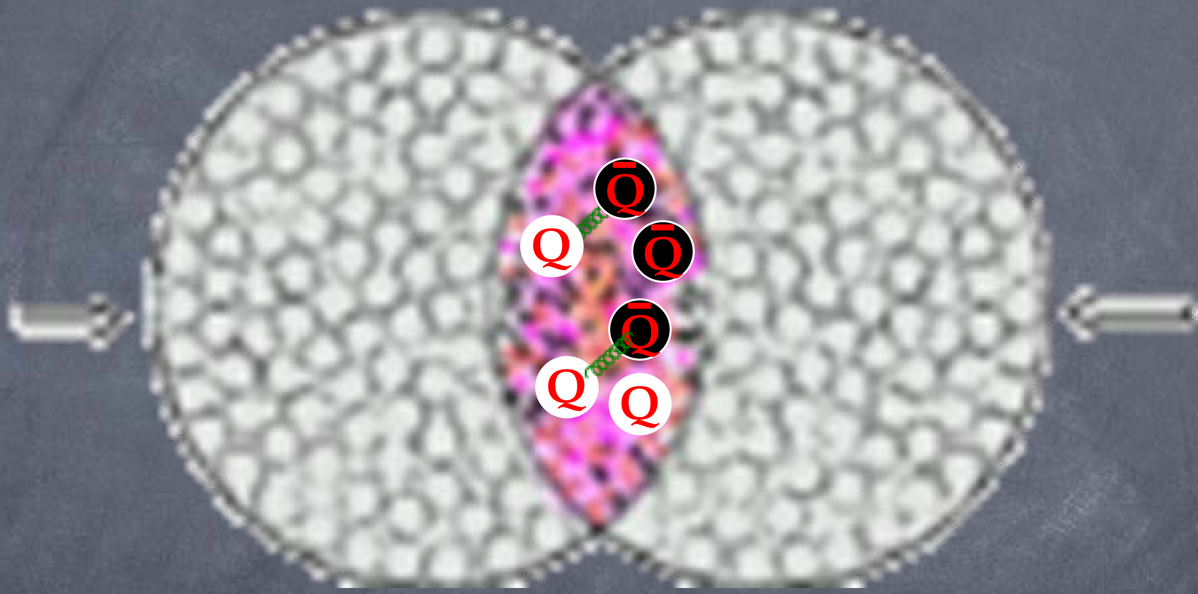
New possible explanation of the small v_2 of the J/ψ's in heavy ion collisions at RHIC

H. Berrehrah

In collaboration with: **P.B. Gossiaux & J. Aichelin**

28^{th} June 2012

Toward a Complete Description of J/ψ in QGP & Hadronic Medium



- **Cold Nuclear Matter Effects**

- 1^{st} J/ψ suppression: Nuclear absorption, Cronin effect, ...

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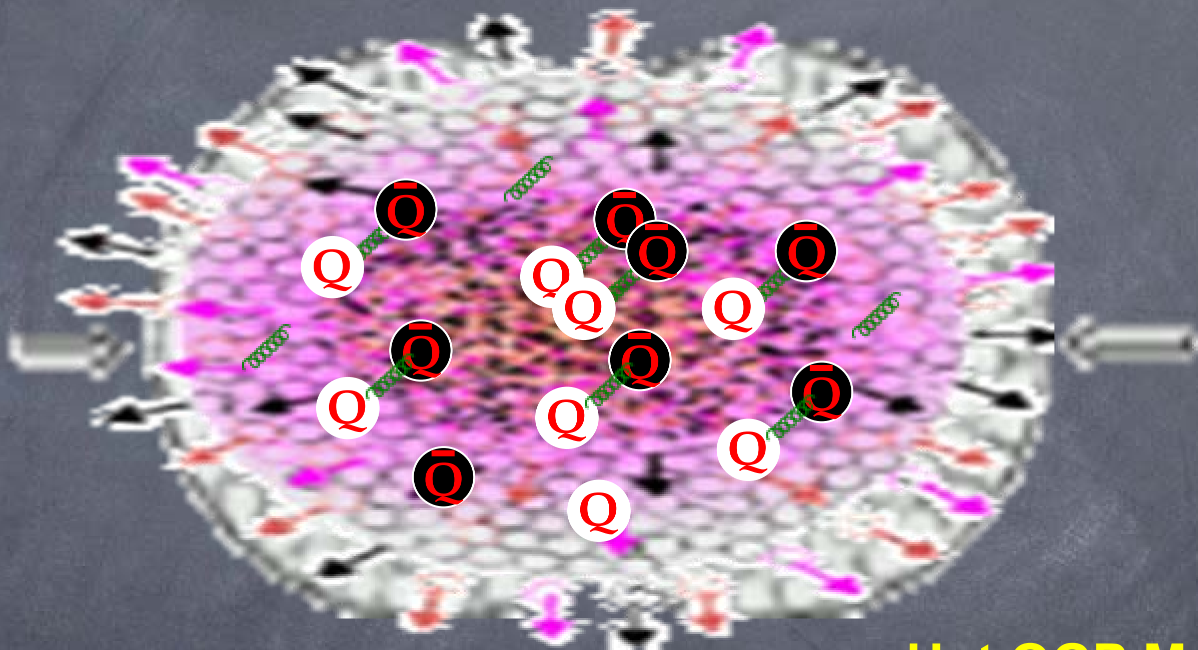
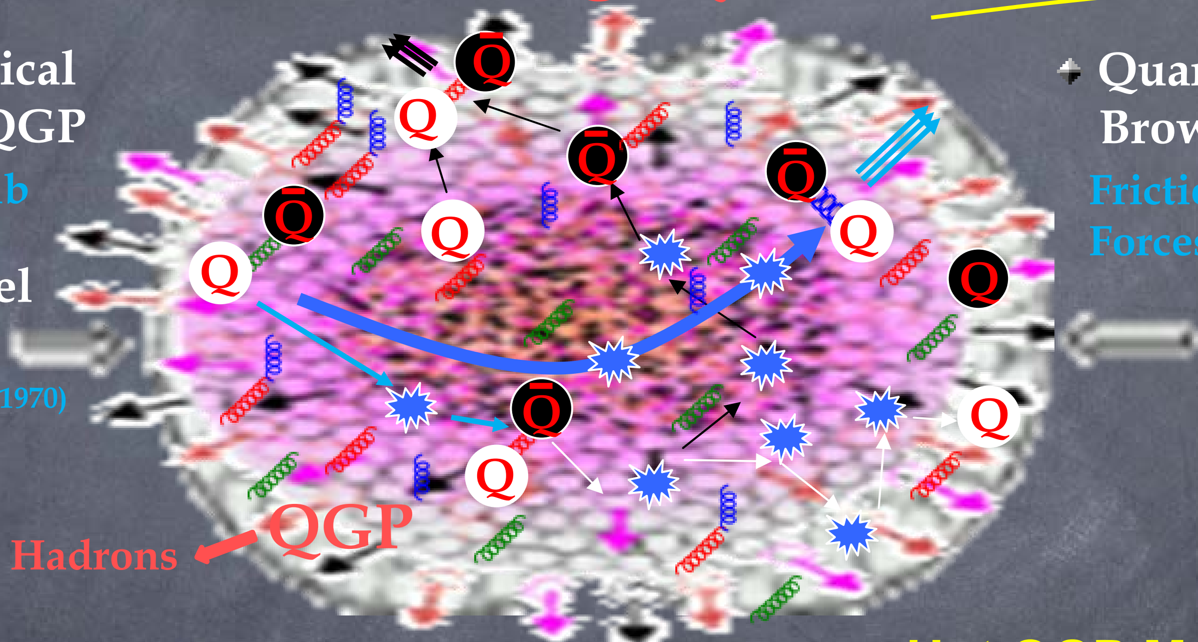


- **Cold Nuclear Matter Effects**
  - 1st J/ψ suppression: Nuclear absorption, Cronin effect, ...

- **Hot QGP Matter Effects**
  - Sequential suppression
  - Recombination
  - ...

Toward a Complete Description of J/ψ in QGP & Hadronic Medium

MC@sHQ



Medium Description

- Hydrodynamical description of QGP
  U. Heinz & P. Kolb
- Glauber model initial state
  (Nucl.Phys., B21:135157, 1970)

$Q\bar{Q}$ Stochastic Evolution

- Quarkonia as Brownian particles
  Friction & Stochastic Forces
- In *MC@sHQ*:
  … sampling the distributions of Langevin forces

Cold Nuclear Matter Effects

- 1st J/ψ suppression: Nuclear absorption, Cronin effect, …

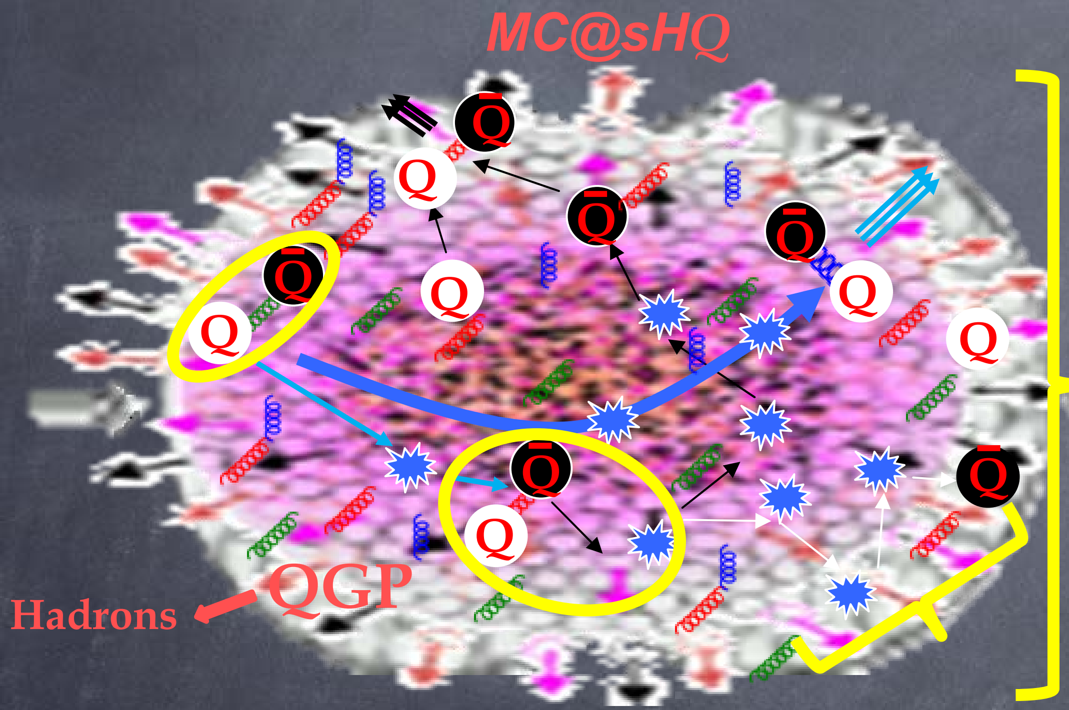
R. Granier De Cassagnac parametrization

(QM2006, J.Phys.G, G34:S955958,2007)

Hot QGP Matter Effects

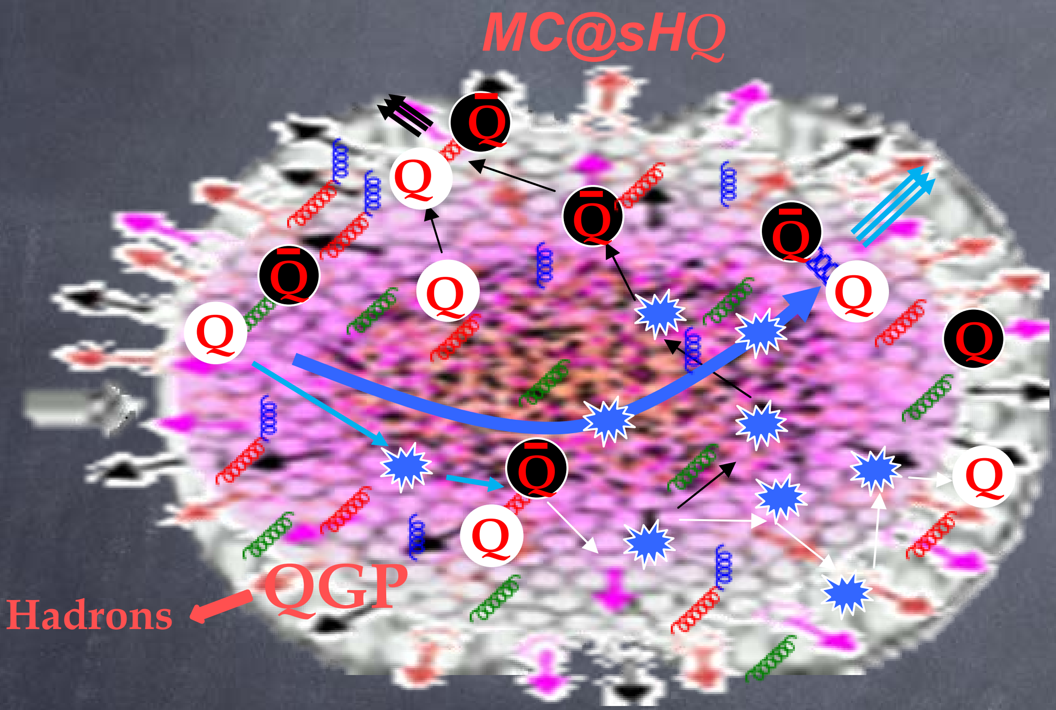
- Instantaneous melting/thermal excitation
- Q-$\bar{Q}$ → Quarkonia fusion (recombination)
- Hard gluon dissociation à la **Bhanot-Peskin**
- Elastic scattering & stochastic propagation
- …

Elastic Scattering: The undersides of quarkonia propagation and collectivity in the QGP



- IV. Stochastic Transport_& collective behaviour of Q
- III. Friction & Stochastic Forces Calculations
- II. $Q\bar{Q}$ –Partons/ Hadrons Elastic Scattering Processe
- I. $Q\bar{Q}$ in a Static Medium at finite Temperature

Elastic Scattering: The undersides of quarkonia propagation and collectivity in the QGP



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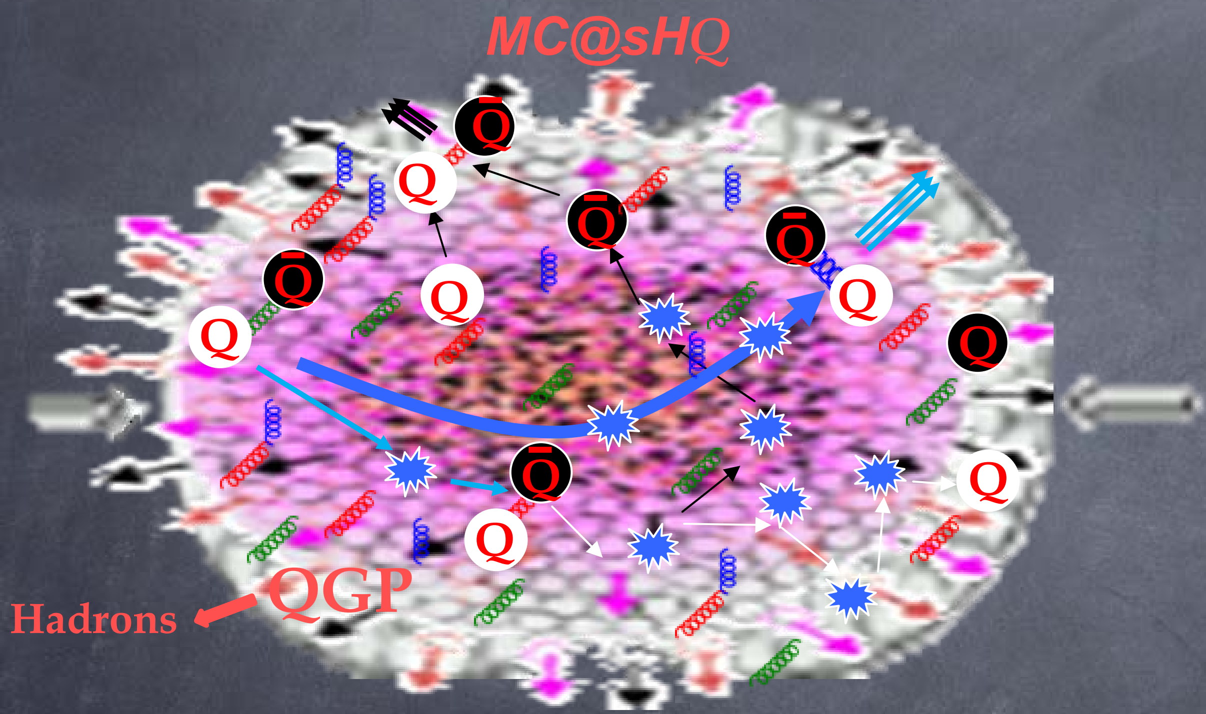


- I. $Q\bar{Q}$ in a Static Medium at finite Temperature

I. $Q\bar{Q}$ in a Static Medium at finite Temperature

Goal

Determine the charmonium and bottomonium spectra and wave functions at zero and finite temperature

How?

1) Model in phenomenology $Q\bar{Q}$ potential *V (r,T)* & resolve the Schrödinger equation
2) Determine the internal energy *U (r,T)* of $Q\bar{Q}$ from lQCD for the corresponding free energy *F(r, T)* using the relation:
$U(r,T) = F(r,T) - T\left(\frac{\partial F(r,T)}{\partial T}\right)$ and solve the Schrödinger equation with *V(r,T) = U (r,T)*
3) Calculate the quarkonium spectrum directly from lQCD at finite T

$Q\bar{Q}$ Potential Models

Φ_i : $Q\bar{Q}$ wave function

E_i : $Q\bar{Q}$ energy

$$\text{Schrödinger equation}: \ \mathcal{H}\ \Phi_i(r,T) = E_i\ \Phi_i(r,T)$$

$$\mathcal{H} = 2m_Q - \frac{\hbar^2 c^2}{m_Q}\nabla^2 + V(r,\,T)$$

Fitted to *U(r, T)* lQCD data

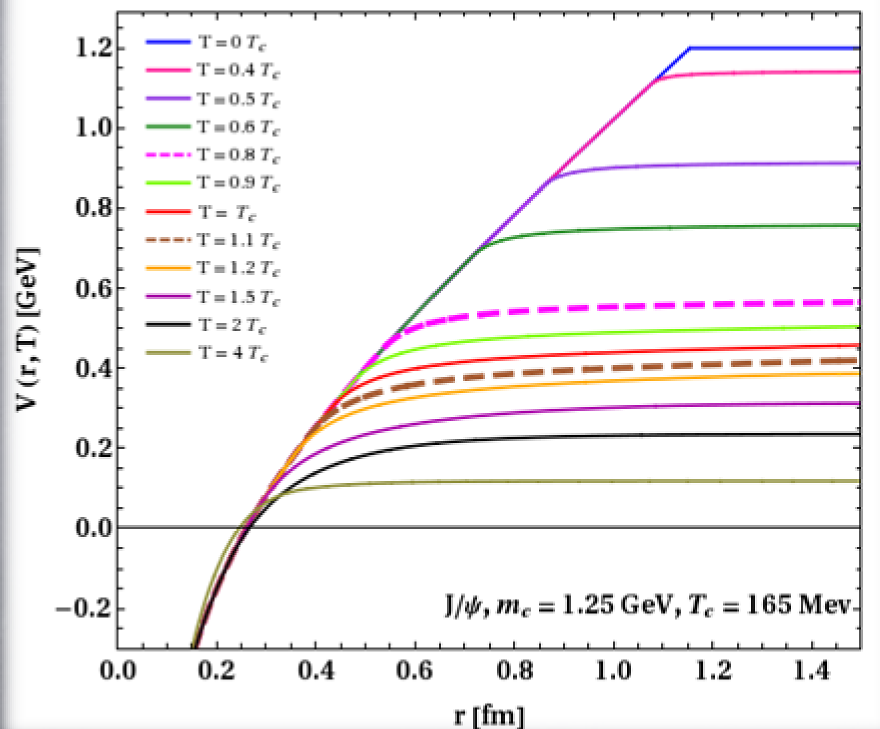
I. $Q\bar{Q}$ in a Static Medium at finite Temperature

Our parametrization of $Q\bar{Q}$ Potential (finite T)

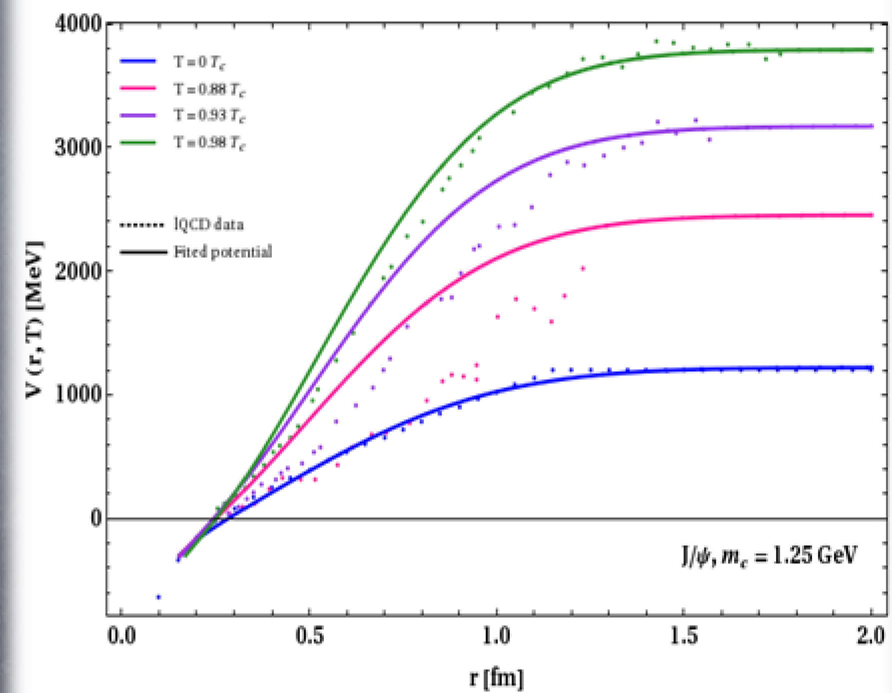
$$U(r,T) = F(r,T) - T\left(\frac{\partial F(r,T)}{\partial T}\right)$$

A. Mocsy and P. Petreczky
arXiv:0705.2559v2[hep-ph], 2007
arXiv:0706.2183v2[hep-ph], 2007

Weakly bound: $F(r,T) < V(r,T) < U(r,T)$



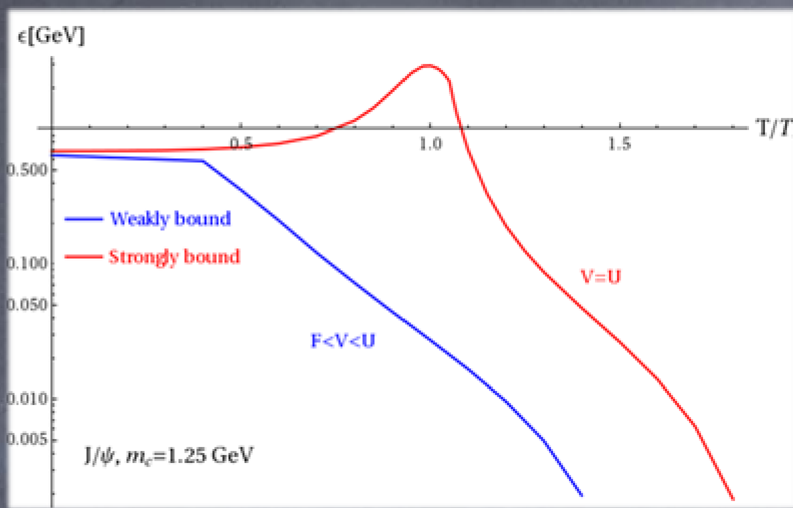
Strongly bound: $V(r,T) = U(r,T)$

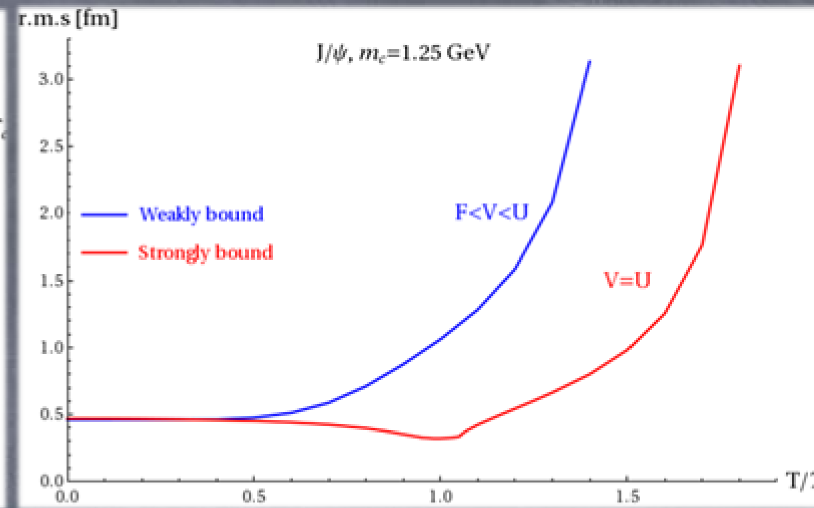


- We obtained $V(r,T)$ for J/Ψ and Υ for different T
- We obtained $V(r,T)$ for J/Ψ and Υ for different $T > T_c$
- We obtained $V(r,T)$ for J/Ψ and Υ for different T
- SB more binding in the medium than in the vacuum

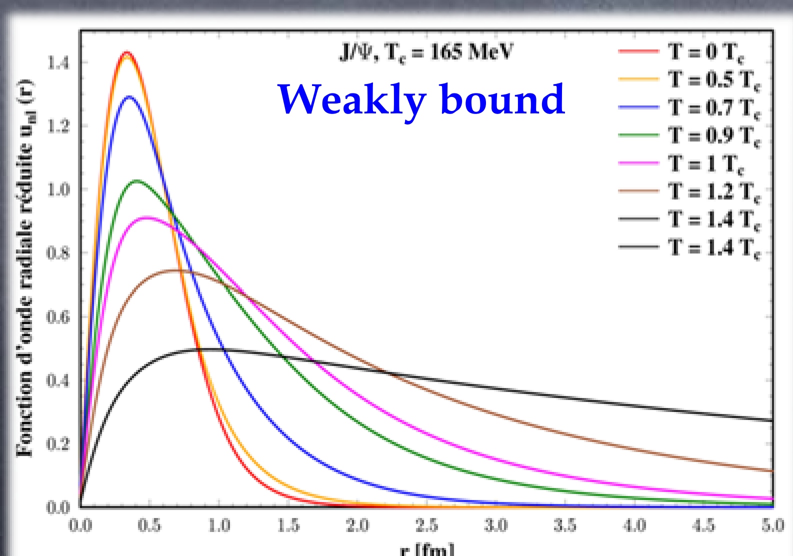
I. $Q\bar{Q}$ in a Static Medium at finite Temperature

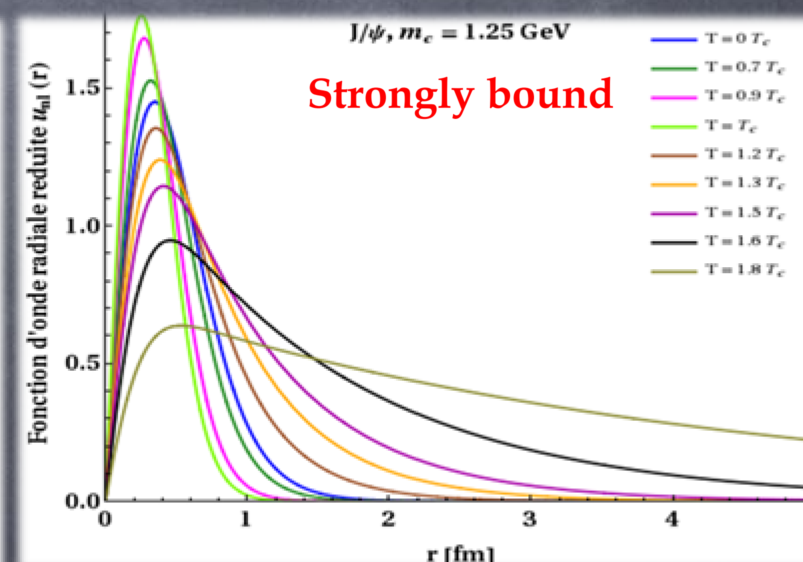
- **J/ψ binding energy (ϵ) & mean square radius (r.m.s)**





- **J/ψ wave functions**





I. $Q\bar{Q}$ in a Static Medium at finite Temperature

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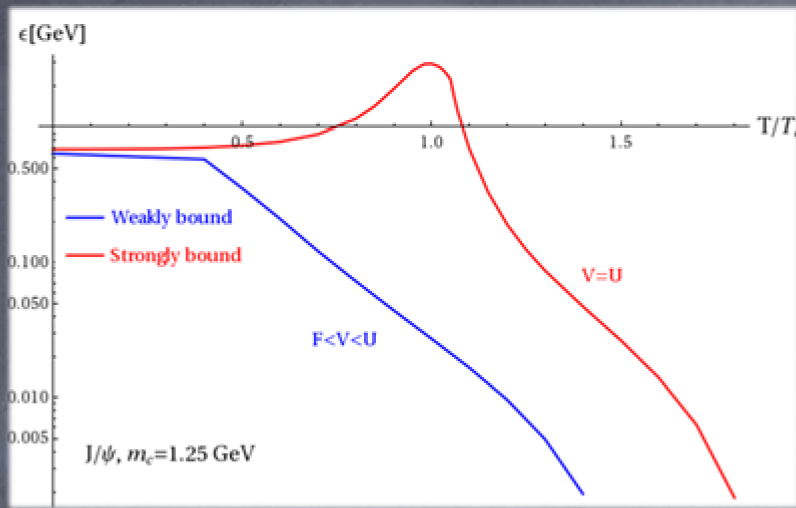
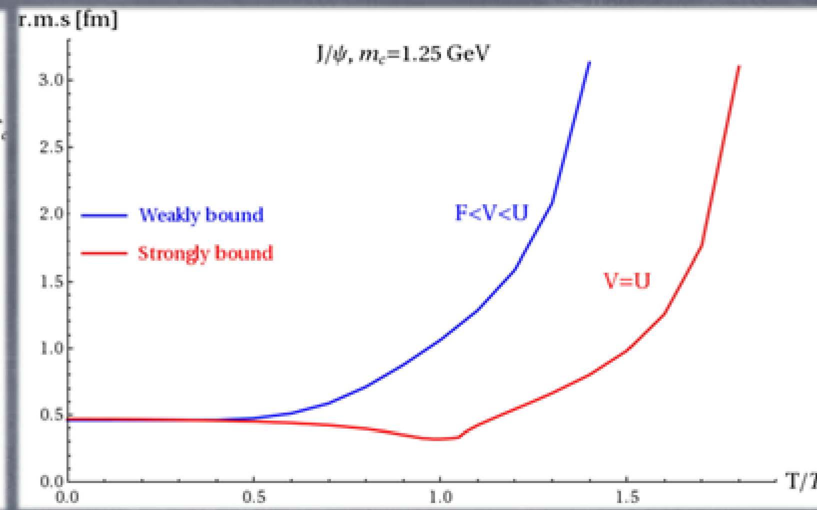


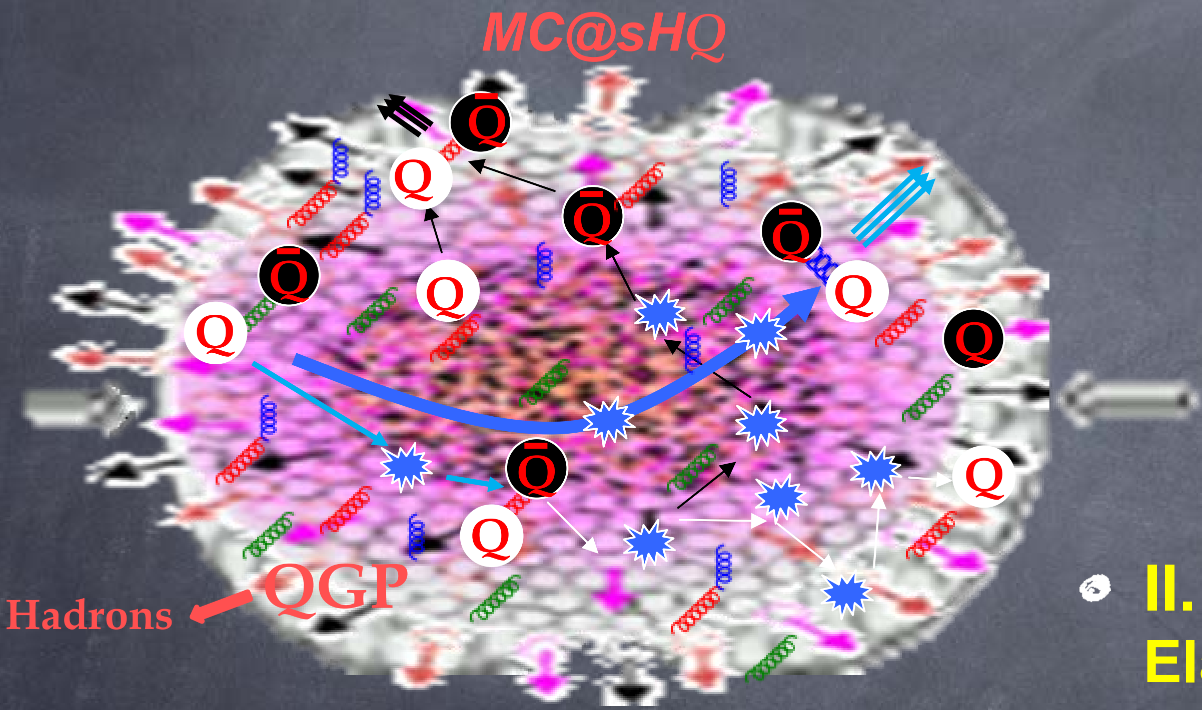


Lessons:

- Quantify the characteristics of charmonium and bottomonium *vs* temperature for two parameterizations of the potential (WB) and (SB)
- The survival of J/ψ and ϒ is related to the medium conditions
- The dissociation points and wave functions for the first state of charmonium and bottomonium system states are determined

But: No treatment of dynamic aspects of $Q\bar{Q}$ pair in the QGP (interactions with partons)

Elastic Scattering: The undersides of quarkonia propagation and collectivity in the QGP



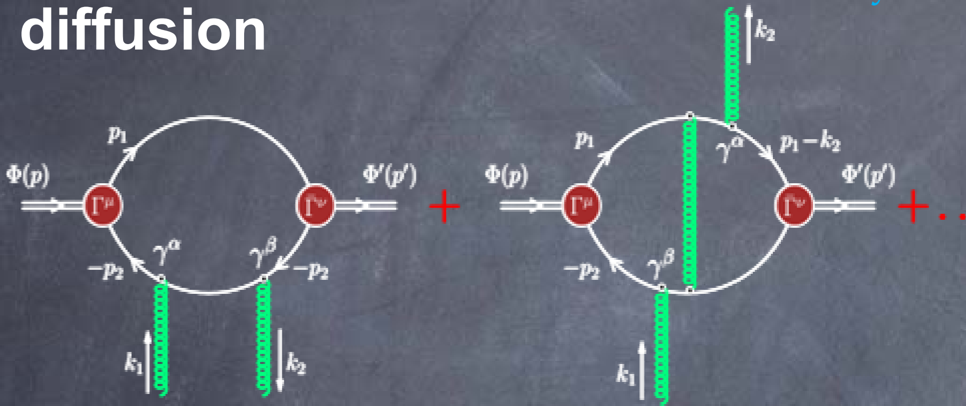
- **II. $Q\bar{Q}$ –Partons/ Hadrons Elastic Scattering Processe**
- **I. $Q\bar{Q}$ in a Static Medium at finite Temperature**

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

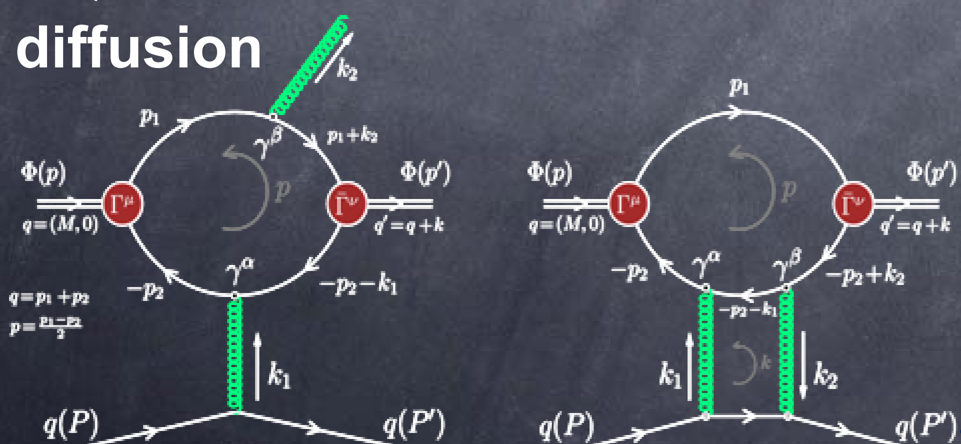
Elastic Processes

Compton diffusion

Dominant process for collectivity



Quark/hadron diffusion

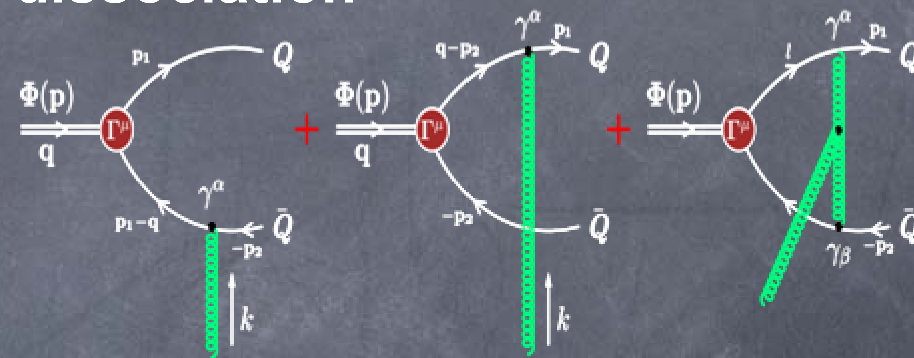


(a) (b)

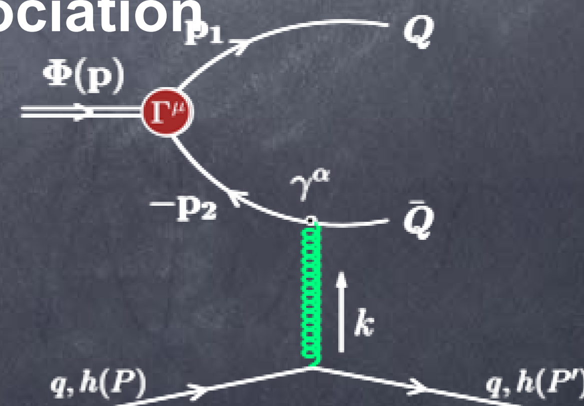
Inelastic Processes

Gluon dissociation

Dominant process for suppression



Quark/hadron dissociation



II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

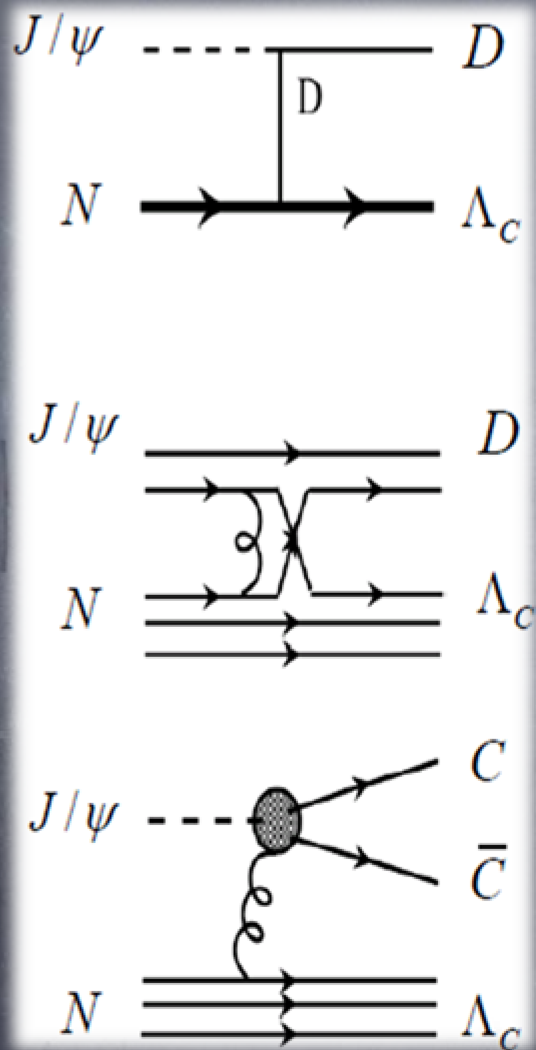
- σ_{inel} **calculation:**

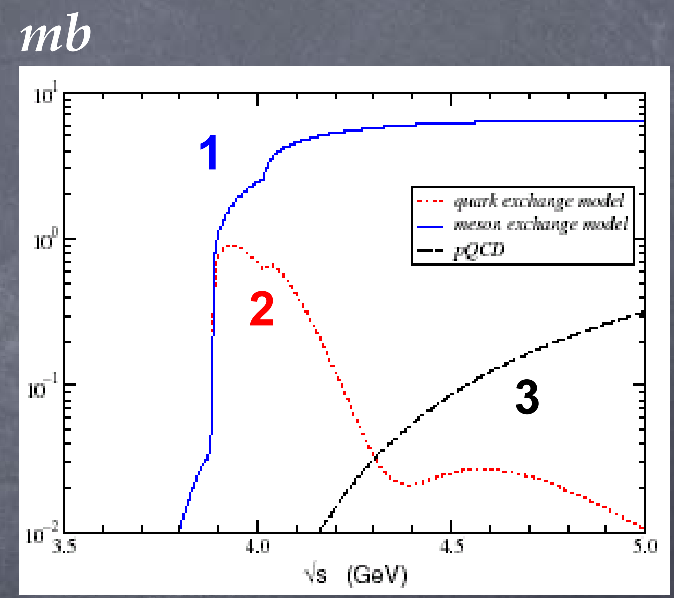
How?

1. **Effective model**
   - *Hadronic models*
   - *Model dependent*

2. **Quark exchange model**
   - *[Povh & Hüfner, Zi-wei Lin 02, A. Sibirtsev & al 01..]*

3. **LO pQCD**
   - *[Bhanot and Peskin 79]*





S.Lee (05), Voloshin, R. Rapp (03)

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

- **σ_{elas} calculations**

λ : gluon wavelength
Q : gluon energy

σ_{ela} (Φ-gluon)$_s$

- $\lambda \gg a_0$ (Bohr radius)
  $\rightarrow Q \ll \epsilon_0$ (binding energy)

$\epsilon_0 \approx mg^4$

- $\lambda \approx a_0$ or $\lambda \ll a_0$ (Bohr radius)
  $\rightarrow Q \approx \epsilon_0$ or $Q \gg \epsilon_0$ (binding energy)

Low energy – **Bhanot-Peskin Formalism**

High & intermediate energy – **Bethe-Salpeter Formalism**

Bhanot G and Peskir
Nucl. Phys., B156, 19
Nucl. Phys., B156:39

r et H. A. Bethe
34:1232_1242, Dec 1951
,2, 1952

	σ_{inel} (Φ-g/h)	σ_{elas} (Φ-g/h)
"Φ" Coulombic	G. Bhanot and M.E. Peskin (BP)	G. Bhanot and M.E. Peskin (BP)
"Φ" Non Coulombic	F. Arleo, J. Cugnon and Y. Kalinovsku (ACK)	H. Berrehrah, PB. Gossiaux & J. Aichelin (BGA)

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

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Low energy

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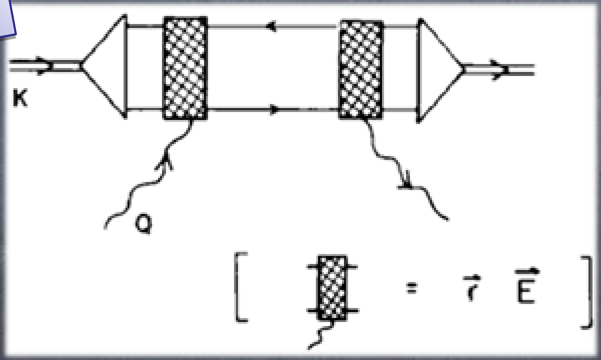
High & intermediate energy

Bethe-Salpeter Formalism

a) From OPE *(operator product expansion)*

b) Binding energy = ε_0 » L_{QCD}

(short distance QCD calculations)



field-dipole Interaction

$$\mathcal{M}_{\Phi g} = \frac{1}{2}\left[\frac{1}{3}g^2Q^2 < \phi|r^i \frac{1}{\epsilon + H_a - Q} r^i|\phi > + (Q \leftrightarrow -Q)\right].$$

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Bhanot G and Peskin M E
Nucl. Phys., B156, 1979.
Nucl. Phys., B156:391, 1979

High & intermediate energy

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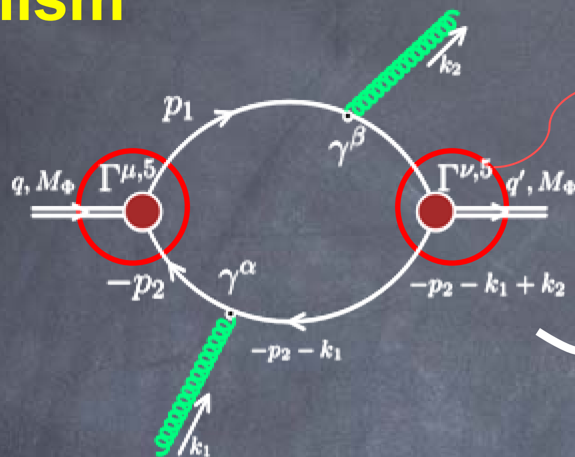
E. E. Salpeter et H. A. Bethe
Phys. Rev., 84:1232_1242, Dec 1951
Phys,Rev 87,2, 1952

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

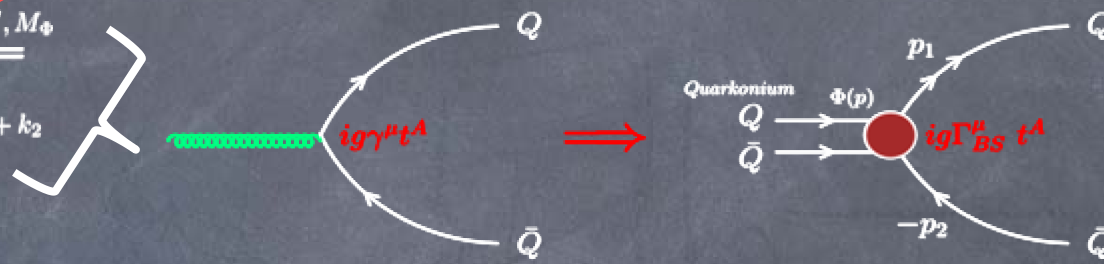
- **Bethe-Salpeter formalism**



- **Goal: Bethe-Salpeter vertex**

Bethe-Salpeter amplitude (vertex)

→ Related to $Q\bar{Q}$ wave function



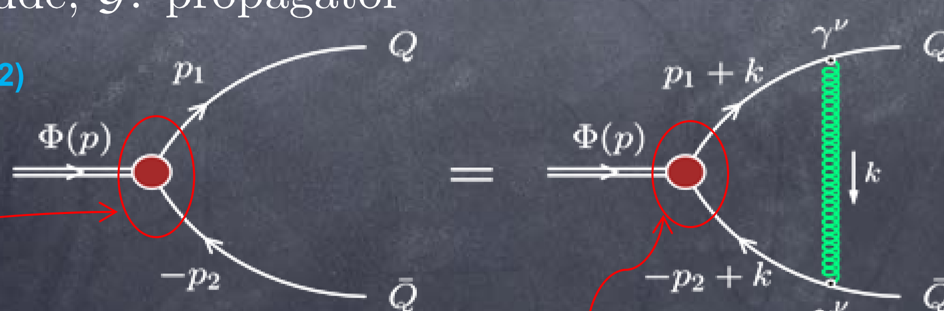
E. E. Salpeter et H. A. Bethe
Phys. Rev., 84:1232_1242, 1951

- **Bethe-Salpeter vertex equation**

$$\mathcal{M} = \mathcal{V} + \int \mathcal{V}\mathcal{G}\mathcal{V} + \int\int \mathcal{V}\mathcal{G}\mathcal{V}\mathcal{G}\mathcal{V} + \cdots + \left(\int \mathcal{V}\mathcal{G}\right)^n + \cdots = \frac{\mathcal{V}}{1 - \int \mathcal{V}\mathcal{G}}$$

$\mathcal{V}$: kernel, $\mathcal{M}$: amplitude, $\mathcal{G}$: propagator

Y. Oh, S. Kim, S. Houng Lee, (2002)



$$\Gamma(p, P) = iC_{color} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\eta}\gamma_\nu \frac{1}{\not{p}_1 + \not{k} - m + i\eta} \Gamma(p + k, P) \frac{1}{-\not{p}_2 + \not{k} - m + i\eta}\gamma_\nu$$

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

Bethe-Salpeter Vertices

- Case of quarkonium in the rest frame

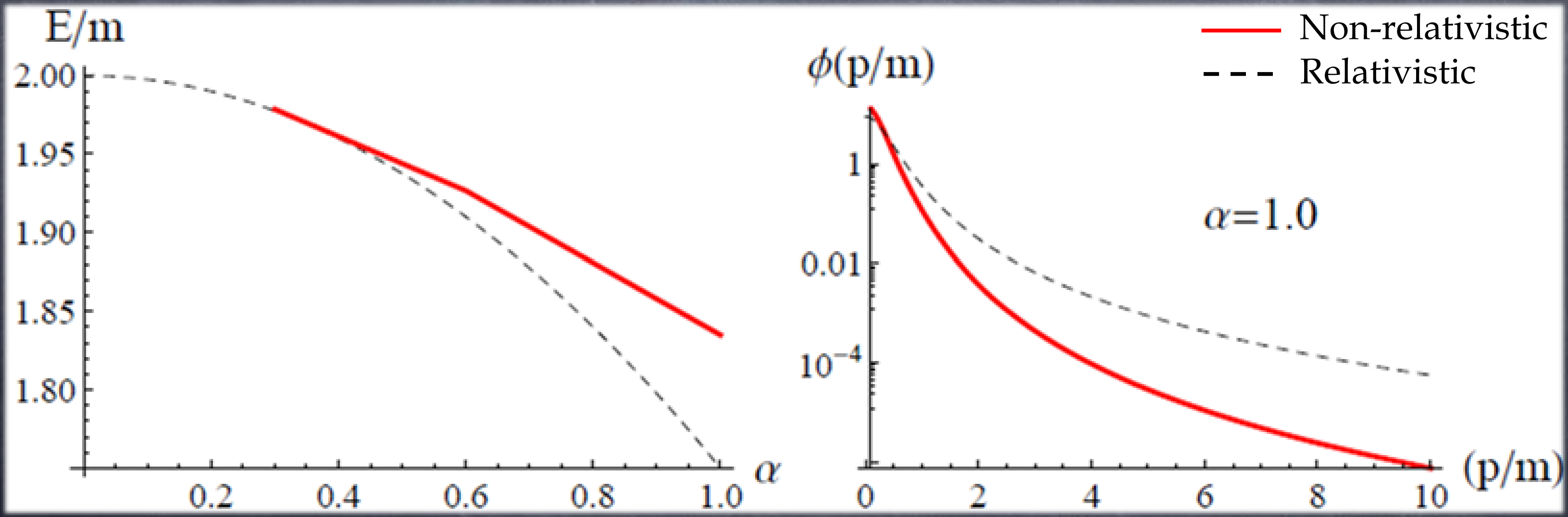
- Instantaneous Interaction

$$\Gamma_I(E,\vec{p}) \approx \frac{-ie_0(2e_0-E)}{\pi E}\,\gamma^0\,.\frac{I+\gamma_0-\vec{p}\!\!/ /m}{2}.\ \phi_{\mathbf{I}}^{+-}(\tilde{\mathbf{p}}).\frac{I-\gamma_0-\vec{p}\!\!/ /m}{2}.\gamma^0$$

$\phi_{\mathbf{I}}^{+-}(\tilde{\mathbf{p}}) = \phi_{\mathbf{space}}(\tilde{\mathbf{p}}) \times \phi_{\mathbf{spin}}^{\mathbf{ij}}$:instantaneous wave function for the bound state

- Retardation effects and hyperfine structure

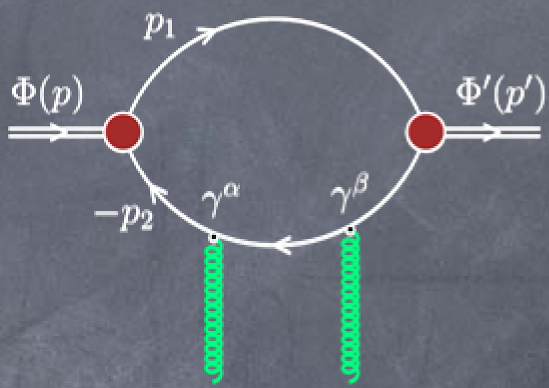
Corrections do not modify Γ_I, but have some influence on the binding energy *E* and on the behaviour of the wave function φ (p) for p ≥ m



II. Q$\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

- **σ_{elas} calculations**

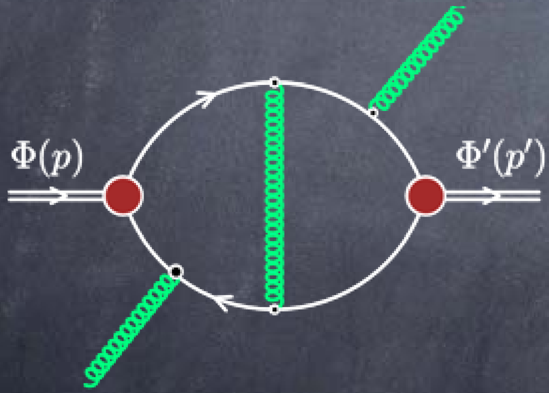
- **Compton diffusion J/ψ-gluon**



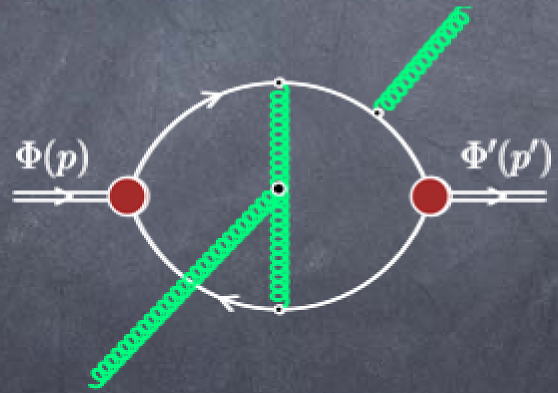
- 2 gluons exchanged, ″LO″

  6 diagrams (bb‖, bbX, tt ‖, ttX, tb, bt)

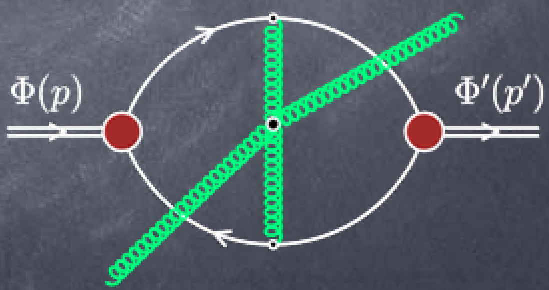
- 3 gluons exchanged, ″SNLO″



4 diagrams
(bb‖, btX, tt ‖, tbX)



7 diagrams
(gluon emited in each line)



1 diagram

II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

σ_{elas} calculations

λ : gluon wavelength
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σ_{ela} (Φ-gluon)s

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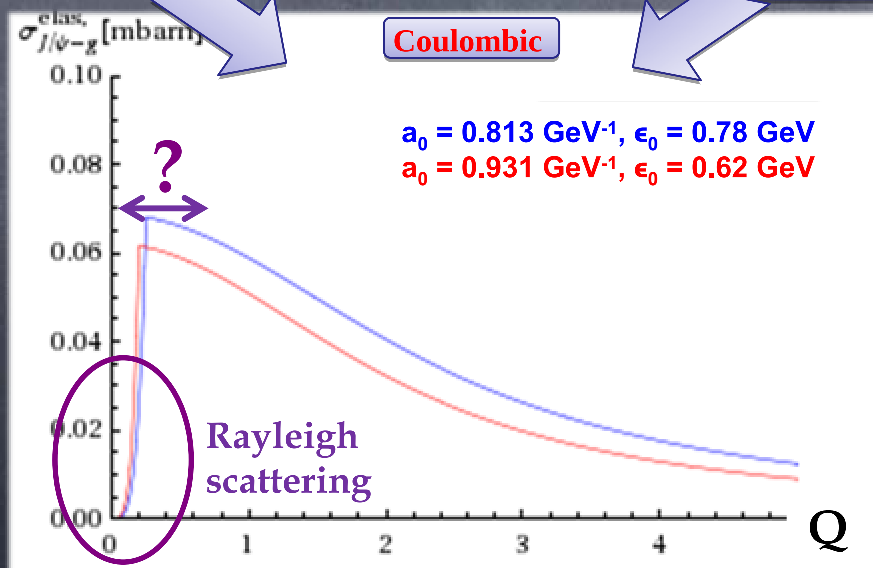
Low energy: Bhanot-Peskin Formalism

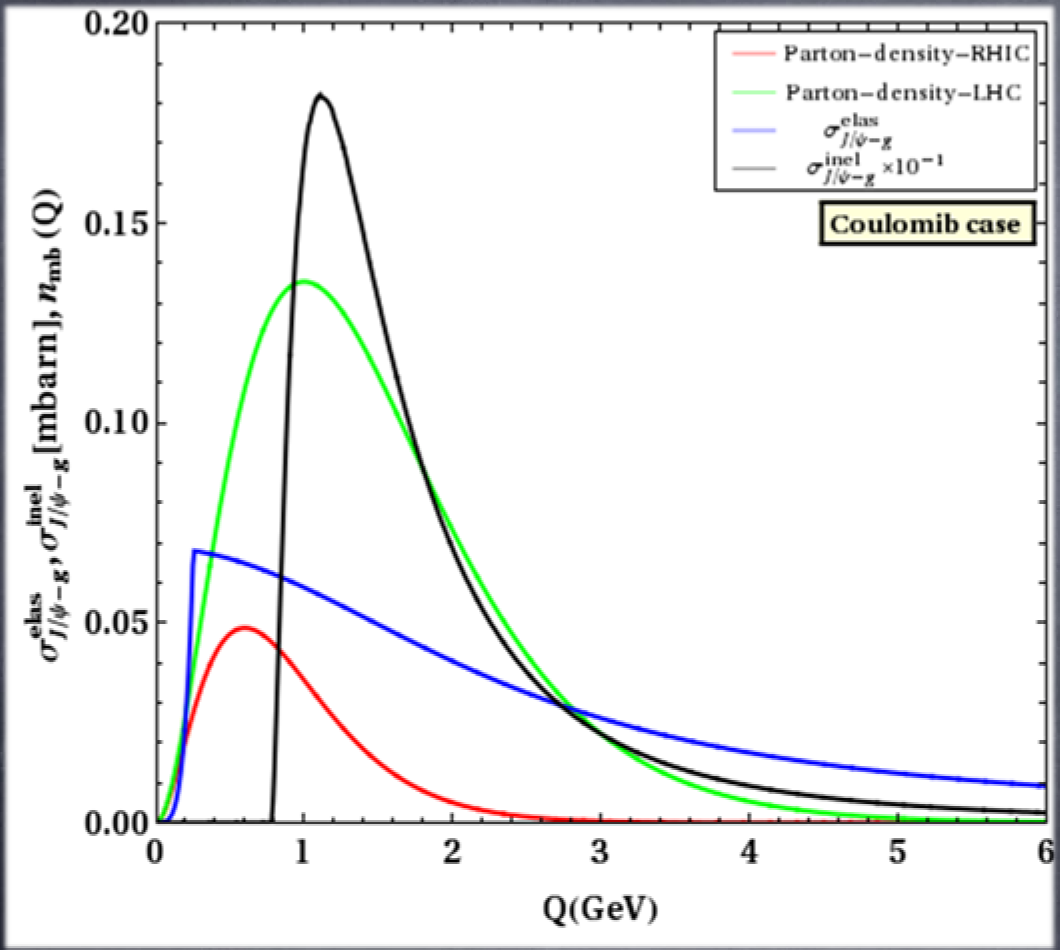
High & intermediate energy: Bethe-Salpeter Formalism



II. $\overline{Q}Q$ – Parton/Hadron Elastic & Inelastic Scattering Processes

J/ψ: Coulombic

- **σ_{elas} Interest & Discussion**
  - J/ψ-gluon: Gluon dissociation *vs* Compton diffusion (LO diagrams)



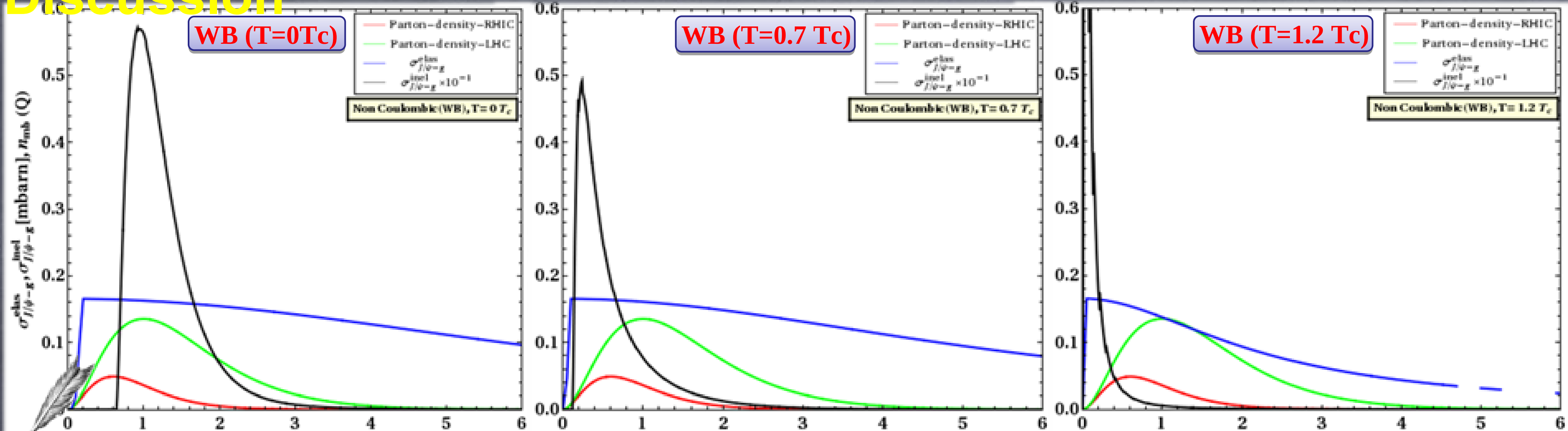
- $n_{mb}(e) = \int dee^2 e^{-e/T}$

- Inelastic cross section has a threshold Y. Oh, S. Kim, S. Houng Lee, (2002)
- Quantities measured are convoluted by n_{mb} (e)
- Overlap σ_{elas} and Maxwell-Boltzmann distribution larger than σ_{inel} and Maxwell-Boltzmann

II. QQ̄ – Parton/Hadron Elastic & Inelastic Scattering Processes

J/ψ: Non Coulombic

- **σ_{elas} Interest & Discussion**



Lessons:

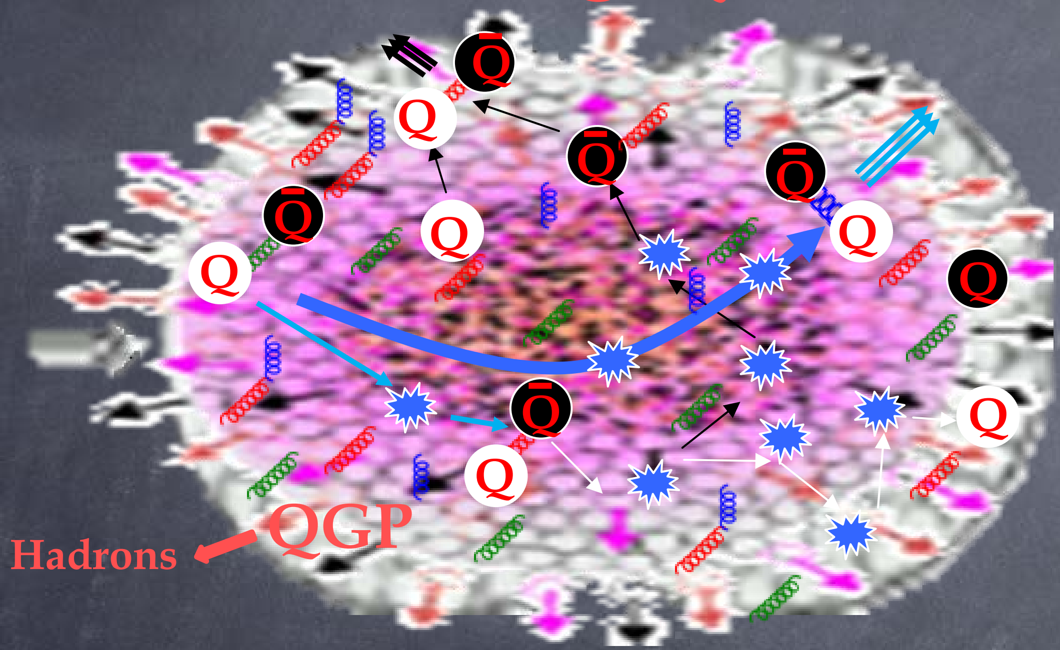
- Evaluation of σ_{elas} (quarkonium-gluon) within BS pQCD formalism
- Comparison between σ_{elas} and σ_{inel} and highlight the interest of σ_{elas}
- Evolution of σ_{elas} & σ_{inel} *vs* temperature (weakly & strongly bounded wave functions)
- Study of QQ̄ bound state (BS) vertex structure & Q, Q̄ interaction inside the quarkonium
- σ_{elas} will be used to evaluate energy loss, FP coefficients, stochastic propagation…

H. Berrehrah, P.B. Gossiaux, J. Aichelin. J. Phys. : Conf. Ser. 270 012036.

P.B. Gossiaux, H. Berrehrah, J. Aichelin.“Perturbative calculation of QED bound states elastic cross section”. In preparation

H. Berrehrah, P.B. Gossiaux, J. Aichelin. “Perturbative calculation of quarkonium-gluon, hadron elastic cross sections in vacuum and in medium”. In preparation”

Elastic Scattering: The undersides of quarkonia propagation and collectivity in the QGP



- III. Friction & Stochastic Forces Calculations
- II. $Q\bar{Q}$ –Partons/ Hadrons Elastic Scattering Processe
- I. $Q\bar{Q}$ in a Static Medium at finite Temperature

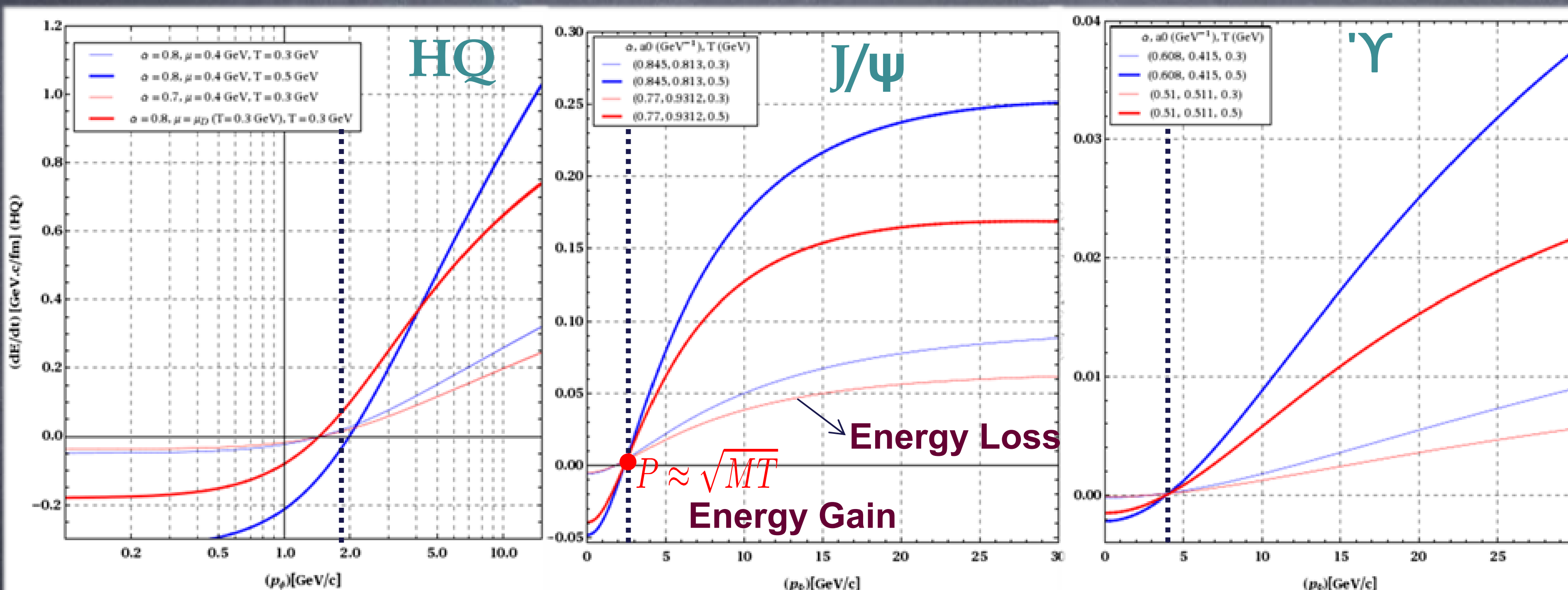
III. Fokker-Planck Coefficients Calculations

- **Energy losses** given by Bjorken (Φ(M, E, p) → "i"(m, e, q))

Collisional-Coulombic

J.D. Bjorken. FERMILAB-Pub-82/59-THY, 1982

$$\frac{dE}{d\tau} = \frac{dE}{dt} \times \frac{E}{M} = \sum_i \int d^3q \; n_i(\vec{q}) \; \frac{\sqrt{(p.q)^2 - M^2 m^2}}{Ee} \int dt \; \frac{d\sigma_{elas}}{dt} \; (E' - E)$$



- Behaviour: Log-increase *vs* ***p***
- Behaviour: decrease at ***p*** ↑
- Behaviour: decrease at ***p*** ↑

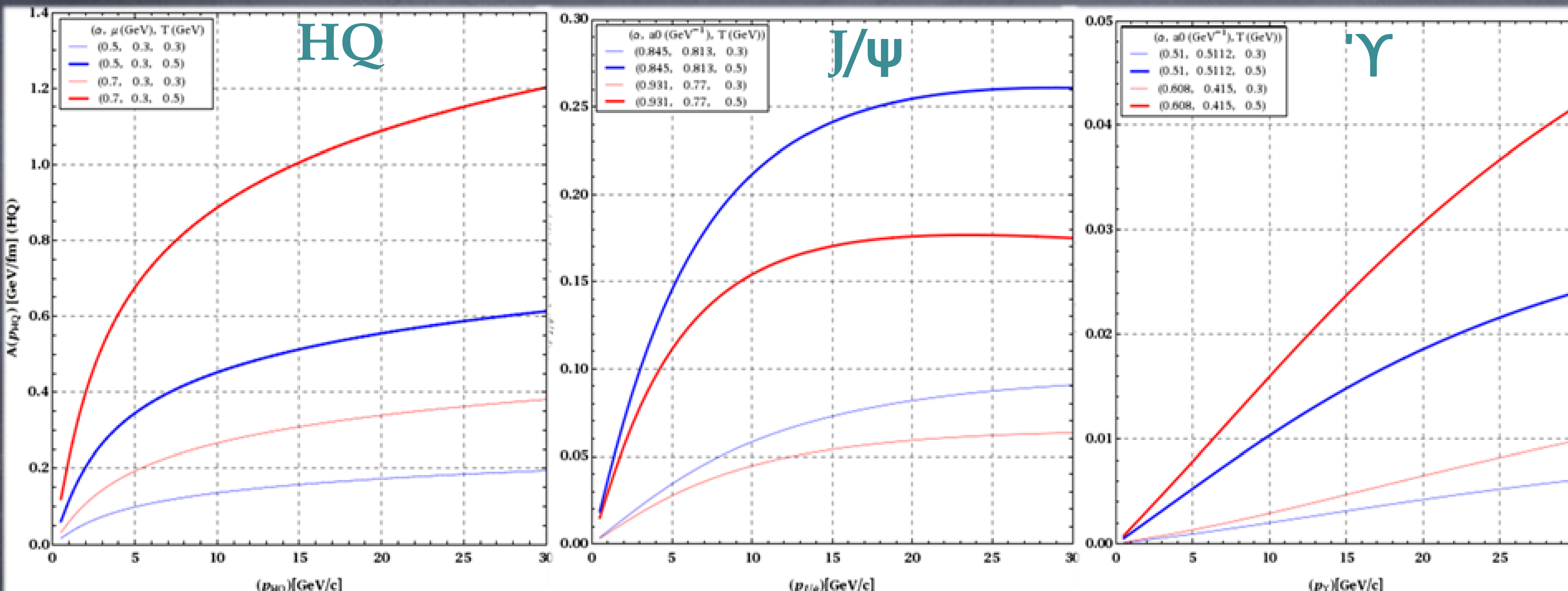
- $\frac{dE}{dt}(HQ) > \frac{dE}{dt}(J/\Psi) > \frac{dE}{dt}(\Upsilon)$
- **For (*HQ, J/ψ, 'ϒ*):** $\frac{dE}{dt} \nearrow$ with T $\nearrow$

III. Fokker-Planck Coefficients Calculations

Collisional-Coulombic

Drag & Diffusion coefficients

$$A_i = \frac{M}{E} \sum_i \int d^3q \; n_i(\vec{q}) \; \frac{\sqrt{(p.q)^2 - M^2 m^2}}{Ee} \int dt \; \frac{d\sigma_{elas}}{dt} \; \frac{\left\langle (\vec{P} - \vec{P}'). \; \vec{P} \right\rangle}{||\vec{P}||}$$



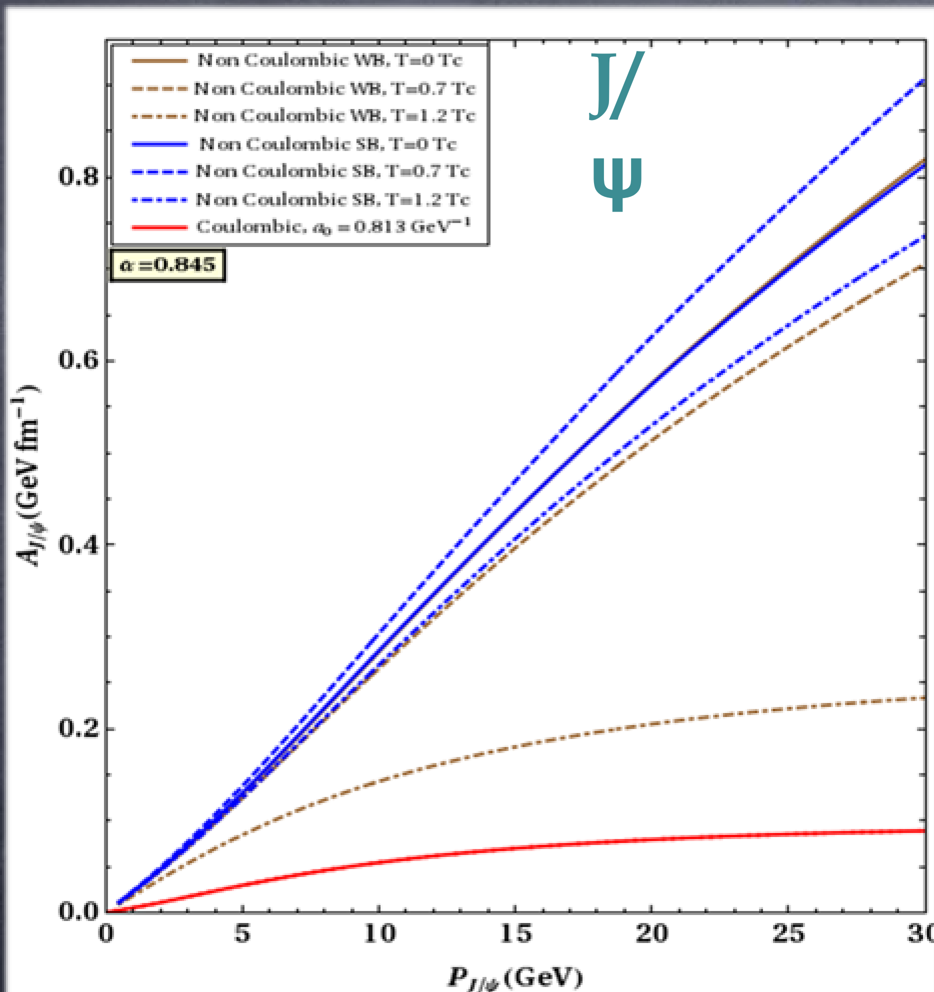
- Same behaviour for HQ, J/Ψ, Υ
- $A(HQ) > A(J/\Psi) > A(\Upsilon)$
- At large p, $A_i \sim dE/dt$
- For (HQ, J/Ψ, Υ): $A_i \nearrow$ with T $\nearrow$
- $B(E) = \int_E^{+\infty} dE' \; A_i(E') \times \frac{E'}{P'} \; e^{-(E'-E)/T}$, with: $B_\perp = B_\parallel = B$, **B ↔ A Einstein relation**

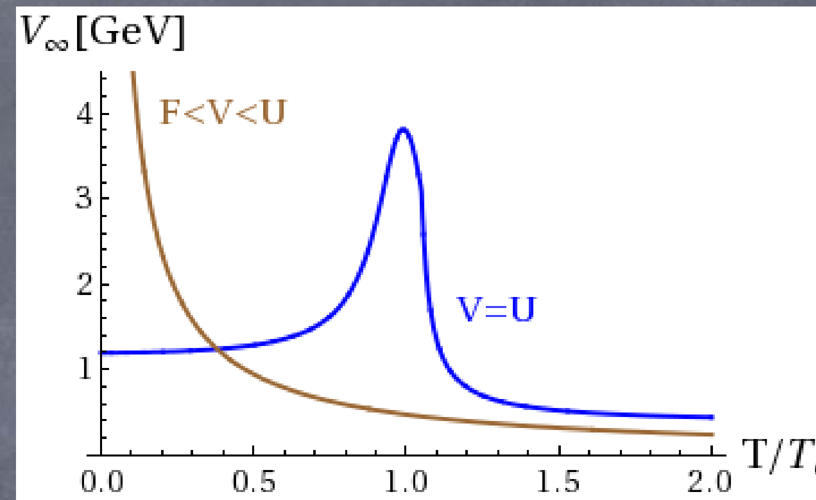
D.B. Walton and J. Rafelski. Phys, Rev Lett, 84(1):3134, 2000

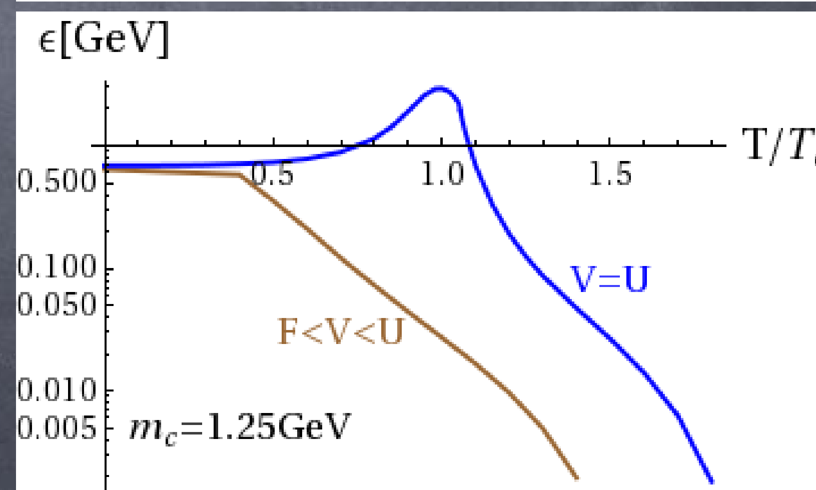
III. Fokker-Planck Coefficients Calculations

- **Wave function influence on dE/dt, Ai, B**

Collisional-Non Coulombic





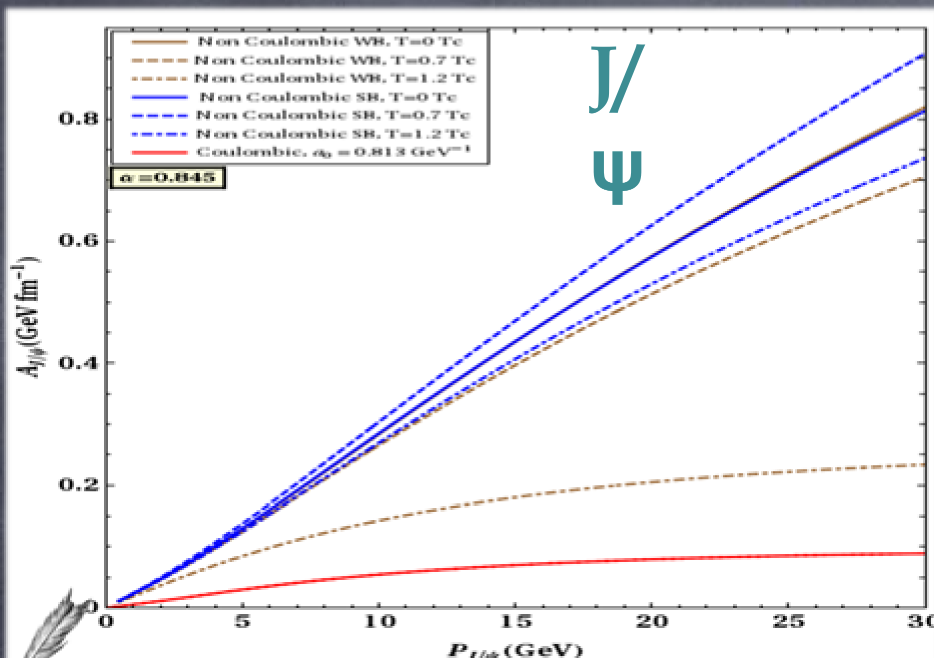


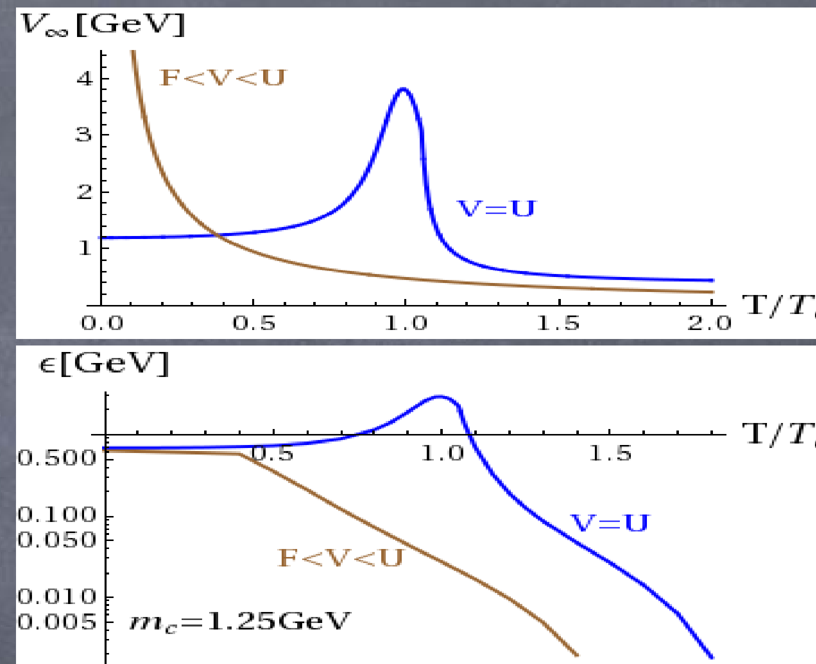
- Weakly bound and strongly bound > coulombic case
- Behavior related to $V_\infty(T)$ and $\epsilon(T)$

III. Fokker-Planck Coefficients Calculations

Wave function influence on dE/dt, Ai, B

Collisional-Non Coulombic

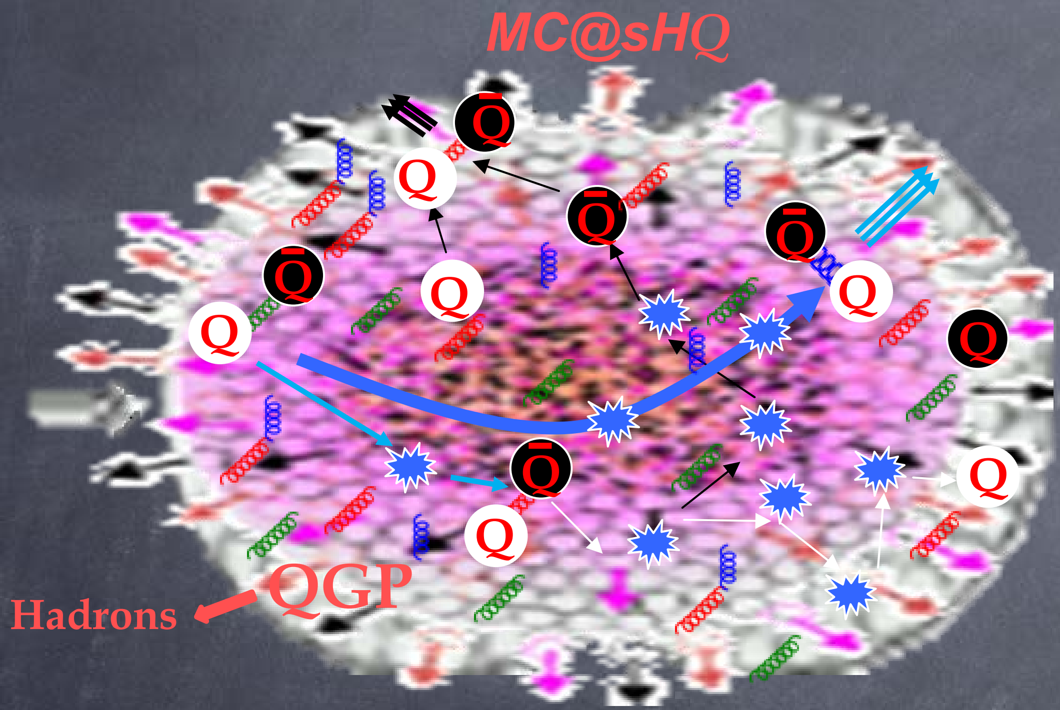




Lessons:

- Quantify the strongest of quarkonia-gluon interaction by evaluating elastic and inelastic rates, collisional energy, transport coefficients, stopping power...
- Study collisional energy loss of quarkonia in the QGP
- Determination of Fokker-Planck coefficients for HQ, J/ψ and 'Υ
- Influence of wave function on Fokker-Planck coefficients
- Evaluated 2 important ingredients for stochastic evolution of $c\bar{c}$ pairs for the next part

Elastic Scattering: The undersides of quarkonia propagation and collectivity in the QGP

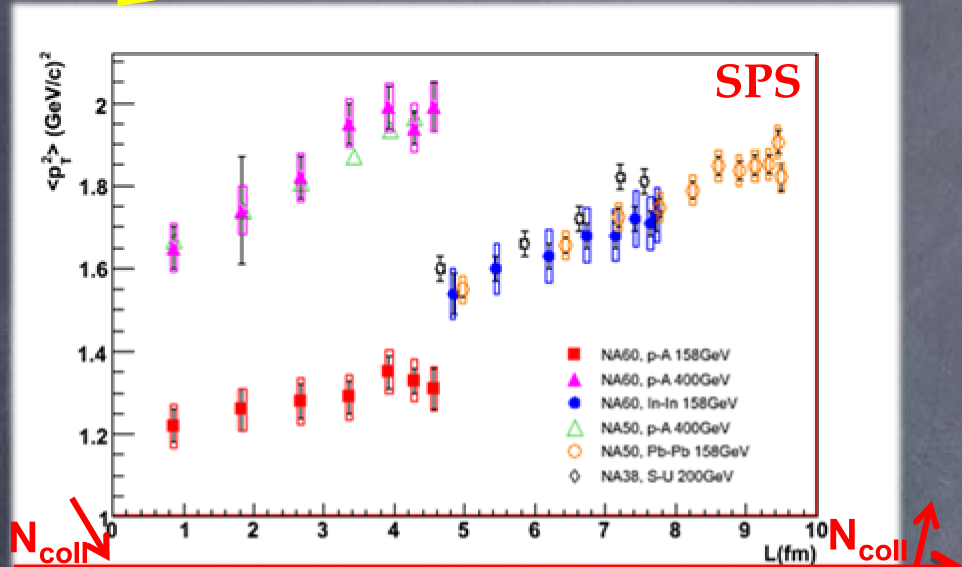


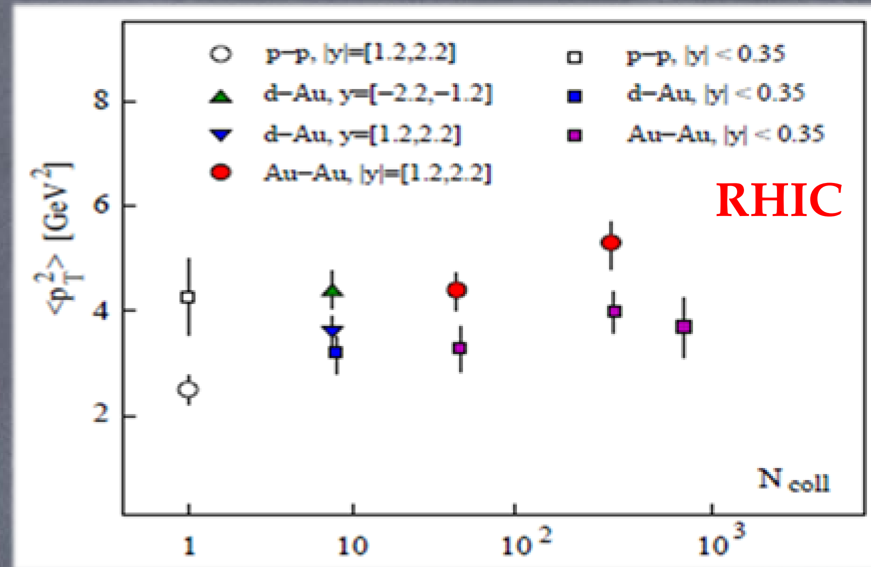
- IV. Stochastic Transport_& collective behaviour of Q
- III. Friction & Stochastic Forces Calculations
- II. $Q\bar{Q}$ –Partons/ Hadrons Elastic Scattering Processe
- I. $Q\bar{Q}$ in a Static Medium at finite Temperature

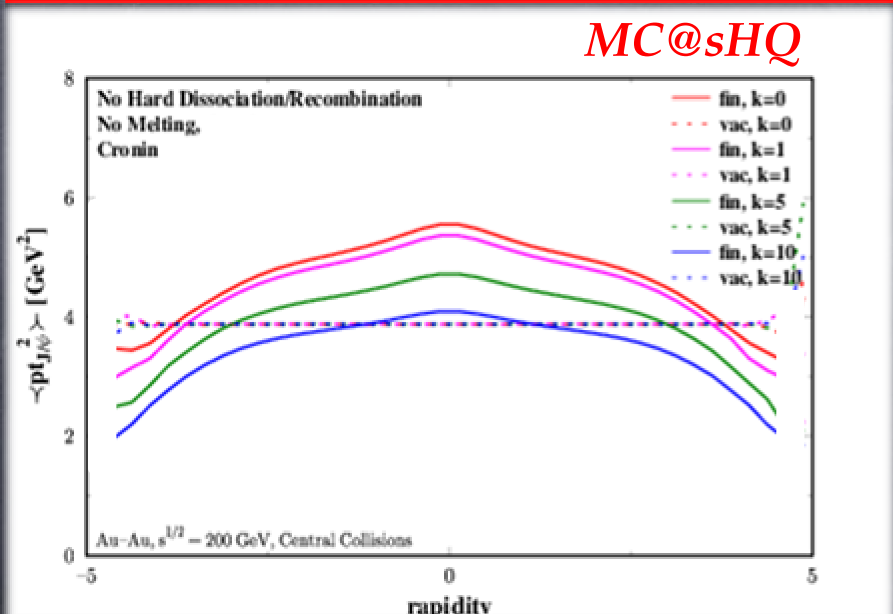
behaviour

- **Mean $<pt^2>^{1/2}$ for J/ψ**

RHIC







- **Broadening of $< p_t^2 >$ *vs* L or N_{coll} :**
  - Initial Effects: cronin effect
  - Final Effects: melting or hard absorption
- **J/ψ Interaction with the medium reduces $< p_t^2 >$**
- **Saturation of J/ψ p_t broadening in SPS and RHIC central collisions (J/ψ cooling) is reproduced with our model for the study of elastic collisions**

behaviour

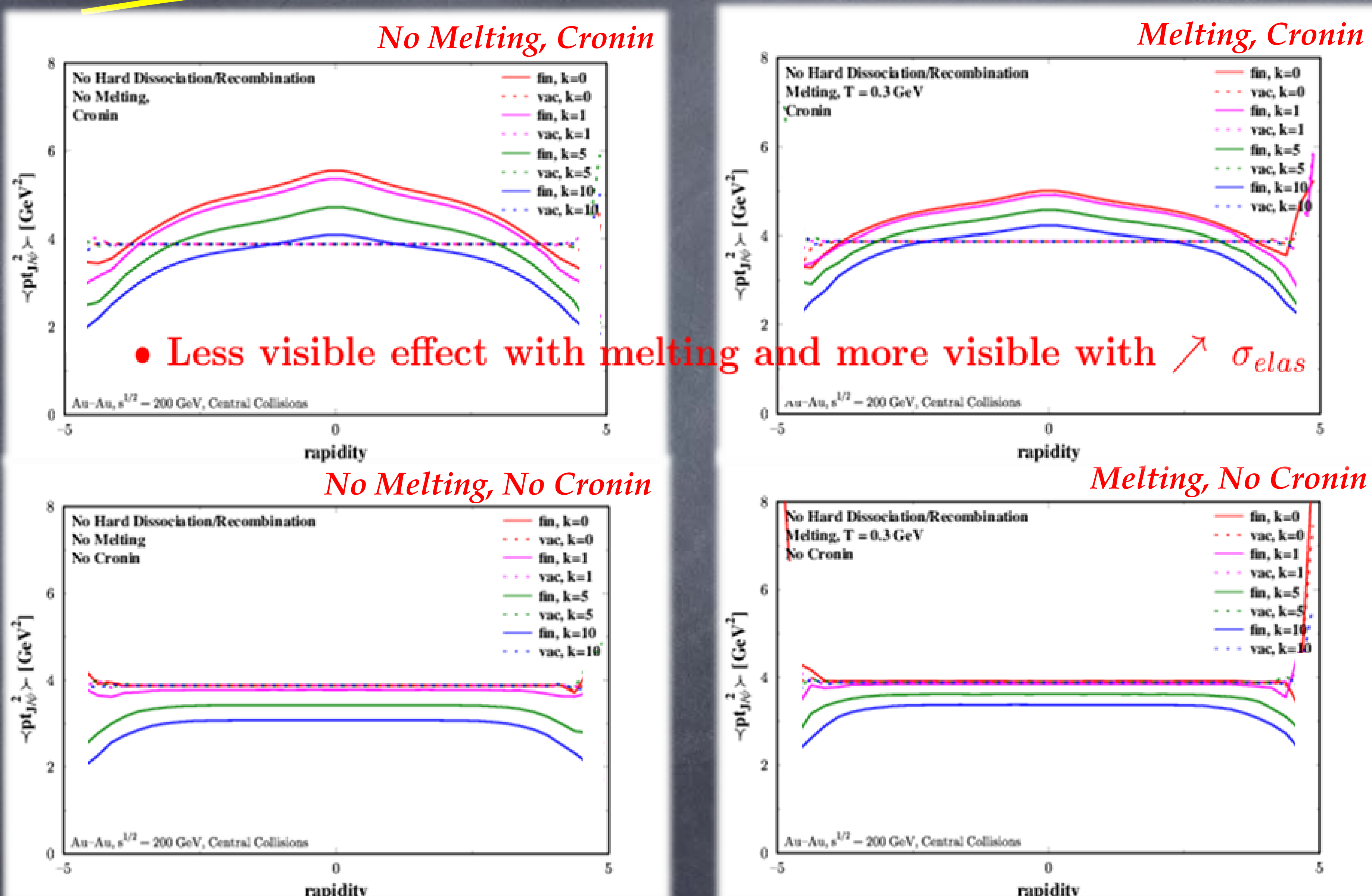
- **Mean $<pt^2>^{1/2}$ for J/ψ**

- Effects on J/ψ 's in our study
  - CNM effects (Cronin)
  - Instantaneous melting/thermal excitation ($T > T_{diss}$)... (Tdiss = ...)
  - Hard gluon dissociation à la Bhanot-Peskin ($T < T_{diss}$)... (Cranck)
  - Q-$\bar{Q}$ → Quarkonia fusion allowed ($T < T_{diss}$) ... (Cranck)
  - J/ψ Elastic scattering processes… (k factor)

Dual Model

behaviour

- **Mean $<pt^2>^{1/2}$ for J/ψ.... MC@sHQ** RHIC

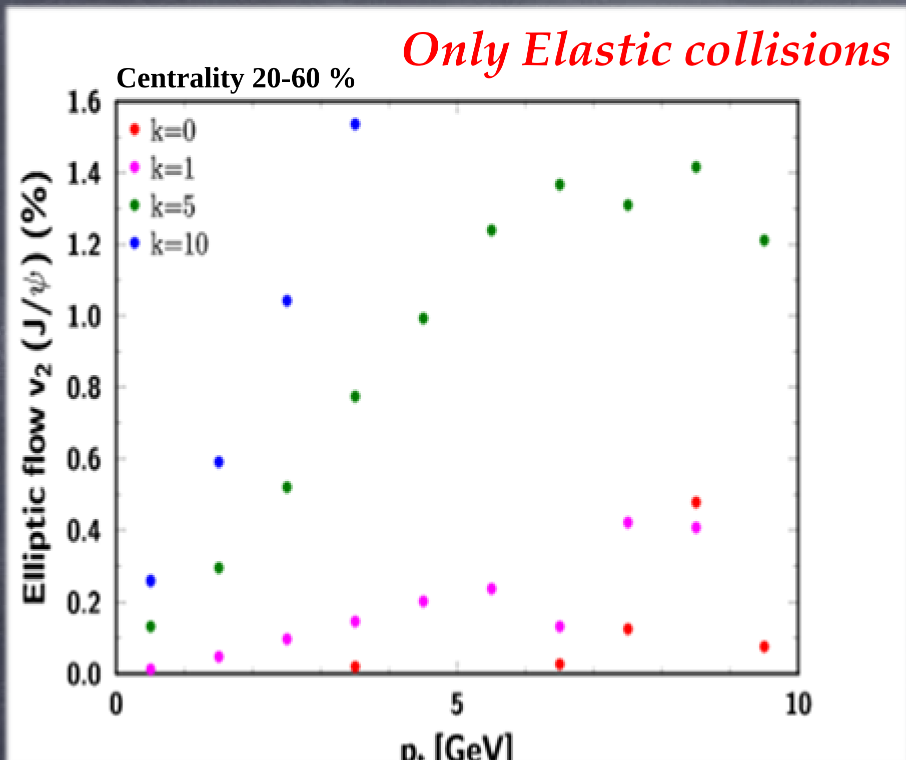


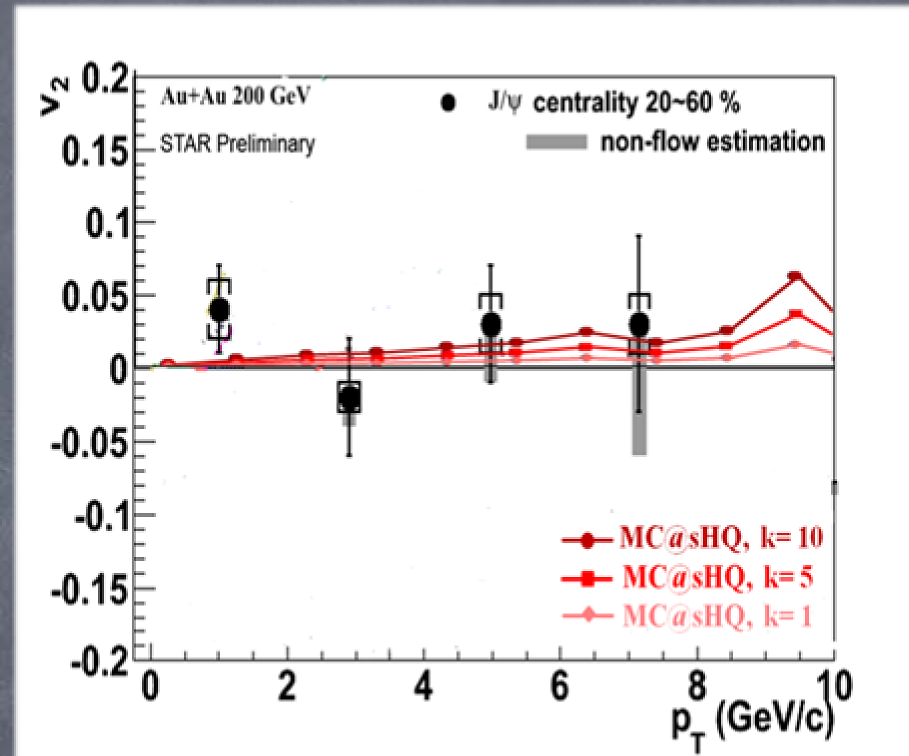
- Less visible effect with melting and more visible with ↗ σ_{elas}

behaviour

- **Elliptic flow $v_2(J/\psi)$**

RHIC



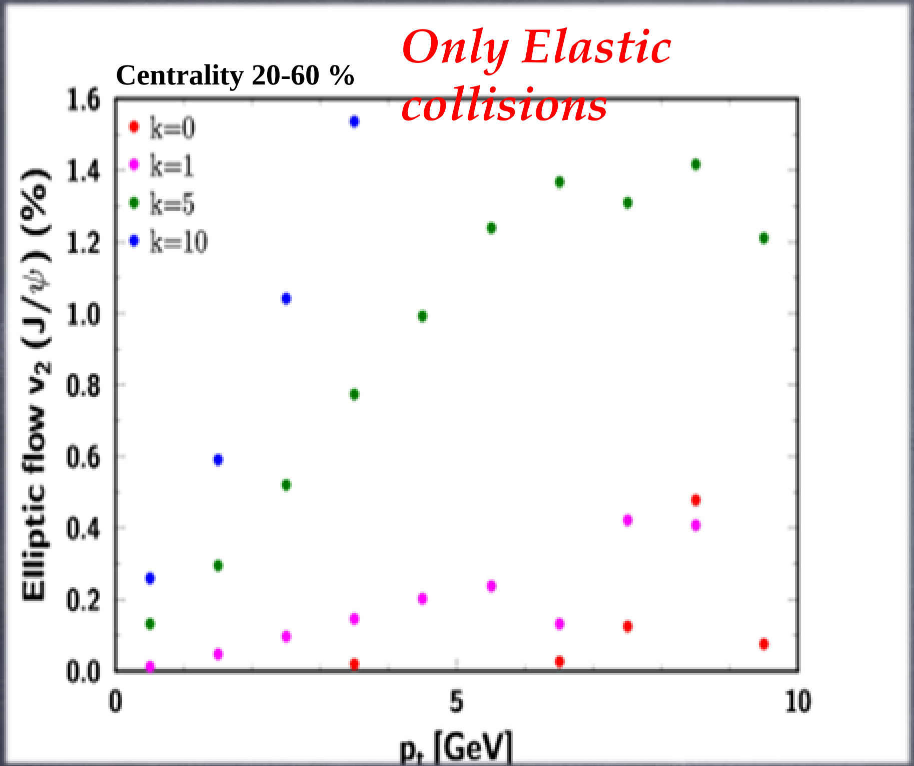


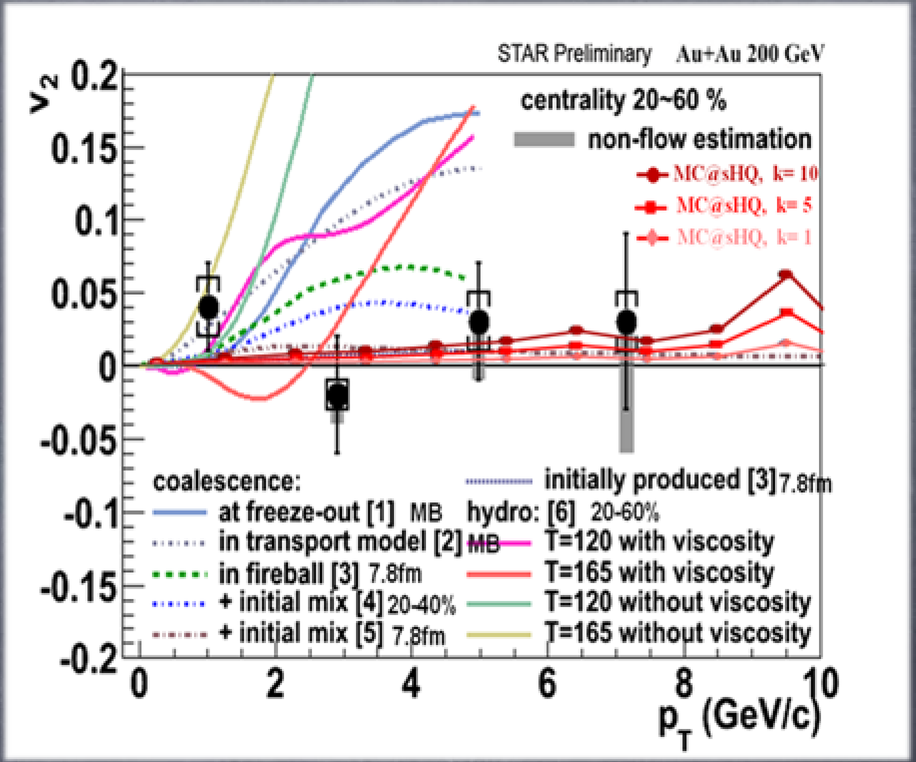
- No zero elliptic flow
- Influence of elastic processes:
  $\rightarrow$ increase of $\sigma_{elas} \rightarrow v_2$ (J/ψ) increases
- Good agreement with preliminary STAR data

(H. Berrehrah, P.B. Gossiaux, J. Aichelin. "Quarkonia collectivity: study of collisional energy loss, elliptic flow and other collective phenomena". In preparation) behaviour

- **Elliptic flow $v_2(J/\psi)$**

RHIC





- No zero elliptic flow
- Influence of elastic processes:
  → increase of σ_{elas} → v_2 (J/ψ) increases

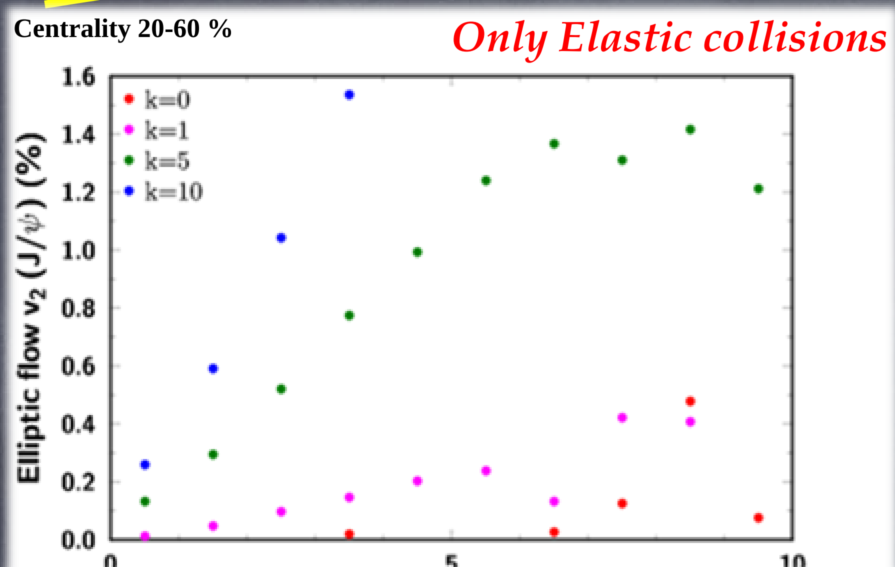
(H. Berrehrah, P.B. Gossiaux, J. Aichelin. "Quarkonia collectivity: study of collisional energy loss, elliptic flow and other collective phenomena". In preparation)

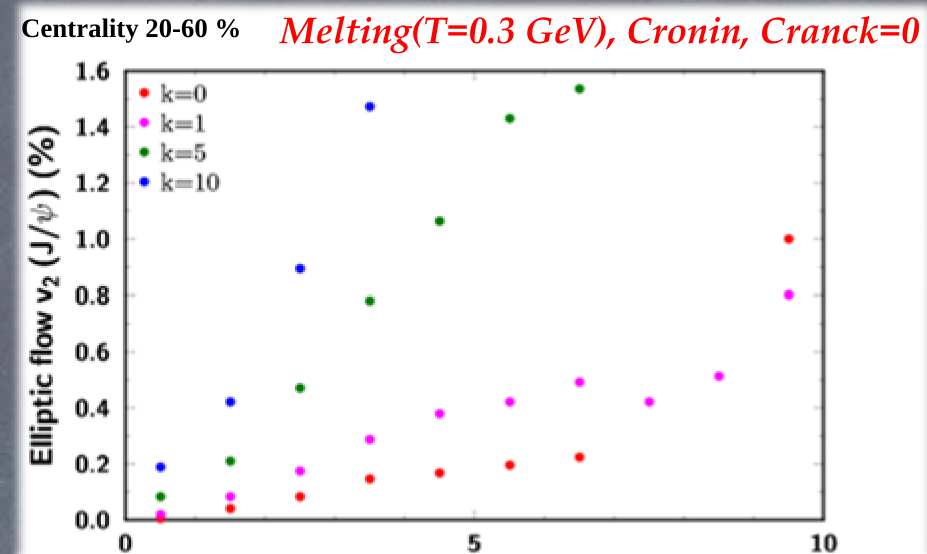
- Good agreement with preliminary STAR data
  - Sequential suppression: v_2 (J/ψ) ≈ 0
  - Hard absorption: v_2 (J/ψ) small but ≠ 0
  - Recombination model: v_2 (J/ψ) high
- Reproduce qualitatively v_2 (J/ψ) value by considering elastic scattering processes

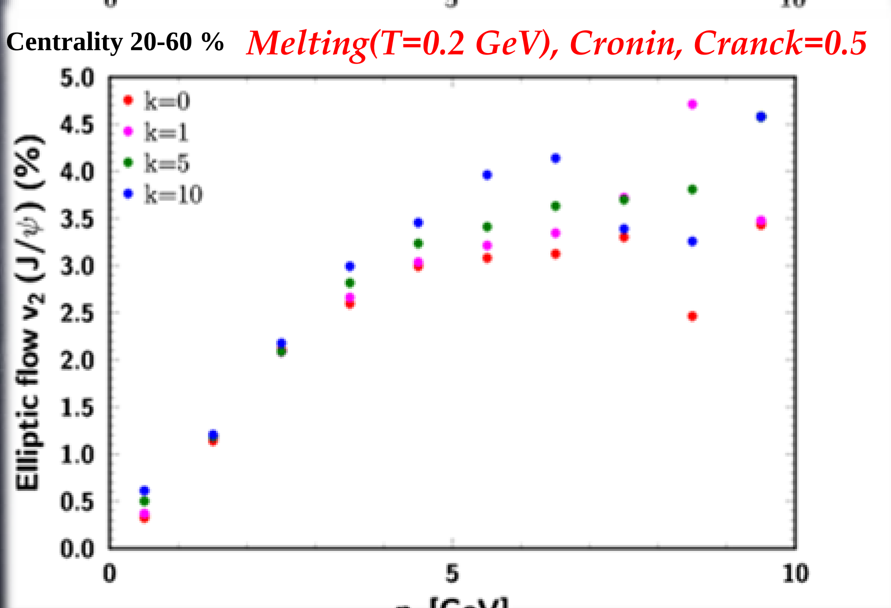
behaviour

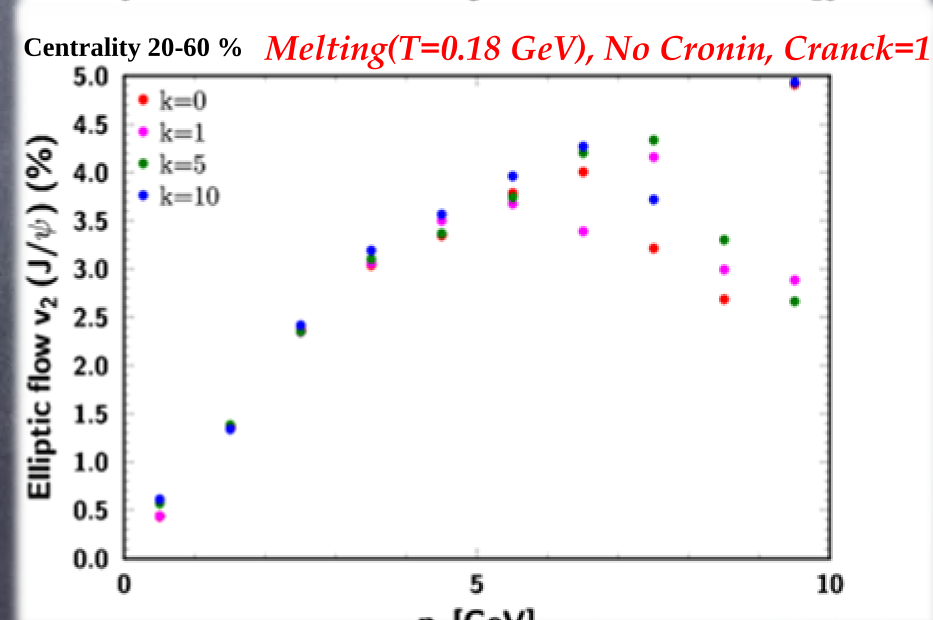
Elliptic flow $v_2(J/\psi)$

RHIC





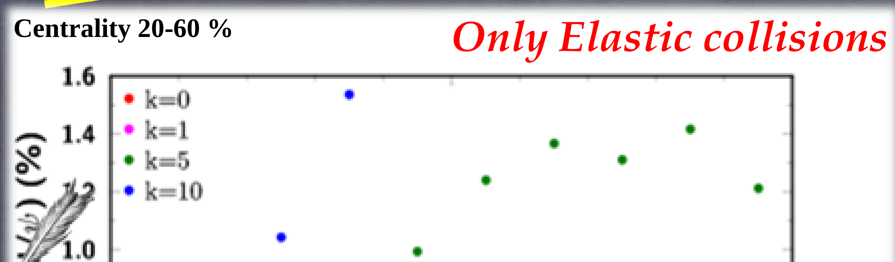


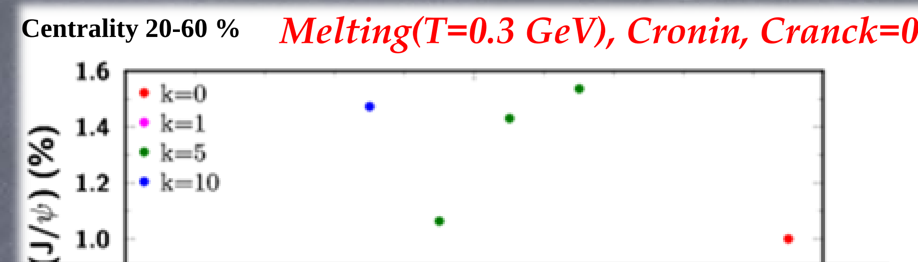


behaviour

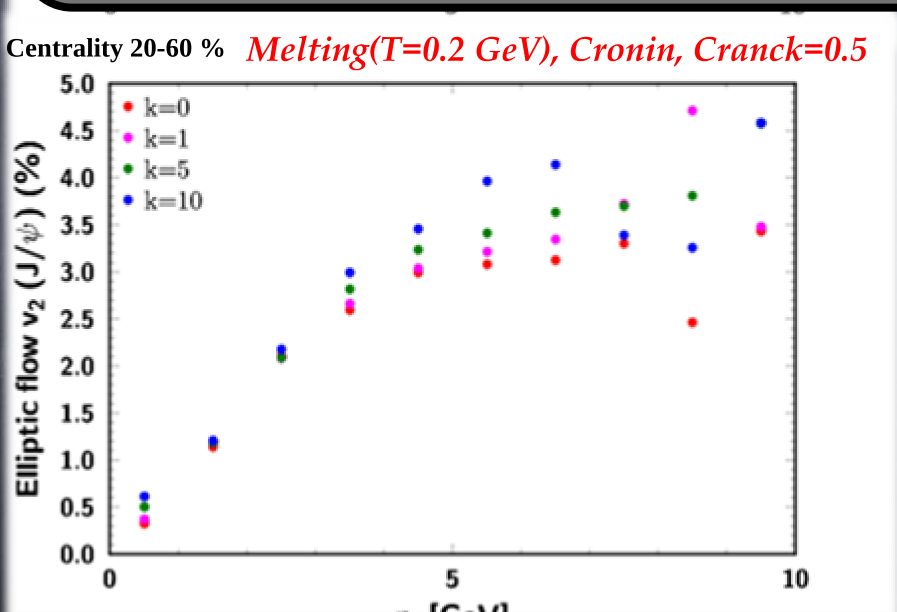
Elliptic flow $v_2(J/\psi)$

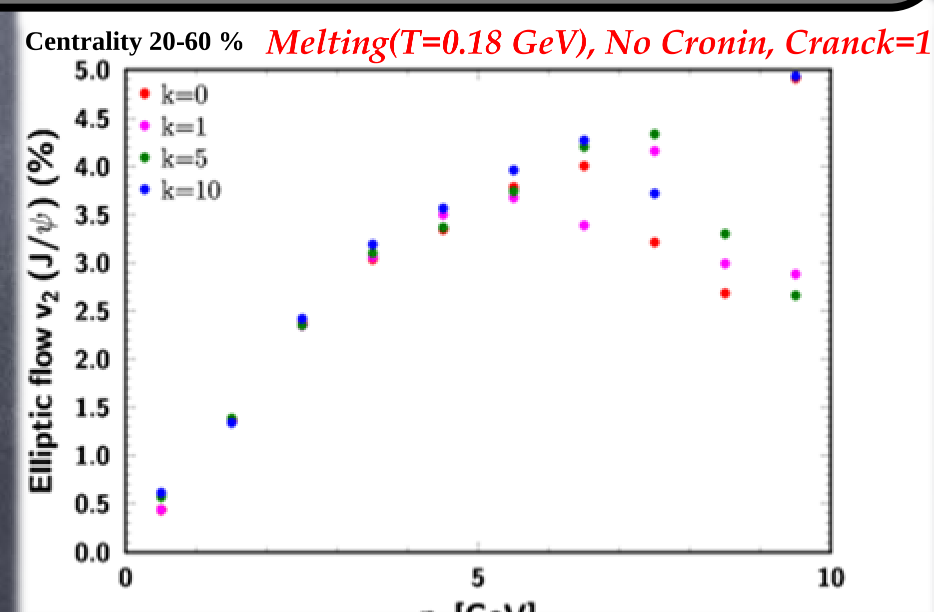
RHIC





Lessons: Our study shows that elastic scattering processes seem to be suitable to describe the collective behaviour of J/ψ in the QGP... But let's wait for final STAR data





Conclusions

- **Project**
  - **Develop a theoretical model to study quarkonia propagation and collectivity**
  - **Highlight the role of elastic scattering processes. These processes were never considered in the literature before**
- **Results**
  - Qualitative and quantitative results on:

Part I: Characterization of the $Q\bar{Q}$ bound state in static hot medium
Binding energy, wave function, r.m.s, T_{diss}, E_{diss}, sequential suppression, ...

Part II: Interaction of the quarkonium with the medium
Bethe-Salpeter structure of $Q\bar{Q}$ vertex,
Elastic and inelastic scattering cross sections interactions in the medium

Part III: Response of the medium to quarkonium propagation
Collisional energy loss and Fokker-Planck coefficients calculations

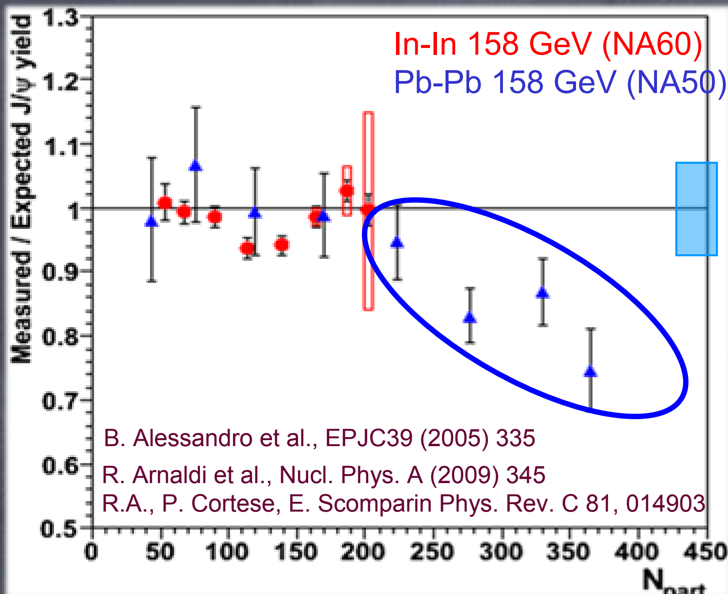
Part IV: Induced phenomena from the quarkonium propagation & collectivity
$Q\bar{Q}$ stochastic propagation in hydrodynamic QGP
Comparison between our model, experimental data and other models

- **Main conclusion**
  - **Elastic processes (forgotten in previous work), should be considered equally with other phenomena studied in the characterization of quarkonia in the QGP, especially in a quantitative analysis**

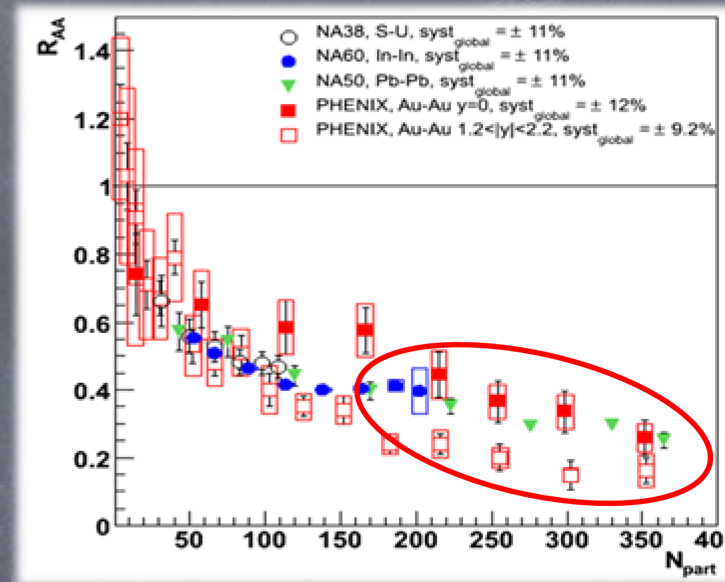
Back up Slides

0. Introduction & Motivations for Elastic Study

o SPS, RHIC...Hunting the QGP



SPS-RHIC



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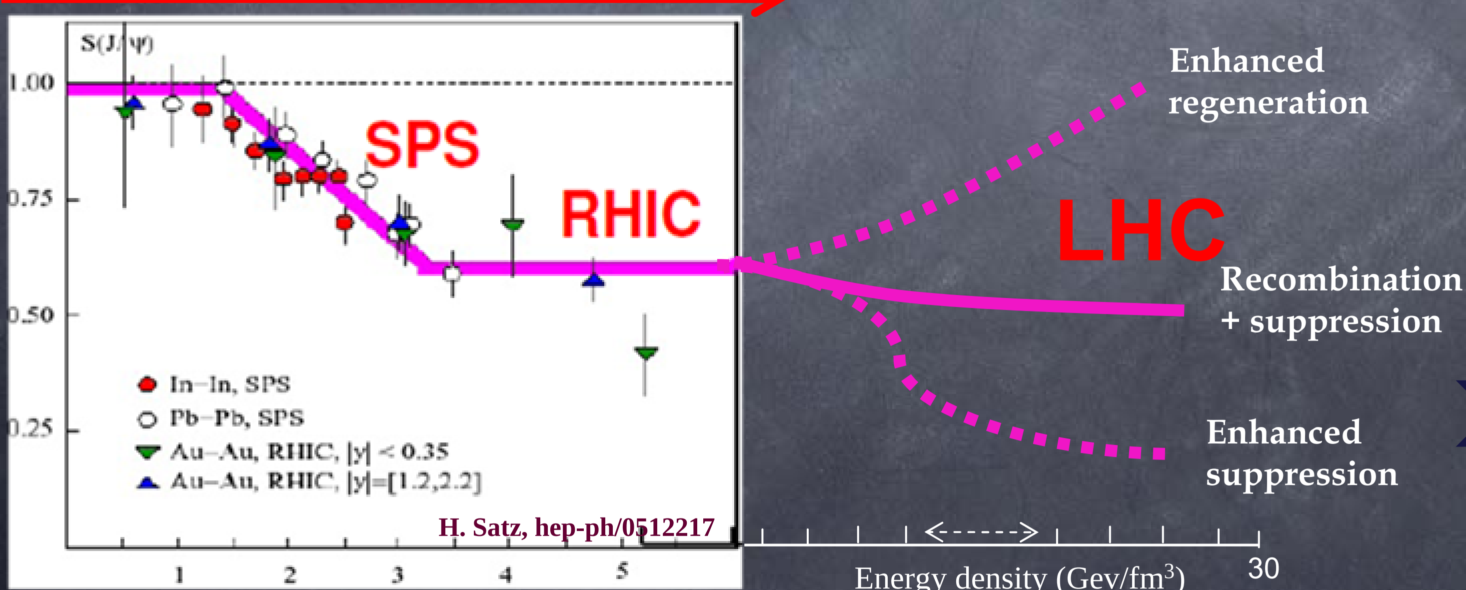

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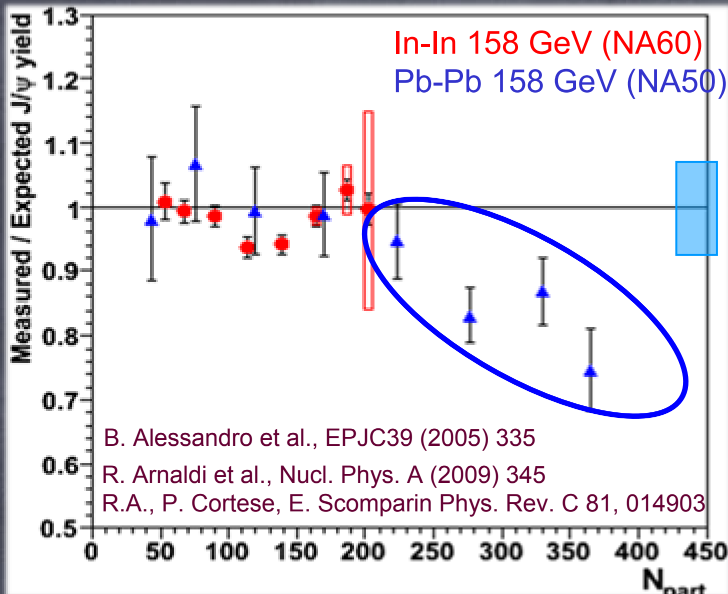
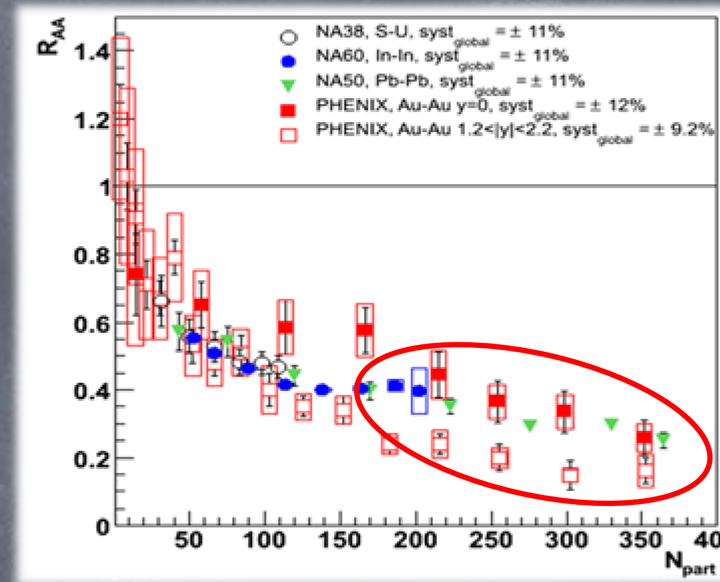
o SPS, RHIC...Hunting the QGP



SPS-RHIC



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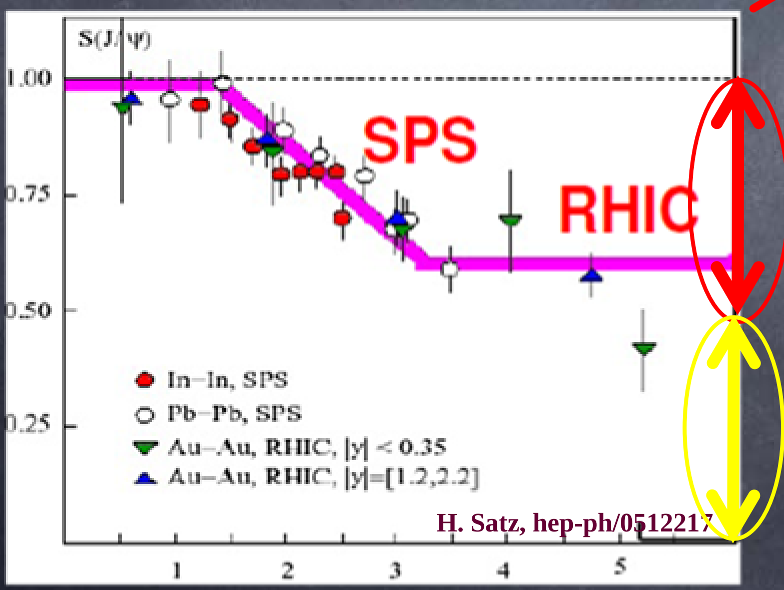
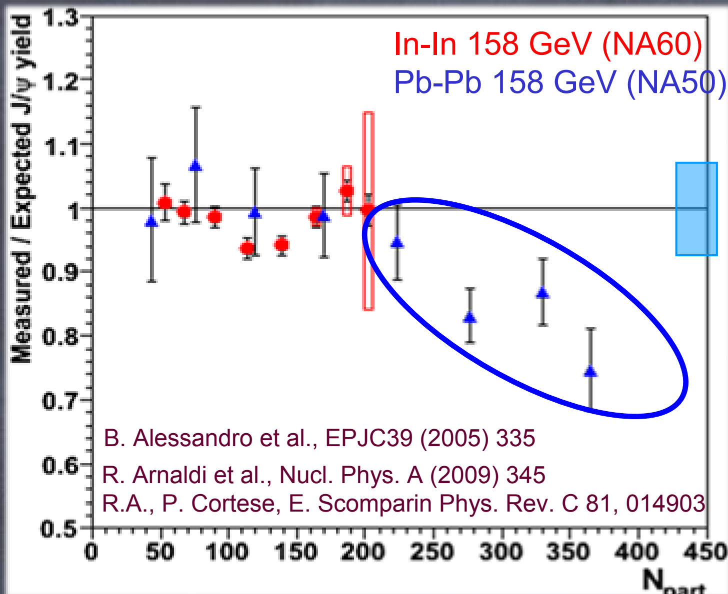


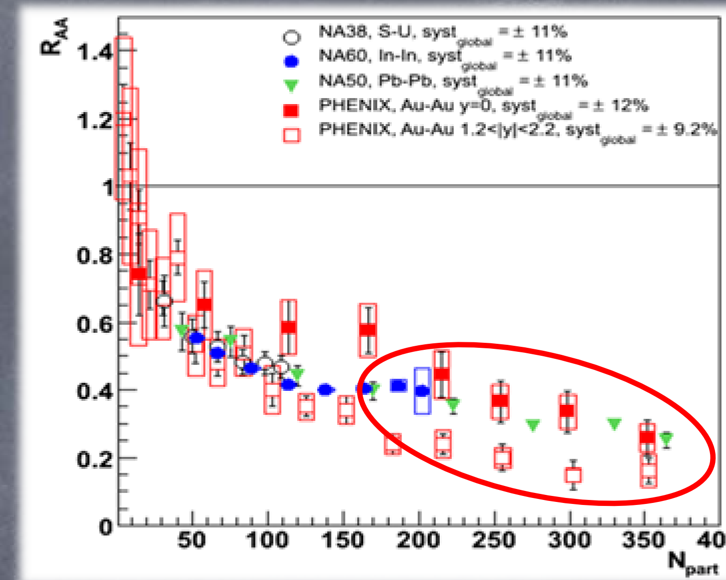
- Suppressed (studied with σ_{inel})
- Focus on remaining J/ψ

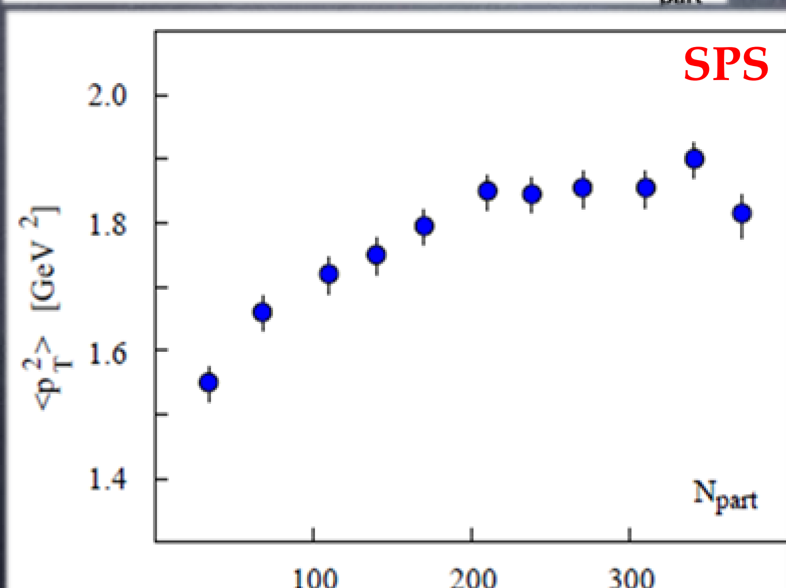
properties modified in the plasma during the scattering J/ψ-hadron, J/ψ-gluons…

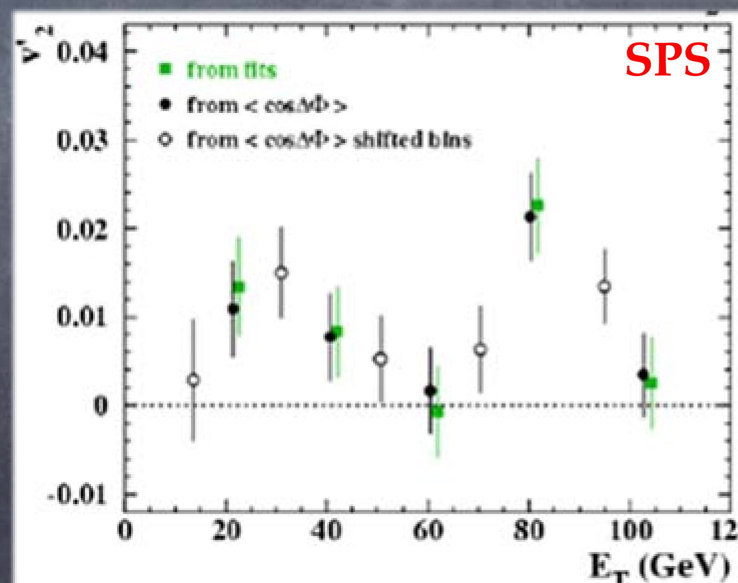
o SPS, RHIC...Hunting the QGP



SPS-RHIC

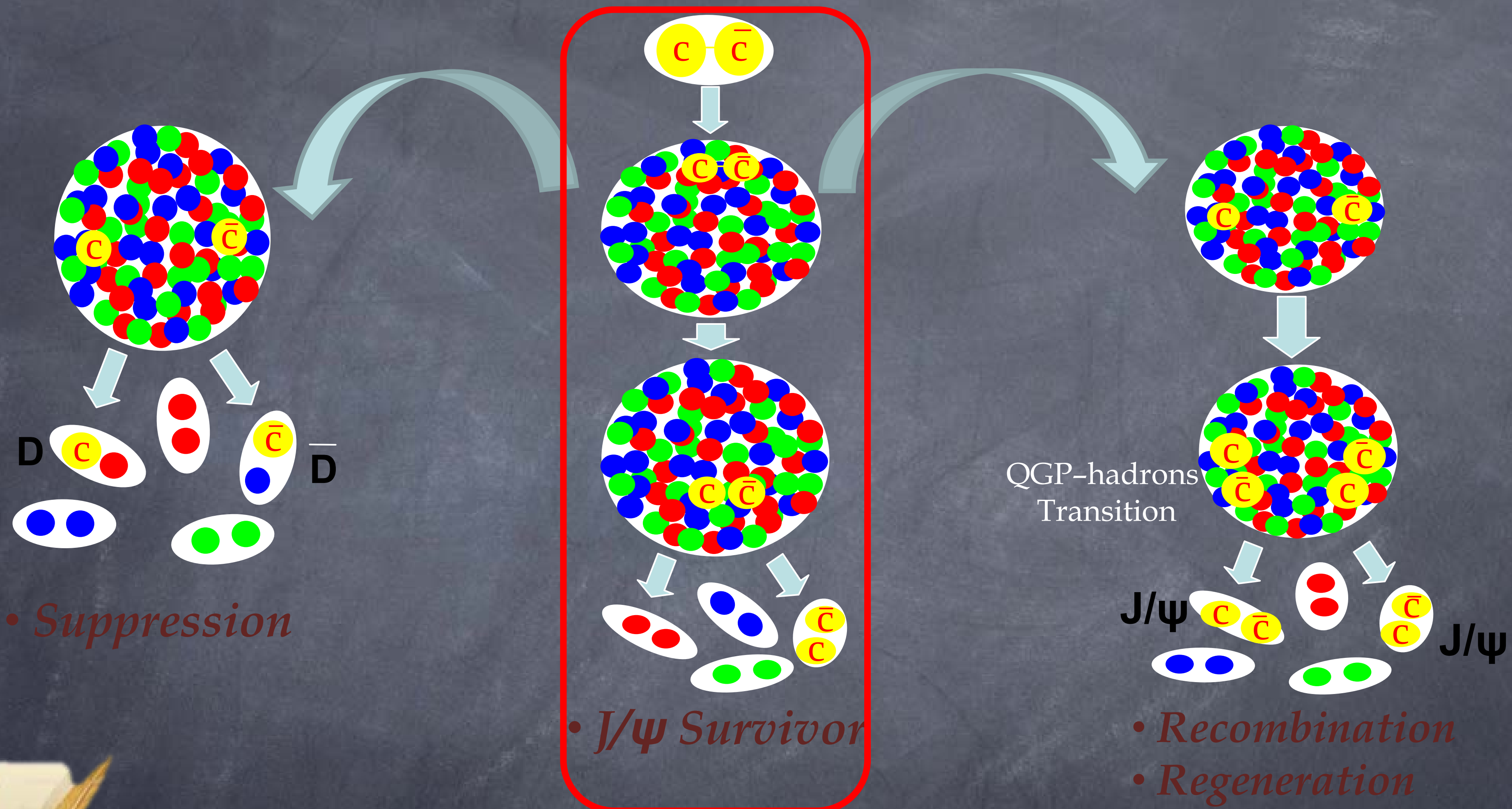






E. Scomparin: International Workshop "Critical Point and Onset of Deconfinement" Firenze, 2006

o ...Hunting the Quarkonia



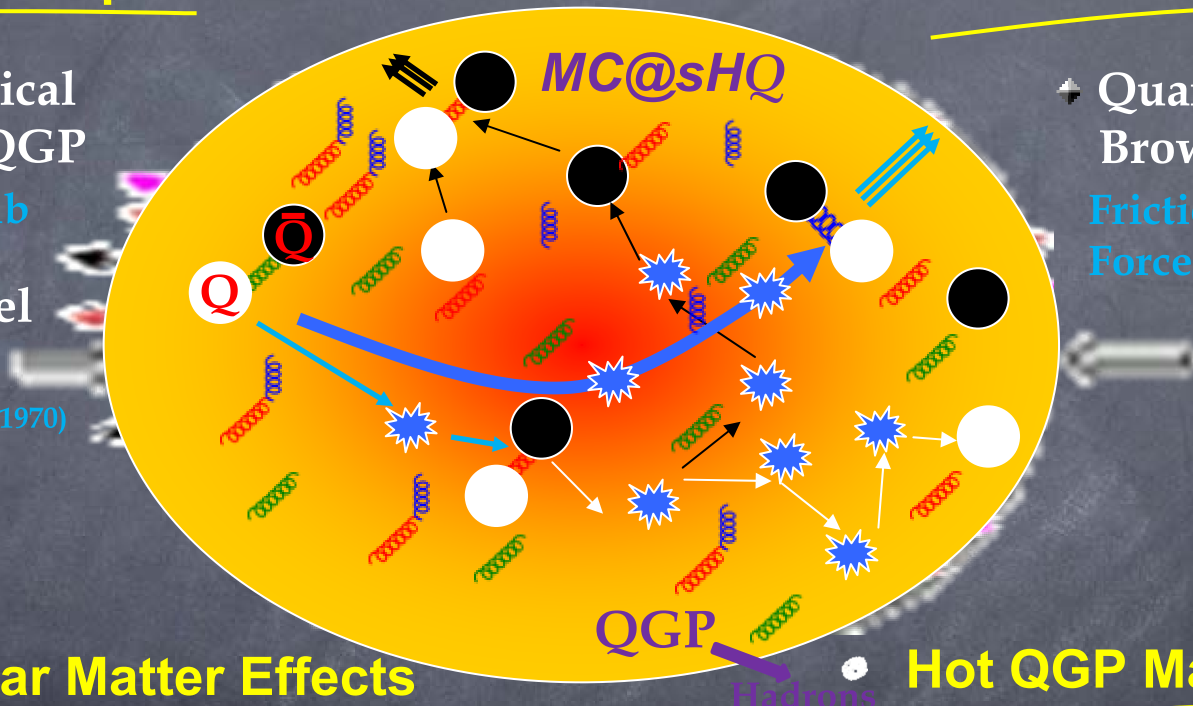
Lessons: - Quarkonia behaviour is a troublemaker probe
- Hunting the QGP → Hunting the quarkonia…

Our Project: - Develop a theoretical model to study the quarkonia propagation & collectivity
- Highlight the role of elastic scattering processes during this propagation

0. Global Project

Toward a Complete Description of J/ψ in QGP and Hadronic Medium

- **Medium Description**
  - Hydrodynamical description of QGP
    U. Heinz & P. Kolb
  - Glauber model initial state
    (Nucl.Phys., B21:135157, 1970)

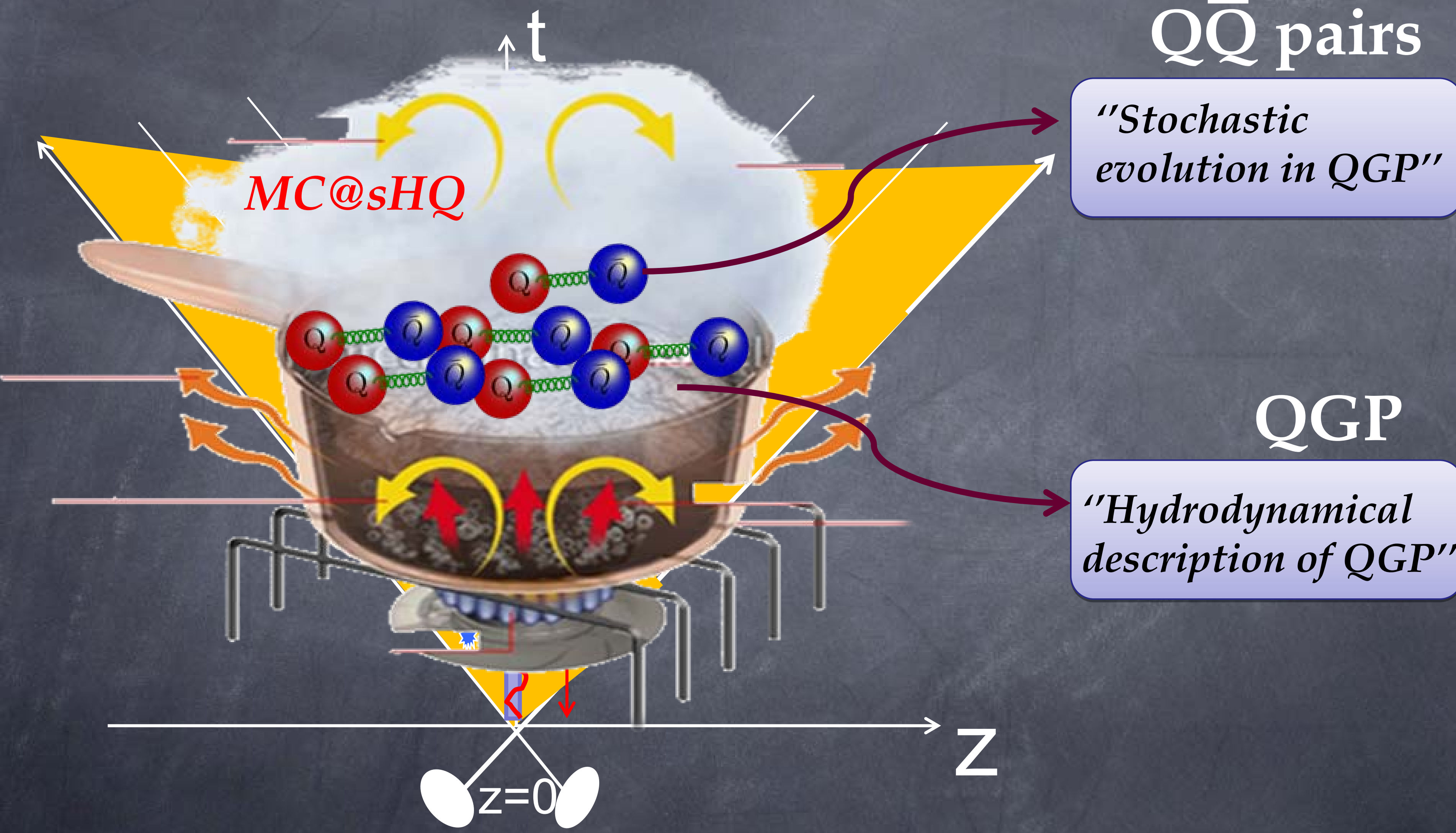


- **Q$\bar{Q}$ Stochastic Evolution**
  - Quarkonia as Brownian particles
    Friction & Stochastic Forces
  - In MC@sHQ:
    …sampling the distributions of Langevin forces

- **Cold Nuclear Matter Effects**
  - 1st J/ψ suppression: **Cronin effect**
    R. Granier De Cassagnac parametrisation
    (QM2006, J.Phys.G, G34:S955958,2007)

- **Hot QGP Matter Effects**
  - Instantaneous melting/thermal excitation
  - Q-$\bar{Q}$ → Quarkonia fusion (recombination)
  - Hard gluon dissociation à la **Bhanot-Peskin**
  - **Elastic scattering & stochastic propagation**
  - …

o *MC@sHQ* in few words



o Hydrodynamical description of the QGP (RHIC)

- **Treatment of quarkonia suppression** (principal ingredient)

The three faces of J/ψ suppression are considered (by cold nuclear matter effects (CNM), by sequential suppression and by inelastic dissociation)

a) Instantaneous melting/thermal excitation

b) No $Q\text{-}\bar{Q} \rightarrow$ Quarkonia fusion

$T > T_{diss}$

a) $Q\text{-}\bar{Q} \rightarrow$ Quarkonia fusion allowed

b) Hard gluon dissociation à la Bhanot-Peskin

$T < T_{diss}$

"Sharp transition"

- **Initial state at RHIC** (other ingredients)

Based on **Glauber** model. It gives the number of c quarks & their distribution

Nucl.Phys., B21:135157, 1970

- **Simulation of plasma phase**

The model used in MC@sHQ is based on U. Heinz and P. Kolb. It uses relativistic hydrodynamics for a perfect fluid, R.C Hwa et X. N Wang : Quark Gluon Plasma 3. 2003

- **Cold nuclear matter effects parametrization**

MC@sHQ use **R. Granier De Cassagnac** for the parametrisation of these effects

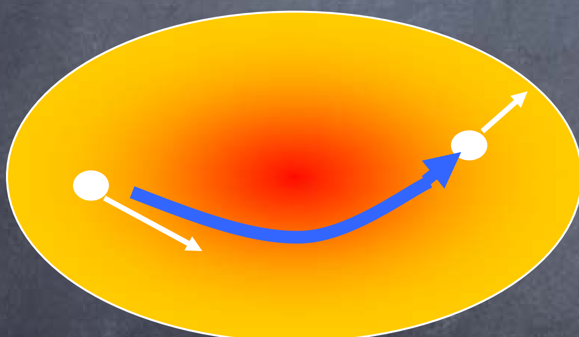
QM2006, J.Phys.G, G34:S955958,2007

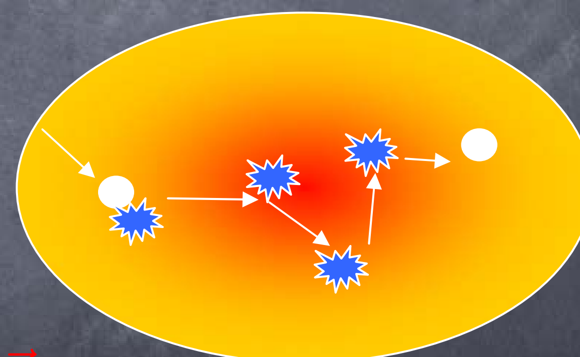
o Stochastic evolution of $Q\bar{Q}$ pairs

- **Quarkonia behaves like Brownian particles**
  - Quarkonia mass □ particles in QGP
  - Quarkonia are rare
  - The high density in QGP implies a mean free path small compared to the size of the quarkonium
    ➡ **Relaxation time □ collision time**

- **Brownian motion**… is the result of two forces which characterise the effect of the QGP on quarkonium

- **Friction Force** (loss of average momentum)
- **Stochastic Force** (diffusion)





- **Langevin equation:** $\frac{d\vec{p}(t)}{dt} = -\vec{A}(\vec{p},T) + \vec{F}_L(t,T)$

$$A_i(\vec{p}(t)) = A(|\vec{p}|(t))\hat{p}_i(t), \quad \overline{F_i(\vec{p}(t))F_j(\vec{p}(t+\tau))} = 2B_{ij}(|\vec{p}|(t))\delta(\tau)$$

- **Quarkonia in MC@sHQ: ...** sampling the distributions of Langevin forces

Hannes Risken: The Fokker-Planck Equation: Methods of Solutions and Applications

I. $Q\bar{Q}$ in a Static Medium at finite Temperature

o Justification of Potential Models

- High mass of **c** and **b** quarks compared to $\boldsymbol{\lambda}_{QCD}$
  - ➡ avoid to deal with the description of QQ by a full and insoluble QFT
- The renormalized mass of **c** and **b** quarks is close to the bare mass and varies slightly depending on the tested scale.
- The binding energies (**ε**) are small compared to the rest mass of **c** and **b** quarks
  - ➡ neglect relativistic and the creation of virtual particles by the Modelisation of (**ε**)

o Our parameterization of QQ Potential: $T = 0$

o Our resolution of Schrödinger equation with *V(r,T)*

$$\left\{ \frac{\partial^2}{\partial r^2} - \frac{l(l+1)}{r^2} + \frac{m_Q}{\hbar^2 c^2}\left(E_{n,l} - V(r,T) - 2m_Q \right) \right\} u_{n,l}(r) = 0$$

with: $R_{n,l}(r,T) = \frac{u_{n,l}(r,T)}{r}$

1)- Find $u_{n,l}(r,T)$ with the corresponding eigenvalue. To procceed :

- Scan, numerically, the values of E_i in an interval around the estimate value
- For each E_i, construct the solution $u^g_{n,l}(r,T)$ from the left and $u^d_{n,l}(r,T)$ from the right
- Propagate these tow solutions until an intermediate commun r_0 (r_0 is taken as:$-\frac{l(l+1)}{r^2} + \frac{m_Q}{\hbar^2 c^2}\left(E_{n,l} - V(r,T) - 2m_Q\right) = 0$

2)- If E_i is an eigenvalue, the solutions $u^g_{n,l}(r,T)$ and $u^d_{n,l}(r,T)$ will connect seamlessly in r_0. This connection is verified if the determinet $\mathcal{D}(E_i)$ is equal to zero

$$\mathcal{D}(E_i) = Det \begin{vmatrix} u^g_{n,l}(r_0,T) & u^d_{n,l}(r_0,T) \\ \frac{\partial u^g_{n,l}(r,T)}{\partial r}\Big|_{r=r_0} & \frac{\partial u^d_{n,l}(r,T)}{\partial r}\Big|_{r=r_0} \end{vmatrix}.$$

3)- To construct the solutions $u^g_{n,l}(r,T)$ and $u^d_{n,l}(r,T)$ step by step of r, one must resolve the second order differential equation on $u^{g,d}_{n,l}(r,T)$. We used the method of **Runge-Kutta** of order 4.

o Our parameterization of QQ Potential (finite T)

$$\mathbf{U(r,T)} = \mathbf{F(r,T)} - \mathbf{T}\left(\frac{\partial \mathbf{F(r,T)}}{\partial \mathbf{T}}\right)$$

- **Weakly bound:** $F(r,T) < V(r,T) < U(r,T)$
  - **Short range** (r < r_{short} = 0.43 fm T_c/T)

    $$V_{short}(r,T) = -\frac{\alpha}{r} + \sigma r + V_{correl}(m_Q, r)$$

    with: $\alpha = \pi/12$, $\sigma = (1.65 - \pi/12)/0.5^2$
  - **Long range** (r > r_{long} = 1.25 fm T_c/T)

    $$V_{long}(r,T) = V_{\infty} - \frac{4}{3}\frac{\alpha_1}{r}\ e^{-\sqrt{4\pi\tilde{\alpha}_1}T\ r}$$

    with: $V_{\infty} = \sigma\ r_{short}$; $\alpha_1, \tilde{\alpha}_1$: fit on lQCD data
  - **Average range** (r_{short} < r < r_{long})

    $$V_{int}(r,T) = \frac{V_{short}(r_{short},T) + g_1(r - r_{short}) + g_2(r - r_{short})^2}{1 + g_3(r - r_{short}) + g_4(r - r_{short})^2}$$

    g_1, g_2, g_3, g_4: must satisfy the junctions conditions

- **Strongly bound:** $V(r,T) = U(r,T)$

  Fit on lQCD data of $U(r, T) \equiv U_{fit}(r,T)$

  $$U_{fit}(r,T) = \left(-\frac{\alpha}{r} + V_{correl}(m_Q, r) + \sigma\ r\right)e^{-(\frac{\mu r}{\hbar c})^2} + U_{fit}^{\infty}(T_{red})\left(1 - e^{-(\frac{\mu r}{\hbar c})^2}\right)$$

  1. Fit on **Kaczmarek-Zantow**, data for $U_{fit}^{\infty}(T_{red})$
  2. Fit on lQCD data of **Kaczmarek-Zantow**, with the form $U_{fit}^{1}(r,T)$. (μ, σ are determined)
  3. Fit again the data with μ, σ obtained in 2). The functional forms for μ, σ are taken as :

  $$\mu(T_{red}) = \sqrt{a_0' + a_2'\ T_{red}^2}$$

  $$\sigma(T_{red}) = \frac{a_0 + a_1 T_{red} + a_2 T_{red}^3}{1 + b_1 T_{red} + b_2 T_{red}^2 + b_3 T_{red}^3}$$

  with a_0'-a_2', a_0-a_2 et b_1-b_3 obtained from the fit.

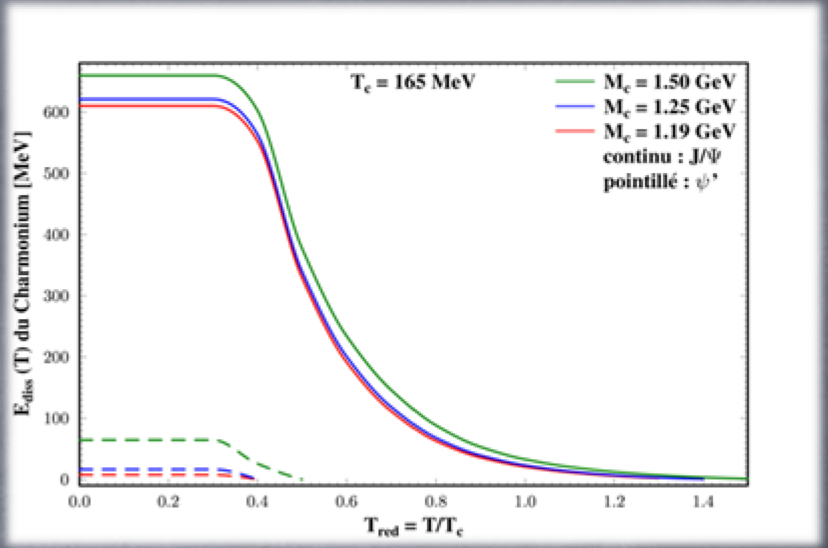
o Charmonium at finite temperature

- **J/ψ energy (E_{diss}) & Temperature (T_{diss}) dissociation**

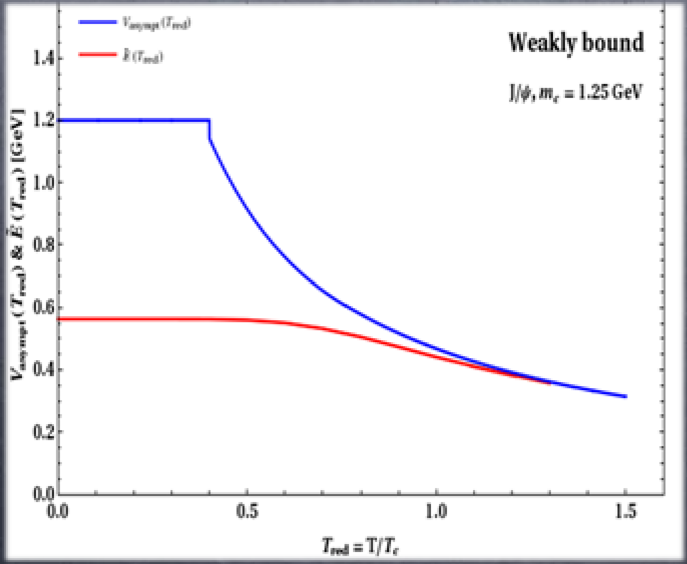
$$E_{diss}(T) = V(r \to \infty, T) + 2m_c - E_{nl}$$

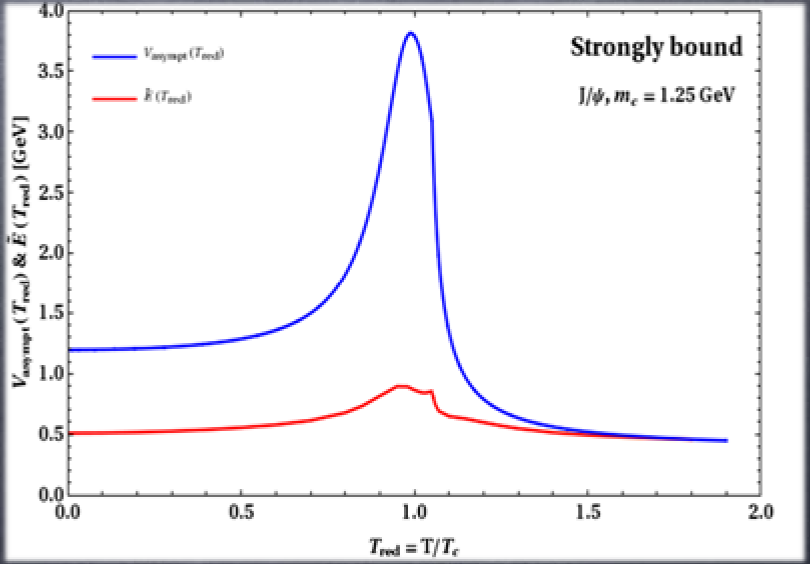
T_{diss} : Temperature for E_{diss} (T) = 0

Quarkonium	m_c	État lié	T_{diss} (faiblement lié)	T_{diss} (fortement lié)
Charmonium	1.25 GeV	$n=1, l=0$ (J/Ψ)	1.45 T_c	1.85 T_c
		$n=2, l=0$ (ψ')	0.40 T_c	1.10 T_c
		$n=1, l=1$ (χ_c)	0.48 T_c	1.20 T_c



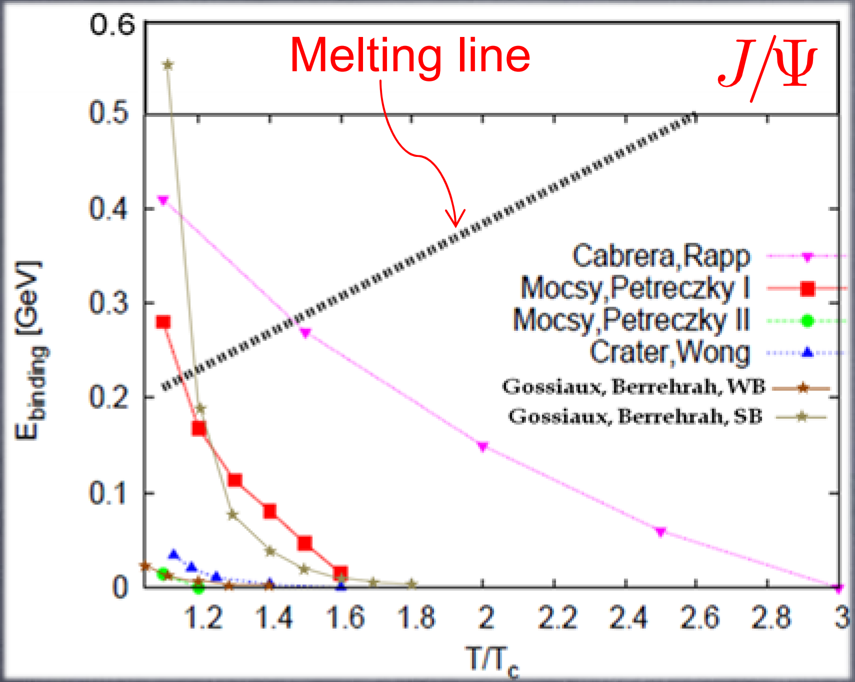
- **V∞ (T) & E(T)**

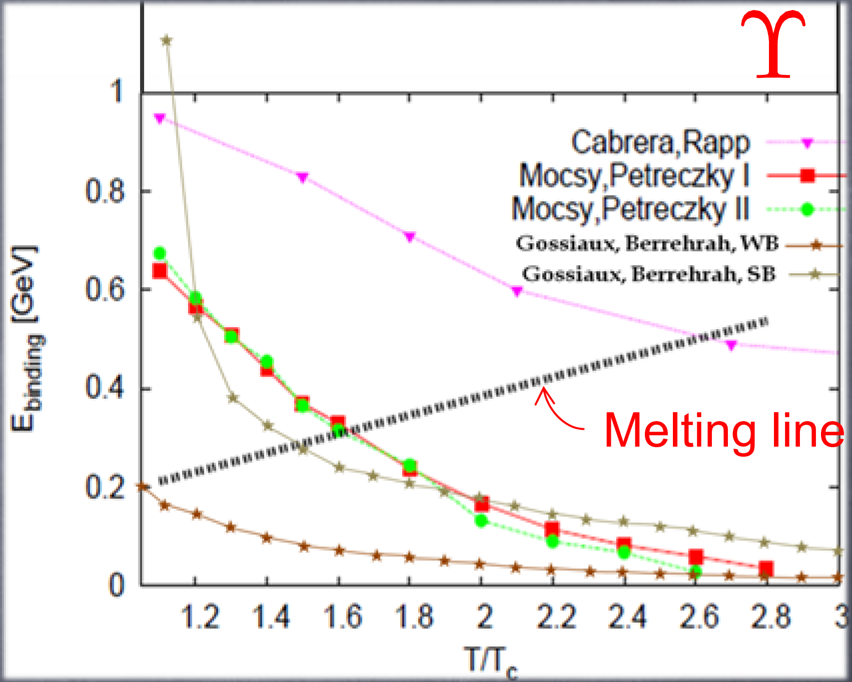




o $c\bar{c}$ and $b\bar{b}$: WB, SB and literature review

- Binding energy ($E_{binding}$)





- Dissociation Temperature (T_{diss})

Modèle	T_{diss}/T_c							
	J/Ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
Potentiel faiblement liant	1.45	0.48	0.4	3.55	0.95	0.8	0.55	0.5
Potentiel fortement liant	1.85	1.20	1.10	4.45	1.65	1.45	1.18	1.2
DIGAL et al, II.[22]	1.1	0.74	0.1-0.2	2.31	1.13	1.1	0.83	0.75
ALBERICO et al, II.[23]	1.78-1.92	1.14-1.15	1.11-1.12	≥ 4.4	1.6-1.65	1.4-1.5	~ 1.2	~ 1.2
WONG, II.[24]	~ 1.42	~ 1.05	-	~ 3.3	~ 1.22	~ 1.18	-	-
SATZ, II.[1]	2.1	1.16	1.12	> 4.0	1.76	1.60	1.19	1.17

II. $Q\bar{Q}$ – Partons/Hadrons Elastic & Inelastic Scattering Process

o σ_{elas}, σ_{inel} calculation: *How?*

Processus de diffusion	Méthode/Modèle	Processus étudié	Ref.
Hadron dissociation	pQCD : short-distance	$(J/\Psi, \Upsilon) + N$	III.[7, 90]
		$J/\Psi + \pi$	III.[7]
		$J/\Psi + \pi$	III.[27]
	pQCD : color dipole	$J/\Psi + N$	III.[6–8, 52, 90]
		$J/\Psi + \pi, N$	III.[6–8, 11, 52]
		$(J/\Psi, \psi') + N$	III.[24, 26, 52]
	pQCD : Bethe-Salpeter	$J/\Psi + N$	III.[24, 30]
	Light-cone dipole	$J/\Psi + N$	III.[26]
	échange de méson	$J/\Psi + N$	III.[29, 36, 47, 52, 97]
		$J/\Psi + \pi$	III.[29, 31–34, 91]
		$J/\Psi + \pi, \rho$	III. [32, 40, 92–94, 101, 102]
		$J/\Psi + K$	III.[103]
		$J/\Psi + \pi, K, \rho, N$	III.[32, 37, 95, 96, 122]
		$J/\Psi + \pi, K, \rho, \eta, \omega, \phi, K^*$	III.[122]
	échange de quark	$J/\Psi + \pi$	III.[39, 40, 91]
		$J/\Psi + \pi, \rho$, $\psi' + \pi, \rho$	III.[52, 98, 100]
		$(J/\Psi, \psi', \chi_c) + (\pi, \rho, K)$	III.[123, 124]
		$J/\Psi + \pi, N$, $\psi' + \pi, N$	III.[104]
		$J/\Psi + \rho$	III.[40]
	QCD sum rules	$J/\Psi + N, \pi$	III.[52, 109, 110]
		$J/\Psi + h$	III.[27, 56, 59]
	MQ	$J/\Psi + N$	III.[105–107]
		diffusion multiple J/Ψ-N	III.[105–107]
Gluon dissociation	pQCD	$J/\Psi + g \rightarrow c\bar{c}$	III. [5–7, 24, 29, 55, 59, 61–63, 90]
		$J/\Psi + g \rightleftharpoons c\bar{c}g$	III.[80, 81]
		$J/\Psi + g \rightarrow c\bar{c}g$ (quasifree)	III.[53, 54]
	pQCD : Bethe-Salpeter	$J/\Psi + g \rightarrow c\bar{c}g$	III.[23, 24, 30, 87]
Diffusion élastique avec des **Hadron**	pQCD	$J/\Psi + \pi$	III.[7, 90]
		$J/\Psi + N$	III.[111–119]
	échange de méson	$J/\Psi + N$	III.[95, 122]
		$J/\Psi + \pi, \omega$	III.[95, 122]
		$J/\Psi + \pi, K, \rho, \eta, \omega, \phi$	III.[122]
	MQ	$J/\Psi + p \rightarrow J/\Psi + p$	III.[105–107]
		diffusion multiple J/Ψ-N	III.[105–107]
Diffusion élastique avec des **Gluons**	pQCD : OPE	$J/\Psi + g \rightarrow J/\Psi + g$	III. [7, 90]
	pQCD : Bethe-Salpeter	$J/\Psi + g \rightarrow J/\Psi + g$	III.[87, 88]

o σ_{elas} Results & discussion

λ : gluon wavelength
Q : gluon energy

σ_{elas} (Φ-gluon)

$\epsilon_0 \approx mg^4$

- $\lambda \gg a_0$ (Bohr radius)
  $\rightarrow Q \ll \epsilon_0$ (binding energy)

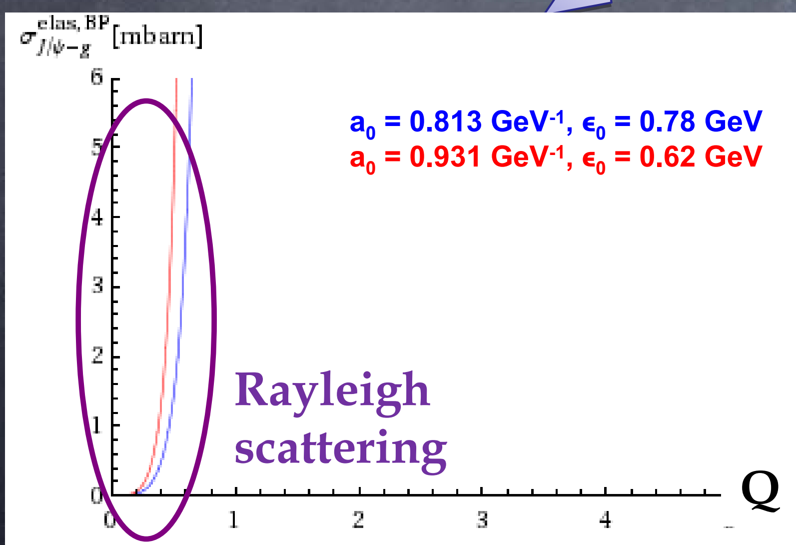
Low energy

Bhanot-Peskin Formalism

- $\lambda \approx a_0$ or $\lambda \ll a_0$ (Bohr radius)
  $\rightarrow Q \approx \epsilon_0$ or $Q \gg \epsilon_0$ (binding energy)

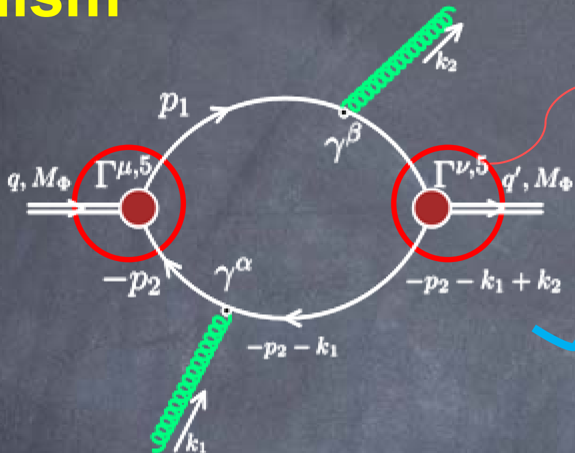
High & intermediate energy

Bethe-Salpeter Formalism



II. $Q\bar{Q}$ – Parton/Hadron Elastic & Inelastic Scattering Processes

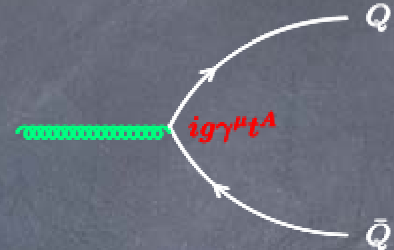
- **Bethe-Salpeter formalism**



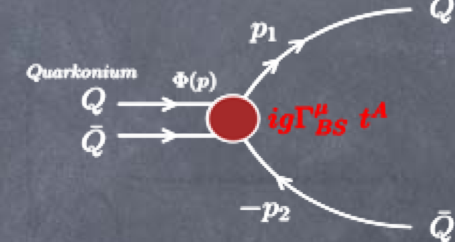
- **Goal: Bethe-Salpeter vertex**

Bethe-Salpeter amplitude (vertex)

→ *Related to $Q\bar{Q}$ wave function*



$\Longrightarrow$

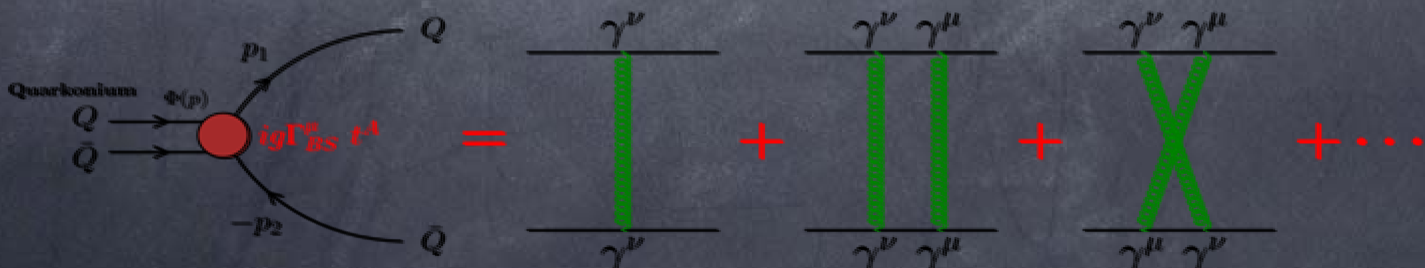


E. E. Salpeter et H. A. Bethe
Phys. Rev., 84:1232_1242, 1951

- **Bethe-Salpeter vertex equation**

$$\mathcal{M} = \mathcal{V} + \int \mathcal{V}\mathcal{G}\mathcal{V} + \int\int \mathcal{V}\mathcal{G}\mathcal{V}\mathcal{G}\mathcal{V} + \cdots + \left(\int \mathcal{V}\mathcal{G}\right)^n + \cdots = \frac{\mathcal{V}}{1-\int \mathcal{V}\mathcal{G}}$$

$\mathcal{V}$: kernel, $\mathcal{M}$: amplitude, $\mathcal{G}$: propagator

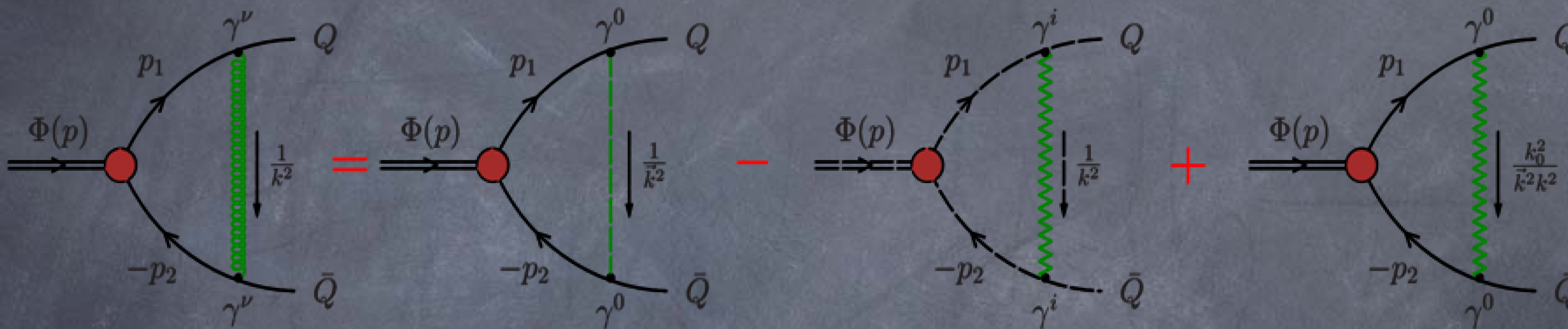


Bound states: produce a pole in $\mathcal{M} \Rightarrow \mathcal{M}$ eigenvector Γ satisfies: $\Gamma = \int_k \mathcal{V}(p,k,P)\mathcal{G}(k,P)\Gamma(k,P)$

Bethe-Salpeter formalism

Goal: Bethe-Salpeter vertex

$$\frac{\gamma_1^{\nu}\gamma_2^{\nu}}{\mathrm{k}^2} = \frac{\gamma_1^{0}\gamma_2^{0} - \vec{\gamma}_1\vec{\gamma}_2}{\mathrm{k}^2} = -\frac{\gamma_1^{0}\gamma_2^{0}}{\vec{k}^2} + \left(-\frac{\vec{\gamma}_1\vec{\gamma}_2}{k^2} + \frac{\gamma_1^{0}\gamma_2^{0}k_0^2}{k^2\vec{k}^2} \right)$$



Instontanous Interaction (dominants)

hyperfine Effects (spin-spin, spin-orbit...)

Retarded Interaction

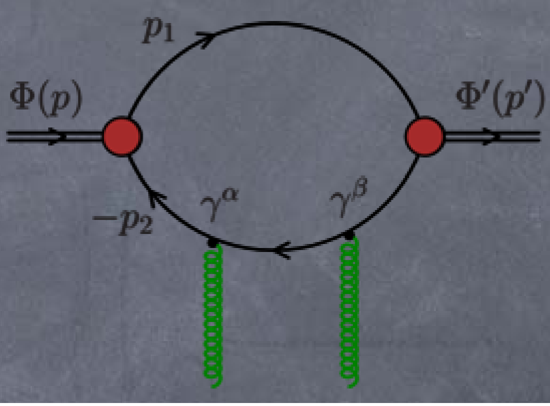
Bethe Salpeter Vertex =

Terms at $O(m\,\alpha^2)$ order Dominant Term in the vertex $\Gamma_I(E,\vec{p}),\ \Gamma_I^{NR}(E,\vec{p})$	Terms at $O(m^2\,\alpha^4)$ order ~ 102 MeV for J/ψ → ψ ' (ε_0 = 0.78 GeV, m_c = 1.94 GeV)	Terms at $O(m^2\,\alpha^3)$ order ~ few MeV for J/ψ → ψ ' (ε_0 = 0.78 GeV, m_c = 1.94 GeV)

o σ_{elas} Results & discussion

❑ Compton diffusion process J/ψ-gluon

- **2 gluons exchanged, "LO"**



➡ **6 diagrams** (bb∥, bbX, tt ∥, ttX, tb, bt)

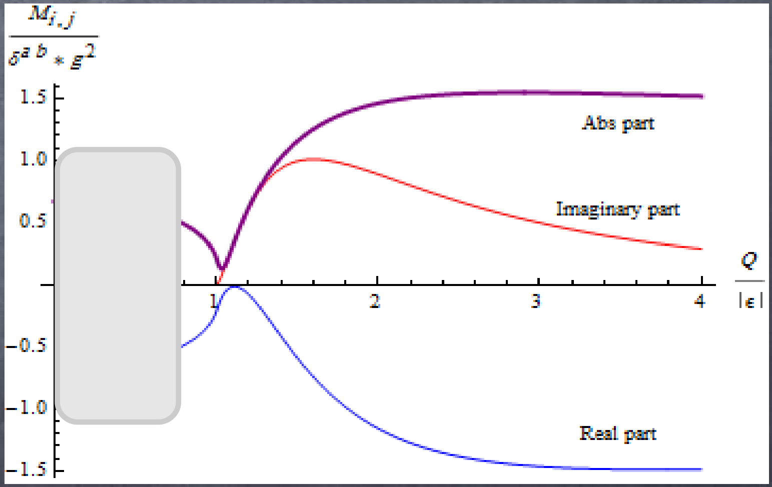
- **"LO" Amplitude**

❑ Soft Gluons ($Q \approx m\, g^4$) o Coulombic case

$$\mathcal{M}(Q \approx mg^4) \approx -2\alpha g^2 \frac{\delta^{ab}}{2N_c}\, \epsilon_{\lambda 1}(k_1).\epsilon_{\lambda 2}(k_2)$$

✓ Opening of the imaginary part for Q > |ε|

✓ Opening of the inelastic channel for Q = |ε|



❑ Hard Gluons ($Q \approx m\, g^2$)

$$\mathcal{M}(Q \equiv mg^2) \approx -\frac{4g^2\delta_{ij}\delta^{ab}}{N_c} \times \frac{1}{\left(1+\left(\frac{a_0|\mathbf{k_1}-\mathbf{k_2}|}{4}\right)^2\right)^2}$$

Form Factor

o σ_{elas} Results & discussion

λ : gluon wavelength
Q : gluon energy

σ_{elas} (Φ-gluon)

- $\lambda \gg a_0$ (Bohr radius)
  $\rightarrow Q \ll \epsilon_0$ (binding energy)

$\epsilon_0 \approx mg^4$

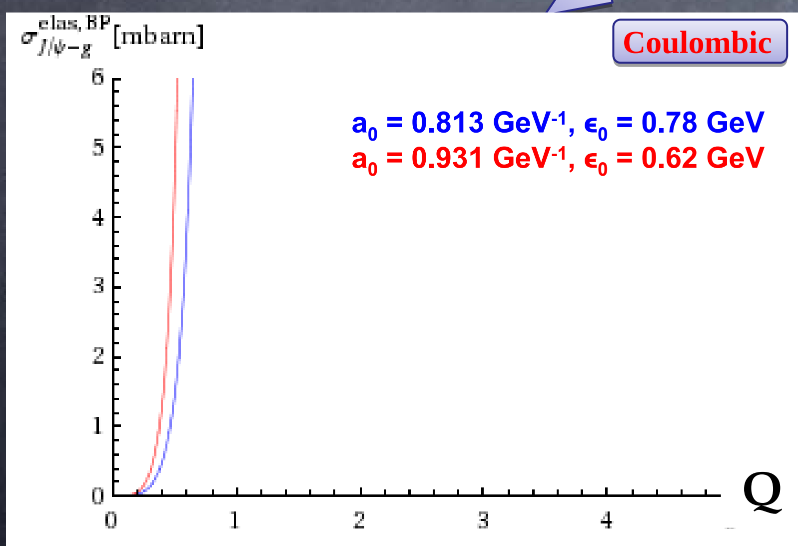
- $\lambda \approx a_0$ or $\lambda \ll a_0$ (Bohr radius)
  $\rightarrow Q \approx \epsilon_0$ or $Q \gg \epsilon_0$ (binding energy)

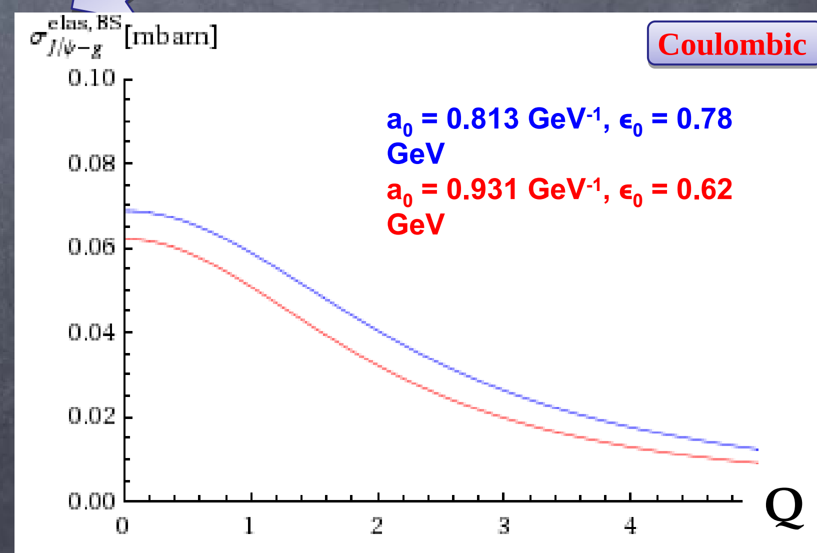
Low energy

Bhanot-Peskin Formalism

High & intermediate energy

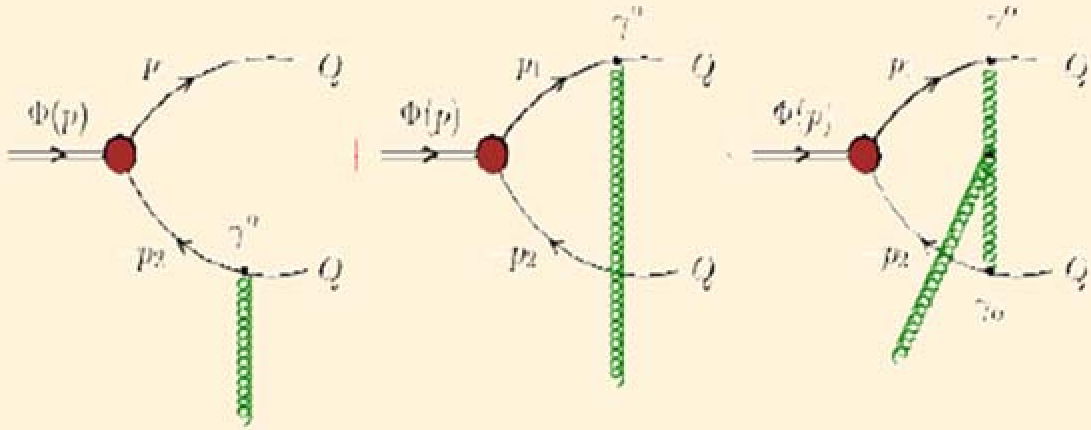
Bethe-Salpeter Formalism

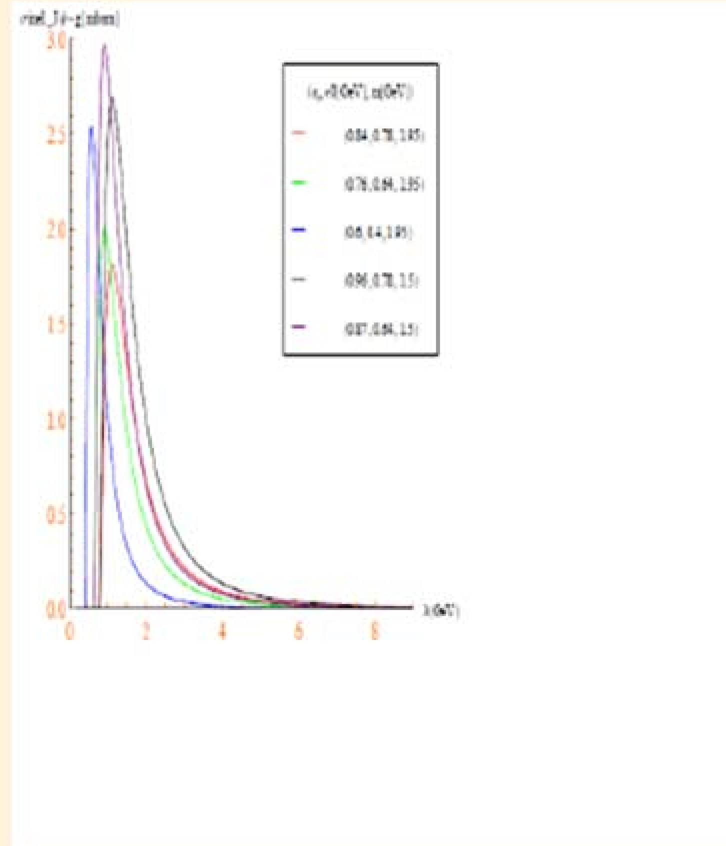




σ_{inel} *Results & Discussion*

- *Gluon Dissociation Process* $J/\psi - g$





$$|\mathcal{M}|^2 = \frac{4g^2 m^2 M_\phi k_0^2}{3N_c} |\nabla \psi(\vec{p})|^2$$

$$\Downarrow$$

$$\sigma_{\phi g}(\lambda) = \frac{128 g^2}{3N_c} a_0^2 \frac{(\lambda/\varepsilon_0 - 1)^{3/2}}{(\lambda/\varepsilon_0)^5}$$

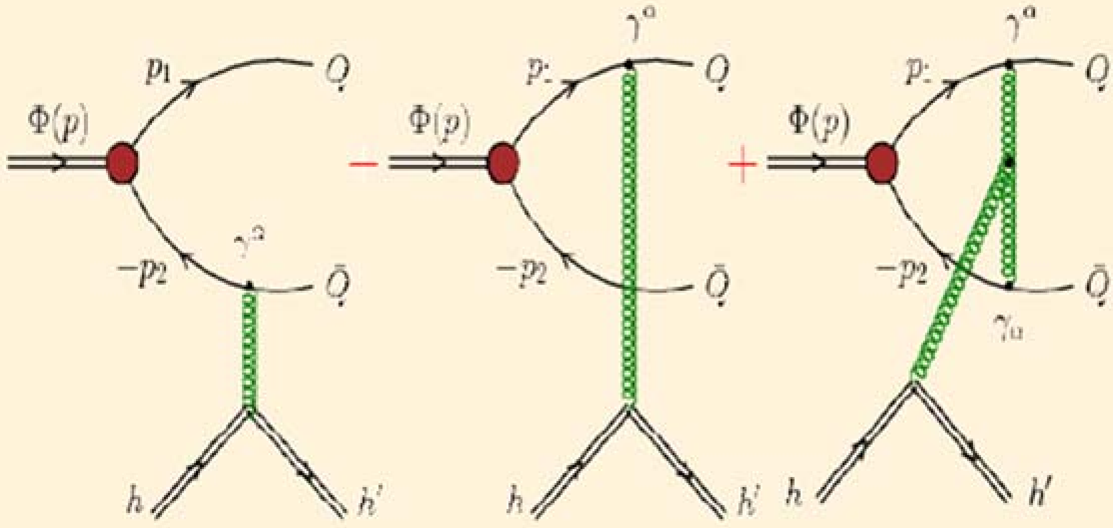
with : $\lambda = \frac{q.k}{M_\phi} = \frac{s - M_\phi^2}{2M_\phi}$

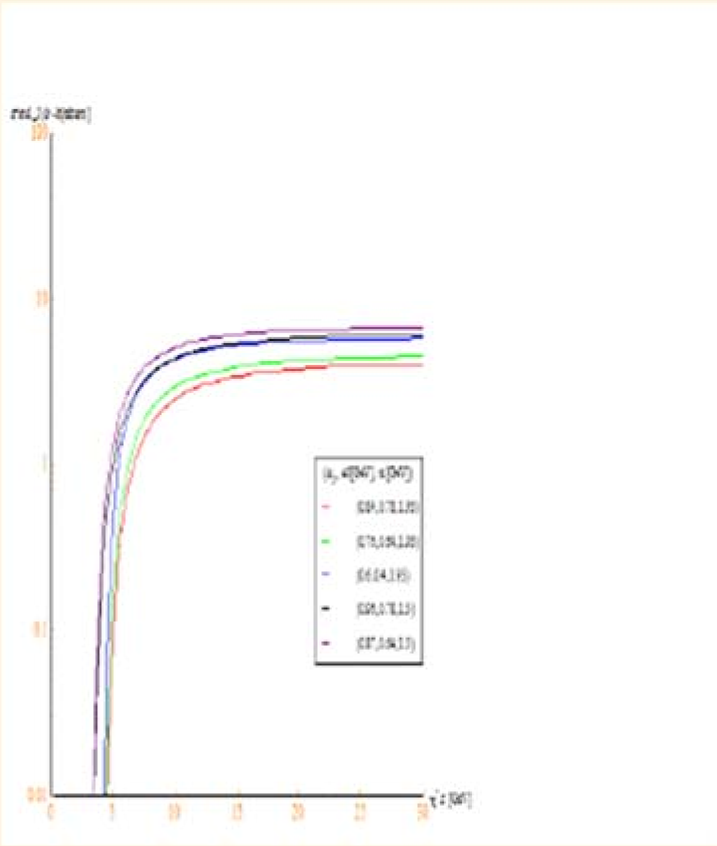
- *Dependence* σ_{inel} *vs* ε *et* m

Oh, S. Kim, S. Houng Lee,(02)

σ_{inel} *Results & Discussion*

❷ *Hadron Dissociation Process* $J/\psi - h$





Factorization Theorem

$$\sigma_{\phi h}(\nu) = \int_0^1 dx\, \sigma_{\phi g}(x\nu) \times g(x)$$

- *Cross section of* $J/\psi - h$
- *Cross section of* $J/\psi - g$
- *GDF* : $g(x) = 0.5(\eta+1)\dfrac{(1+x)^\eta}{x}$, $\eta = 5(BP)$

- *Dependence* σ_{inel} *vs* ε *et* m

Oh, S. Kim, S. Houng Lee,(02)

III. Fokker-Planck Coefficients Calculations

o **Energy losses**, given by Bjorken ($\varphi(M, E, p) \rightarrow$ "i"(m, e, q)) **Collisionel-Coulombic**

$$\frac{dE}{d\tau} = \frac{dE}{dt} \times \frac{E}{M} = \sum_i \int d^3q\, n_i(\vec{q})\, \frac{\sqrt{(p.q)^2 - M^2m^2}}{Ee} \int dt\, \frac{d\sigma_{elas}}{dt}\, (E' - E)$$

- $\hat{Q}(s) = \int \frac{d\sigma_{elas}}{dt}\, t\, dt$ $\propto$ Transport coefficient
- $(E' - E) = \frac{t}{2M}\left(\frac{E_{cell}}{M} + \left(1 + \frac{q}{M}\right)\frac{\vec{p}_{cell}.\vec{q}}{||\vec{q}||^2}\right)$ Energy loss term

o **Drag coefficient**, for ($\varphi(M, E, p) \rightarrow$ "i"(m, e, q)) **Collisionel-Coulombic**

$$A_i = \frac{M}{E} \sum_i \int d^3q\, n_i(\vec{q})\, \frac{\sqrt{(p.q)^2 - M^2m^2}}{Ee} \int dt\, \frac{d\sigma_{elas}}{dt}\, \frac{\left\langle (\vec{P} - \vec{P}').\ \vec{P} \right\rangle}{||\vec{P}||}$$

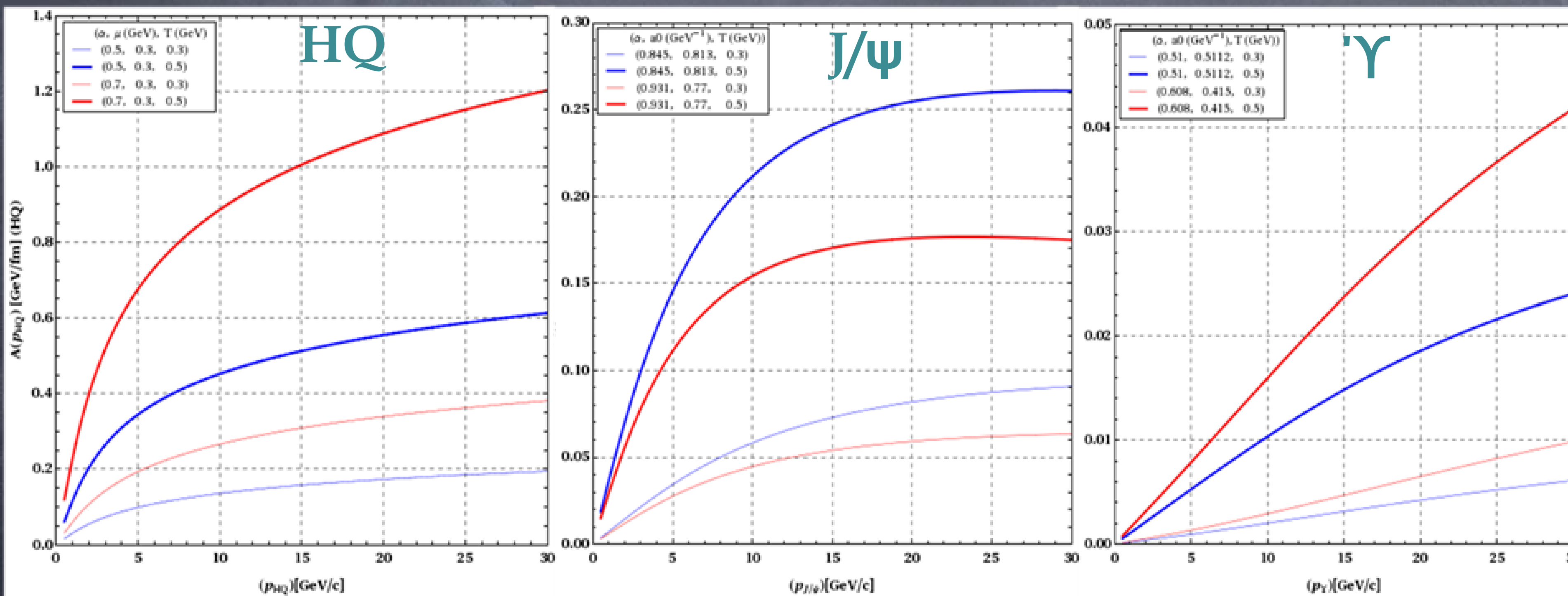
- $\hat{Q}(s) = \int \frac{d\sigma_{elas}}{dt}\, t\, dt$ $\propto$ Transport coefficient
- $\frac{\left\langle (\vec{P} - \vec{P}').\ \vec{P} \right\rangle}{||\vec{P}||} = \frac{t}{2P}\left(-1 + \frac{E}{M}\left(\frac{E_{cell}}{M} + \left(1 + \frac{q}{M}\right)\frac{\vec{p}_{cell}.\vec{q}}{q^2}\right)\right)$

III. Fokker-Planck Coefficients Calculations

o **Drag coefficient**, for (Φ(M, E, p) → "i"(m, e, q))

Collisional-Coulombic

$$A_i = \frac{M}{E}\sum_i \int d^3q\; n_i(\vec{q})\; \frac{\sqrt{(p.q)^2 - M^2m^2}}{Ee} \int dt\; \frac{d\sigma_{elas}}{dt}\; \frac{\left\langle (\vec{P}-\vec{P}').\;\vec{P}\right\rangle}{\|\vec{P}\|}$$



- Same behaviour for HQ, J/Ψ, Υ
- At large p, $A_i \sim dE/dt$
- $A(HQ) > A(J/\Psi) > A(\Upsilon)$
- For (HQ, J/Ψ, Υ): $A_i \nearrow$ with T $\nearrow$

III. Fokker-Planck Coefficients Calculations

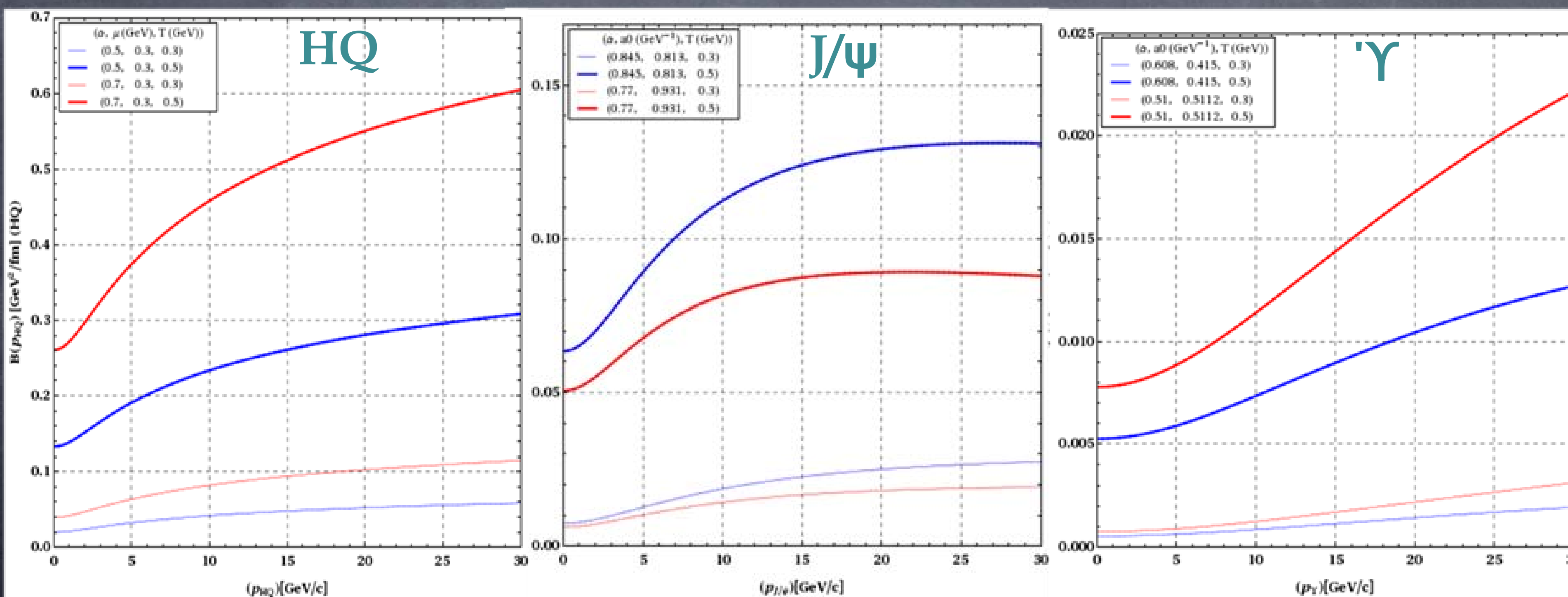
o **Diffusion coefficient**, for (φ(M, E, p) → "i"(m, e, q))

Collisionel-Coulombic

D.B. Walton and J. Rafelski. Phys, Rev Lett, 84(1):3134, 2000

$$B(E) = \int_E^{+\infty} dE' \, A_i(E') \times \frac{E'}{P'} \, e^{-(E'-E)/T}, \text{ with: } B_\perp = B_\parallel = B, \quad B \leftrightarrow A \text{ relation}$$

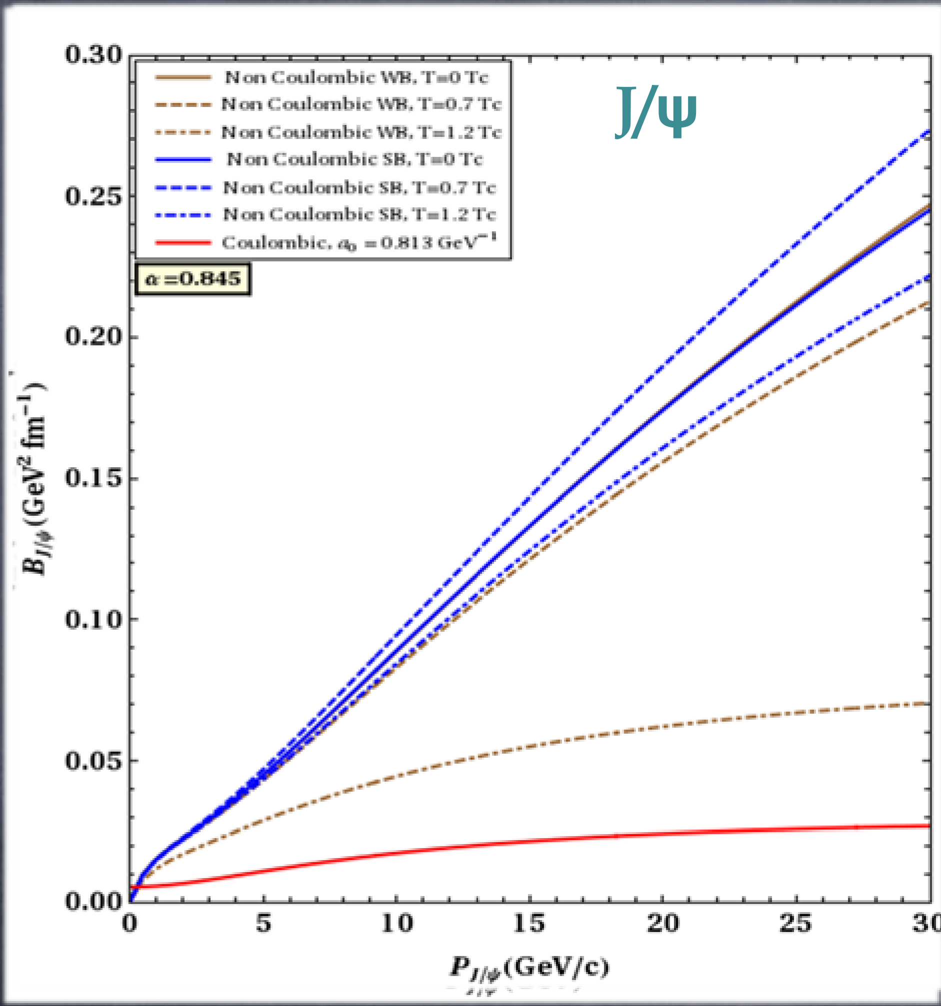
- Fokker-Planck equation: $\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(A_i f + \frac{\partial}{\partial p_i} B_{ij} f\right) = -\vec{\nabla}_p . \, \vec{\rho}$, (homogenous background)
- Einstein relation: $\left[\vec{A} f + \vec{\nabla}_p (B f)\right]_i = 0, \; f = e^{-E/T}$, (stationary case)

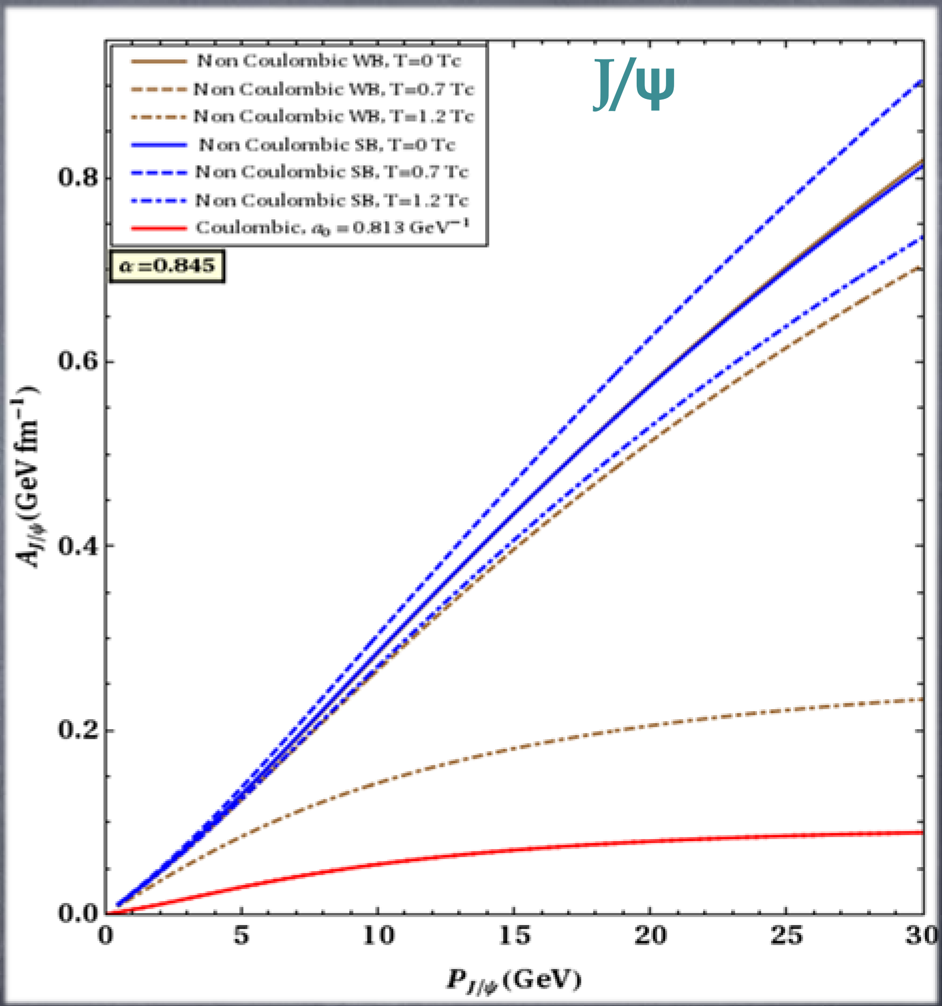


- Same behaiouvor for HQ, J/Ψ, Υ
- For (HQ, J/Ψ, Υ): $B \nearrow$ with T $\nearrow$
- $B(HQ) > B(J/\Psi) > B(\Upsilon)$

o Wave function influence on dE/dt, A_i, B

Collisionel-Non Coulombic



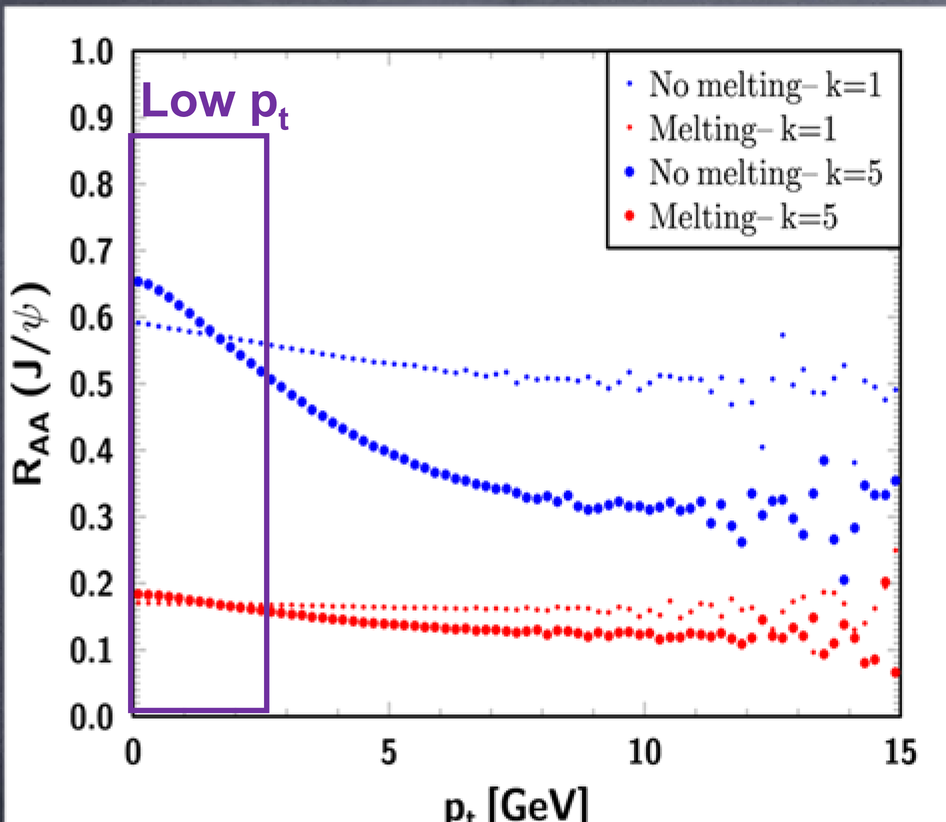


- Weakly bound and strongly bound > coulombic case
- Behavior related to $V_\infty(T)$ and $\epsilon(T)$

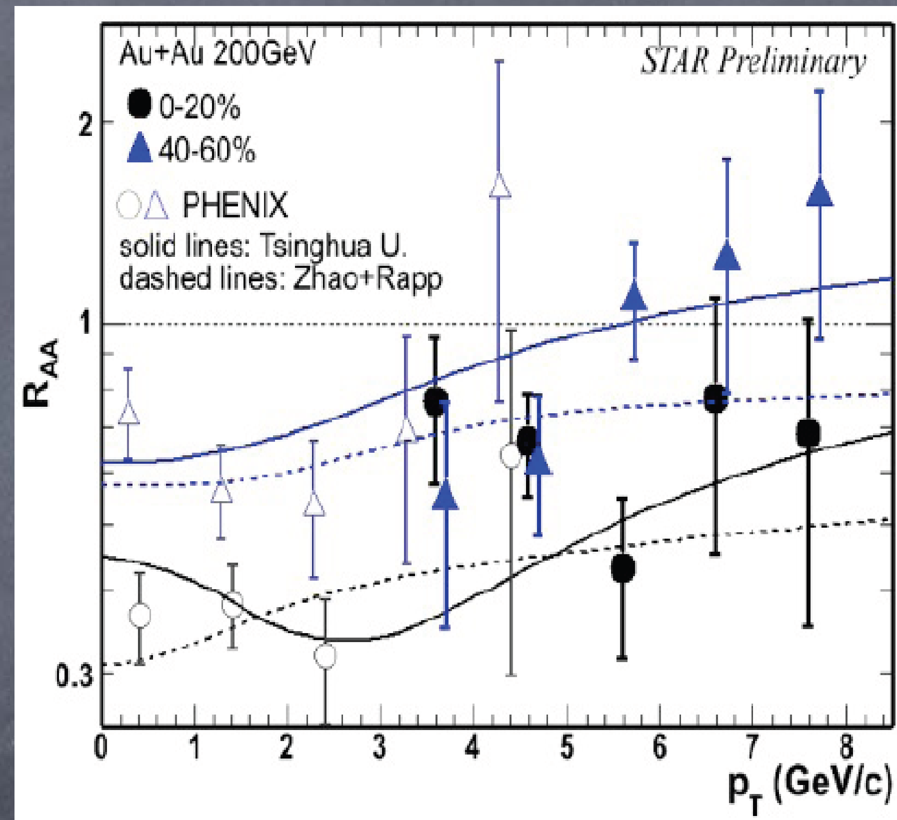
IV. Observables for Stochastic Transport & collective behaviuor of J/ψ’s

behaviour

$R_{AA}(J/\psi)$ Nuclear modification factor



$Au-Au, \sqrt{s} = 200\ GeV$, Min bias collisions

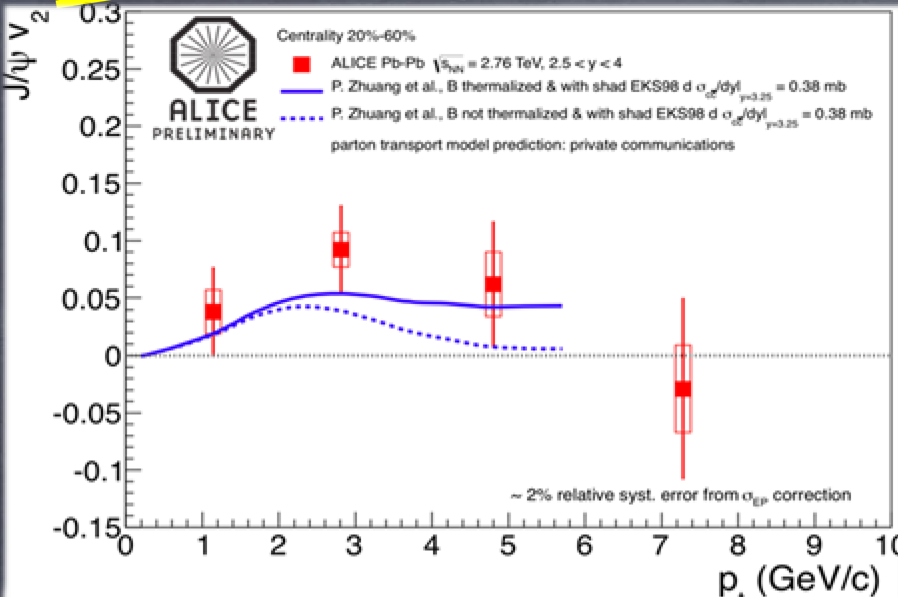


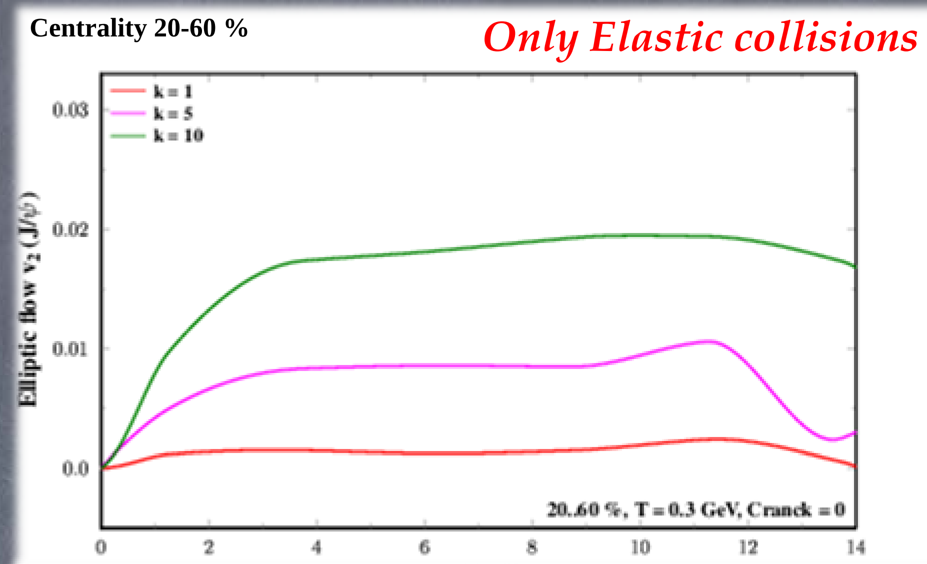
- Elastic scattering reduces the J/ψ momentum. The coupling plasma-J/ψ is sufficiently strong and elastic collisions are sufficiently important
- Part of R_{AA} is due to elastic scattering processes
- Some ingredients left in our model at high p_t in order to reproduce data

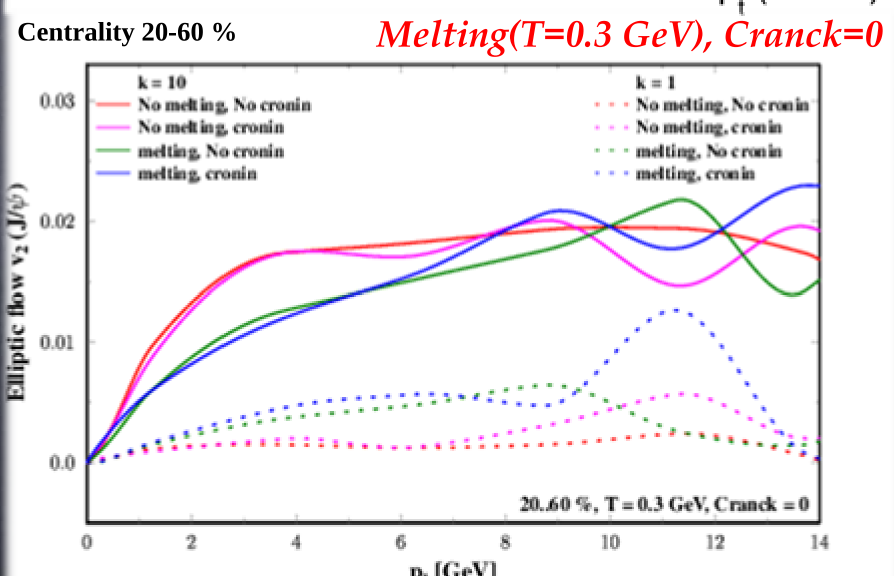
IV. Observables for J/ψ Stochastic Transport & collective ~~behaviour~~

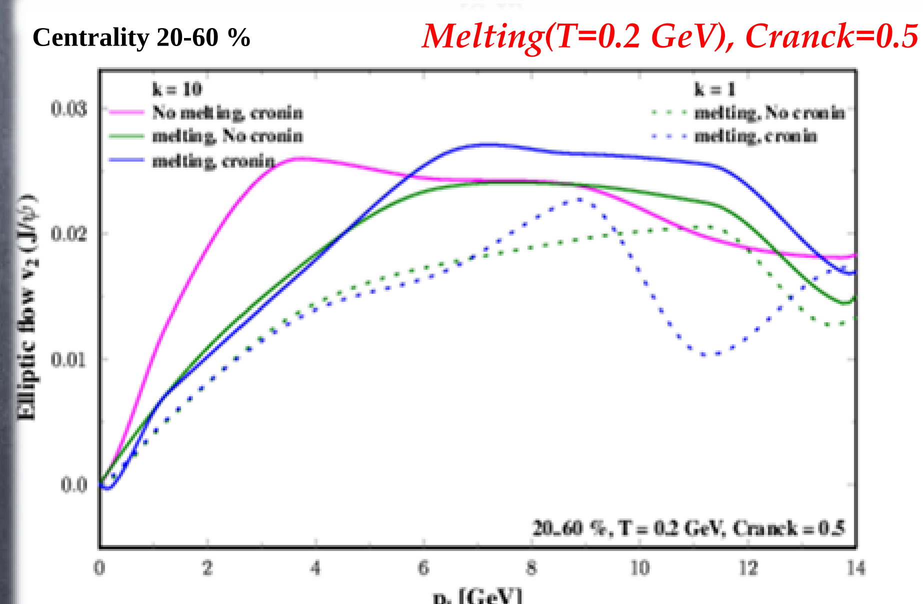
- **Elliptic flow $v_2(J/\psi)$**

LHC

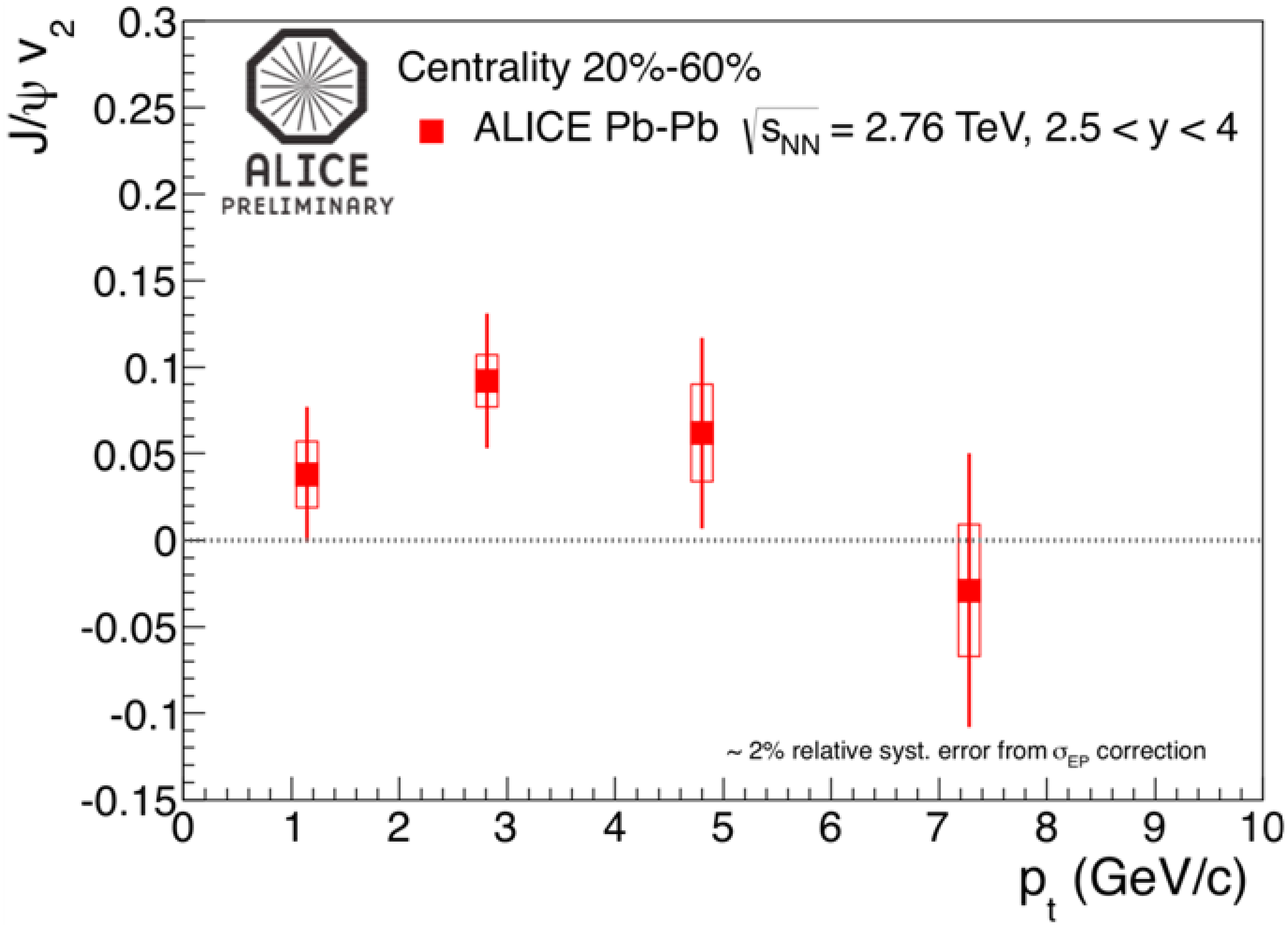






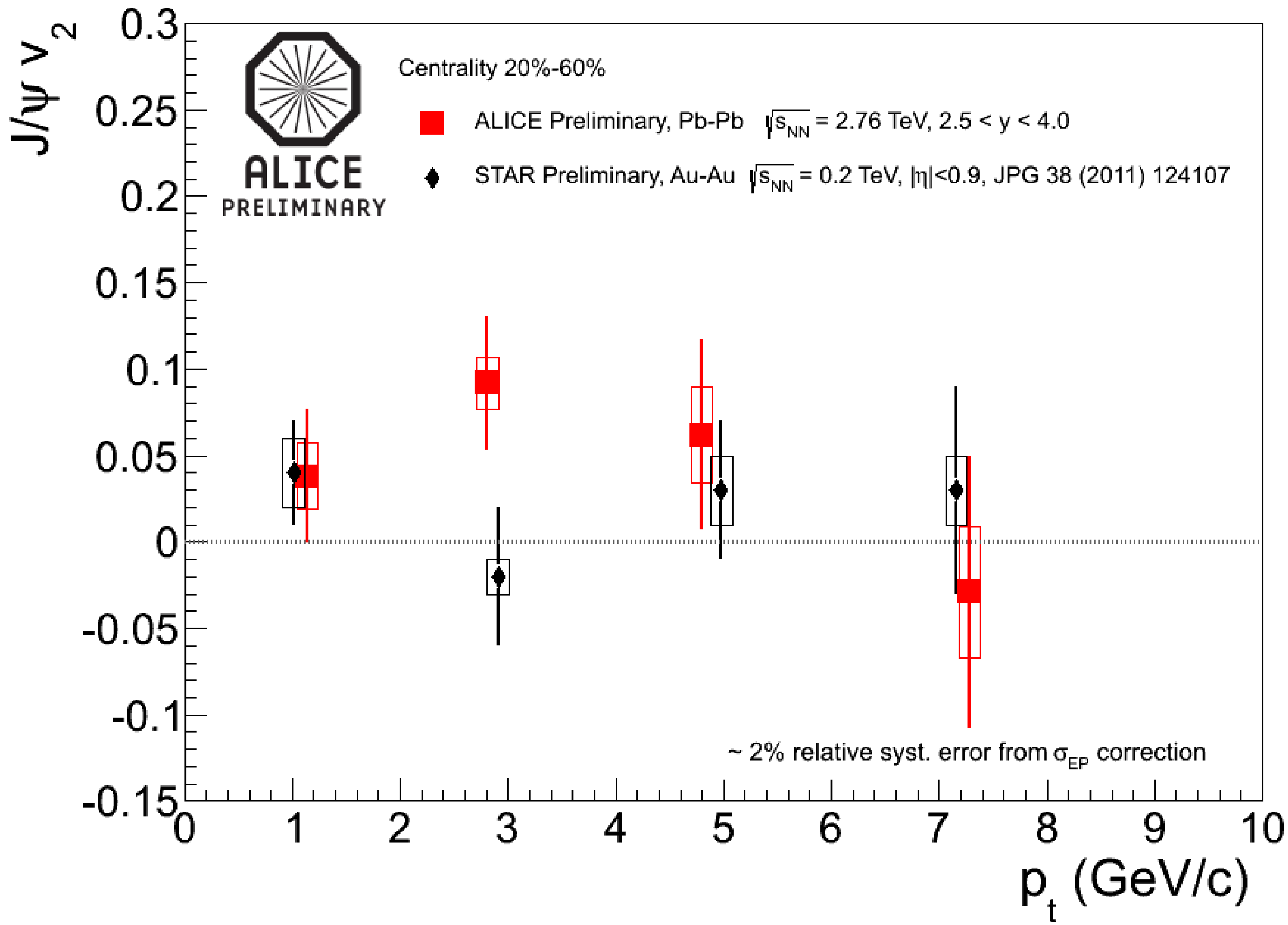


Results : p_T-differential J/ψ v_2



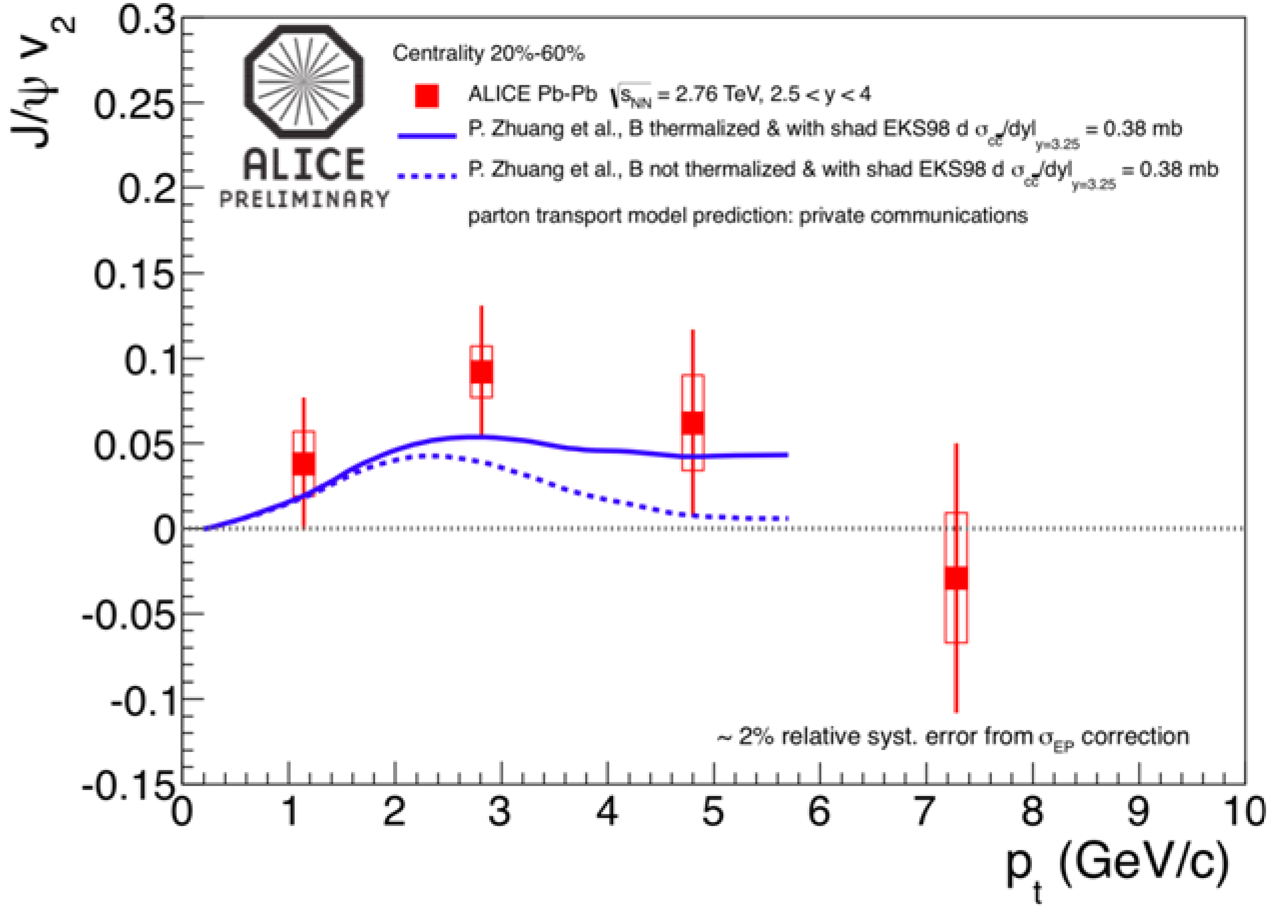
- Hints for non-zero J/ψ v_2 measured in the centrality range 20-60% and in the p_T range 2-4 GeV/c with a significance of 2.2σ
- Statistical error is dominant

p_T-differential J/ψ v_2 (comparison with STAR)



- Different behaviour observed between STAR and ALICE in the p_t range 2-4 GeV/c
  (reminder : 2.2σ deviation from zero for ALICE J/ψ v_2 in that p_t bin)

p_T-differential J/ψ v_2 (comparison with theory)



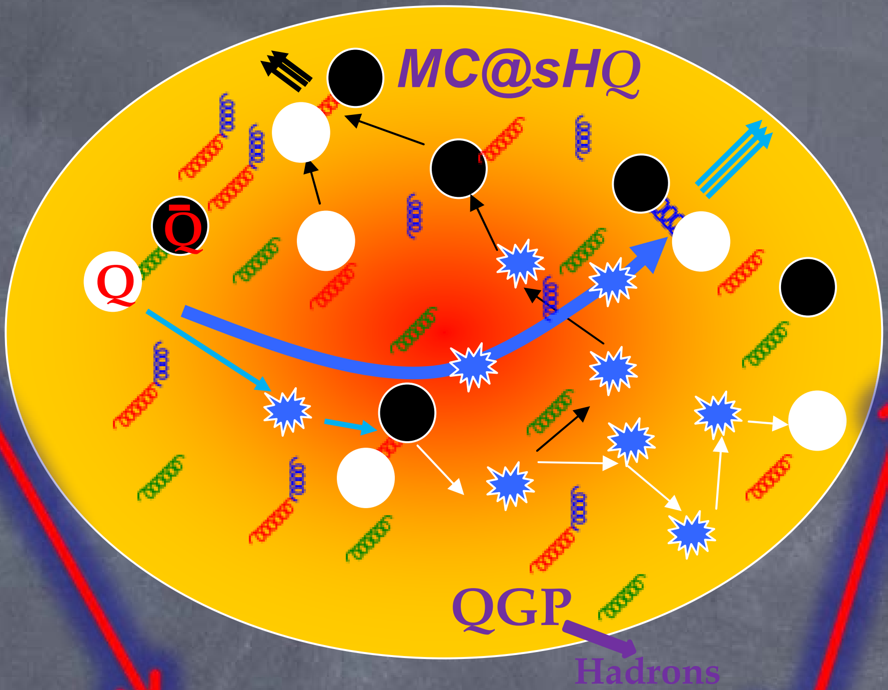
- Parton transport model :
  - Charm production cross section : 0.38mb (between pp data and FONLL calculations)
  - Shadowing effects included
  - Thermalized or unthermalized b quark assumption
  - if unthermalized b quark → small contribution to J/ψ v_2

- This model qualitatively describes the J/ψ R_{AA} versus centrality and p_t

See talk by Jens Wiechula this afternoon and Christophe Suire plenary talk on wednesday

V. Conclusions & Perspectives

o Perspectives



Part I

Try other parametrisation of $Q\bar{Q}$ potential,...

Part II

- Extend our BS formalism for the σ_{elas} (Φ-gluons) at low energy and introduce NLO Feynman diagrams (3 gluons)
- Refine the BS vertex (fine & hyperfine structure, cross diagram in retarded interaction)
- Elastic cross section for Φ -quark interactions
- Apply our study to QED bound states (positronium)

Part III

- Fokker-Planck coefficients for elastic Φ-quark interaction process
- Direct calculation of B...

Part IV

- Systematic studies at RHIC
- Study of J/ψ and 'Υ at LHC energies
- Study of J/ψ at SPS
- Include viscosity in MC@sHQ, recent CNM

o Perspectives (1/2)

❑ Direct applications of our formalism

VI

- The study presented in part IV of J/ψ at RHIC energies has to be extended
- Proceed to systematic studies (several centralities, Cu-Cu, …)
- Study of J/ψ and ϒ propagation at LHC energies (all the ingredients are available)
- Study of J/ψ at SPS energies (same FP coefficients), introduce QGP description
- Take into account the temperature dependence of FP coefficients in MC@sHQ transport code (instead of k factor)

II

- Apply our study to QED bound states (positronium and muonium)

❑ Extensions of our formalism

II

- Extend our BS formalism for the calculation of σ_{elas} (quarkonia-gluons) at low energy
- Introduce NLO Feynman diagrams (with 3 gluons...)
- Elastic cross section for quarkonium-quark interactions
- Refine the BS vertex (fine & hyperfine structure, cross diagram in retarded interaction)

III

- Fokker-Planck coefficients for elastic quarkonia-quark interaction process

VI

- Include viscosity (η) in MC@sHQ (η is deduced from σ_{elas} calculations)
- Include recent improvement in QGP description (CNM effects, quarkonia suppression...)

o Perspectives (2/2)

❑ Integration of our formalism in parallel developments

❑ Final Project

- Full characterization of the study of quarkonia in the QGP. This project includes our study, but should cover other aspects to reach a good physical understanding of this QGP probe and especially how to use it to probe the QGP.

❑ Stochastic localisation and QQ dynamic studies

$$\left[\left(\frac{\partial}{\partial t}+\vec{p}\,\frac{\partial}{\partial \vec{x}}\right)-\frac{2}{\hbar}\sin\left(\frac{\hbar}{2}\frac{\partial}{\partial \vec{p}}\frac{\partial}{\partial \vec{x}}\right)V(\vec{x})\right]F(\vec{x},\vec{p},t)=0$$

Wigner-Moyal equation

Quantum treatment, realistic stochastic forces deduced from our calculations

- The aim of this model is to determine QQ survival probability *vs* time and QGP scale. The influence of dynamics on J/ψ statistical weight *vs* time will be modelled (preliminary results showed that J/ψ-dynamics increases its survival probability especially at high temperature)

➡ Possible interpretation of the suppression of J/ψ-suppression