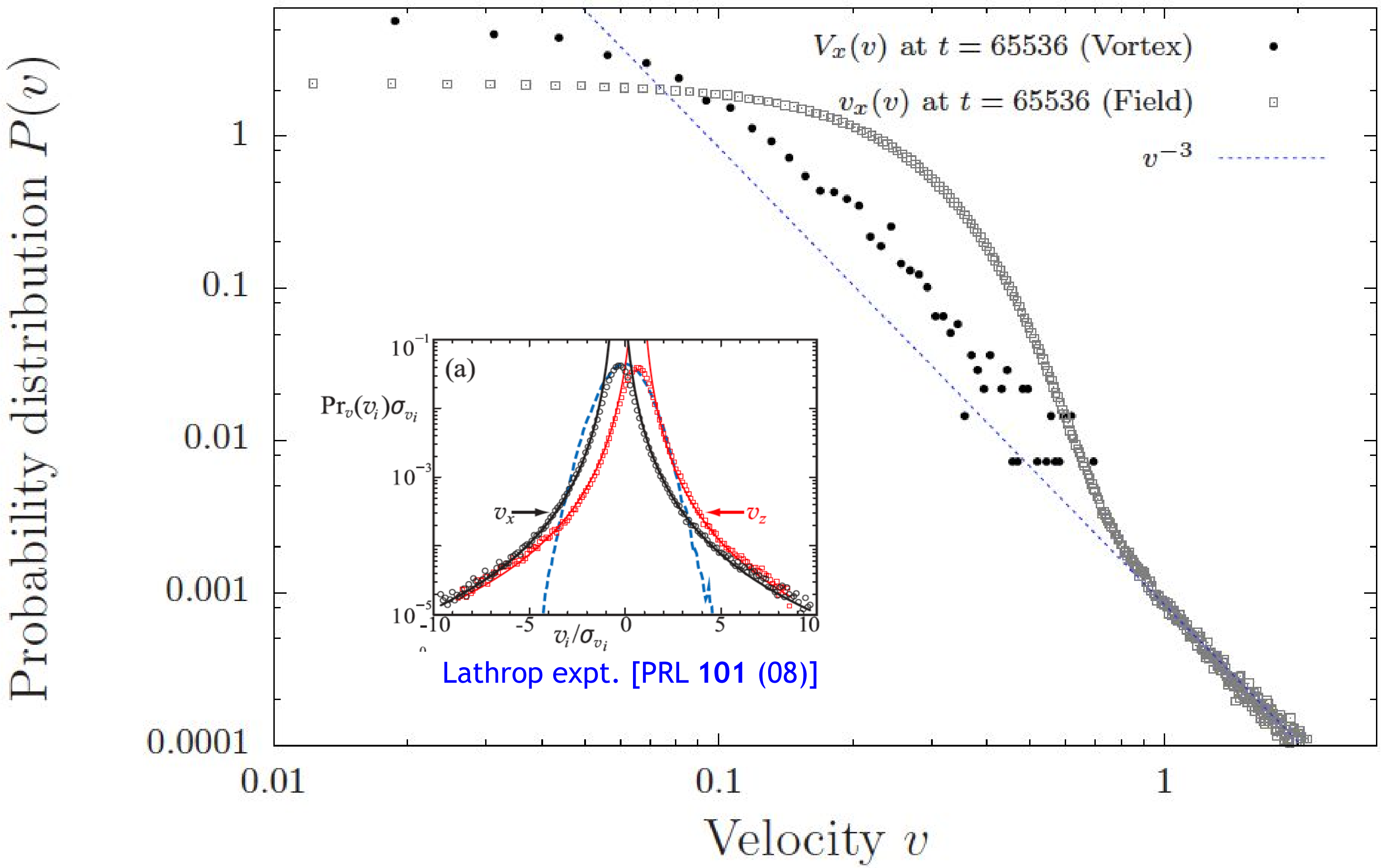
Line vortex model in 3+1 D



Simulations in 3+1 D



Vortex velocity distribution



J. Schole, B. Nowak, D. Sexty, TG (unpublished)
s. also C.F. White et al., PRL **104** (10); I.A. Min, Phys. Fluids **8** (96)

Velocity distributions

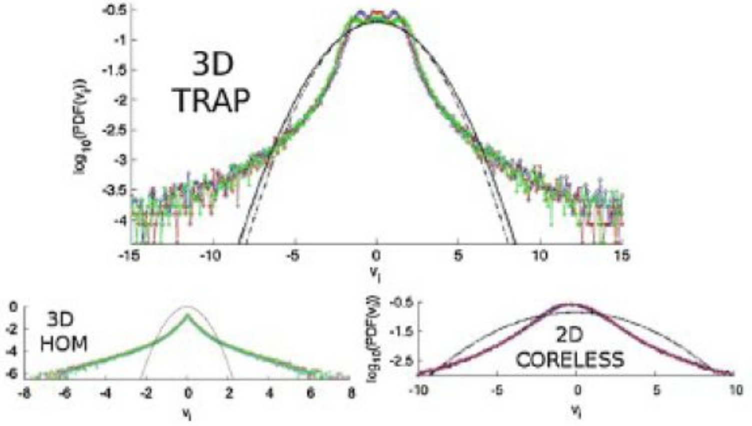
Paoletti et al. PRL 101, 154501 (2008):

Power law tails distinguish classical turbulence from classical turbulence.

Min et al. Phys. Fluids 8, 1169 (1996), White et al. PRL 104, 075301 (2010):

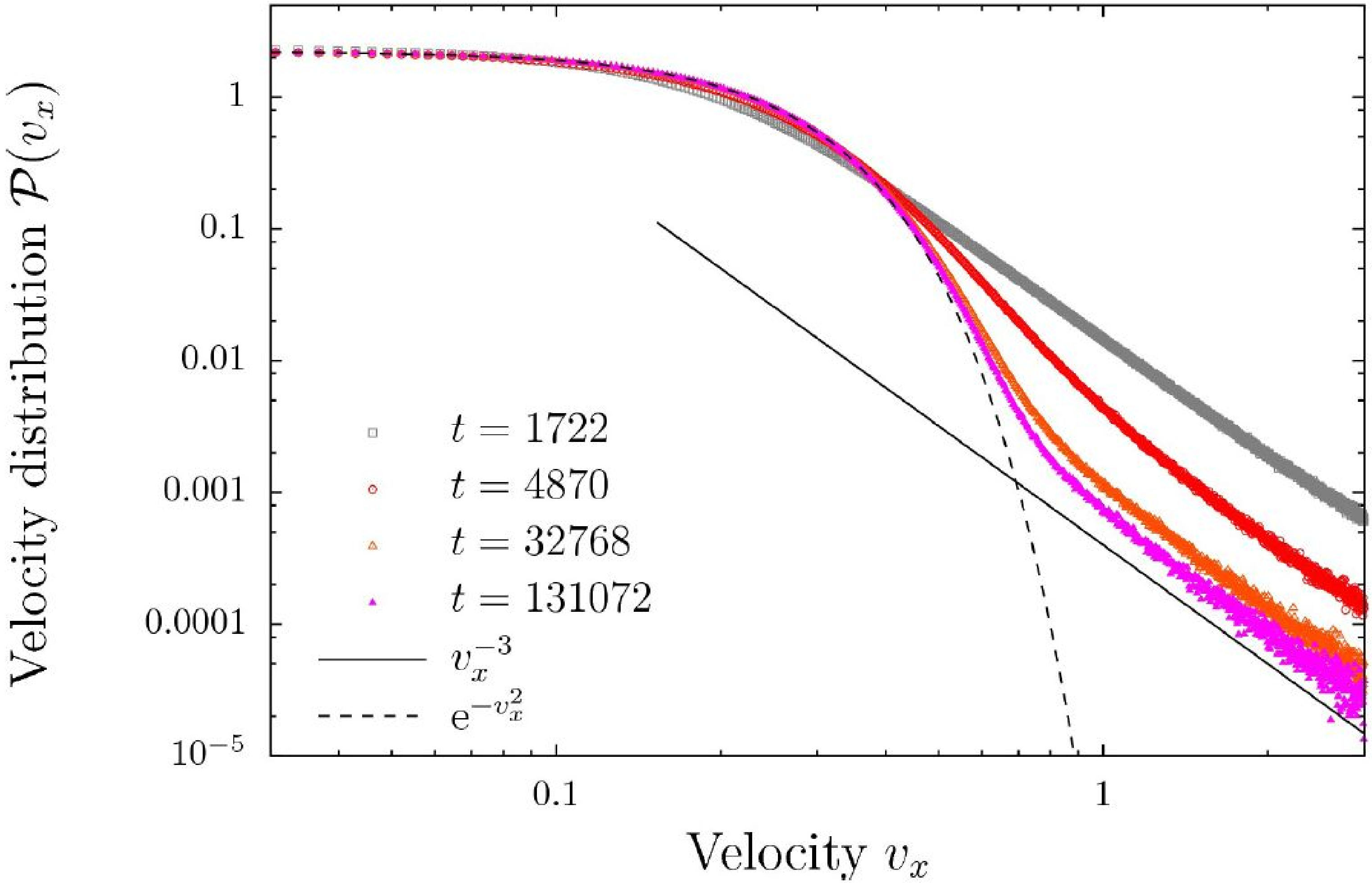
Point vortices: Power law tails

Vorticity patches: Gaussian distributions



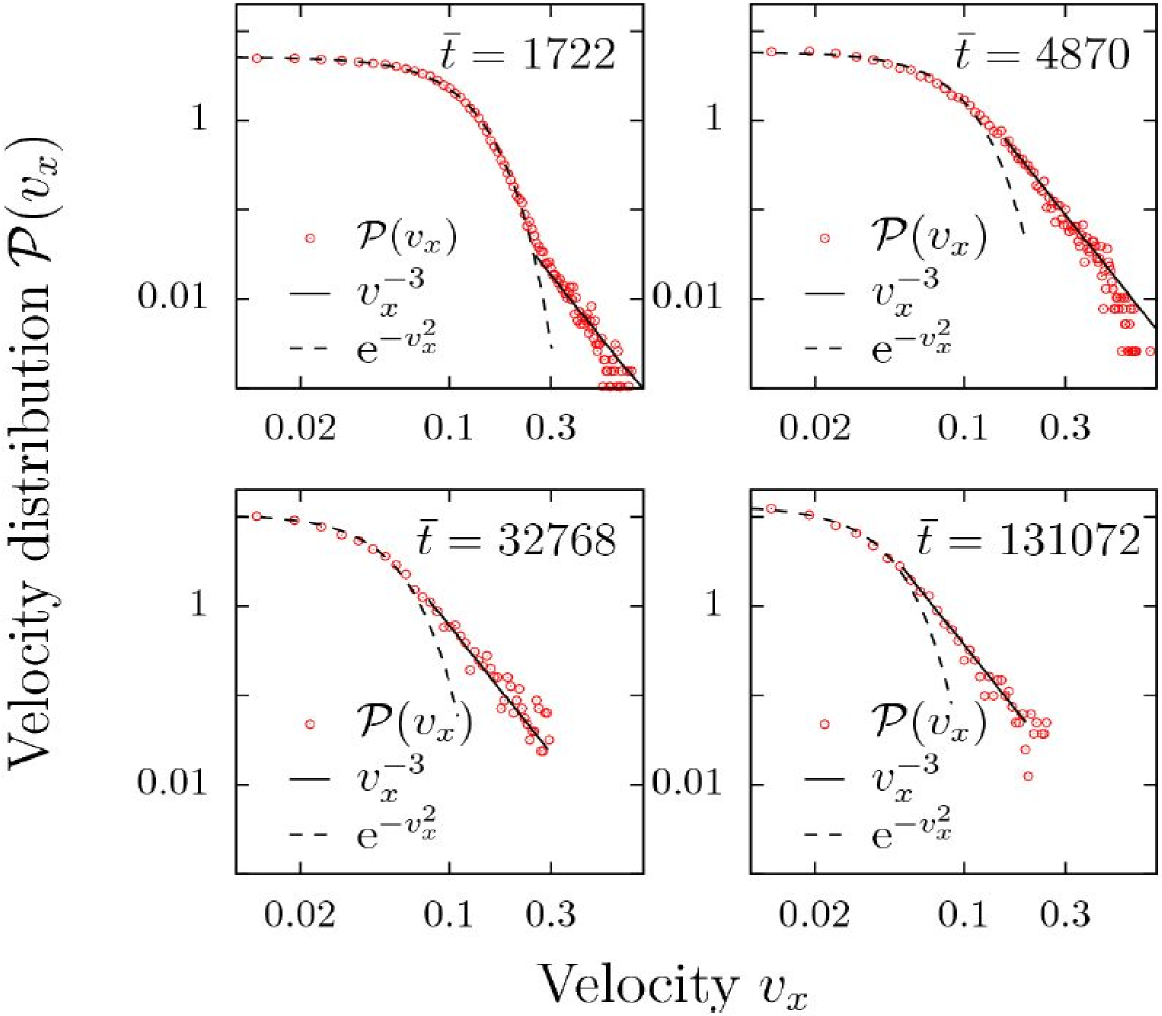
White et al. PRL 104, 075301 (2010)

Velocity distributions (Field)



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Velocity distributions (Vortices)

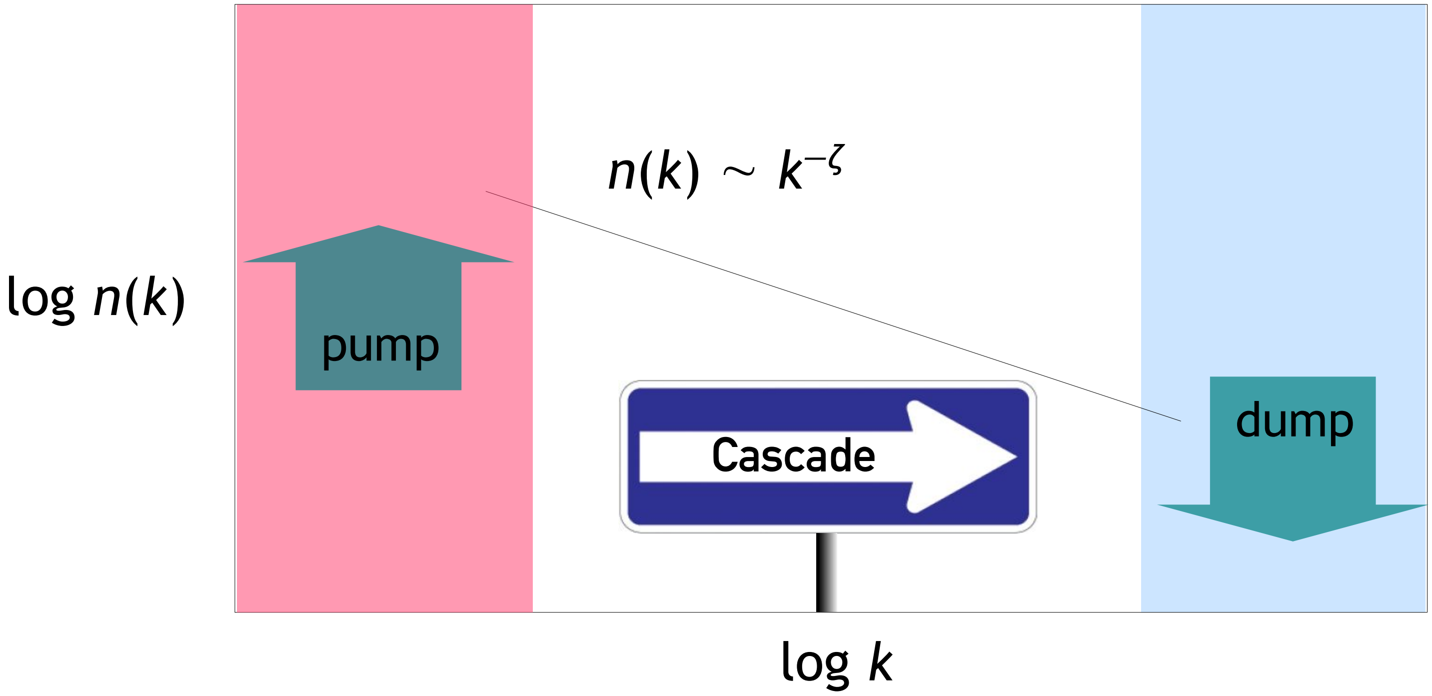


J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Wave turbulence
& vortex formation
in an ultracold Bose gas

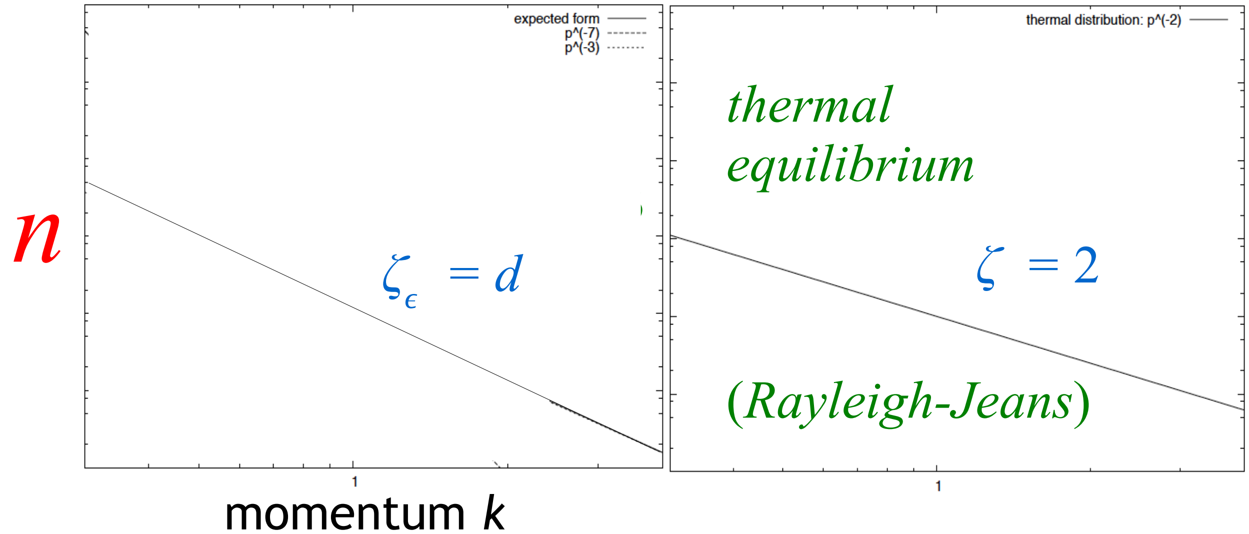
Wave turbulence

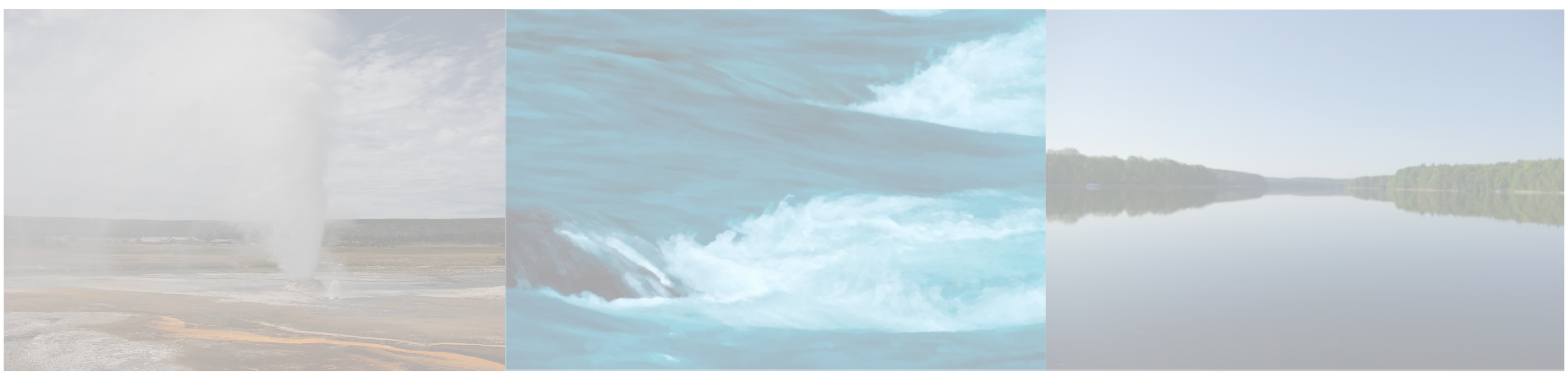
Stationary scaling $n(k)$ within inertial region:



Zakharov, L'vov, & Falkovich, *Kolmogorov Spectra of Turbulence I* (Springer, 1992)

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$





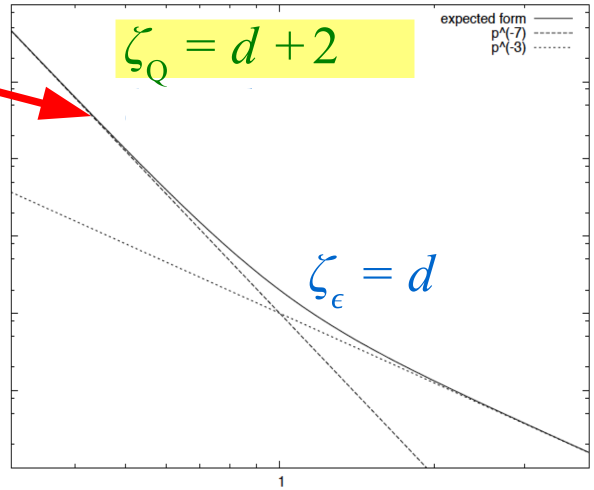
Zakharov, L'vov, & Falkovich, *Kolmogorov Spectra of Turbulence I* (Springer, 1992)

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$

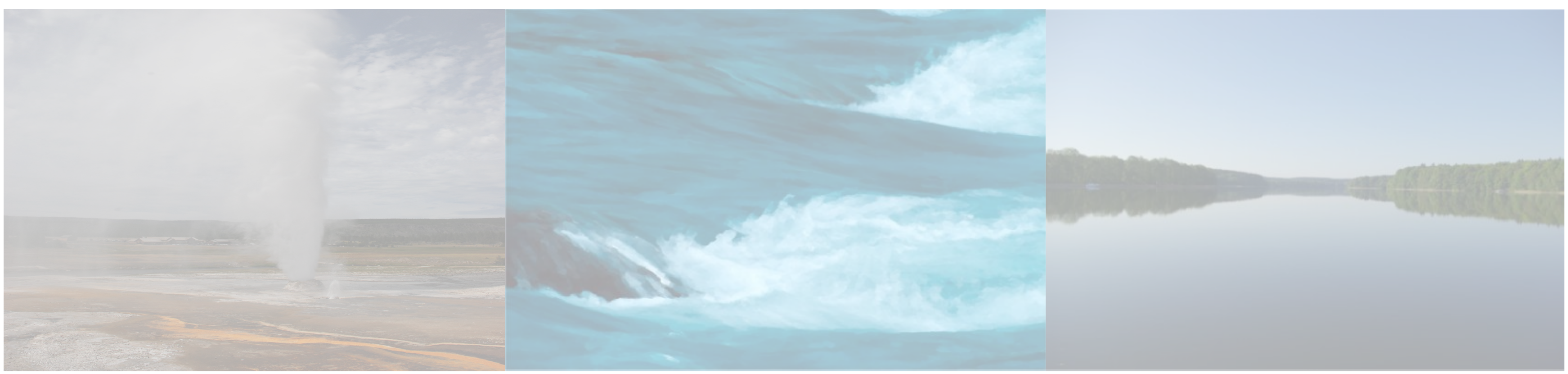
New exponent
beyond
Quantum Boltzmann!

@ Nonthermal Fixed Point

n



momentum k



J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603, J. Berges, G. Hoffmeister, NPB **813** (09) 383
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Bose gas in *d* spatial dimensions $n \sim k^{-\zeta}$

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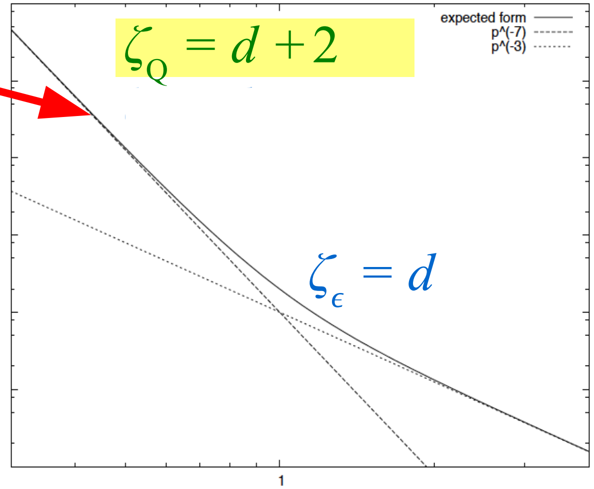
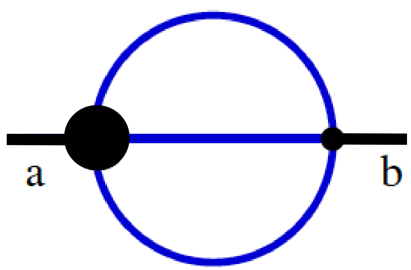
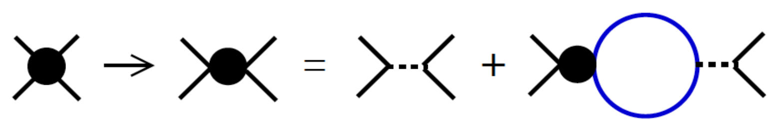


momentum k

$$\Sigma_{ab}(x,y) \;=$$



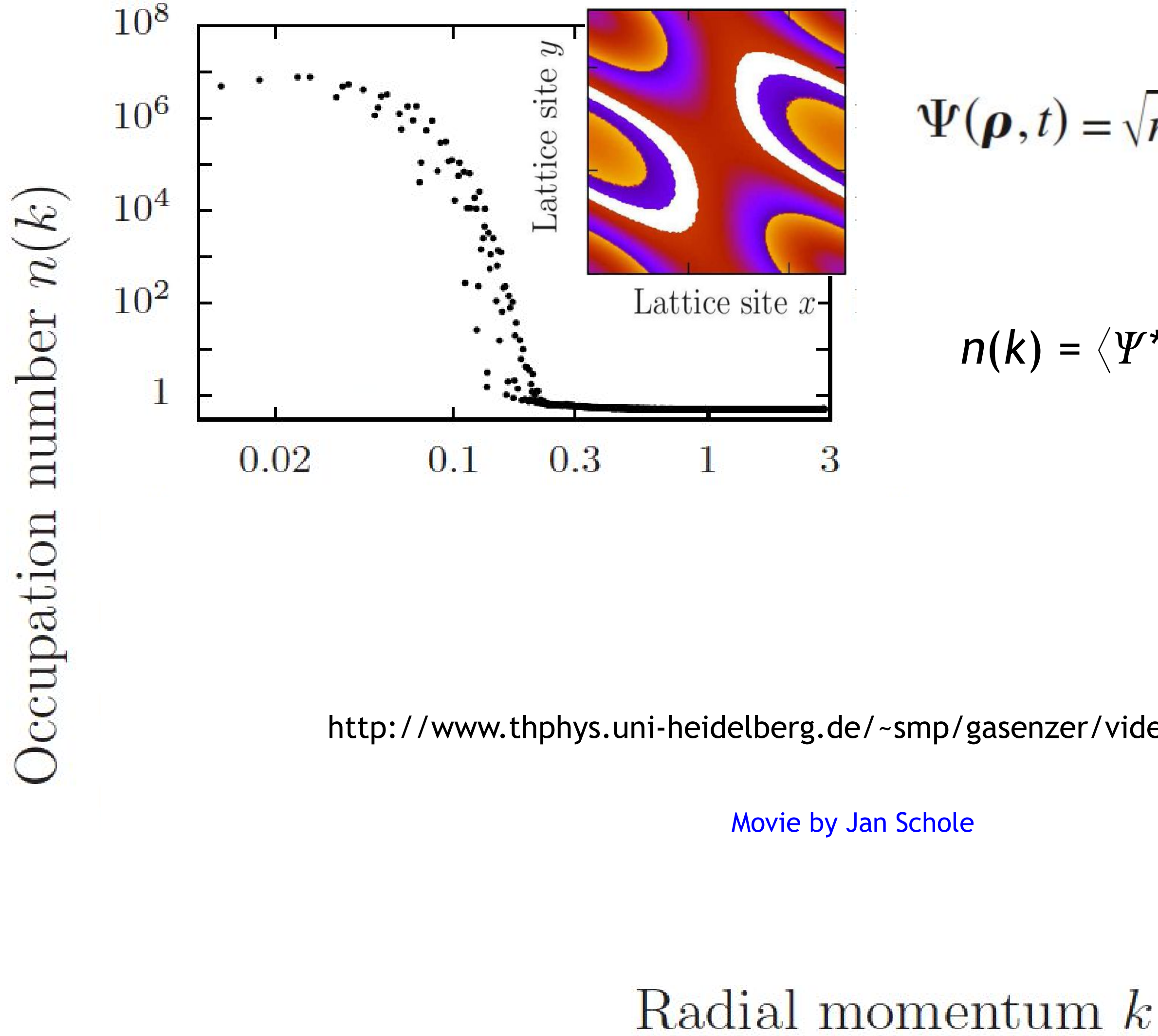
Vertex bubble resummation:
(2PI to NLO in $1/N$)



J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603, J. Berges, G. Hoffmeister, NPB **813** (09) 383
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Cascades in 2+1 D

B. Nowak, D. Sexty, TG, PRB 84 (11), 020506(R);
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127, PRA to appear (12)



$$\Psi(\boldsymbol{\rho},t) = \sqrt{n(\boldsymbol{\rho},t)}\exp[i\varphi(\boldsymbol{\rho},t)]$$

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$

http://www.thphys.uni-heidelberg.de/~smp/gasenzer/videos/boseqt.html

Movie by Jan Schole

Superfluid hydro of Bose-condensed Gas

The Gross-Pitaevskii Equation, ($g = 4\pi a_0/m$)

$$i\frac{\partial\Psi(\boldsymbol{\rho},t)}{\partial t} = \left(-\frac{\nabla^2}{2} + g|\Psi(\boldsymbol{\rho},t)|^2\right)\Psi(\boldsymbol{\rho},t)$$

using the defs.

$$\Psi(\boldsymbol{\rho},t) = \sqrt{n(\boldsymbol{\rho},t)}\exp[i\varphi(\boldsymbol{\rho},t)]$$

$$Q = gn \qquad \mathbf{u}(\boldsymbol{\rho},t) = \nabla\varphi(\boldsymbol{\rho},t)$$

can be written as

$$\frac{\partial}{\partial t}n + \nabla\cdot(n\mathbf{u}) = 0$$

Continuity equation

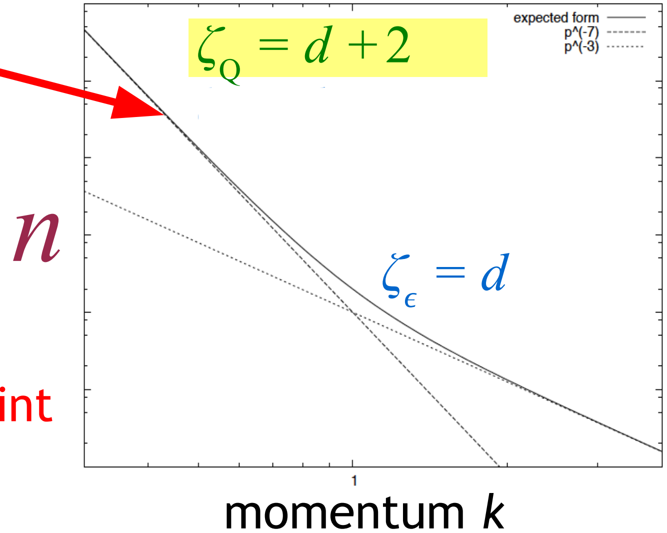
$$\frac{\partial}{\partial t}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla Q$$

Euler equation

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$

New exponent
beyond
Quantum Boltzmann!

@ Nonthermal Fixed Point



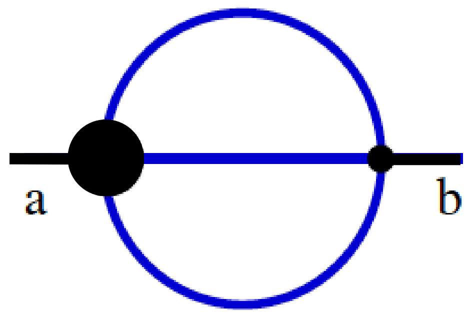
J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Dyn. QFT: Resummed Vertex

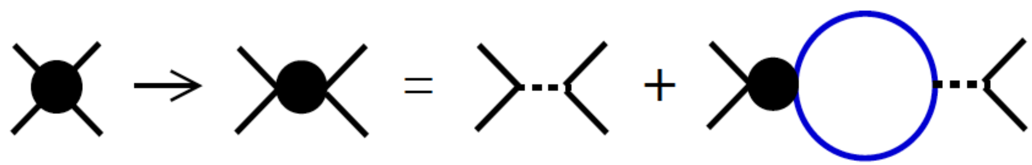
$p = (p_0, \mathbf{p})$:

$$J(p) := \Sigma^{\rho}_{ab}(p) F_{ba}(p) - \Sigma^{F}_{ab}(p)\rho_{ba}(p) \overset{!}{=} 0$$

$\Sigma_{ab}(x,y)$ =

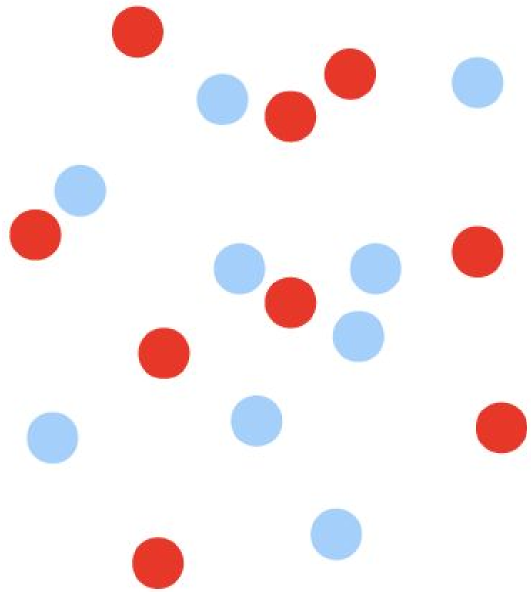


Vertex bubble resummation:
(e.g. 2PI to NLO in $1/N$)

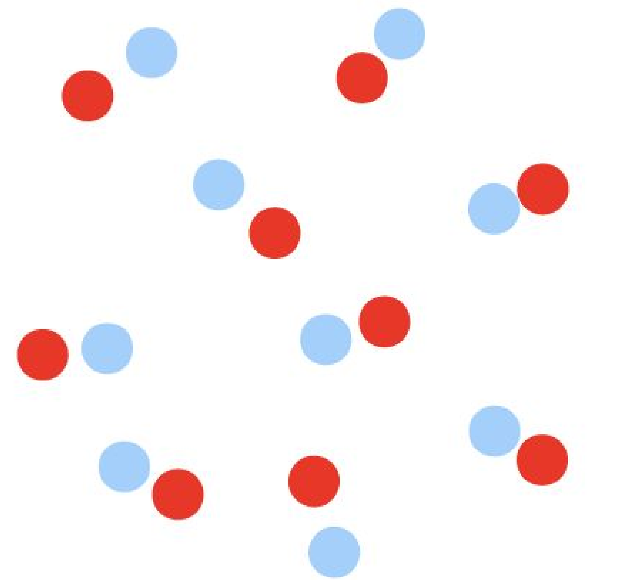


[relativ. QF *Dynamics*: Berges, Aarts (02-), Borsanyi, Serreau, et al.;
Ultracold gases: A. M. Rey et al. (04-), TG et al. (05-)
Nonth. fixed points: J. Berges, A. Rothkopf, J. Schmidt, PRL (08)]

Point vortex model



$$n_k \sim k^{-4}$$

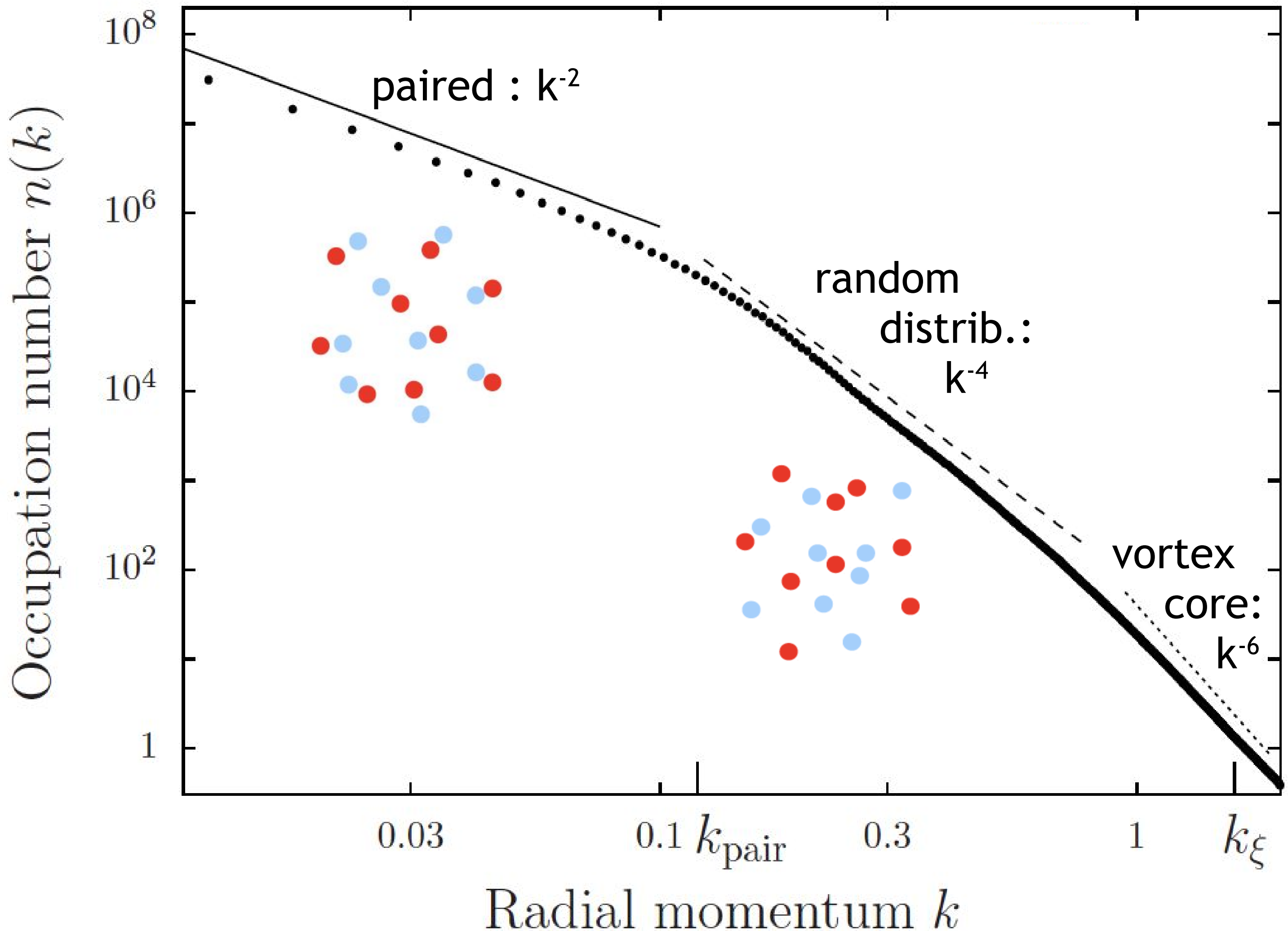


$$n_k \sim k^{-2}, \;\; k < k_{\text{pair}}$$

$$n_k \sim k^{-4}, \;\; k > k_{\text{pair}}$$

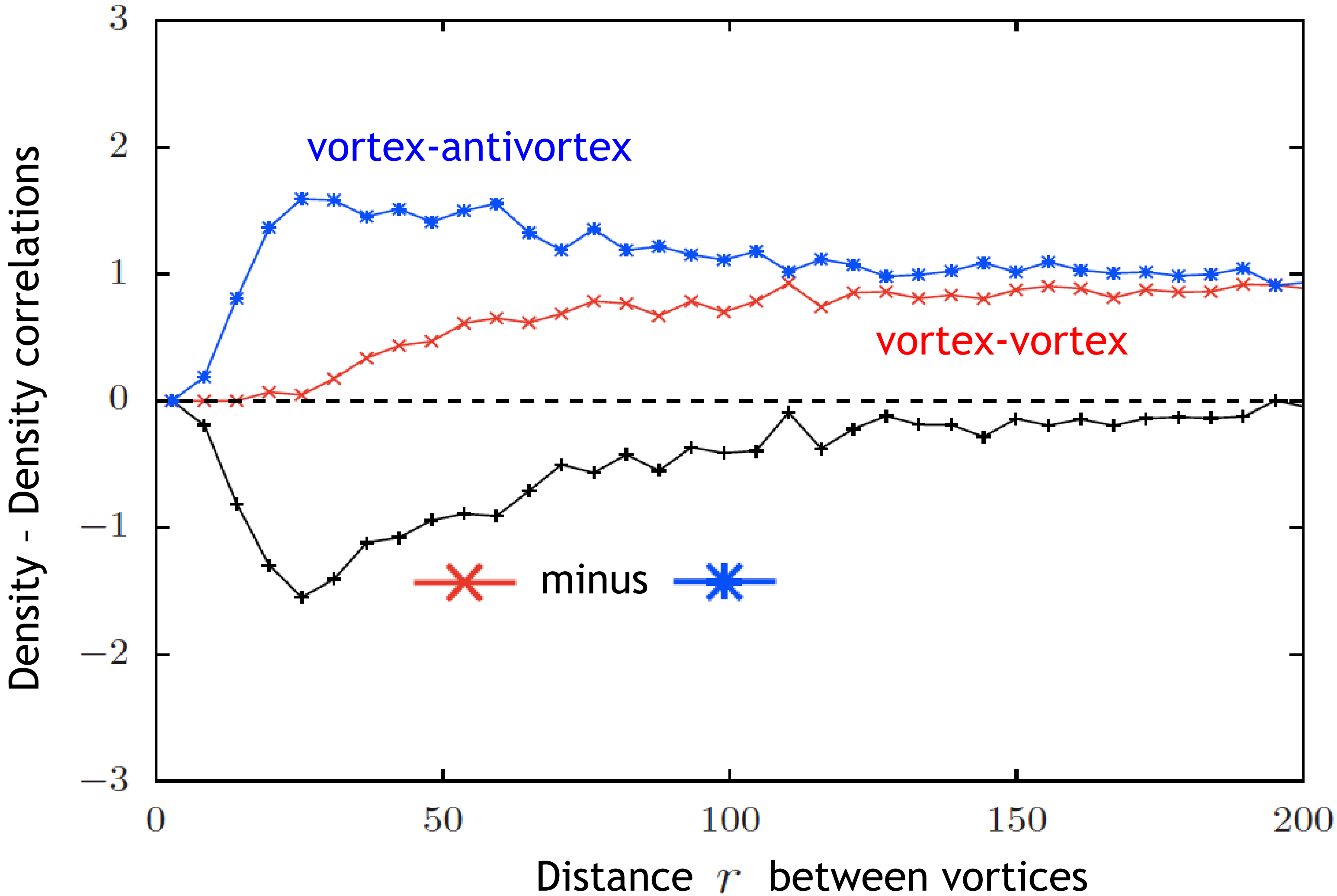
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127

Point vortex model in 2+1 D



B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127

Vortex position correlations



B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127