How far can we go with Hydro?

- Hidden Coarse-Graining Size

Takeshi Kodama
ExtreMe Matter Institute – EMMI
Frankfurt Institute for Advanced Study – FIAS
Federal University of Rio de Janeiro - UFRJ

EMMI ExtreMe Matter Institute EMMI

FIAS Frankfurt Institute for Advanced Studies

How far can we go with Hydro?

- Hidden Coarse-Graining Size

With
Ph. Mota – UFRJ, FIAS
R. D. Souza – UNICAMP
Jun Takahashi - UNICAMP

ExtreMe Matter Institute EMMI

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Common Statement we hear frequently:

Success of (almost ideal) hydrodynamic description in Relativistic Heavy Ion Collisions

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Expectations we hear also frequently :

- Determination of Properties of Matter (EoS, Transport coefficieints)
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- Key for the QCD dynamics…

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Local Thermal Equilibirum

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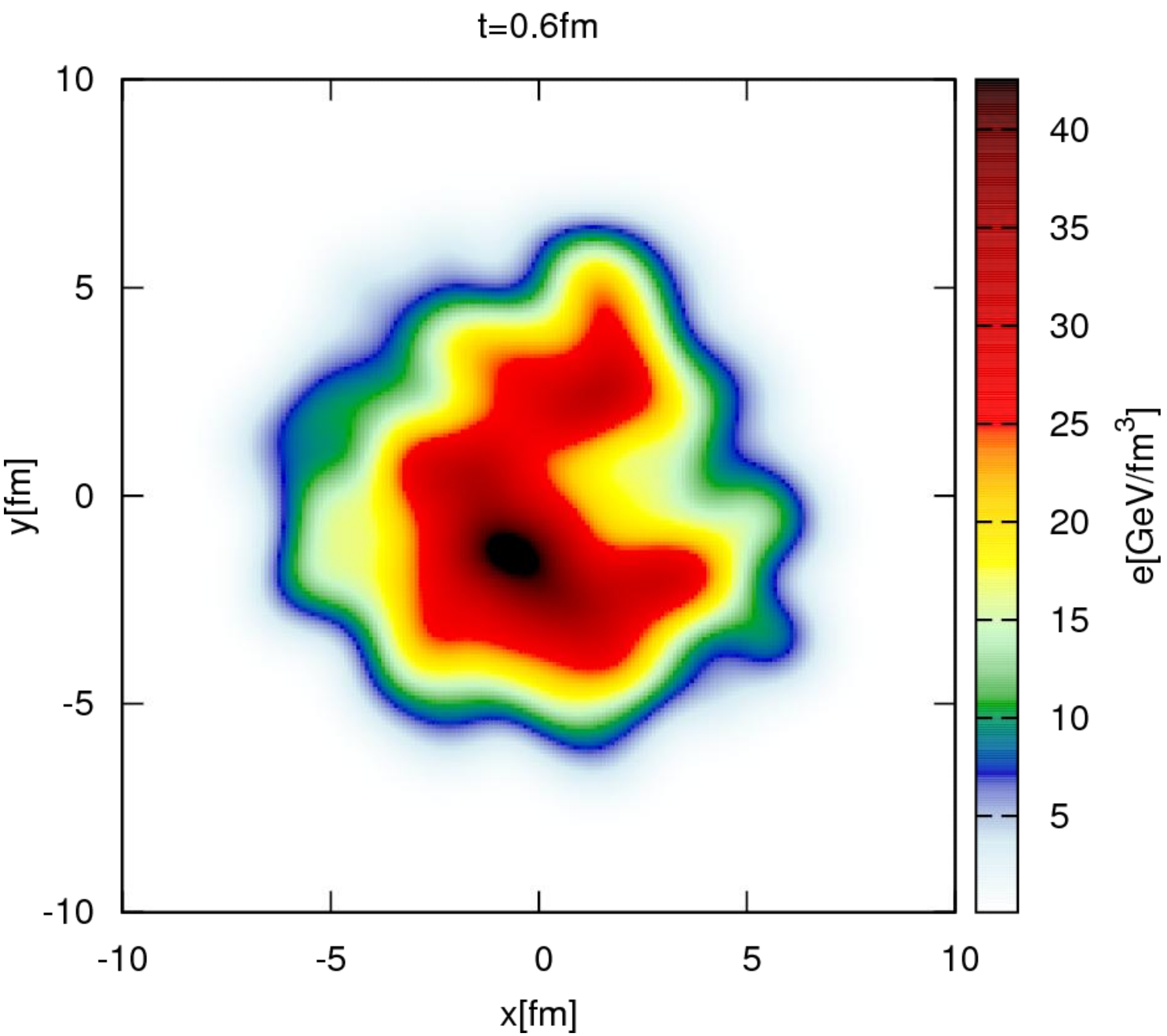
Relativistic Hydrodynamics as Covariant Local Classical Field Theory

- Local Thermal Equilibrium is considered as a necessary condition
- Very difficult…. even Conflicting, if it is really local.

Structure of Relativistic Hydrodynamics

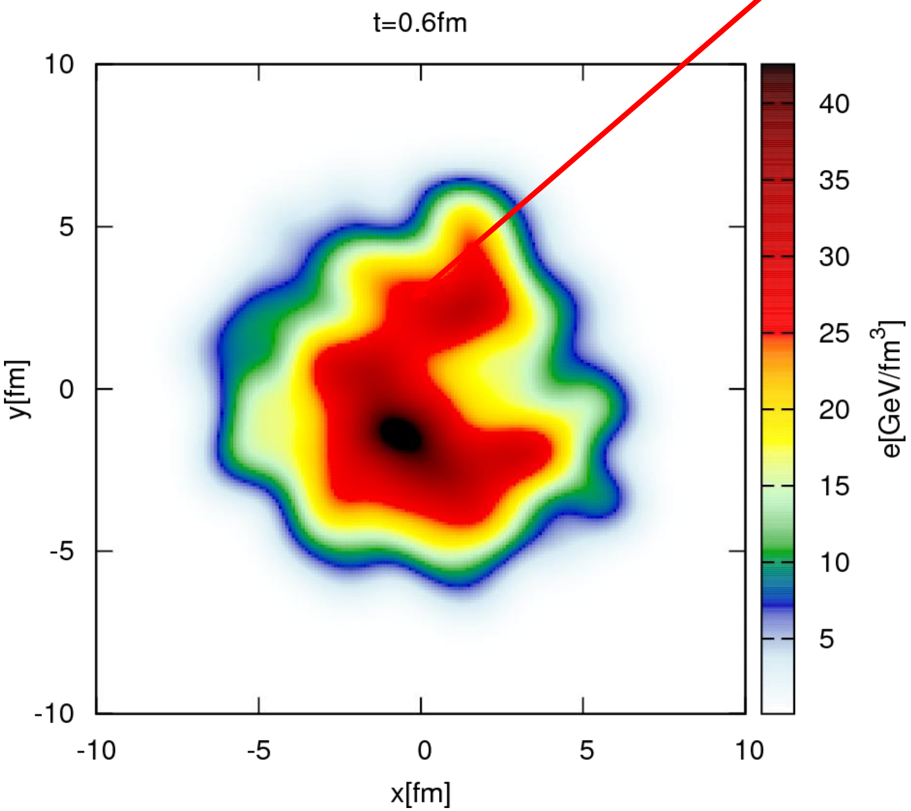
Structure of Relativistic Hydrodynamics



Structure of Relativistic Hydrodynamics

$$^{\forall}x = (t, \vec{r}),$$

$$^{\exists}T^{\mu\nu}(x),\ n^{\mu}(x)$$



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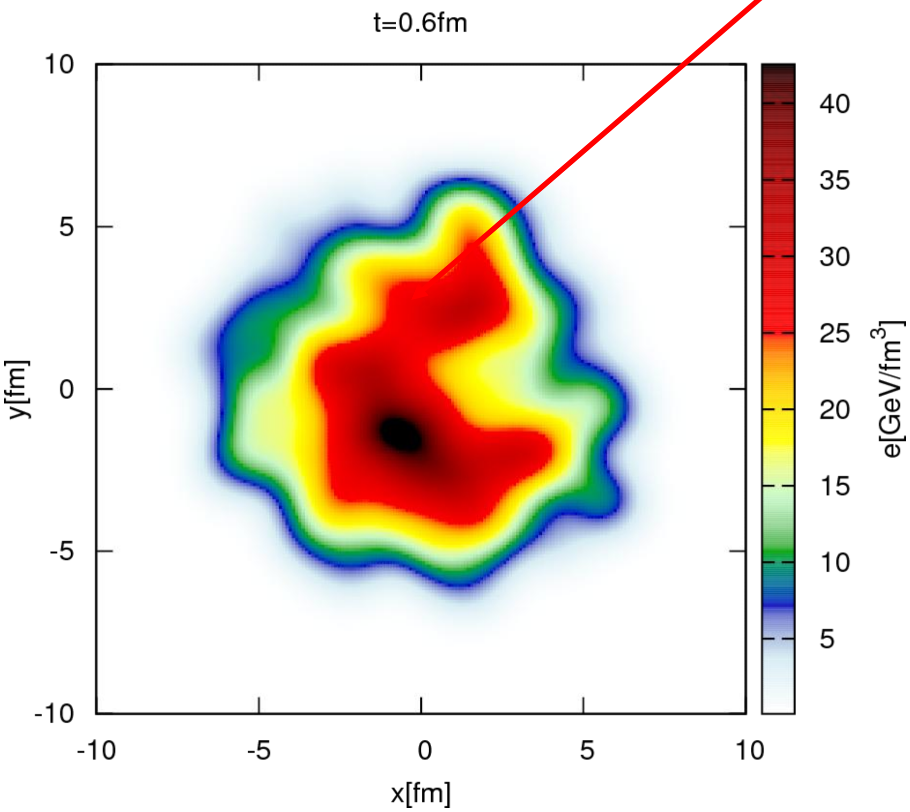


$$^{\forall}x = (t, \vec{r}),$$

$$^{\exists}T^{\mu\nu}(x),\ n^{\mu}(x)$$

with

$$\partial_{\mu}T^{\mu\nu}(x) = 0,$$

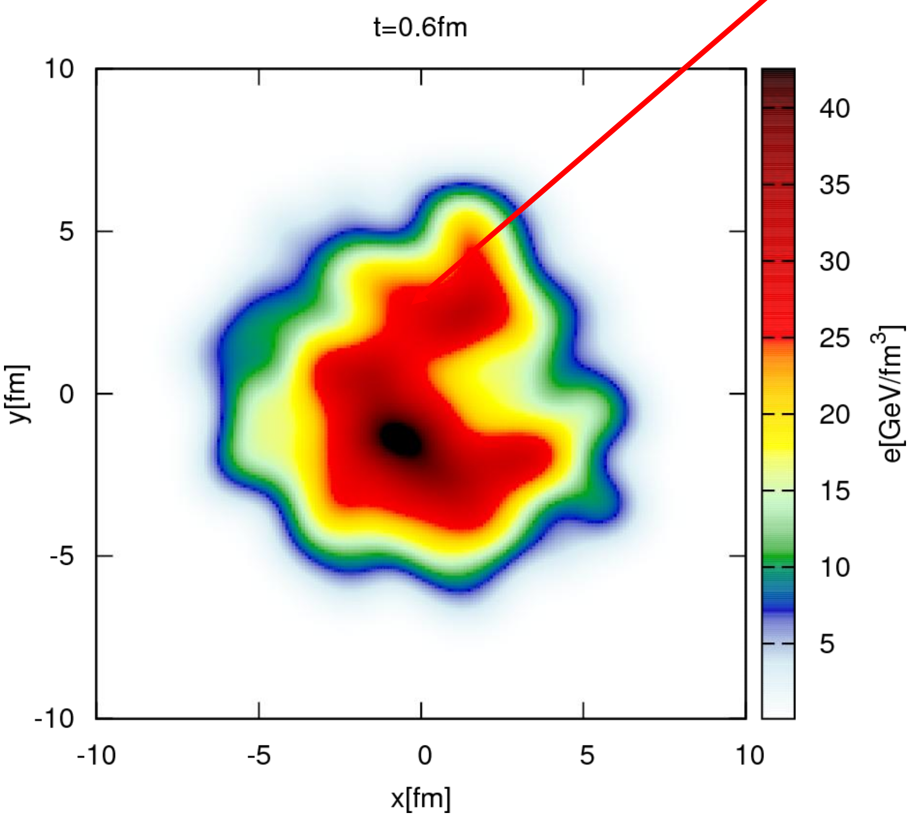
$$\partial_{\mu}n^{\mu}(x) = 0.$$

Structure of Relativistic Hydrodynamics

14 unknowns

$^{\forall}x = (t, \vec{r}),$

$^{\exists}T^{\mu\nu}(x),\ n^{\mu}(x)$



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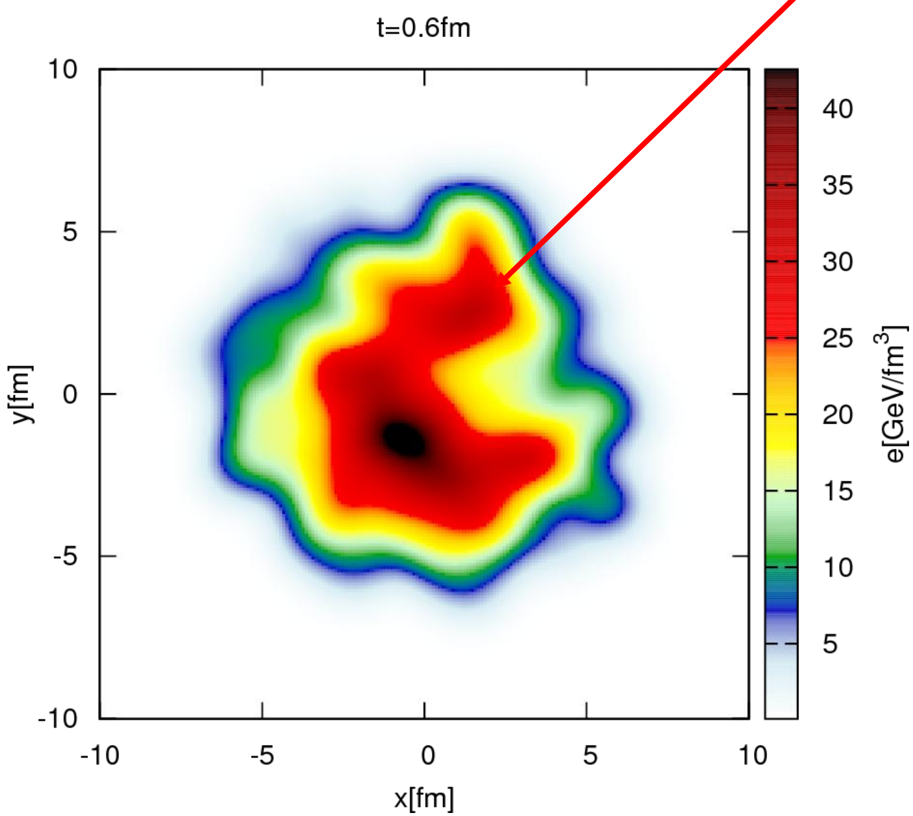
5 equations

Structure of Relativistic Hydrodynamics

Local Rest Frame (Landau)

$^{\forall}x,$

$$T^{\mu\nu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} \varepsilon & 0 \\ 0 & T \end{pmatrix}$$



Structure of Relativistic Hydrodynamics

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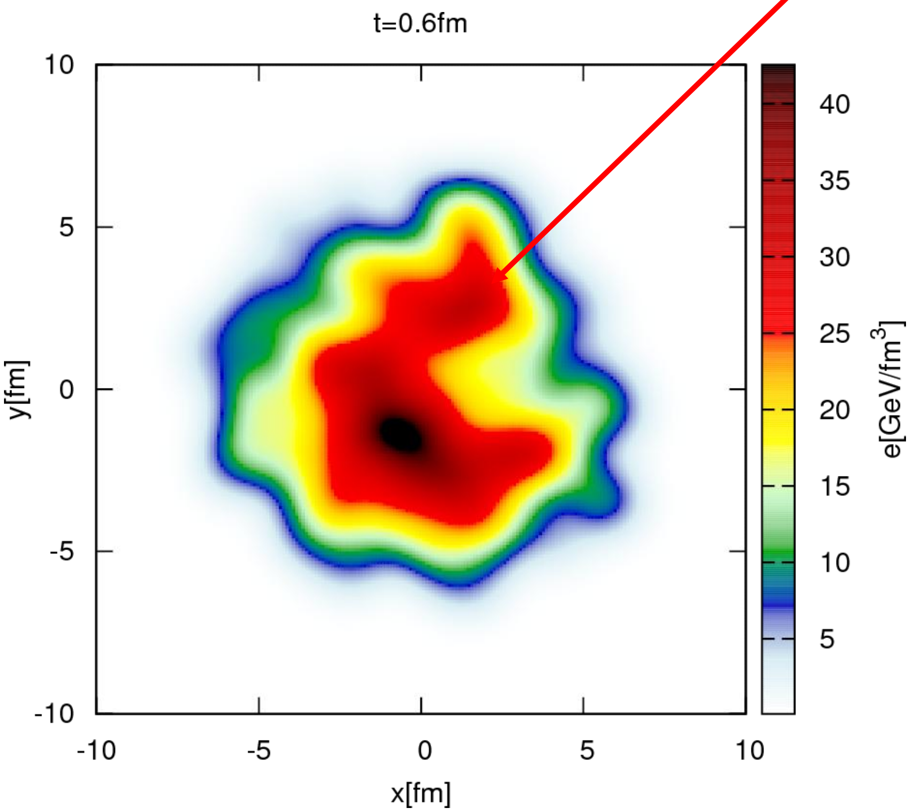

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14-5

Structure of Relativistic Hydrodynamics

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14-5 -3

Structure of Relativistic Hydrodynamics

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Additional conditions:

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$$T^{\mu\nu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} \varepsilon & 0 \\ 0 & T \end{pmatrix}$$

and

14-5 -3 -1 = 5

$$n^{\mu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} n \\ \vec{j}_{Diff} \end{pmatrix}$$

- ε is strongly correlated with n.

Structure of Relativistic Hydrodynamics

Ideal fluid case

$^{\forall}x,$

Additional conditions:

- T is isotropic
- No diffusion $\vec{j}_{Diff} = 0$

$$T^{\mu\nu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} \varepsilon & 0 \\ 0 & T \end{pmatrix}$$

and

14-5 -3 -1 = 5 = No. of Eqs.!!

$$n^{\mu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} n \\ \vec{j}_{Diff} \end{pmatrix}$$

- ε is strongly correlated with n.

Structure of Relativistic Hydrodynamics

Local Thermal Equilibrium

$^\forall x,$

$$T^{\mu\nu}(x) \xrightarrow{\Lambda(u)} \begin{pmatrix} \varepsilon & 0 \\ 0 & T \end{pmatrix}$$

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- ε is strongly correlated with n.

Questions for Local Thermal Equilibrium

- It is a sufficient condition for Ideal Fluid dynamics. But is it a necessary condition?

- How local?

Can not be strictly local (compatibility with the thermodynamics).

- If not local, how the local covariant theory can emerge?

- How much can we say about the inhomogeneous nature of the initial conditions?

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Relativity x Thermodynamics

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X

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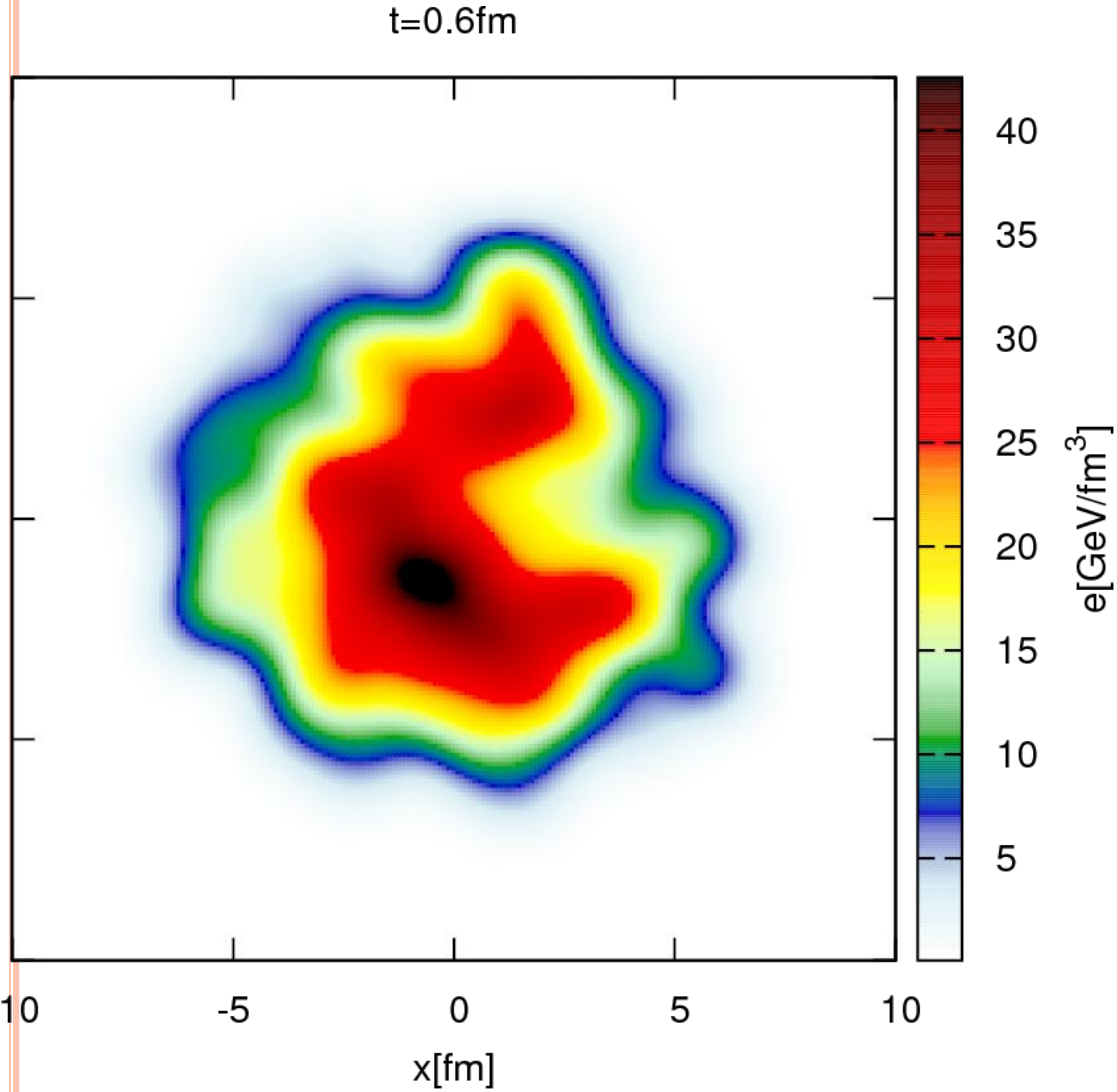
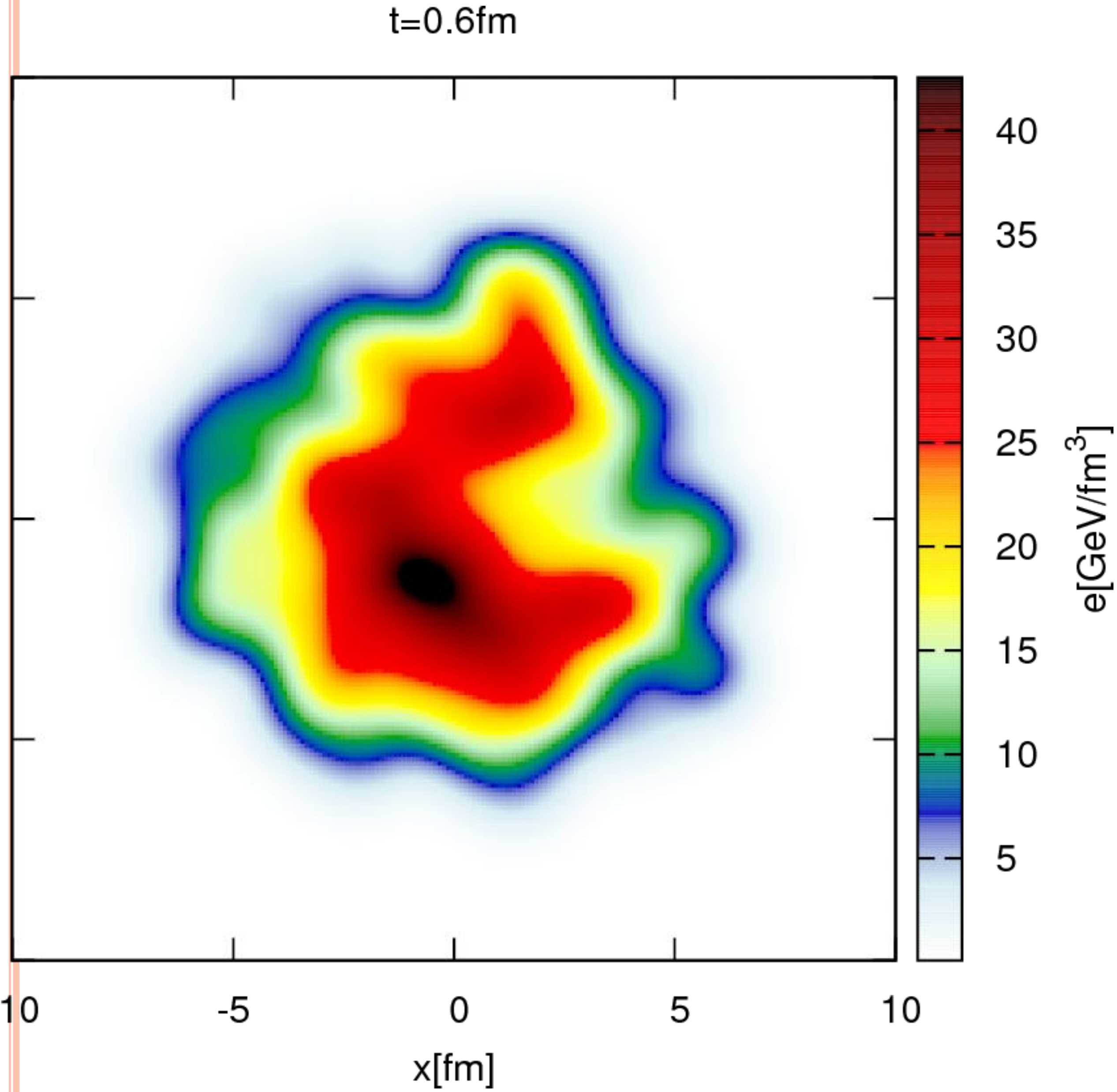
Coarse Graining and resolution



Coarse Graining and resolution



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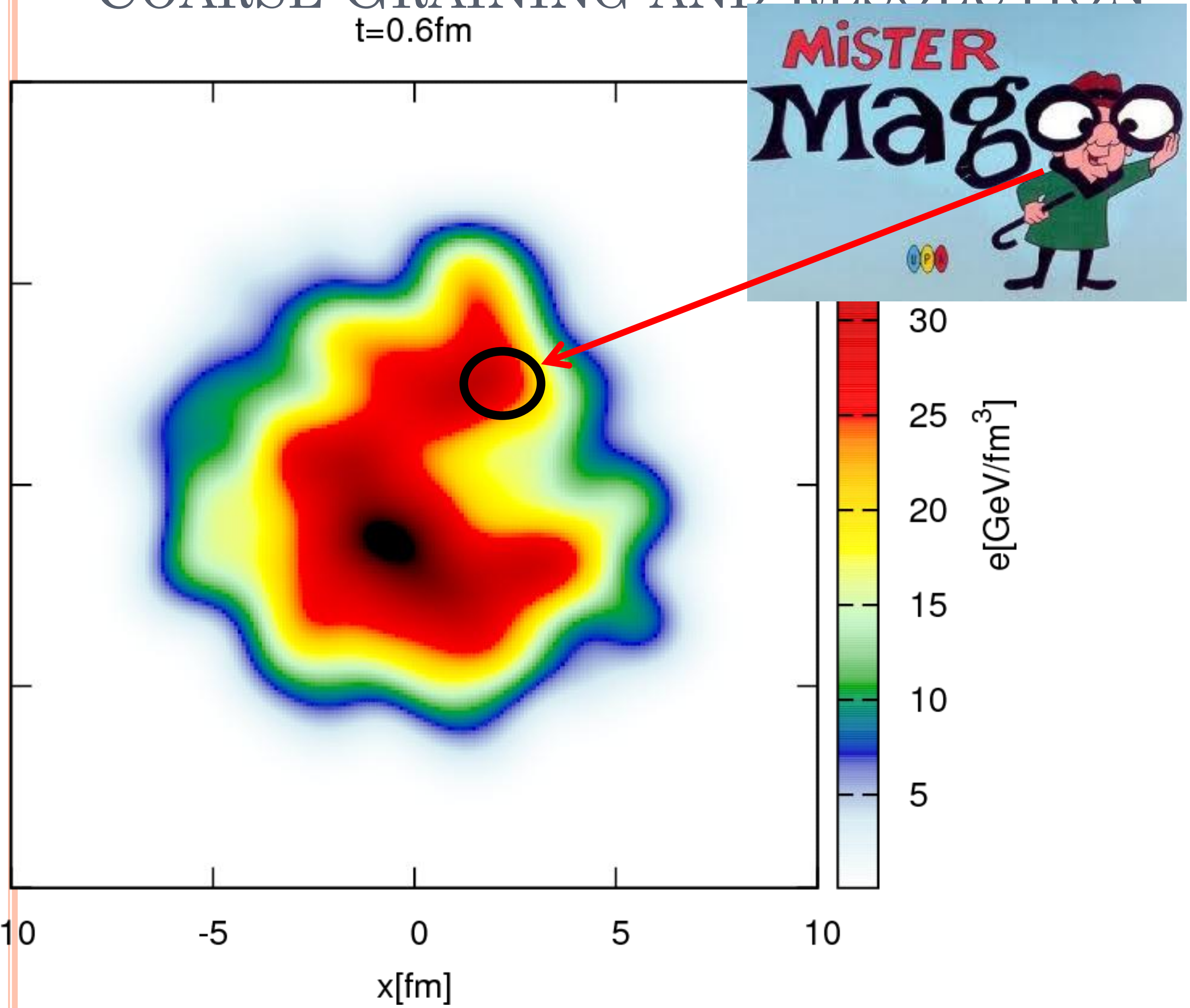
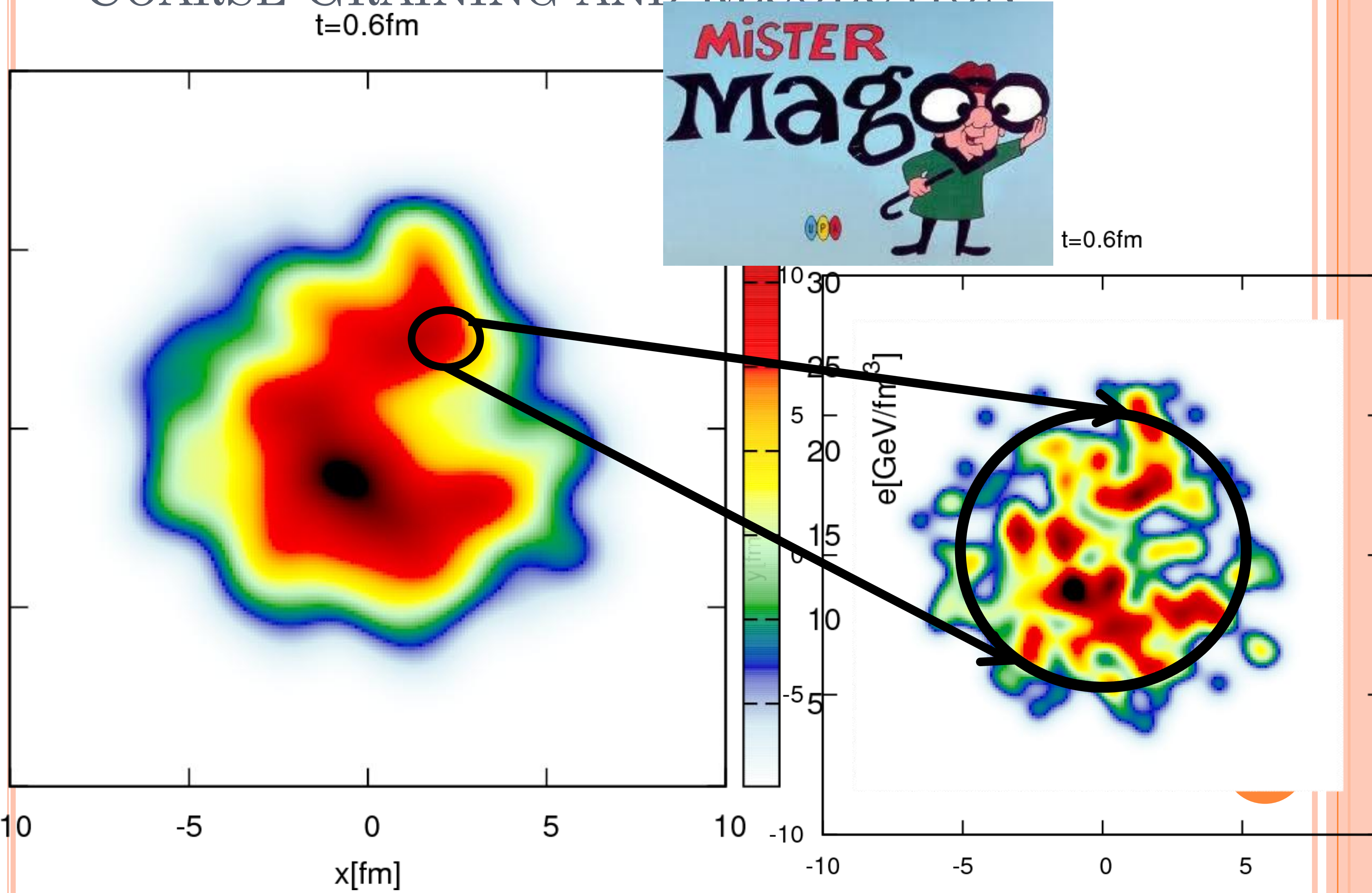


Coarse Graining and resolution



EXAMPLE:

- Matter density expressed in terms of Lagrange Coordinates:

$$n^*(t,\vec{r}) = \int d^3\vec{R}\; n_0(\vec{R})\; \delta(\vec{r} - \vec{r}_R(t))$$

EXAMPLE:

- Matter density expressed in terms of Lagrange Coordinates:

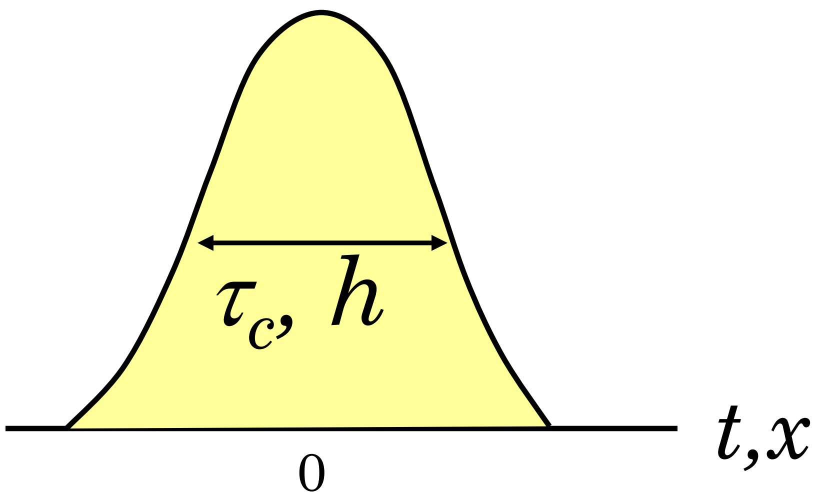
$$n^*(t,\vec{r}) = \int d^3\vec{R}\; n_0(\vec{R})\; \delta(\vec{r} - \vec{r}_R(t))$$

- When we don't have space-time resolution,

$$n^*(t,\vec{r}) \rightarrow$$

$$\int dt' d^3\vec{R}\; n_0(\vec{R})\; U_{\tau_c}(t'-t) W_h(\vec{r} - \vec{r}_R(t))$$

$$U_{\tau_c}(t), W_h(\mathbf{x}) \leftrightarrow \text{smoothing kernel}$$



$$\int U(t)dt = \int W(x)dx = 1$$

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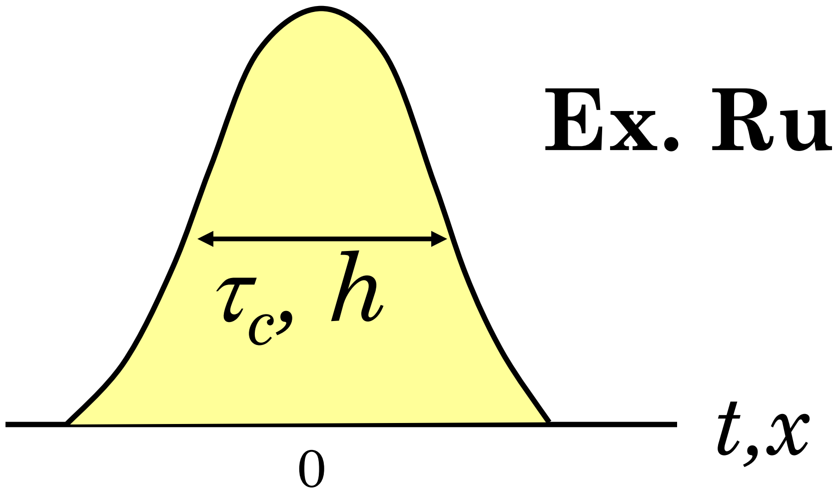

Ex. Rudy Marty's Gaussian

$$\int U(t)dt = \int W(x)dx = 1$$

$$n^*(t,\vec{r}) = \int dt' d^3\vec{R}\, n_0(\vec{R})\, U_{\tau_c} W_h$$

$$\vec{j}(t,\vec{r}) = \int dt' d^3\vec{R}\, n_0(\vec{R})\, U_{\tau_c} W_h \frac{d\vec{r}_M}{dt'}$$

$$U_{\tau_c} = U_{\tau_c}(t'-t)$$

$$W_h = W_h(\vec{r} - \vec{r}_R(t'))$$

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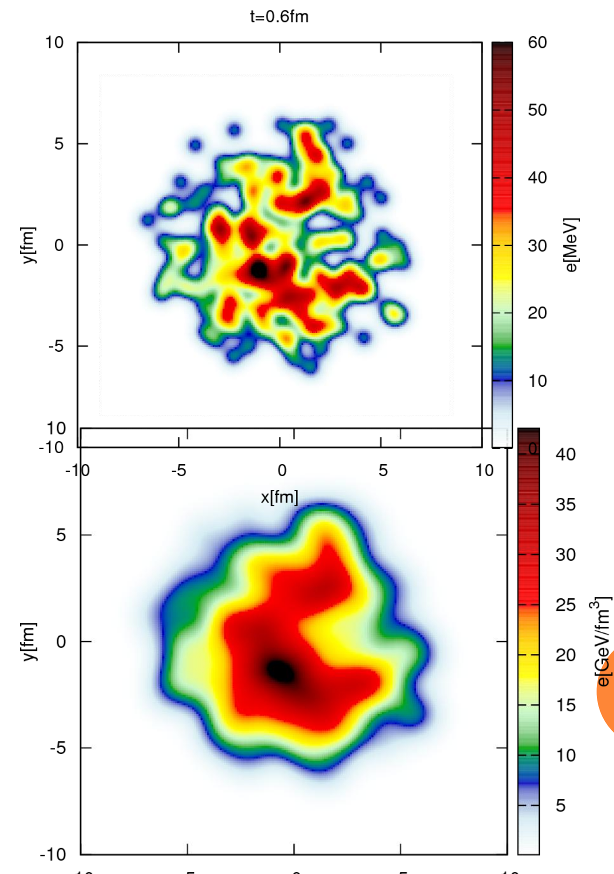

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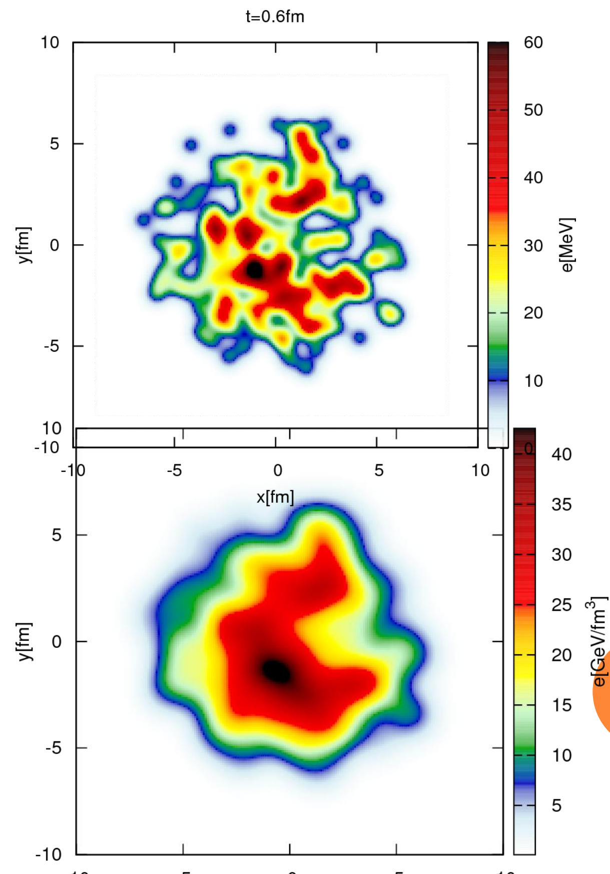
Not exactly local .. but

$$\partial_t n^*(t,\vec{r}) + \nabla\cdot\vec{j}(t,\vec{r}) = 0$$



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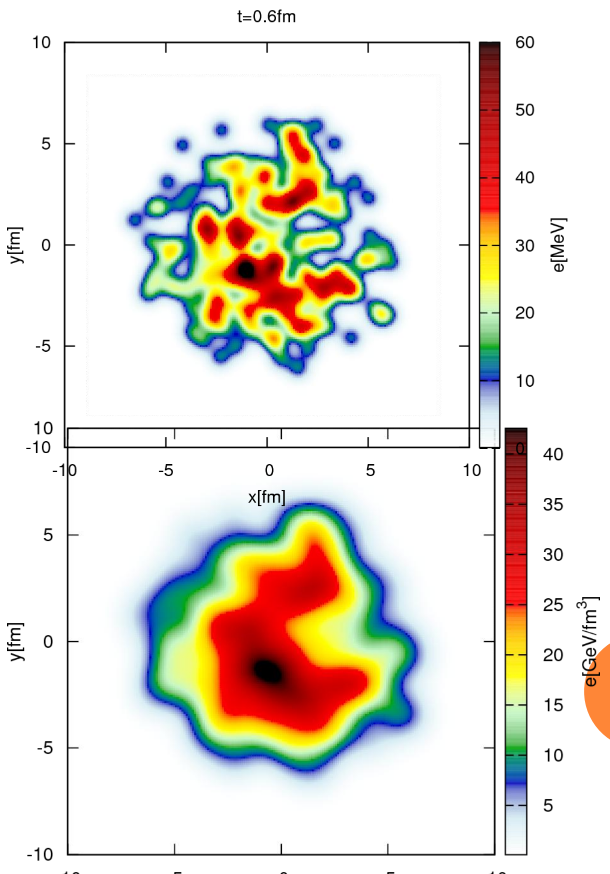
Even we can write

$$n^\mu = (n^*, \vec{j}),$$

$$\partial_\mu n^\mu = 0.$$



We can do this also for $T^{\mu\nu}(x)$

$$T^{\mu\nu}(x) = \int dt' d^3\vec{x}' \; U_{\tau_c} W_h \, T_M{}^{\mu\nu}(t, \vec{x}')$$

Define $n(t, \vec{r}) = \sqrt{n_\mu n^\mu},$

$$u^\mu(t, \vec{r}) = n^\mu / n,$$

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Physical meaning of ε and n:

"Proper" energy and number densities measured in the local rest frame defined with the coarse-grained quantities.

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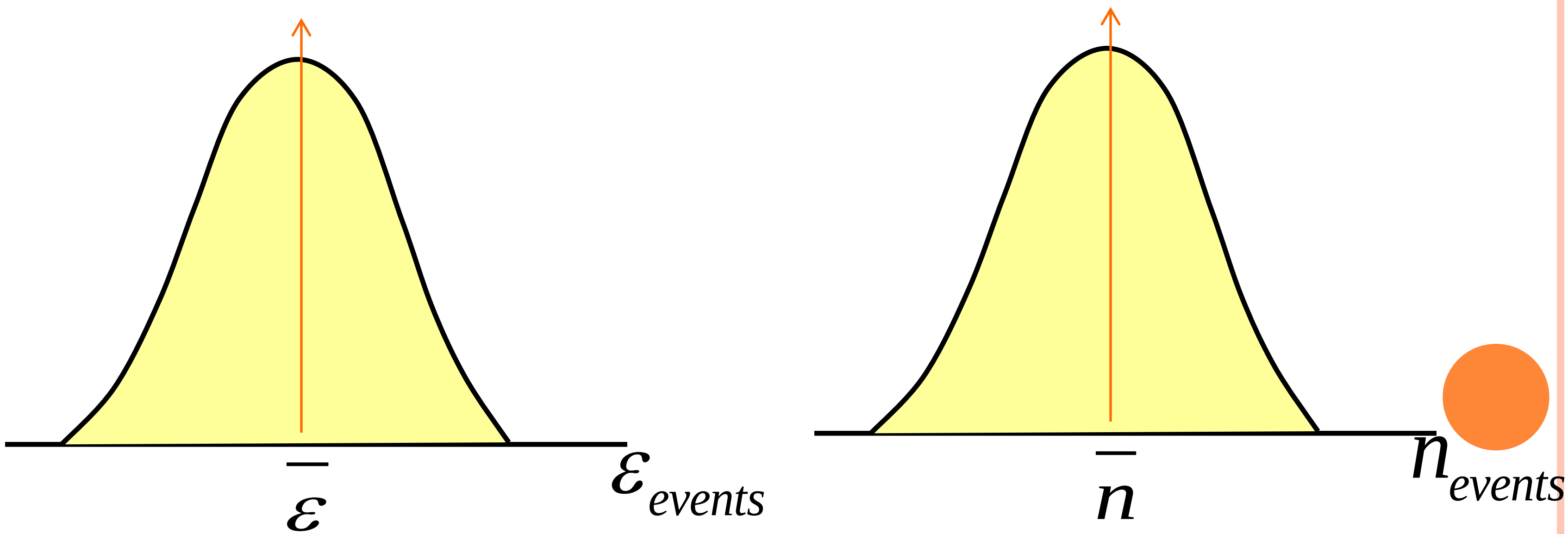
Reminder:

For a given coarse-grained profile $n^\mu(t_0, \vec{r})$ there are many events in microscopic level, that is exists a big statistical ensemble.

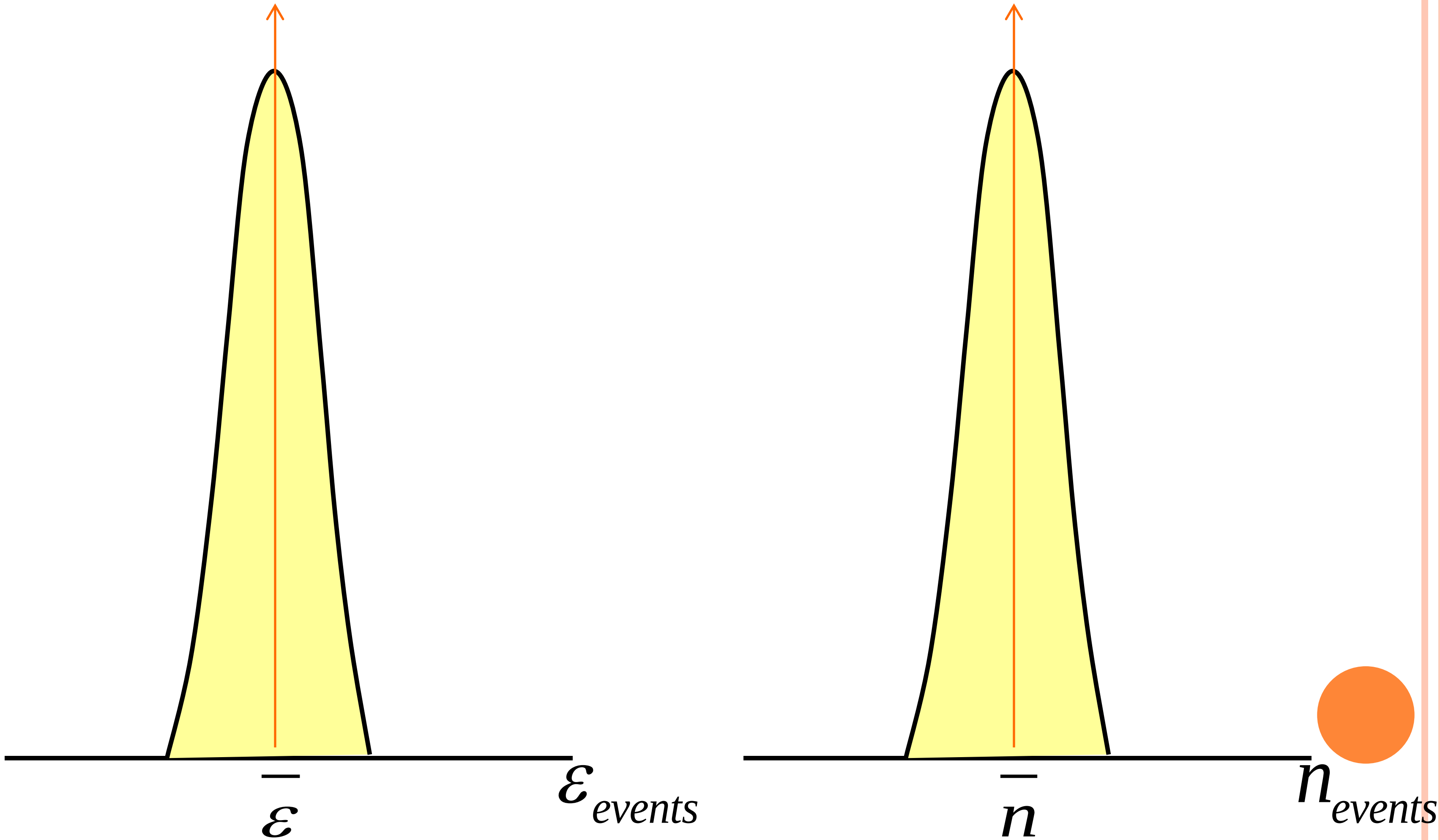
Say, Ω, such an ensemble that,

$$\Omega = \left\{ events \ \middle| \ n^{\mu}(t_0, \vec{r}) = n_0{}^{\mu}(\vec{r}) \right\}.$$

Densities at a given space and time point $(t, \vec{r})$, ε and n fluctuate event by event in this ensemble, Ω.

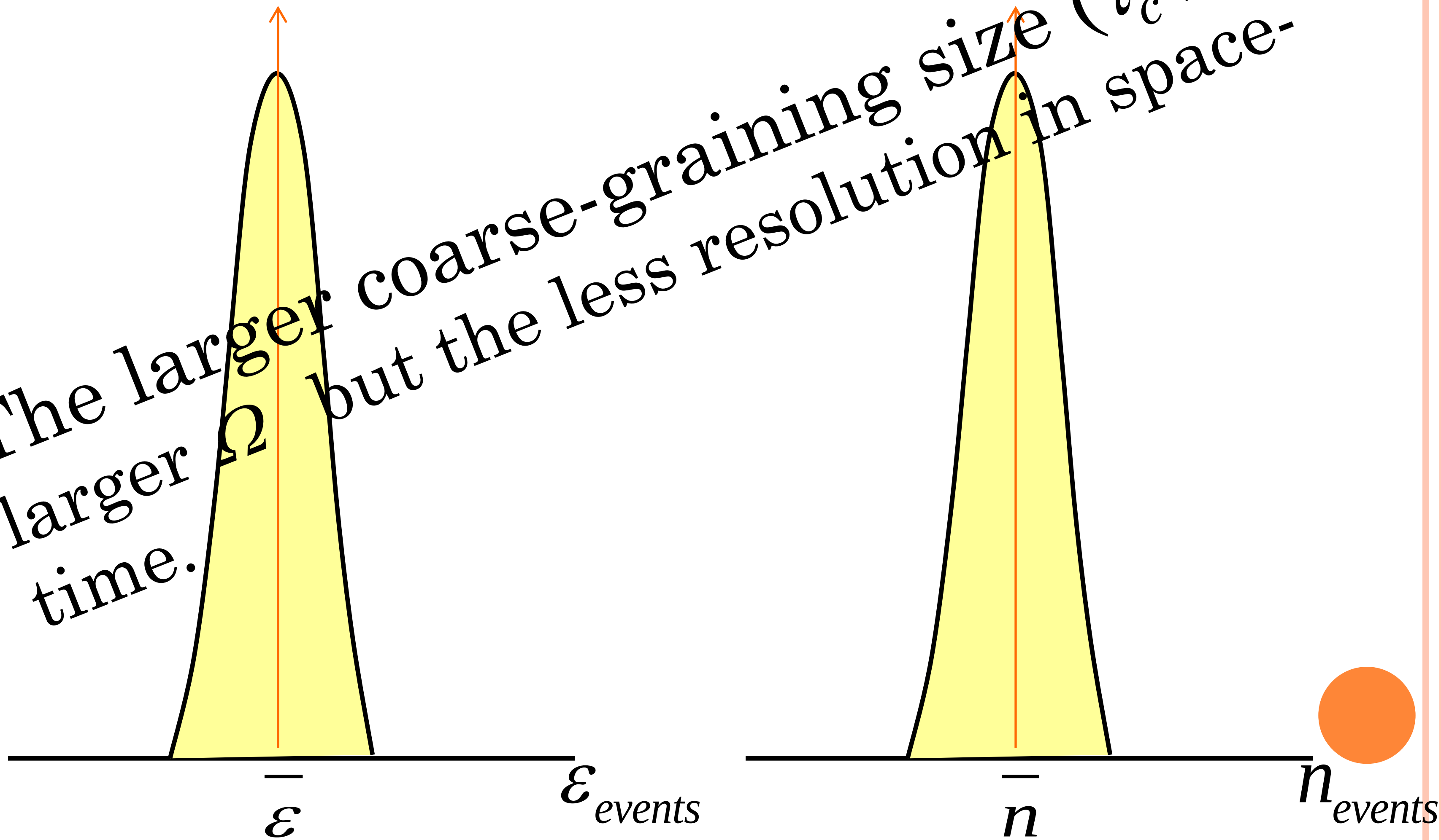


For larger Ω, the width may become small (central limit)



For larger Ω, the width may become small (central limit)

The larger coarse-graining size (τ_c, h) larger Ω but the less resolution in space-time.



Hydrodynamic modeling via Variational Principle

1. $\bar{\varepsilon}$ and $\bar{n}$ are strongly correlated so that

$$\bar{\varepsilon} = \bar{\varepsilon}(\bar{n})$$

2. Dynamics in terms of coarse-grained variable, $\bar{n}^{\mu}$ is determined by the action,

$$I = -\int d^4x\, \bar{\varepsilon}(\bar{n}(x))$$

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(continuum generalization of the Lagrangian for a particle)

$$L = -m\sqrt{1-\vec{v}^{\,2}}$$

Hydrodynamic modeling via Variational Principle

$$\delta I = -\delta \int d^4x \, \overline{\varepsilon}(\overline{n}(x)) = 0$$

with respect to

$$\overline{n}^{\mu} = (\overline{n}^{*}, \ \overline{n}^{*} \vec{v})$$

subject to the constraint

$$\overline{n}_{\mu} \overline{n}^{\mu} = \overline{n}^{2}$$

H-T. Elze, Y. Hama, T. K and J. Rafelski,.J. PhysG:25(9):1935, 1999

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leads

Relativistic Euler Eqs.

$$\partial_{\mu} \left\{ (\bar{\varepsilon} + P) u^{\mu} u^{\nu} - P g^{\mu\nu} \right\} = 0, \qquad P = \frac{d\bar{\varepsilon}}{dn} \bar{n} - \bar{\varepsilon},$$

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Hydrodynamic modeling via Variational Principle

When the fluctuation is not negligible;

$$\delta I = -\delta \int d^4x\, \varepsilon(n(x)) = 0$$

for stochastic variable leads to

Navier-Stokes Eqs. for a viscous fluid, in non-relativistic limit !

T. Koide and T. K, .J. PhysA: 45(25):255204

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$$\delta I = -\delta \int d^4x\, \varepsilon(n(x)) = 0$$

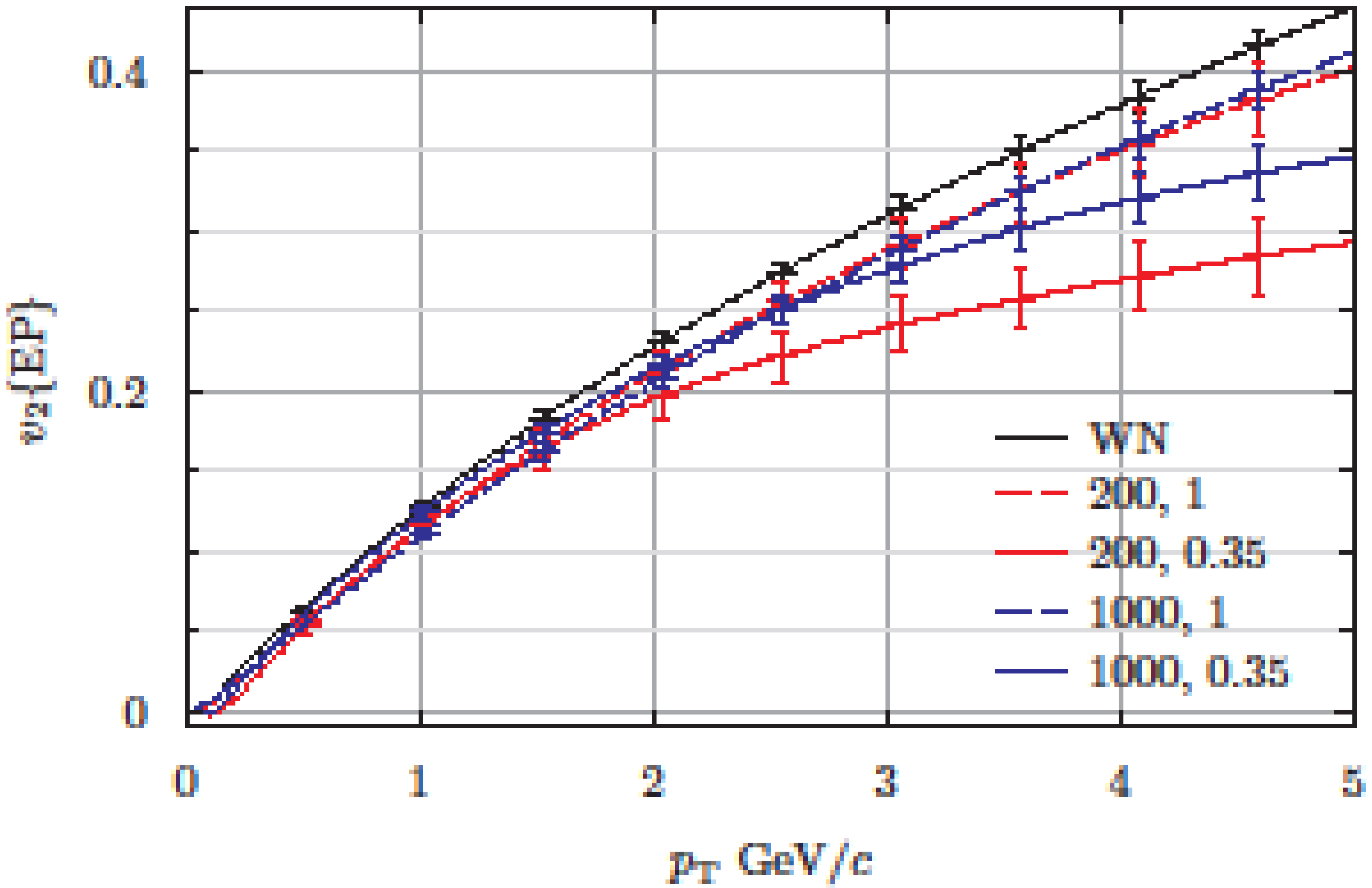
for stochastic variable leads to

Navier-Stokes Eqs. for a viscous fluid, in non-relativistic limit !

In fact, fluctuations in initial conditions gives a similar effect as viscosity

T. Koide and T. K, .J. PhysA: 45(25):255204

Event averaged v_2



R. Andrade, et al,, Phys. Rev. Lett., 97:202302
Ph. Mota et al., Nuclear Physics A, 862:188 , 2011

Now we have problem…..

- Once arrived to the relativistic Euler equation, we cannot tell the coarse-graining scale.

- Transport coefficients, or even effective EoS may depend on this scale.

- Some observables may not be sensitive to this scale. If we see only these, we would conclude that the ideal hydro works well…

Important to study

- Event by Event fluctutations

  S.Paiva, Y. Hama and T.K. Phys. Rev. C, 55:1455 (1997), C.E. Aguiar, Y. Hama, T. K. and T. Osada. Nucl. PhysA, 698, 639 (2002),

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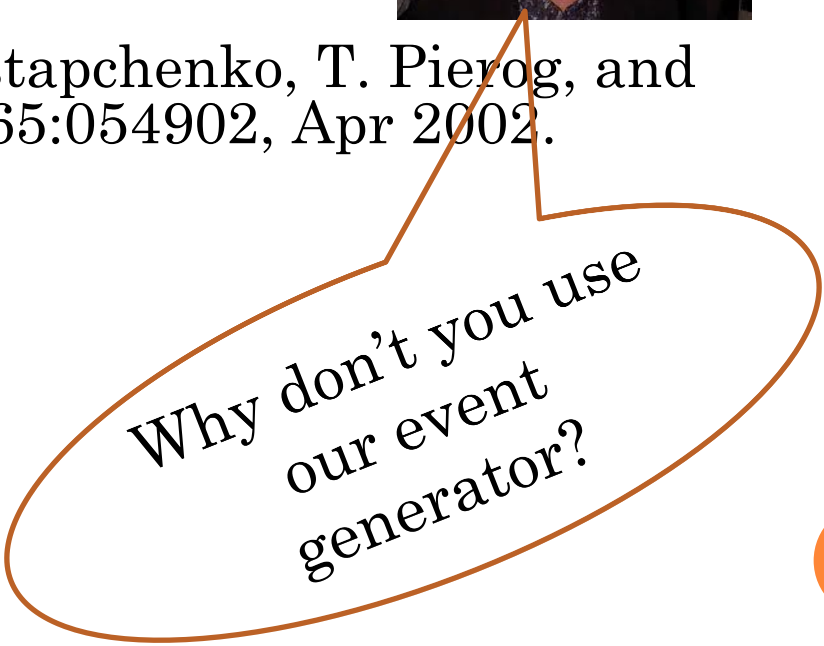

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- Find observables that are sensitive to the genuine hydro signal

Genuine (Local) Hydrodynamic Signal

- Time evolution of hydrodynamic profile.

Genuine (Local) Hydrodynamic Signal

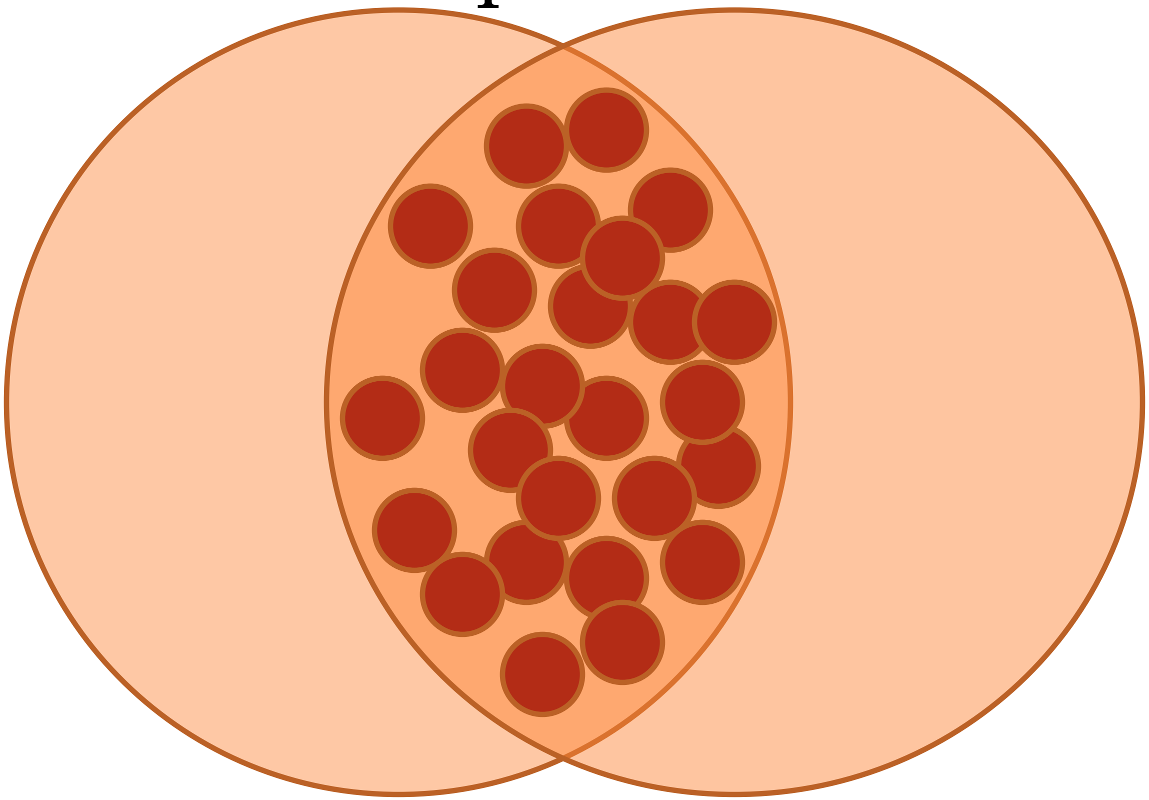
- Time evolution of hydrodynamic profile.

 - Not observable in heavy ion collisions (may be shock wave and its thickness, or Kelvin-Helmholtz instability (L. P. Csernai, D. D. Strottman, and Cs. Anderlik. Phys. Rev. C, 85:054901)

Necessity for Systematic Studies on the effects of granularities in the initial conditions

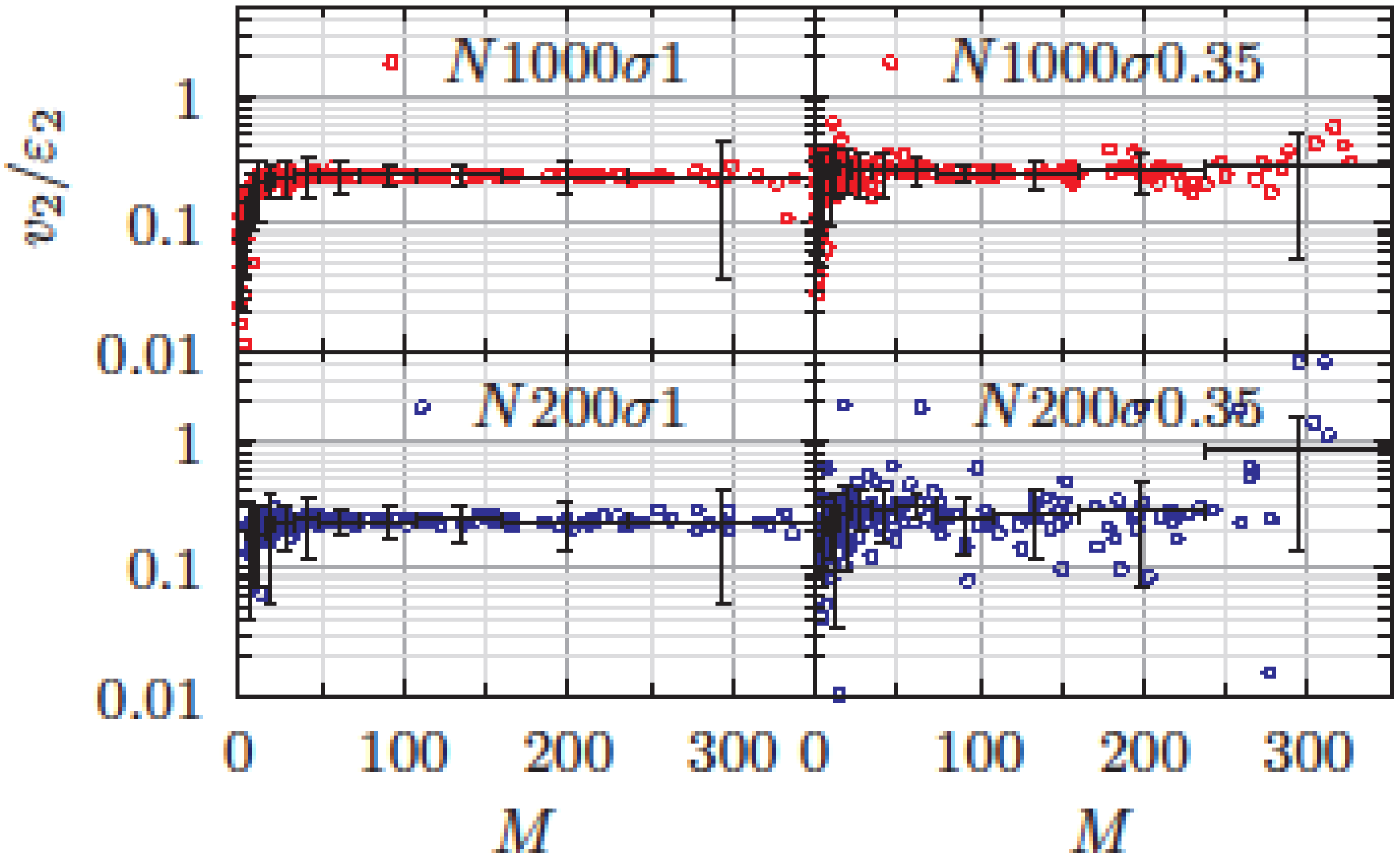
- Multi-flux tube inspired model

Hannu Holopainen's talk



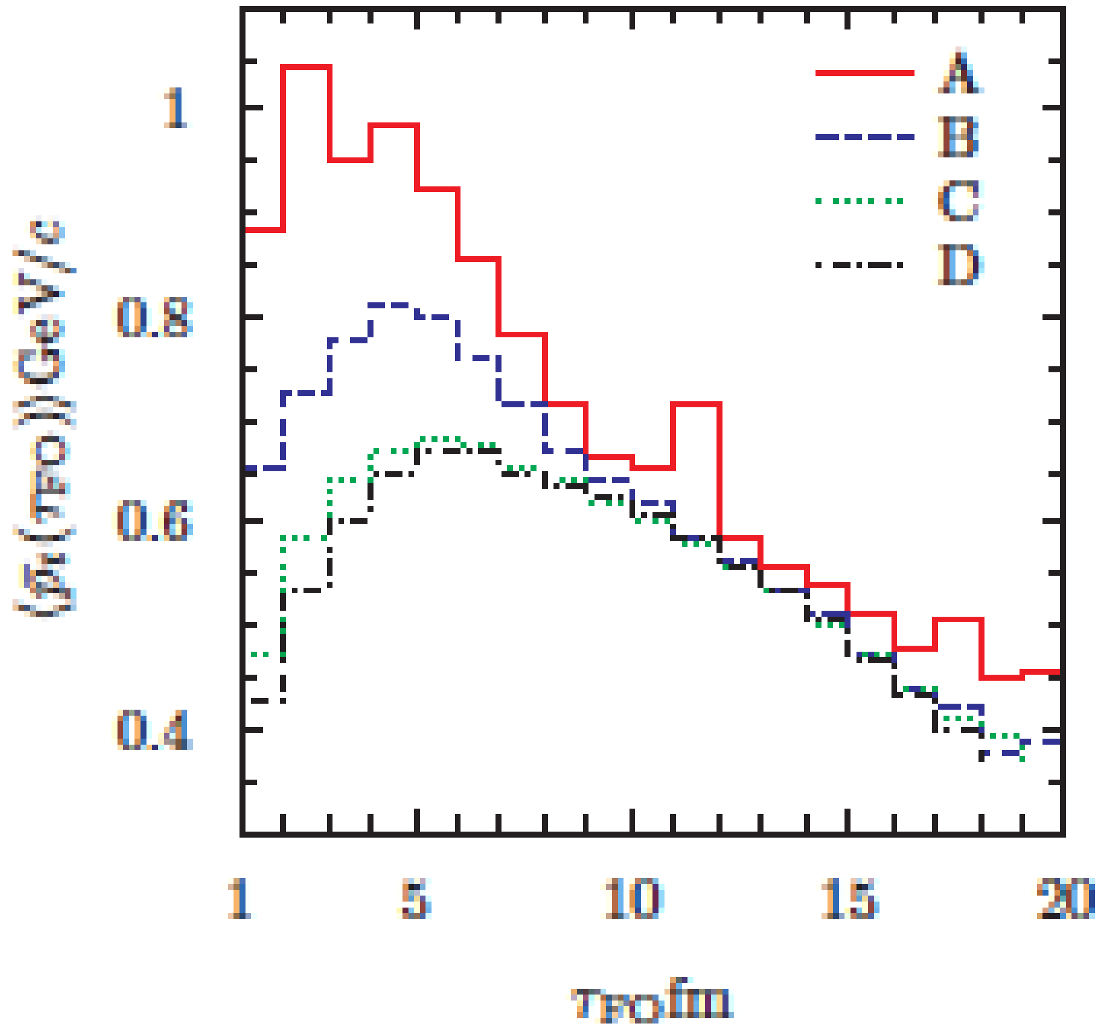
Gaussian with the width σ and the energy $\varepsilon_0 = \varepsilon_T / N$

Sensitivity of v_2 / e_2



Event averaged v_2 / e_2 is not sensitive to the granularity, although almost looses the EbE correlation for high granularity

Average p_T as function of freezeout time

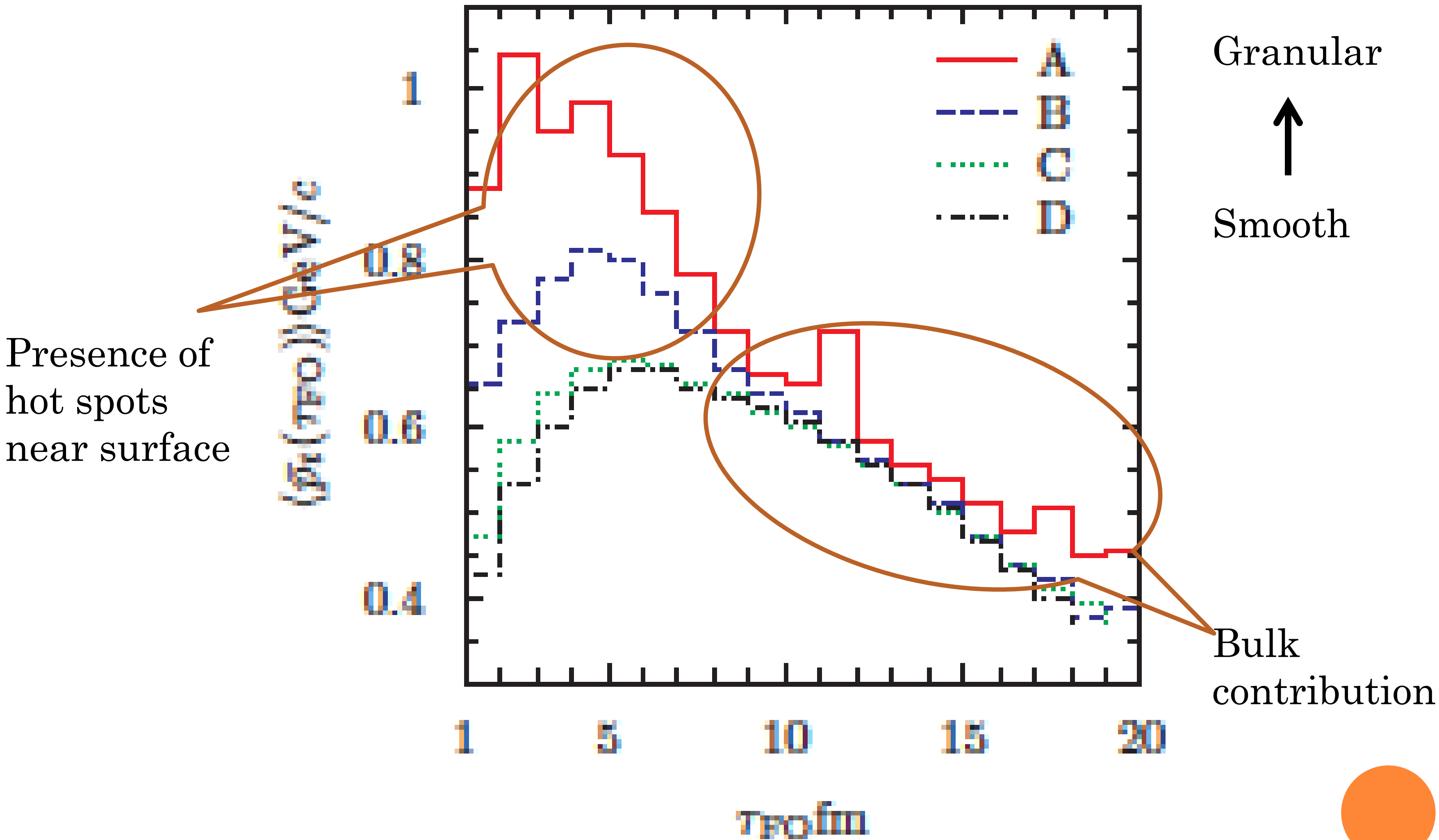


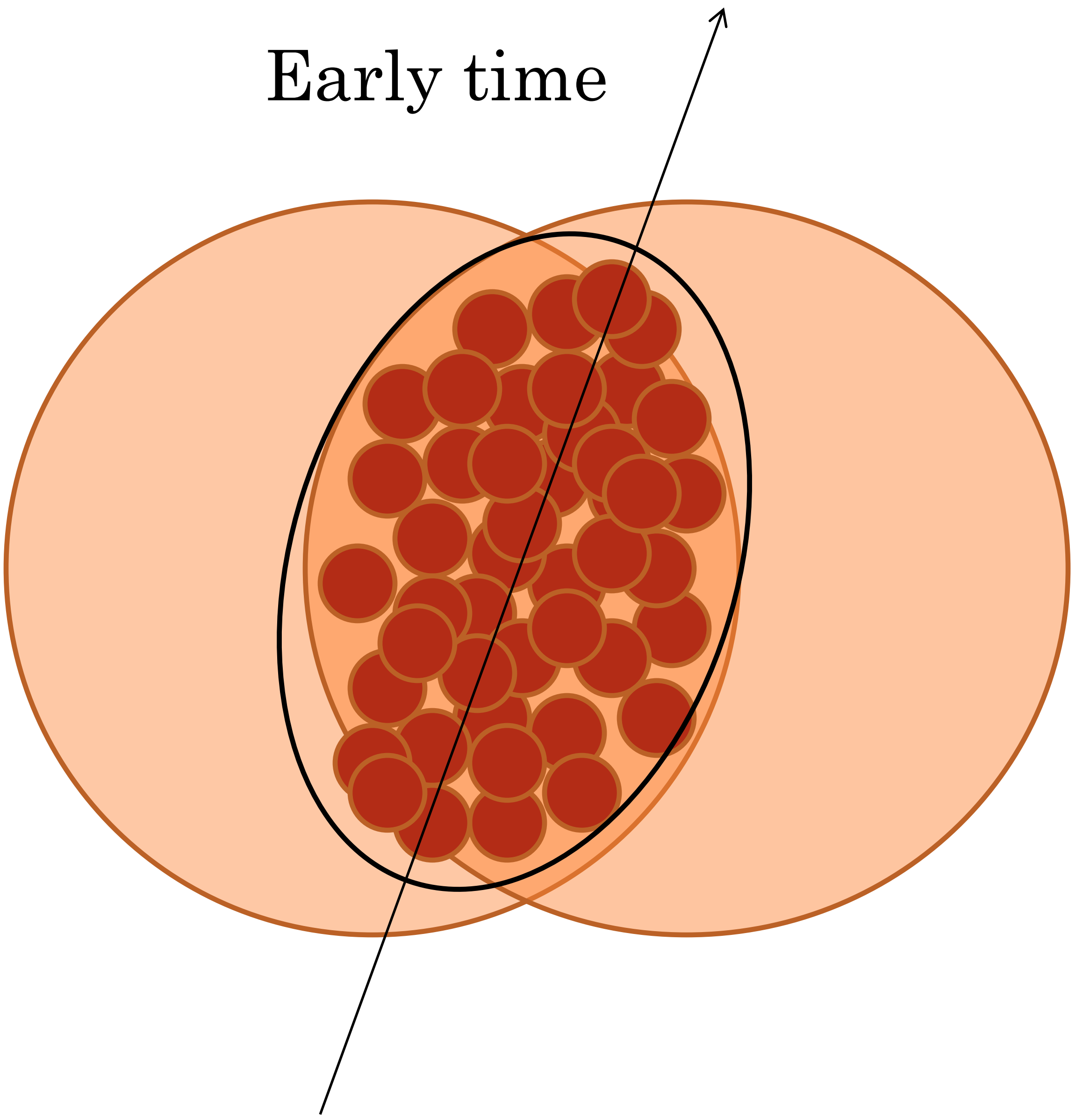
Granular

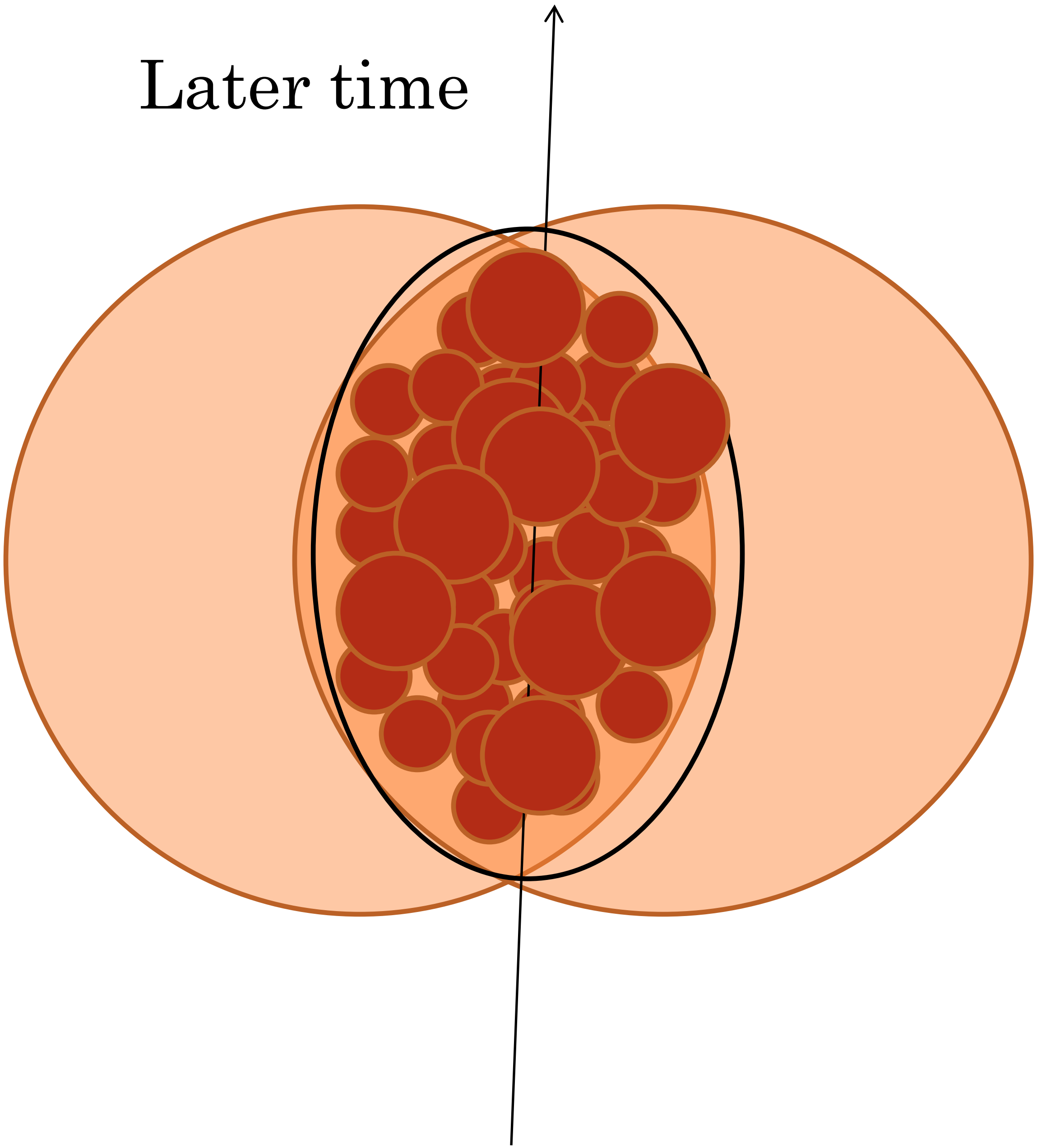
↑

Smooth

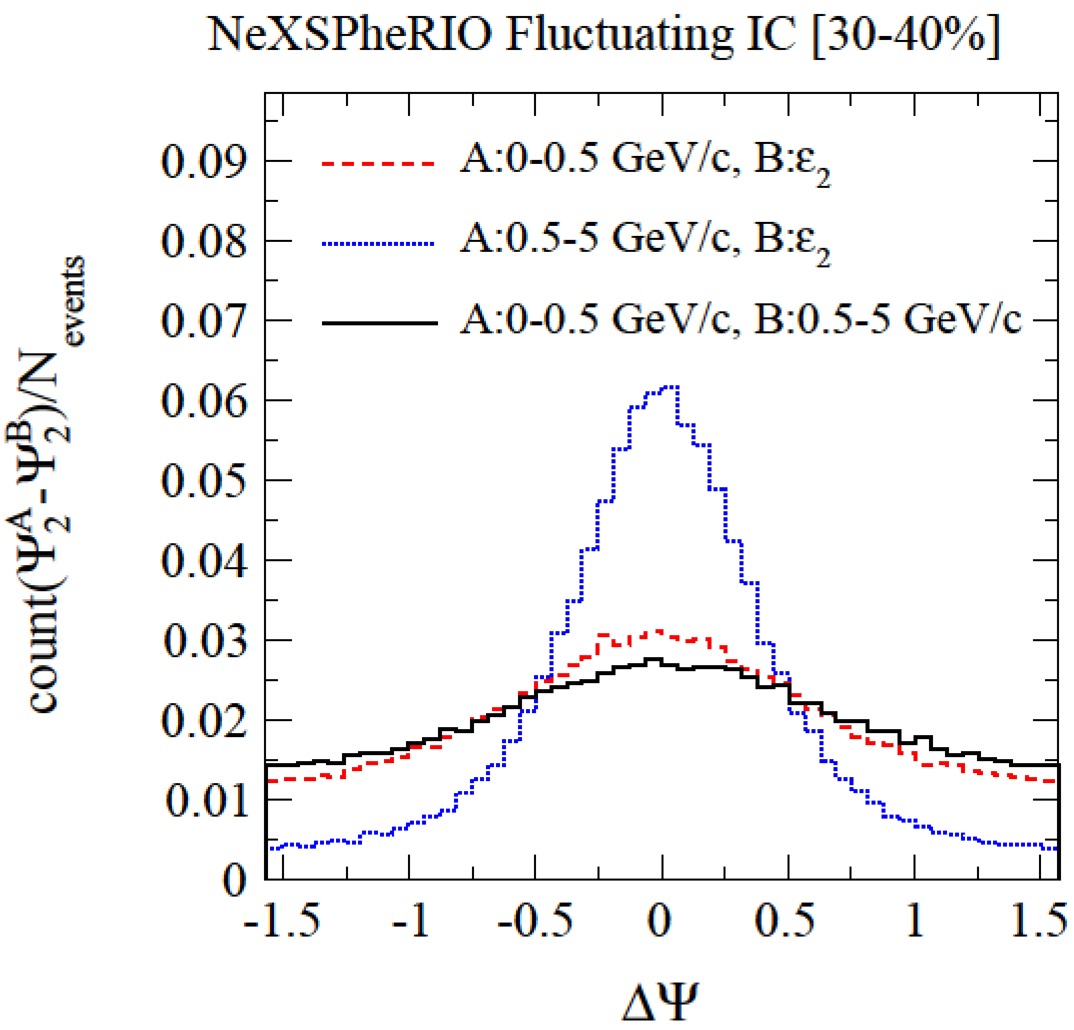
Average p_T as function of freezeout time

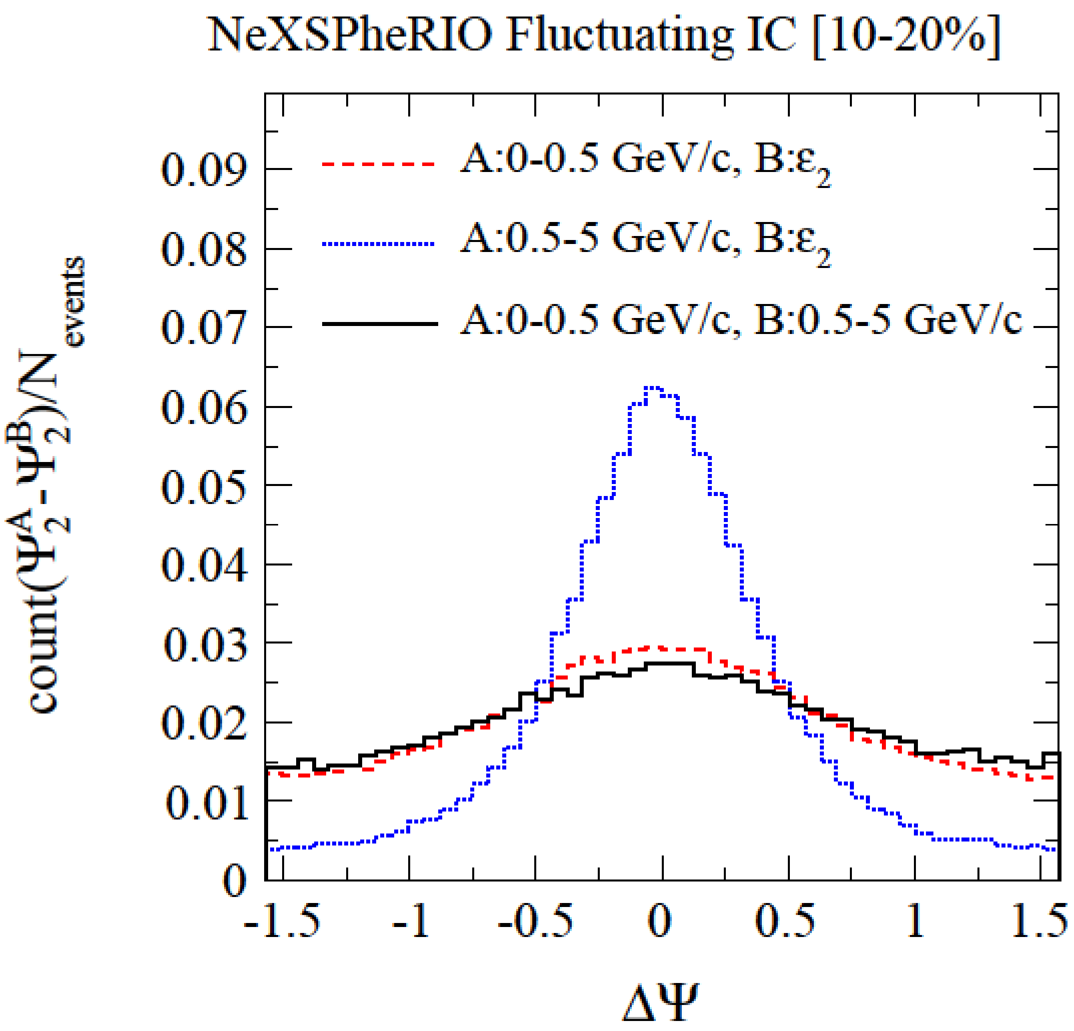


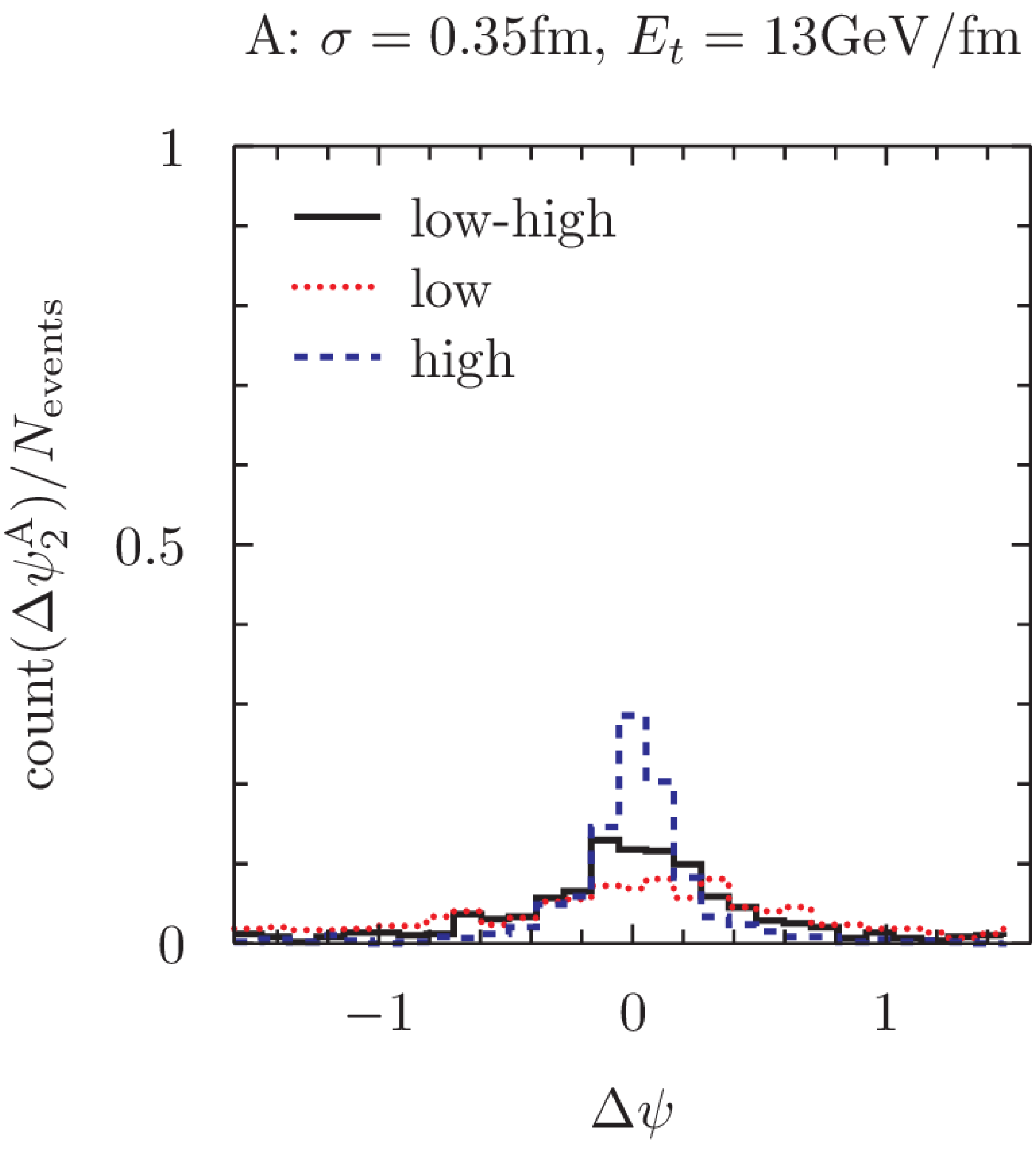

Early time


Later time

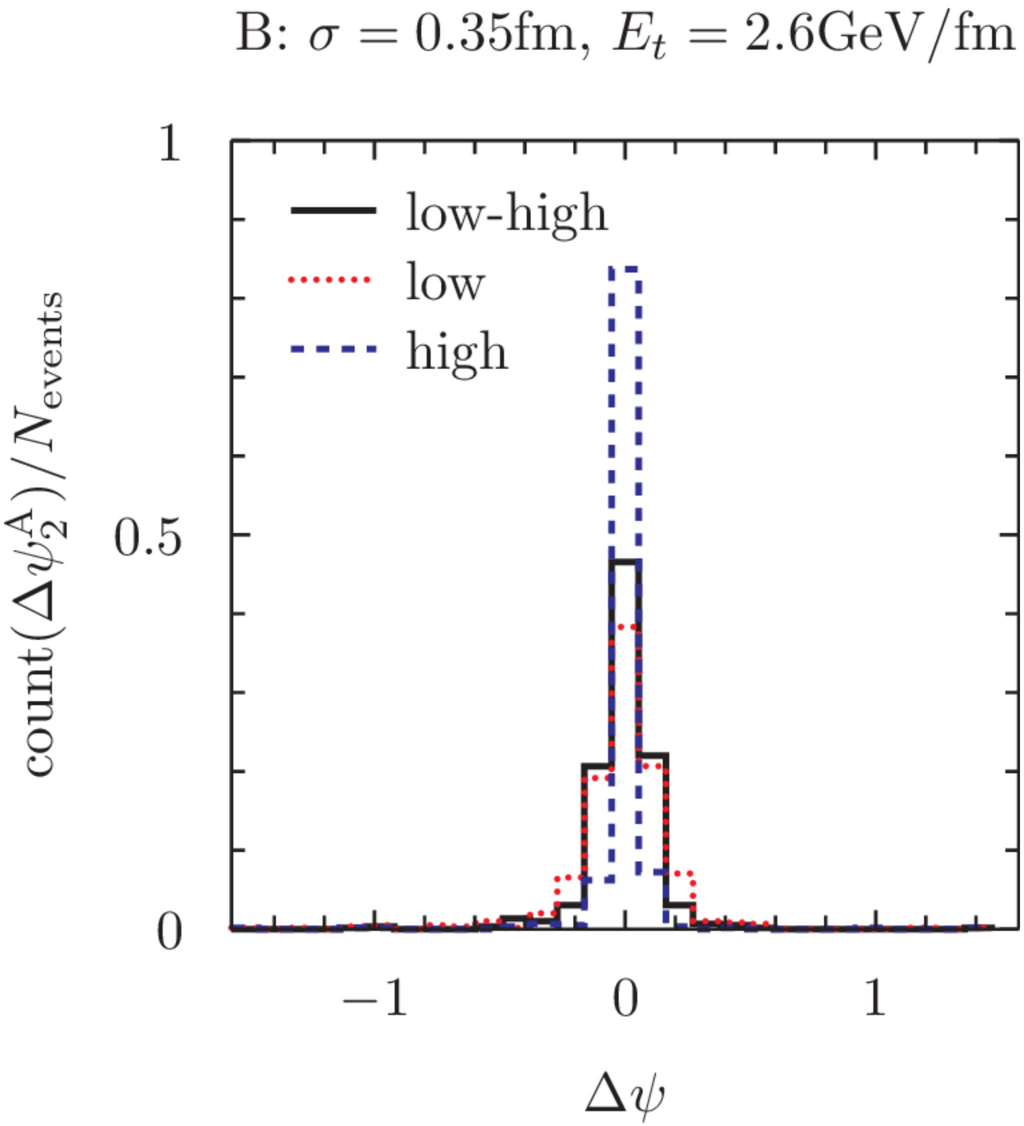
Take a look on the NeXSPherio case

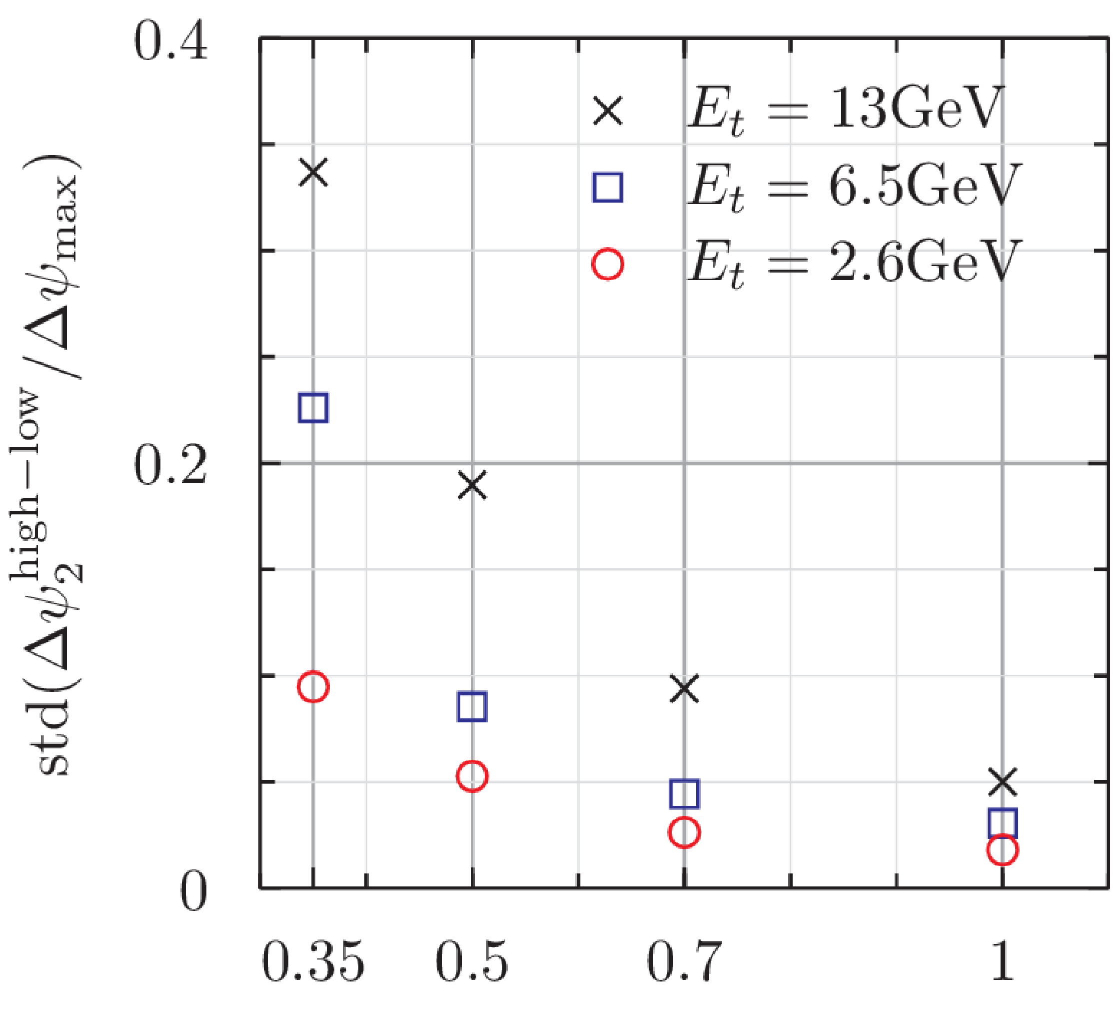



NeXSPheRIO Fluctuating IC [10-20%]
A:0-0.5 GeV/c, B:ε2
A:0.5-5 GeV/c, B:ε2
A:0-0.5 GeV/c, B:0.5-5 GeV/c
count(Ψ2A-Ψ2B)/Nevents
ΔΨ
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-1.5
-1
-0.5
0
0.5
1
1.5

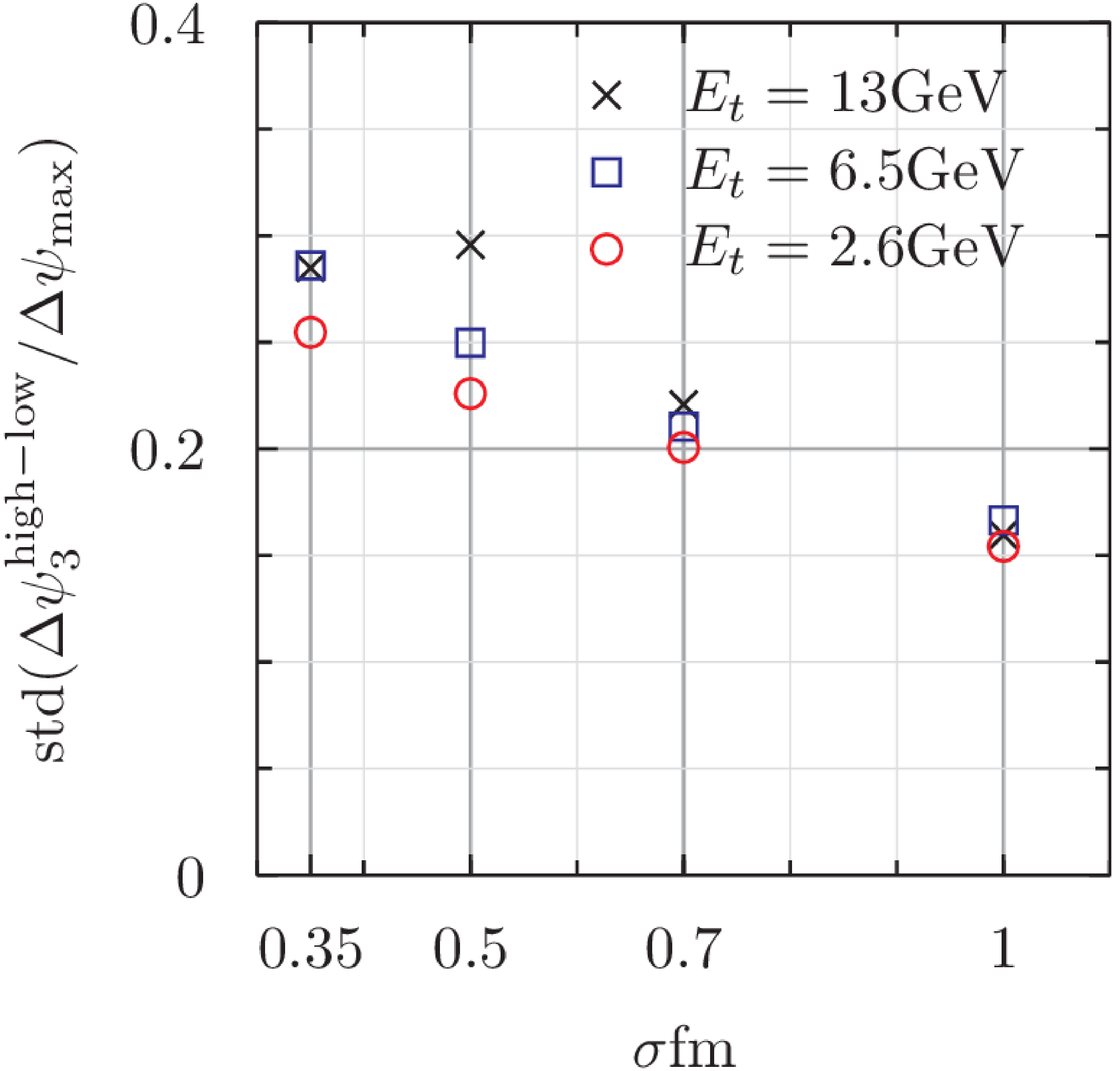


A: $\sigma = 0.35$fm, $E_t = 13$GeV/fm
1
0.5
0
low-high
low
high
count$(\Delta\psi_2^A)/N_{\text{events}}$
−1
0
1
$\Delta\psi$


B: $\sigma = 0.35\text{fm}$, $E_t = 2.6\text{GeV/fm}$
1
0.5
0
−1
0
1
$\Delta\psi$
$\text{count}(\Delta\psi_2^A)/N_{\text{events}}$
low-high
low
high

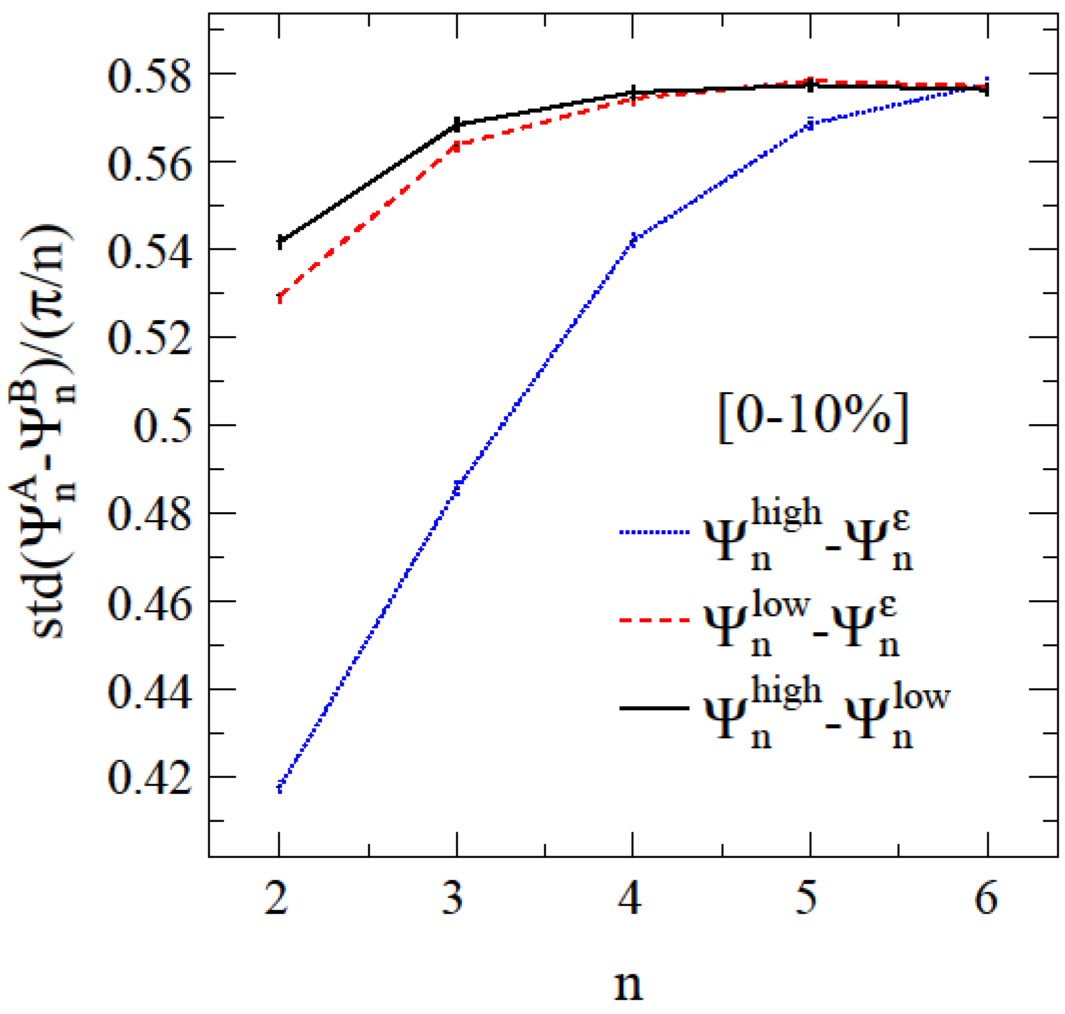


0.4
0.2
0
$E_t = 13\text{GeV}$
$E_t = 6.5\text{GeV}$
$E_t = 2.6\text{GeV}$
$\mathrm{std}(\Delta\psi_2^{\mathrm{high-low}}/\Delta\psi_{\mathrm{max}})$
0.35
0.5
0.7
1
σfm



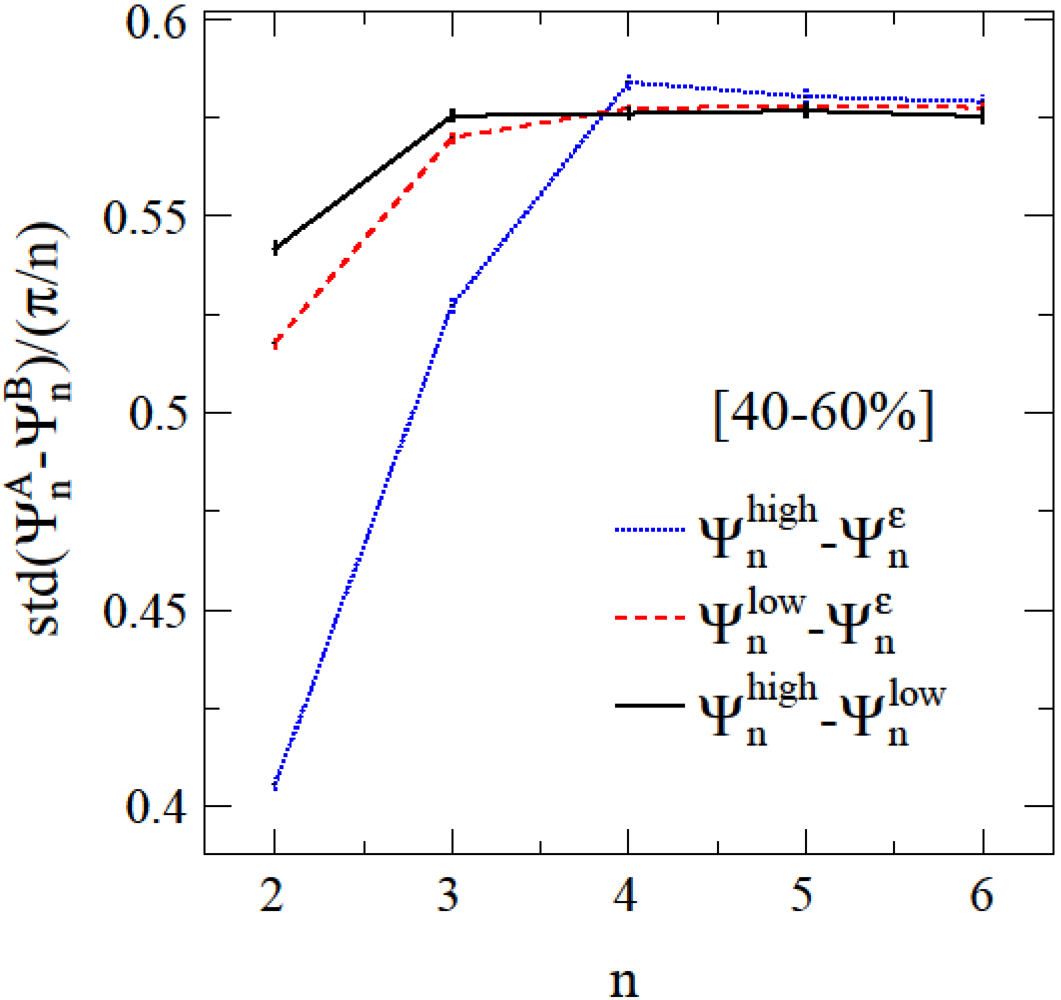
0.4
0.2
0
0.35
0.5
0.7
1
σfm
std(Δψ₃^{high−low}/Δψ_max)
× E_t = 13GeV
□ E_t = 6.5GeV
○ E_t = 2.6GeV

NeXSpheRIO



std(Ψ_n^A-Ψ_n^B)/(π/n)
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0.42
2
3
4
5
6
n
[0-10%]
Ψ_n^{high}-Ψ_n^{ε}
Ψ_n^{low}-Ψ_n^{ε}
Ψ_n^{high}-Ψ_n^{low}

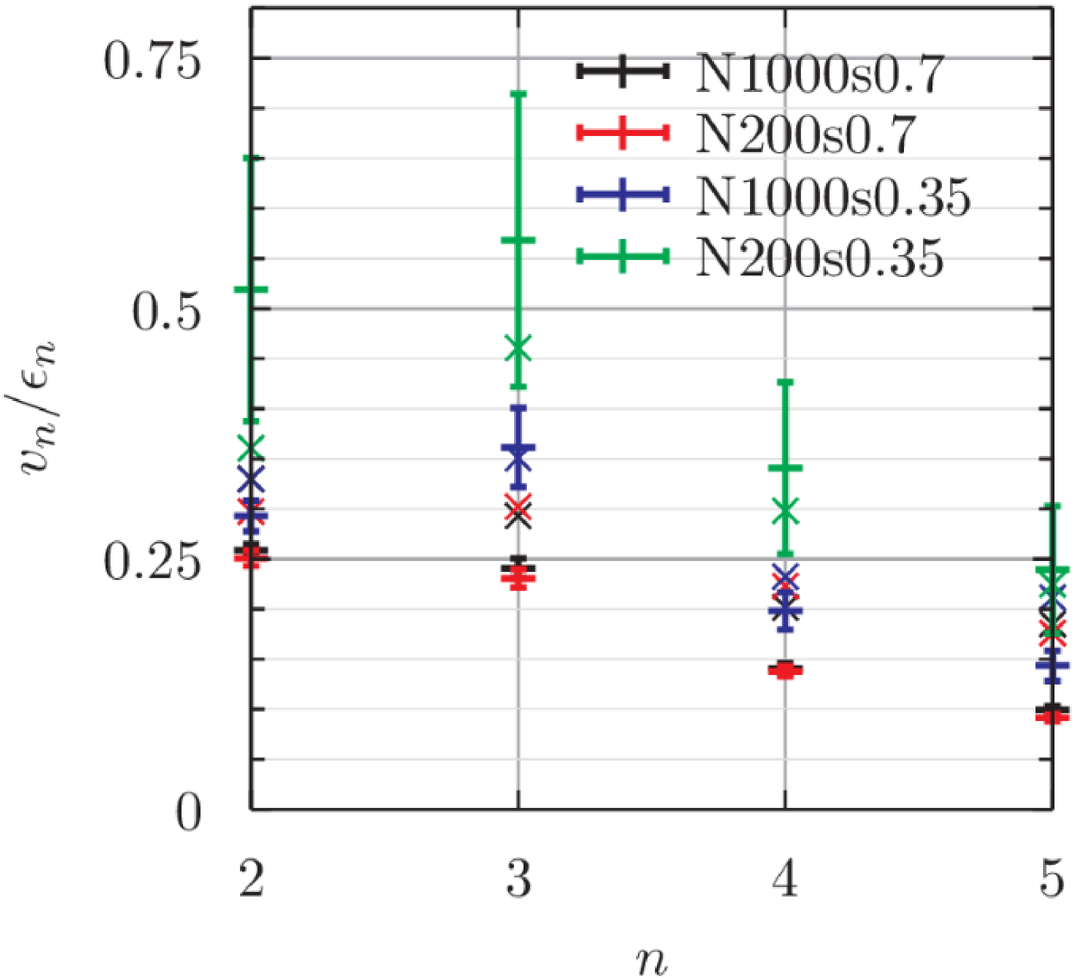
NeXSpheRio

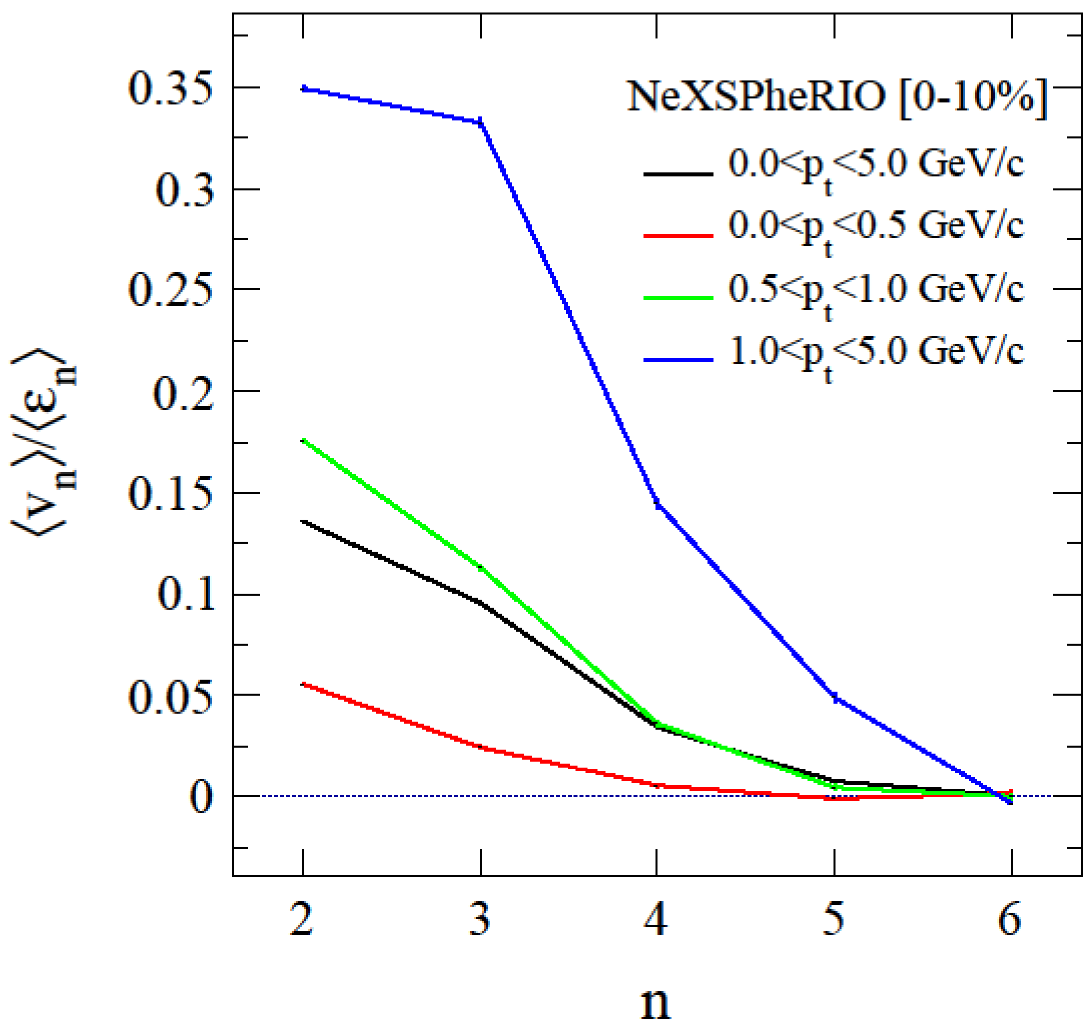


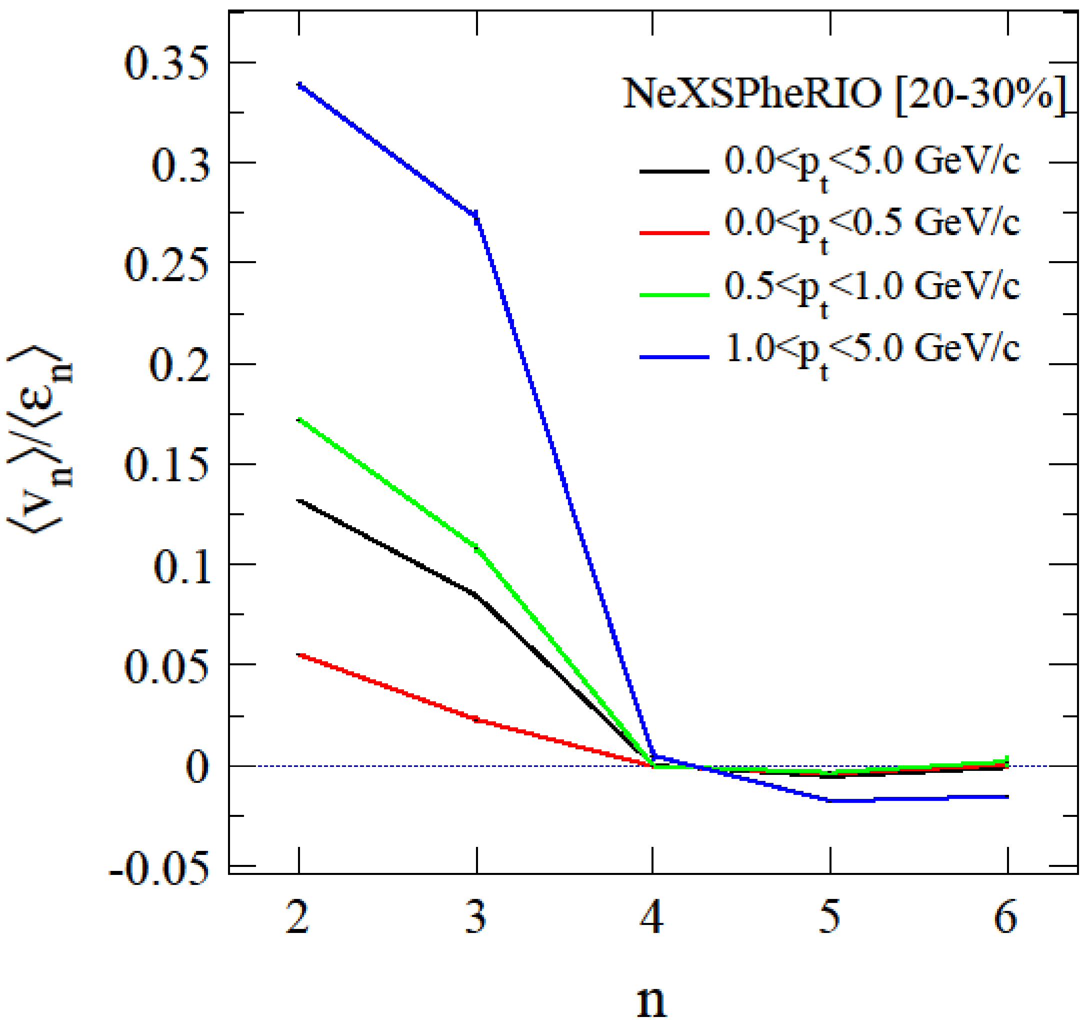
0.6
0.55
0.5
0.45
0.4
2
3
4
5
6
n
$\mathrm{std}(\Psi_n^A - \Psi_n^B)/(\pi/n)$
[40-60%]
$\Psi_n^{high} - \Psi_n^{\varepsilon}$
$\Psi_n^{low} - \Psi_n^{\varepsilon}$
$\Psi_n^{high} - \Psi_n^{low}$

n-dependence of event averaged v_n/ε_n

crosses: $p_t = 0.5 - 5$GeV and 10-50%; bars: 10-50%




NeXSPheRIO [0-10%]
0.0<p_t<5.0 GeV/c
0.0<p_t<0.5 GeV/c
0.5<p_t<1.0 GeV/c
1.0<p_t<5.0 GeV/c
⟨v_n⟩/⟨ε_n⟩
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2
3
4
5
6
n


NeXSPheRIO [20-30%]
0.0<p_t<5.0 GeV/c
0.0<p_t<0.5 GeV/c
0.5<p_t<1.0 GeV/c
1.0<p_t<5.0 GeV/c
⟨v_n⟩/⟨ε_n⟩
n
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
2
3
4
5
6

Summary

- Hydrodynamic model requires the coarse-graning scale, but not easy to discover.
- Effective model based on variational principle?
- Viscosity vs. Fluctuation
- Need genuine hydro signals.
- Pt separation may carry information on time evolution.
- Emission plane (mid rapidity) changes in time.
- How to separate “non-hydro” part? (Klaus’ talk)
- More systematic study is necessary.

Summary

- Hydrodynamic model requires the coarse-graning scale, but not easy to discover.
- Effective model based on variational principle?
- Viscosity vs. Fluctuation
- Need genuine hydro signals.
- Pt separation may carry information on time evolution.
- Emissio …
- How to s …
- More sy …

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