Chiral density waves in a parity doublet model at cold and dense nuclear matter

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Outline

The meson fields

lets build a global $U(2)_R \times U(2)_L$ invariant Lagrangian:

scalar and pseudoscalar fields

$$\Phi = \sum_{a=0}^{3} \phi_a t_a = (\sigma + \imath \eta_N)\, t^0 + (\mathbf{a}_0 + \imath \boldsymbol{\pi}) \cdot \mathbf{t} \,, \quad \Phi^\dagger = \sum_{a=0}^{3} \phi_a t_a = (\sigma - \imath \eta_N)\, t^0 + (\mathbf{a}_0 - \imath \boldsymbol{\pi}) \cdot \mathbf{t}$$

$$\Phi \to U_L \Phi U_R^\dagger \,, \quad \Phi^\dagger \to U_R \Phi^\dagger U_L^\dagger$$

vector and axial-vector fields

$$V^\mu = \sum_{a=0}^{3} V_a^\mu t_a = \omega^\mu t^0 + \boldsymbol{\rho}^\mu \cdot \mathbf{t} \,, \quad A^\mu = \sum_{a=0}^{3} A_a^\mu t_a = f_1^\mu t^0 + \mathbf{a}_1^\mu \cdot \mathbf{t}$$

$$R^\mu \equiv V^\mu - A^\mu \text{ and } L^\mu \equiv V^\mu + A^\mu$$

$$R^\mu \to U_R R^\mu U_R^\dagger \,, \quad L^\mu \to U_L L^\mu U_L^\dagger$$

The Lagrangian for the mesons

$$\mathscr{L}_M = \mathrm{Tr}\left[(D_\mu\Phi)^\dagger\,(D^\mu\Phi) \quad - m^2\Phi^\dagger\Phi \;-\lambda_2\left(\Phi^\dagger\Phi\right)^2\right] - \lambda_1\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2$$
$$+h_0\mathrm{Tr}\left[\Phi^\dagger+\Phi\right] \quad +c\left(\det\Phi^\dagger+\det\Phi\right)$$
$$-\frac{1}{4}\mathrm{Tr}\left[L_{\mu\nu}L^{\mu\nu}+R_{\mu\nu}R^{\mu\nu}\right]+\frac{1}{2}m_1^2\mathrm{Tr}\left[L_\mu L^\mu+R_\mu R^\mu\right]$$
$$+\frac{1}{2}h_1\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\mathrm{Tr}\left[L_\mu L^\mu+R_\mu R^\mu\right]+h_2\mathrm{Tr}\left[\Phi^\dagger L^\mu L_\mu\Phi+\Phi R^\mu R_\mu\Phi^\dagger\right]+2h_3\mathrm{Tr}\left[\Phi R_\mu\Phi^\dagger L^\mu\right]$$
$$-2\imath g_2\left(\mathrm{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\}+\mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}\right)$$
$$-2g_3\left(\mathrm{Tr}[(\partial_\mu R_\nu+\partial_\nu R_\mu]\{R^\mu,R^\nu\}+\mathrm{Tr}[(\partial_\mu L_\nu+\partial_\nu L_\mu]\{L^\mu,L^\nu\}\right)$$
$$+g_4\{\mathrm{Tr}[L^\mu L^\nu L_\mu L_\nu]+\mathrm{Tr}[R^\mu R^\nu R_\mu R_\nu]\}+g_5\{\mathrm{Tr}[L^\mu L_\mu L^\nu L_\nu]+\mathrm{Tr}[R^\mu R_\mu R^\nu R_\nu]\}$$
$$+g_6\mathrm{Tr}[R^\mu R_\mu]\mathrm{Tr}[L^\nu L_\nu]+g_7\{\mathrm{Tr}[R^\mu R_\mu]\mathrm{Tr}[R^\nu R_\nu]+\mathrm{Tr}[L^\mu L_\mu]\mathrm{Tr}[L^\nu L_\nu]\}\ ,$$

spontaneous symmetry breaking, explicit symmetry breaking, trace anomaly.

with the covariant derivative:

$$D^\mu\Phi = \partial^\mu\Phi - \imath c_1(\Phi R^\mu - L^\mu\Phi)$$

The mirror assignment

mirror assignment

$$\psi_{1,R} \to U_R\ \psi_{1,R}\ , \qquad \psi_{1,L} \to U_L\ \psi_{1,L},$$
$$\psi_{2,R} \to U_L\ \psi_{2,R}\ , \qquad \psi_{2,L} \to U_R\ \psi_{2,L}$$

baryon Lagrangian

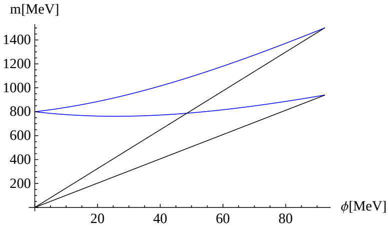
$$\begin{aligned}\mathscr{L}_B = {} & \bar{\psi}_{1,L} \imath \not{D}_{1,L} \psi_{1,L} + \bar{\psi}_{1,R} \imath \not{D}_{1,R} \psi_{1,R} + \bar{\psi}_{2,L} \imath \not{D}_{2,L} \psi_{2,L} + \bar{\psi}_{2,R} \imath \not{D}_{2,R} \psi_{2,R} \\ & - \hat{g}_1 \left(\bar{\psi}_{1,L} \Phi \psi_{1,R} + \bar{\psi}_{1,R} \Phi^\dagger \psi_{1,L} \right) - \hat{g}_2 \left(\bar{\psi}_{2,L} \Phi^\dagger \psi_{2,R} + \bar{\psi}_{2,R} \Phi \psi_{2,L} \right) \\ & + m_0 \left(\bar{\psi}_{2,L} \psi_{1,R} - \bar{\psi}_{2,R} \psi_{1,L} - \bar{\psi}_{1,L} \psi_{2,R} + \bar{\psi}_{1,R} \psi_{2,L} \right)\end{aligned}$$

$D^\mu_{1,R} = \partial^\mu - \imath c_1 R^\mu$, $D^\mu_{1,L} = \partial^\mu - \imath c_1 L^\mu$, $D^\mu_{2,R} = \partial^\mu - \imath c_2 R^\mu$, and $D^\mu_{1,L} = \partial^\mu - \imath c_2 L^\mu$.

arXiv:hep-ph/9805306v1

mirror assignment

not just the nucleon N but also their chiral partners N^*
chiral eigenstates are not equal to the mass eigenstates
$\Rightarrow$ mass eigenstates have to be diagonalized



naive assignment ($m_0 = 0$):

$$m_N = m_{\Psi_1} \propto \varphi$$
$$m_{N^*} = m_{\Psi_2} \propto \varphi$$

mirror assignment ($m_0 \neq 0$):

$$m_N = \frac{1}{2}\sqrt{(\hat{g}_1 + \hat{g}_2)^2 \varphi^2 + 4m_0^2} + \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\varphi$$
$$m_{N^*} = \frac{1}{2}\sqrt{(\hat{g}_1 + \hat{g}_2)^2 \varphi^2 + 4m_0^2} - \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\varphi$$

Further extensions and achievements

further modifications:

- shift of the axial-vector fields with the corresponding pseudoscalar fields
- origin of the m_0 term:
  glueball condensate G_0 or tetraquark condensate χ_0 with
  $m_0 = aG_0 + b\chi_0$

resulting achievements ($m_0 = b\chi_0$):

- scattering length and decay width of the mesons can be described within reasonable errors
- πN scattering is also in good agreement with experiment
- at finite density (within a mean field approach), conditions for nuclear matter are fullfilled

arXiv:1003.4934, arXiv:1105.5003v1, arXiv:0907.5084, arXiv:1103.3238

CDW and implementation

Model is simplified to the very basic level:

- most of the mesons beside $\sigma, \vec{\pi}$ and ω^μ are ignored
- higher order interactions of vector mesons are ignored
- m_0 treated as a constant
- no aim (for the moment) to describe vacuum phenomenology
- all calculations are done within a mean field approach

minimal Baryonic Lagrangian

$$\begin{aligned}\mathscr{L} = & \bar{\psi}_1 \imath \partial\!\!\!/ \psi_1 + \bar{\psi}_2 \imath \partial\!\!\!/ \psi_2 - g_\omega^{(1)} \bar{\psi}_1 \imath \gamma_\mu \omega^\mu \psi_1 - g_\omega^{(2)} \bar{\psi}_2 \imath \gamma_\mu \omega^\mu \psi_2 \\ & - \frac{1}{2} \hat{g}_1 \bar{\psi}_1 \left(\sigma + \imath \gamma_5 \vec{\tau} \cdot \vec{\pi} \right) \psi_1 - \frac{1}{2} \hat{g}_2 \bar{\psi}_2 \left(\sigma - \imath \gamma_5 \vec{\tau} \cdot \vec{\pi} \right) \psi_2 \\ & + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 \right) + \mathscr{L}_M\end{aligned}$$

arXiv:nucl-th/0608044v2

The chiral density wave

Ansatz for the chiral density wave:

$$\langle\sigma\rangle \sim \varphi \cos(2\ f\ x)\ , \qquad \langle\pi_0\rangle \sim \varphi \sin(2\ f\ x)$$

$$\begin{aligned}
\mathscr{L}_B =& \bar{\psi}_1 \imath \partial\!\!\!/ \psi_1 + \bar{\psi}_2 \imath \partial\!\!\!/ \psi_2 + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2\right) \\
& - \frac{1}{2} \hat{g}_1 \varphi \bar{\psi}_1 \left[\cos(2fx) + \imath \gamma_5 \tau_3 \sin(2fx)\right] \psi_1 - \frac{1}{2} \hat{g}_2 \bar{\psi}_2 \left[\cos(2fx) - \imath \gamma_5 \tau_3 \sin(2fx)\right] \psi_2 \\
& + \dots \\
=& \bar{\psi}_1 \imath \partial\!\!\!/ \psi_1 + \bar{\psi}_2 \imath \partial\!\!\!/ \psi_2 + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2\right) \\
& - \frac{1}{2} \hat{g}_1 \varphi \bar{\psi}_1 \exp\left(+\imath 2 \gamma_5 \tau_3 f x\right) \psi_1 - \frac{1}{2} \hat{g}_2 \varphi \bar{\psi}_2 \exp\left(-\imath 2 \gamma_5 \tau_3 f x\right) \psi_2 \\
& + \dots
\end{aligned}$$

recall:

$\exp(\imath a \tau_3) = \cos(a) + \imath \tau_3 \sin(a)$ and $\exp(\imath a \gamma_5 \tau_3) = \cos(a) + \imath \gamma_5 \tau_3 \sin(a)$

arXiv:1102.4049v1

Towards the grand canonical potential

transformation of the fermion fields

$$\bar{\psi}_1 \to \bar{\psi}_1 \exp[-\imath\gamma_5\tau_3 fx], \quad \psi_1 \to \exp[-\imath\gamma_5\tau_3 fx]\psi_1$$
$$\bar{\psi}_2 \to \bar{\psi}_2 \exp[+\imath\gamma_5\tau_3 fx], \quad \psi_2 \to \exp[+\imath\gamma_5\tau_3 fx]\psi_2$$

- $\bar{\psi}_1 \exp[+\imath\gamma_5\tau_3 2fx]\psi_1 \to \bar{\psi}_1\psi_1, \qquad \bar{\psi}_2 \exp[-\imath\gamma_5\tau_3 2fx]\psi_2 \to \bar{\psi}_2\psi_2$
- $\bar{\psi}_1\gamma_\mu\psi_1 \to \bar{\psi}_1\gamma_\mu\psi_1, \qquad \bar{\psi}_2\gamma_\mu\psi_2 \to \bar{\psi}_2\gamma_\mu\psi_2$
- $\bar{\psi}_1\imath\partial\!\!\!/\psi_1 \to \bar{\psi}_1\imath\partial\!\!\!/\psi_1 + \bar{\psi}_1\gamma_1\gamma_5\tau_3 f\psi_1, \quad \bar{\psi}_2\imath\partial\!\!\!/\psi_2 \to \bar{\psi}_2\imath\partial\!\!\!/\psi_2 - \bar{\psi}_2\gamma_1\gamma_5\tau_3 f\psi_2$

$\Rightarrow$ the explicit space dependence transformed to an additional momentum dependence

The mesonic Lagrangian

$$\mathscr{L}_M = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 + h_0\sigma + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$

within the mean field approximation:

$$F_{\mu\nu}F^{\mu\nu} \to 0$$
$$\omega_\mu\omega^\mu \to \bar{\omega}_0^2$$
$$\sigma^2+\vec{\pi}^2 \to \varphi^2$$
$$\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi} \to 4f^2\varphi^2$$

$$V_M = 2f^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - \frac{1}{2}m^2\varphi^2 - h_0\varphi - \frac{1}{2}m_\omega^2\bar{\omega}_0^2$$

Grand canonical potential

$$\frac{\Omega}{V} = 2f^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - \frac{1}{2}m^2\varphi^2 - \epsilon\varphi - \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \sum_{k=1}^{4}\frac{2}{(2\pi)^2}\int d^3p \left(\sqrt{\vec{p}^2 + \bar{m}_k(p_1)^2} - \mu^*\right)\Theta\left(\mu^* - \sqrt{\vec{p}^2 + \bar{m}_k(p_1)^2}\right)$$

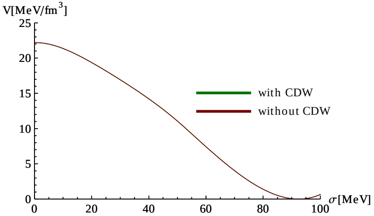
with the short notation $\mu^* = \mu - g_\omega\bar{\omega}_0$

mean meson fields are obtained by minimizing Ω

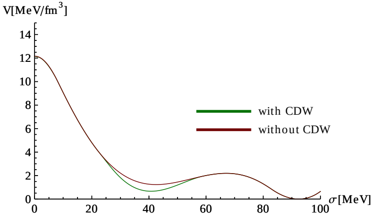
$$0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial\varphi},\ 0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial\bar{\omega}_0},\ 0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial f}$$

Potential in the chiral limit

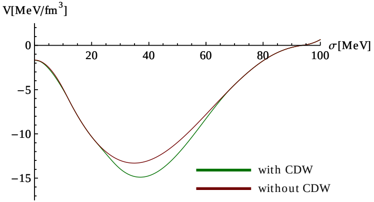
$\mu_B = 800$ MeV, vacuum



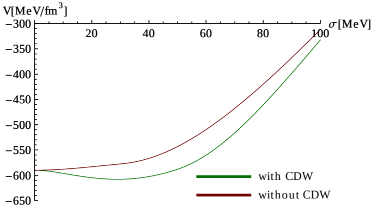
$\mu_B = 900$ MeV, vacuum



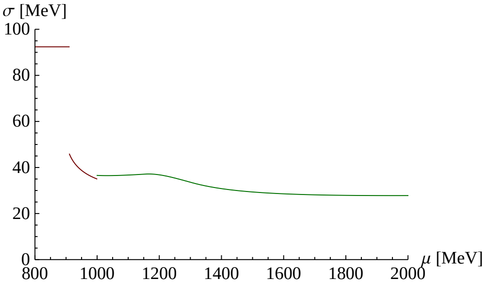
$\mu_B = 950$ MeV



$\mu_B = 1500$ MeV



Condensate φ at finite μ_B and finite m_π

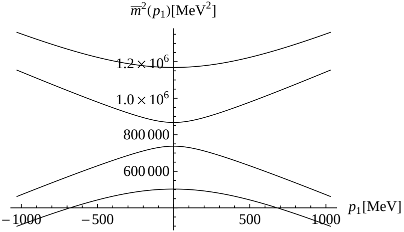


red line: homogeneous condensation

green line: inhomogeneous condensation

- nuclear matter is possible
- for moderate μ_B crystalline phase is realized
- chiral symmetry is never restored, indeed value increases
- increase of g_ω: inhomogeneous phase is realized for higher μ_B
- decrease of $m_0 \to 0$: intermediate homogeneous state disappears

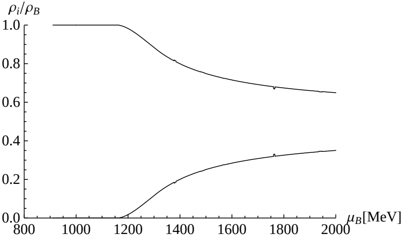
Dispersion relation and relative density



$\mu = 1000$ MeV, $\varphi = 36.6$ MeV, $\bar{\omega}_0 = 30.9$ MeV,

$f = 183.7$ MeV, and $p_2 = p_3 = 0$

- $E_k = \sqrt{\vec{p}^2 + \bar{m}_k(p_1)^2}$, $k = 1...4$
- shape remains similar even for high μ_B



- for densities far lower than the masses of N and N^* a finite number of nucleons is present
- for high μ_B only two different states are present

Summary and Outlook

- parity doublet model favors inhomogeneous condensation
- crystalline phase has a strong parameter depends
- chiral symmetry will not be restored for asymptotic large μ_B

- extend to more realistic setup
- calculations beyond mean field
- test further inhomogeneous realizations beside the CDW

Thank you