Finite volume corrections to moments of charge fluctuations in HIC

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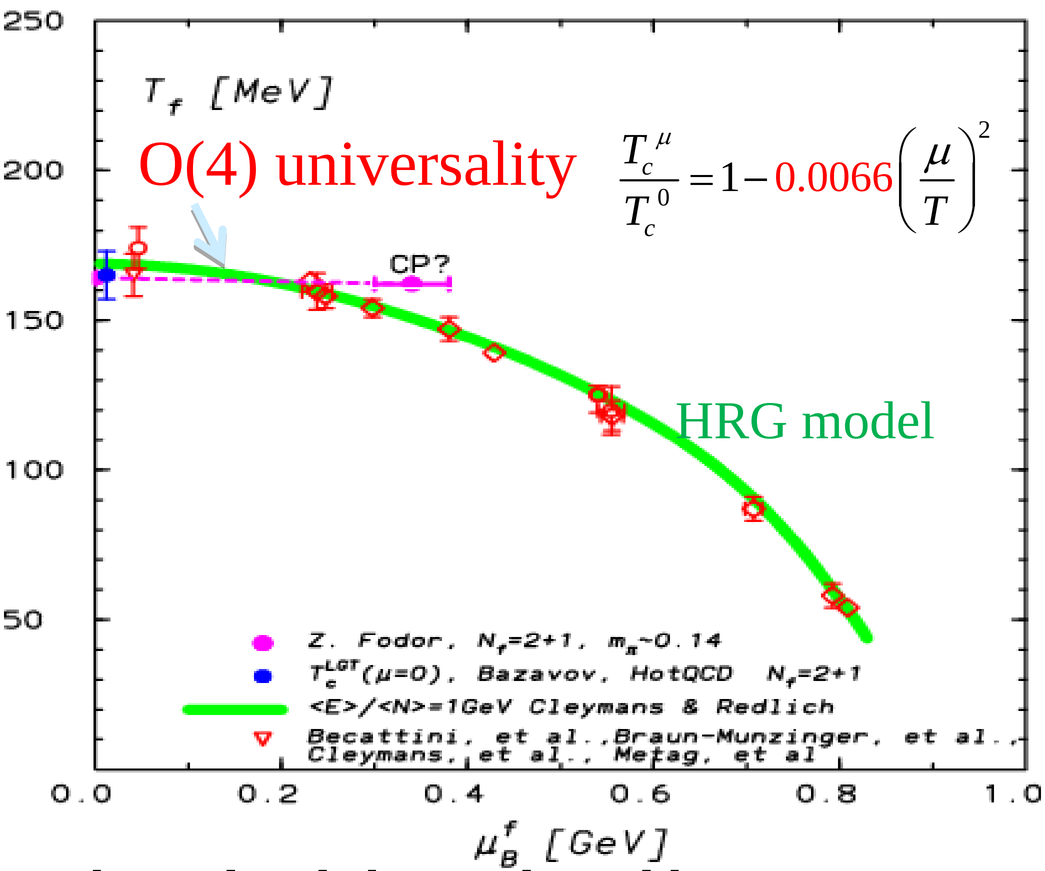
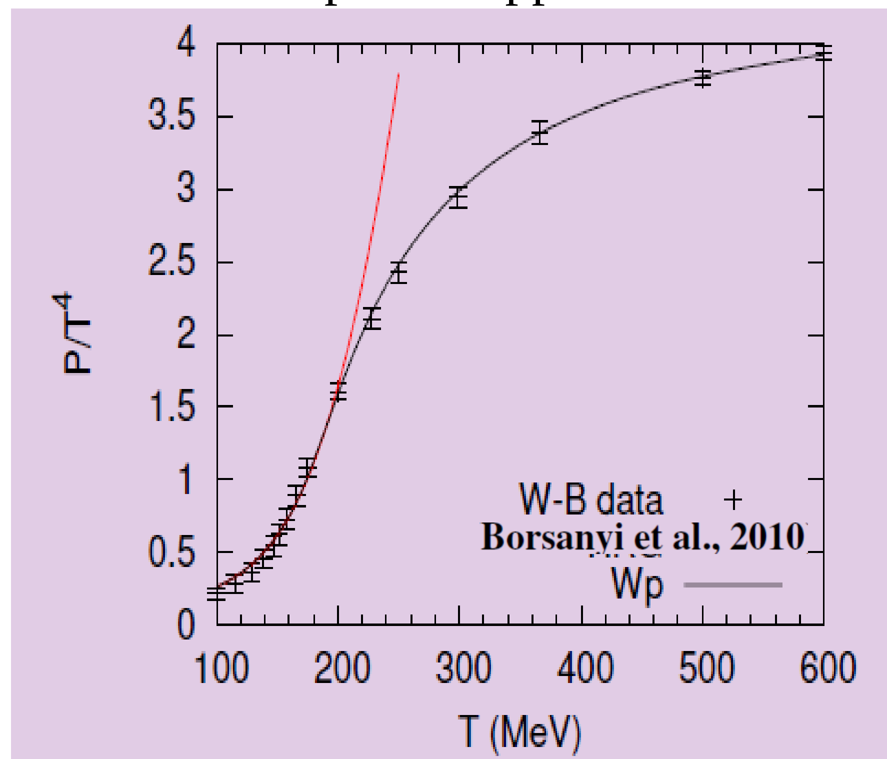
- QCD phase boundary and its O(4) „scaling”
- Higher order moments of conserved charges as probe of QCD criticality in HIC
- Influence of volume fluctuations on charge cumulants
- STAR data & expectations

Collaboration with: P. Braun-Munzinger, B. Friman , F. Karsch,
K. Morita, C. Sasaki & V. Skokov

- Particle yields and their ratio, as well as LGT results at $T < T_c$ are well described by the Hadron Resonance Gas Partition Function .



Budapest-Wuppertal LGT data



Thermal and chemical equilibrium: H. Barz, B. Friman, J Knoll & H.Schultz
P. Braun-Munzinger, J. Stachel, Ch. Wetterich
C. Greiner & J. Noronha-Hostler, ……..

O(4) scaling and critical behavior

- Near T_c critical properties obtained from the singular part of the free energy density

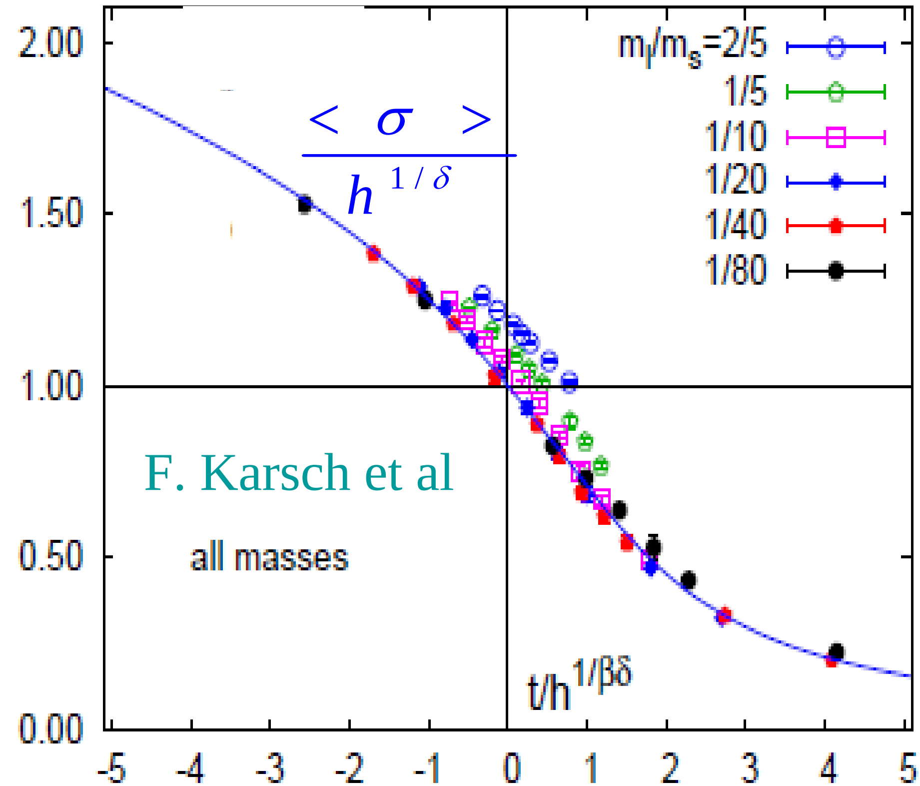
$$F = F_{reg} + F_S \quad \text{with} \quad F_S(t,h) = b^{-d} F(b^{1/\nu} t, b^{\beta\delta/\nu} h)$$

with $t = \frac{T - T_c}{T_c} + \kappa \left(\frac{\mu}{T_c} \right)^2$

- Phase transition encoded in the "equation of state"

$$<\sigma> = -\frac{\partial F_s}{\partial h} \Rightarrow \text{pseudo-critical line}$$

$$\frac{<\sigma>}{h^{1/\delta}} = F_h(z) \ , \quad z = t h^{-1/\beta\delta}$$



O(4) scaling of net-baryon number fluctuations

- The fluctuations are quantified by susceptibilities

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} := c_n \qquad \text{with} \qquad P = P_{reg} + P_{Singular}$$

- From free energy and scaling function one gets

$$\chi_B^{(n)} \approx \chi_r^{(n)} + c\, h^{2-\alpha-n/2} f_{\pm}^{(n/2)}(z) \quad \mathit{for} \;\; \mu=0 \;\; \mathit{and} \;\; n \;\; \mathit{even}$$

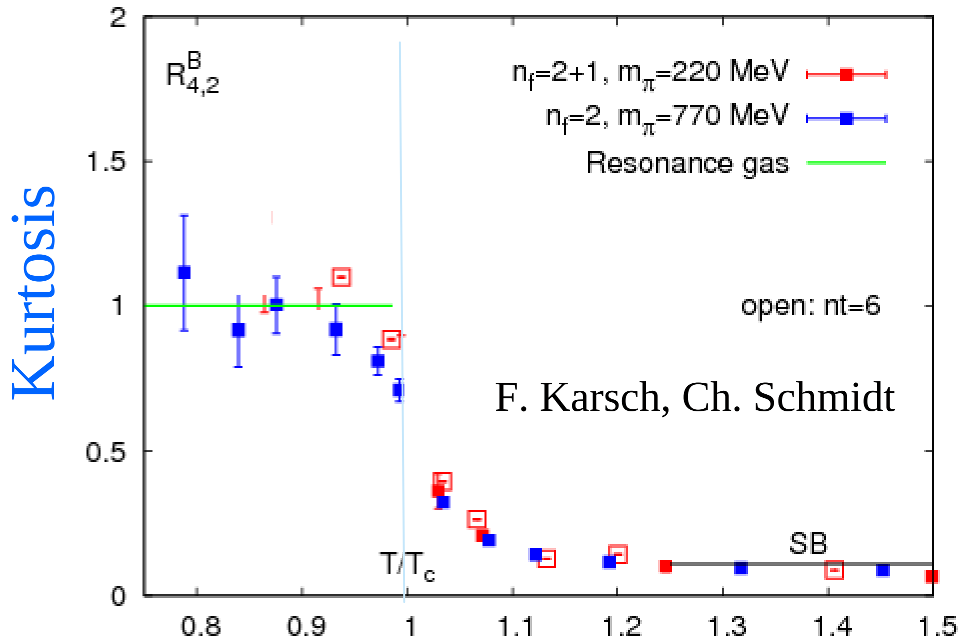
$$\chi_B^{(n)} \approx \chi_r^{(n)} + c_\mu\, h^{2-\alpha-n} f_{\pm}^{(n)}(z) \quad \mathit{for} \;\; \mu \neq 0$$

- Resulting in singular structures in n-th order moments which appear for $n \geq 6 \;\; at \;\; \mu = 0$ and for $n \geq 3 \;\; at \;\; \mu \neq 0$ since $\alpha \approx -0.2$ in O(4) univ. class

Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R.

$$R_{4,2}{}^{B} = \frac{1}{9}\frac{c_4}{c_2}$$



The $R_{4,2}{}^{B}$ measures the quark content of particles carrying baryon number

- HRG factorization of pressure:

$$\boxed{P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)}$$

consequently: $c_4/c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

$$c_4^q / c_2^q = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3<(\delta N_q)^2>$$

excellent probe of deconfinement

Deconfinement is density driven - (percolation)

LGT result shows:

strong dependence of T_c on m_q and N_f, however for $N_f = 2,3$ and for all m_q

$$\varepsilon(T_c) \approx (0.6 \pm 0.3) GeV / fm^3$$

condition for deconfinement

$$n_p \approx \frac{1.22}{V_h} = const.$$

R_h 0.8 fm

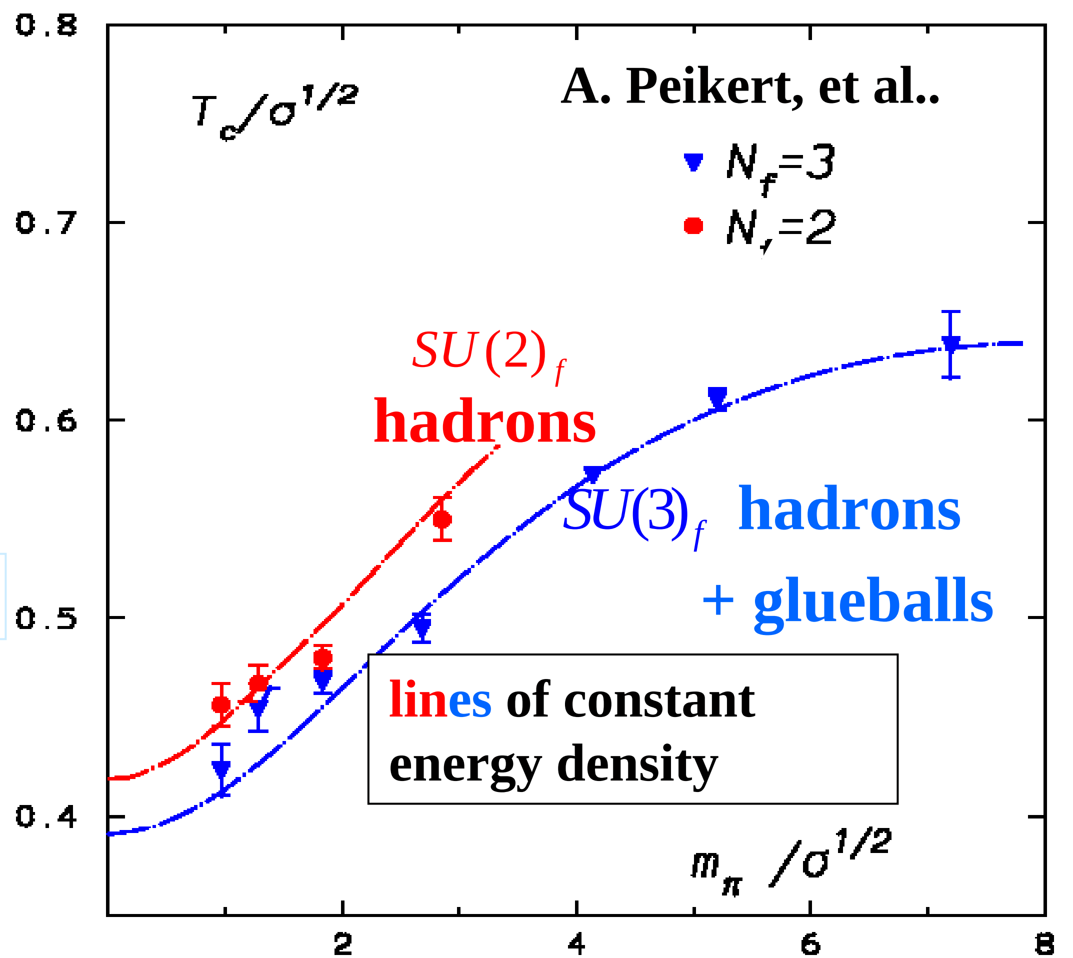
H. Satz

percolation ⟺ **deconfinement**

$$n_P \approx 0.58 \frac{1}{fm^3}$$

$$\varepsilon_c \approx 0.6 \frac{GeV}{fm^3}$$

$$\frac{< E >}{< N >} \approx 1.0 \; GeV$$



Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

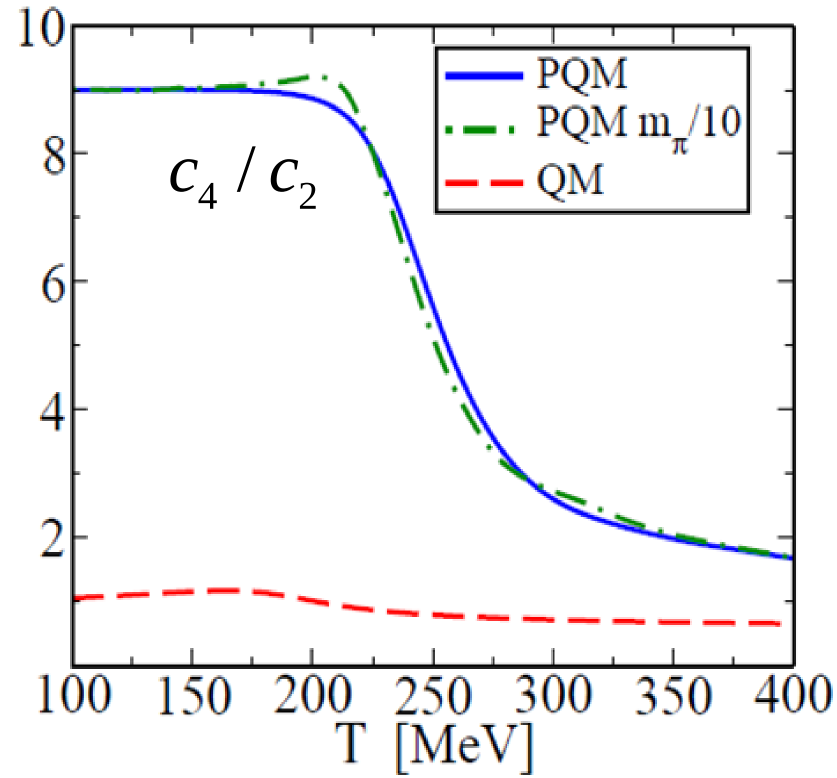
- For $T < T_c$

  the assymptotic value

  due to „confinement" properties

$$\frac{P_{q\bar{q}}(T)}{T^4} \approx \frac{2N_f}{27\pi^2}\left(\frac{3m_q}{T}\right)^2 K_2\left(\frac{3m_q}{T}\right)\cosh\frac{3\mu_q}{T}$$

$$\Rightarrow \quad c_4/c_2 = 9$$



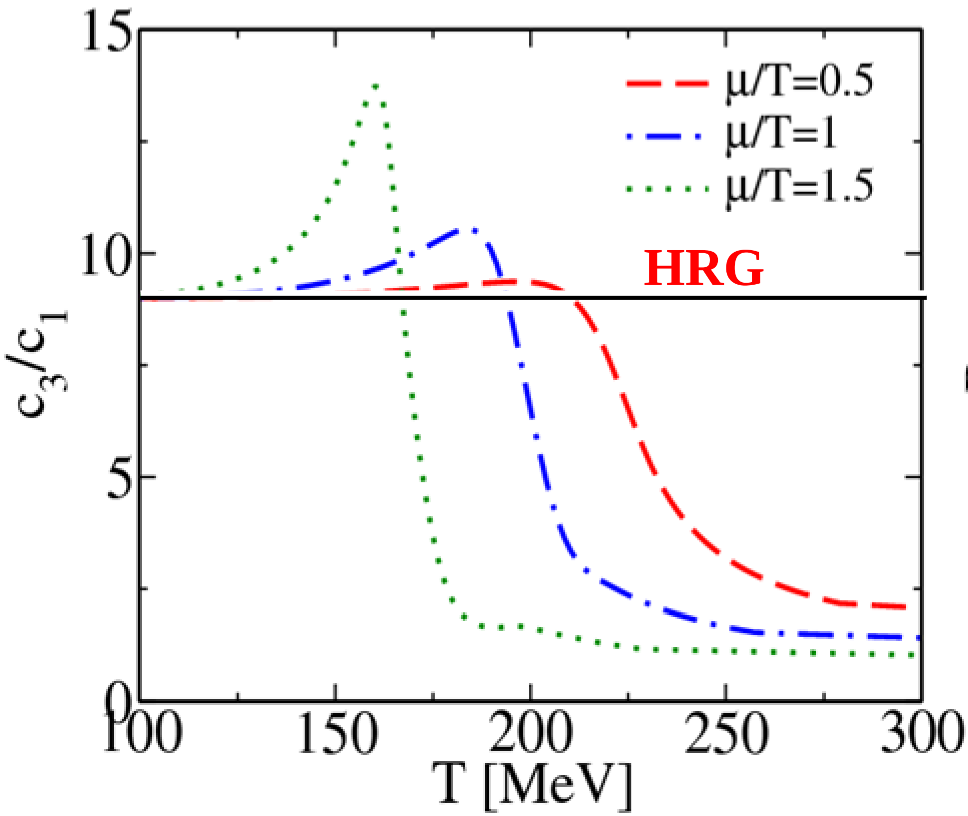
- For $T >> T_c$

$$\frac{P_{q\bar{q}}(T)}{T^4} = N_f N_c\left[\frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu}{T}\right)^2 + \frac{7\pi^2}{180}\right]$$

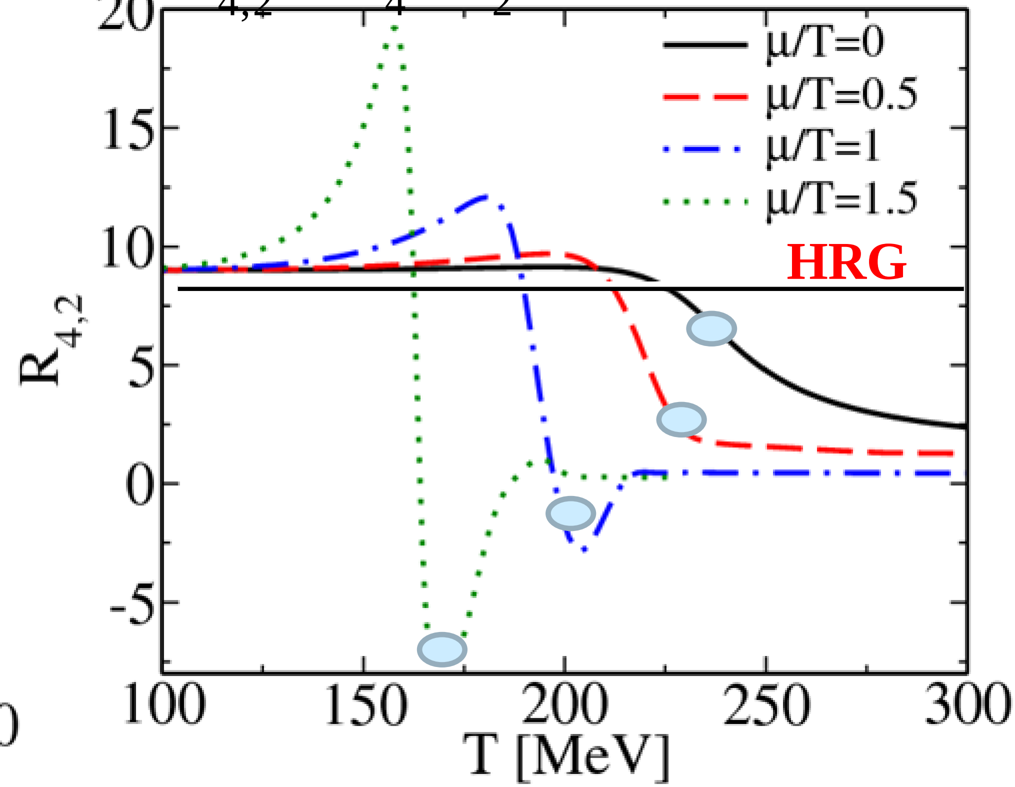
$$\Rightarrow c_4/c_2 = 6/\pi^2$$

- Smooth change with a very weak dependence on the pion mass

Ratios of cumulants at finite density



$R_{4,2} = c_4 / c_2$



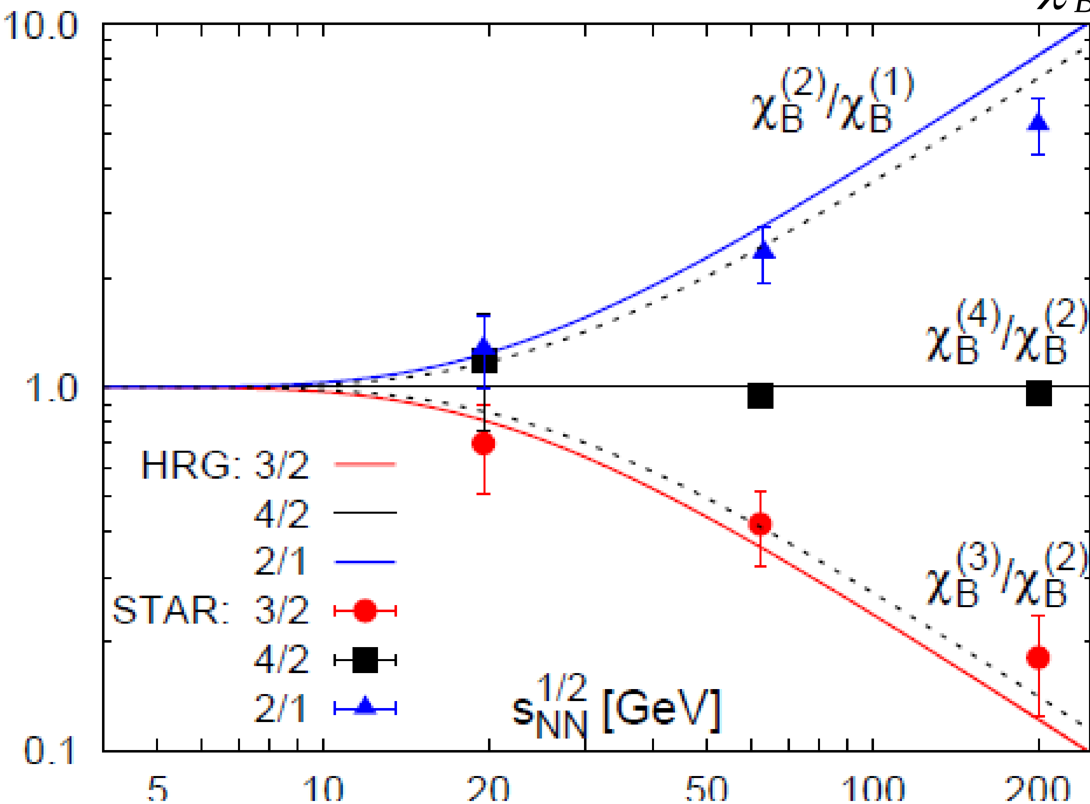
Deviations of the ratios of odd and even order cumulants from their asymptotic, low T-value, $c_4 / c_2 = c_3 / c_1 = 9$ are increasing with μ / T and the cumulant order

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Comparison of the Hadron Resonance Gas Model with STAR data

- Frithjof Karsch & K.R.

$$\frac{\chi_B^{(2)}}{\chi_B^{(1)}} \approx \coth(\mu_B / T)$$



$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1$$

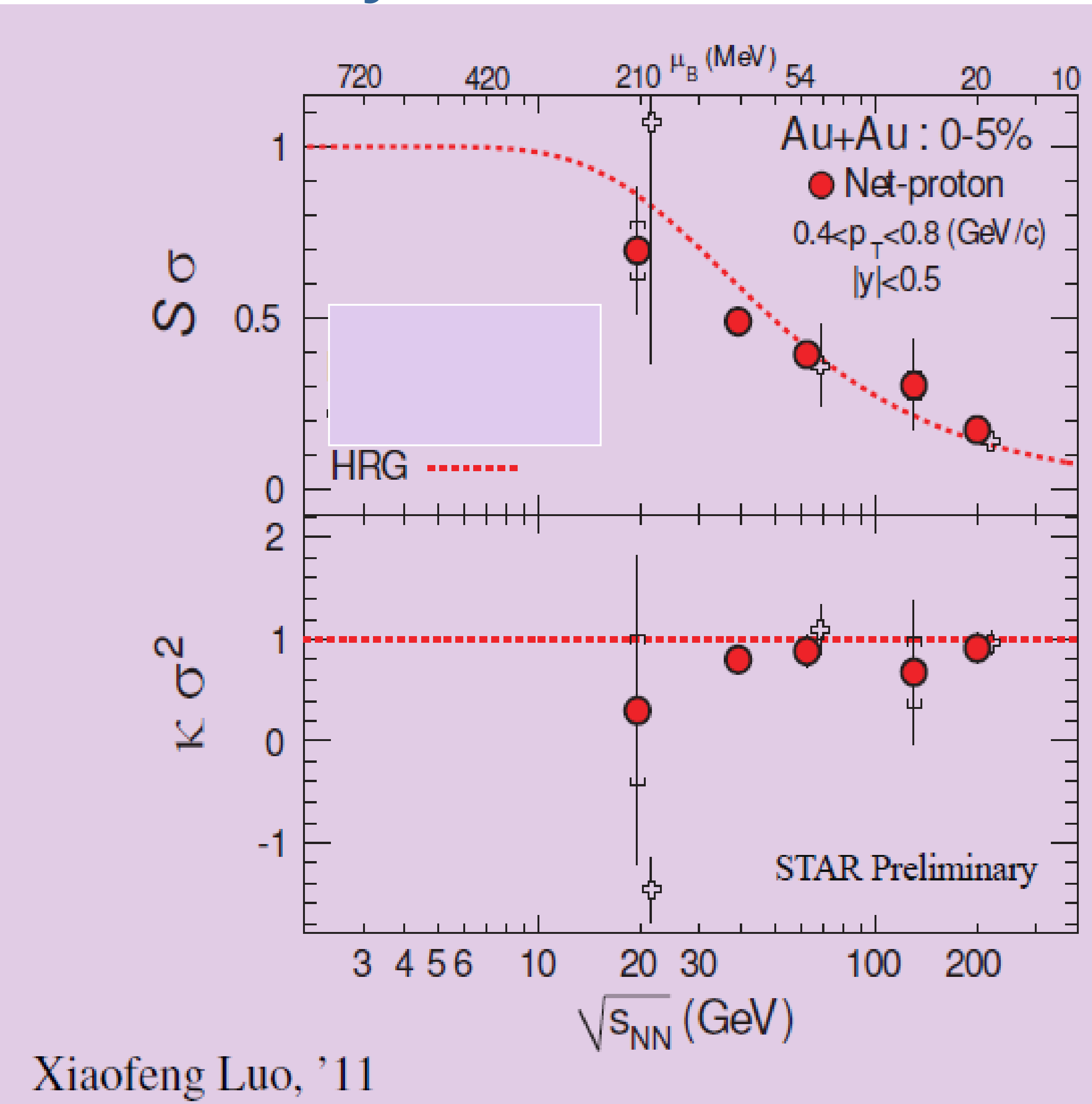
$$\frac{\chi_B^{(3)}}{\chi_B^{(2)}} \approx \tanh(\mu_B / T)$$

- RHIC data follow generic properties expected within HRG model for different ratios of the first four moments of baryon number fluctuations

deviations between HRG model and data for the variance ($\chi_B^{(2)}$)?

Error Estimation for Moments Analysis: Xiaofeng Luo arXiv:1109.0593v

STAR analysis:



Deviations from HRG

- Critical behavior?
- Conservation laws?

M. Bleicher

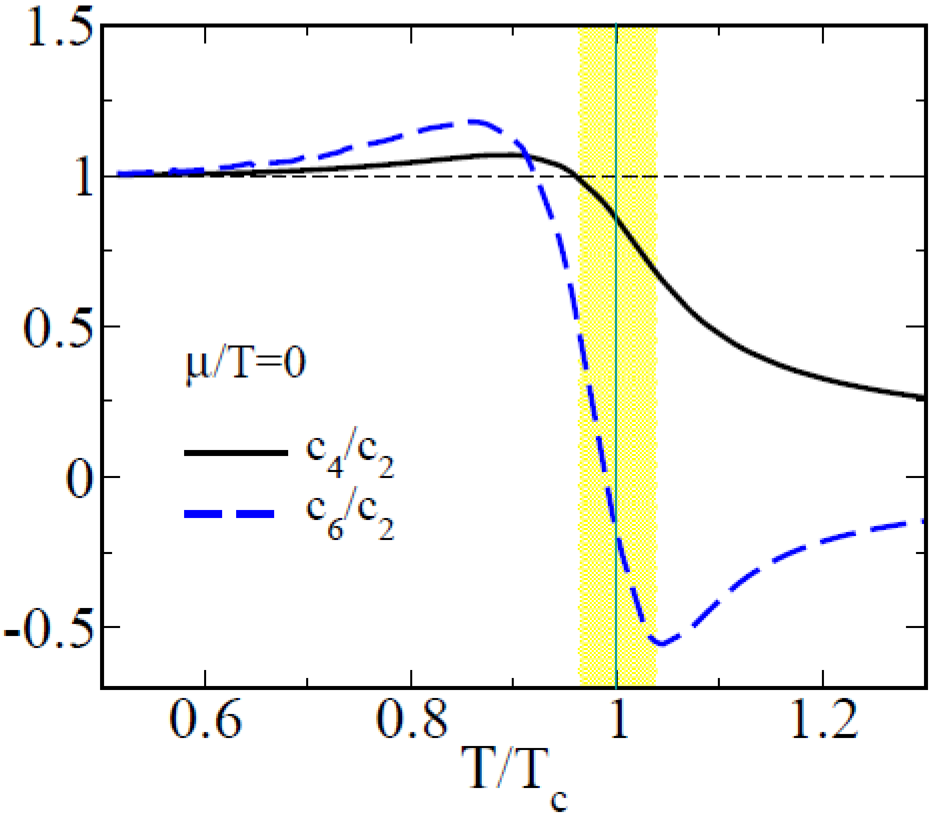
A. Bzdak, V. Koch, V.Skokov

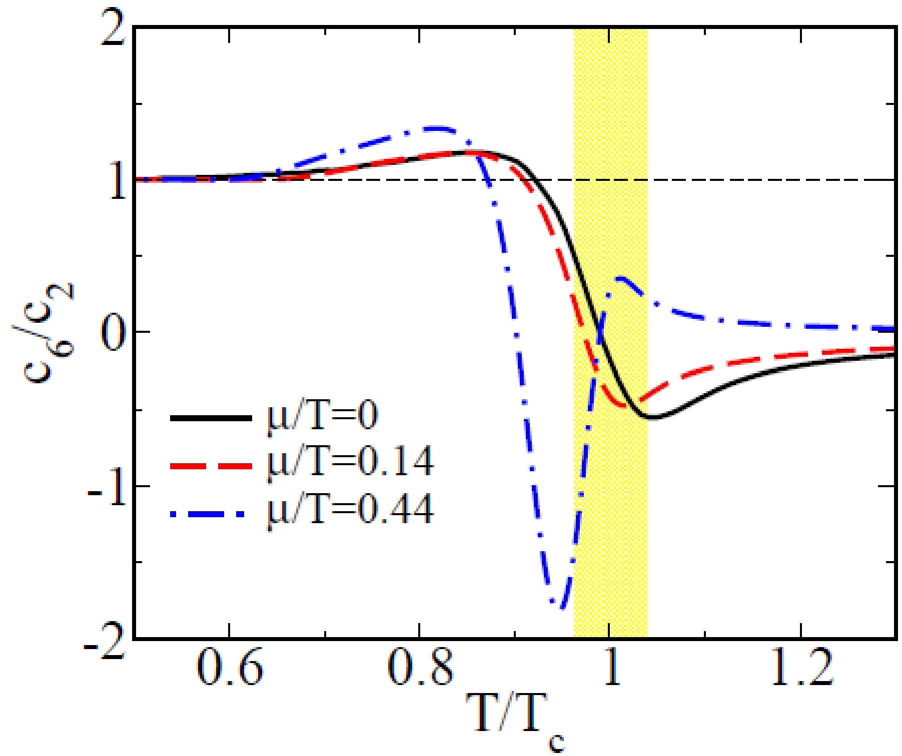
ARXIV:1203.4529

- Volume fluctuations?

B. Friman, V.Skokov &K.R.

Ratio of higher order cumulants





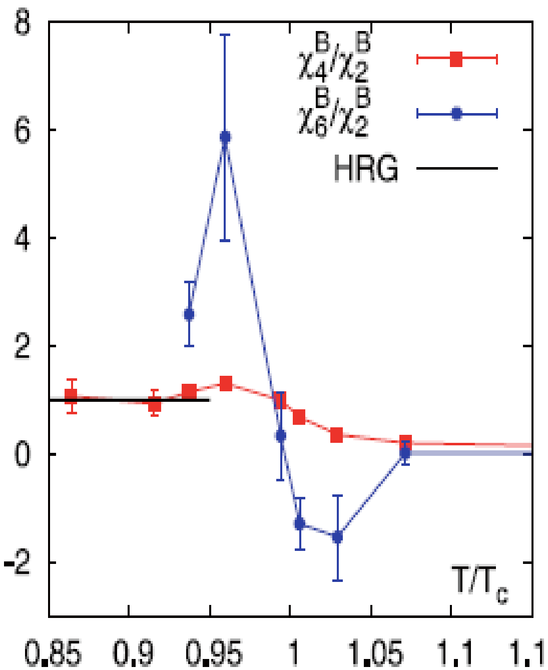
Deviations of the ratios from their asymptotic, low T-value, are increasing with the order of the cumulant

Theoretical predictions and STAR data

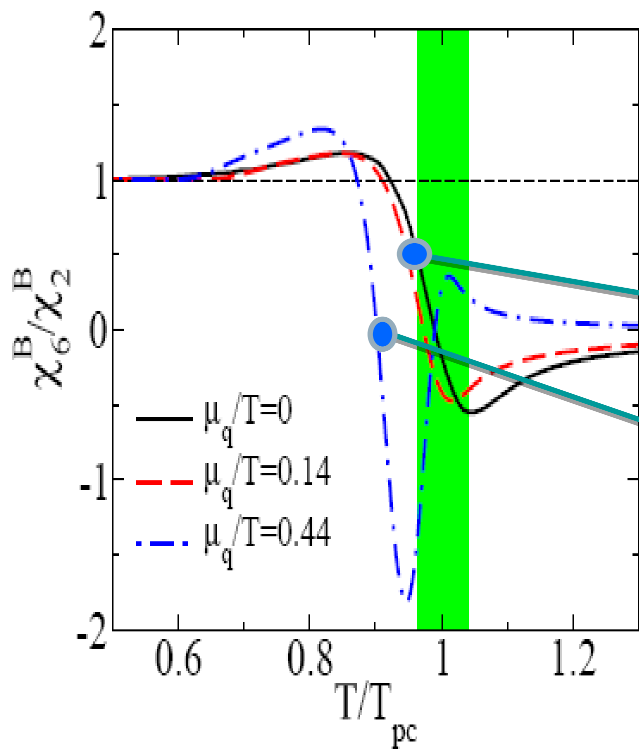
Deviation from HRG if freeze-out curve close to Phase Boundary/Cross over line

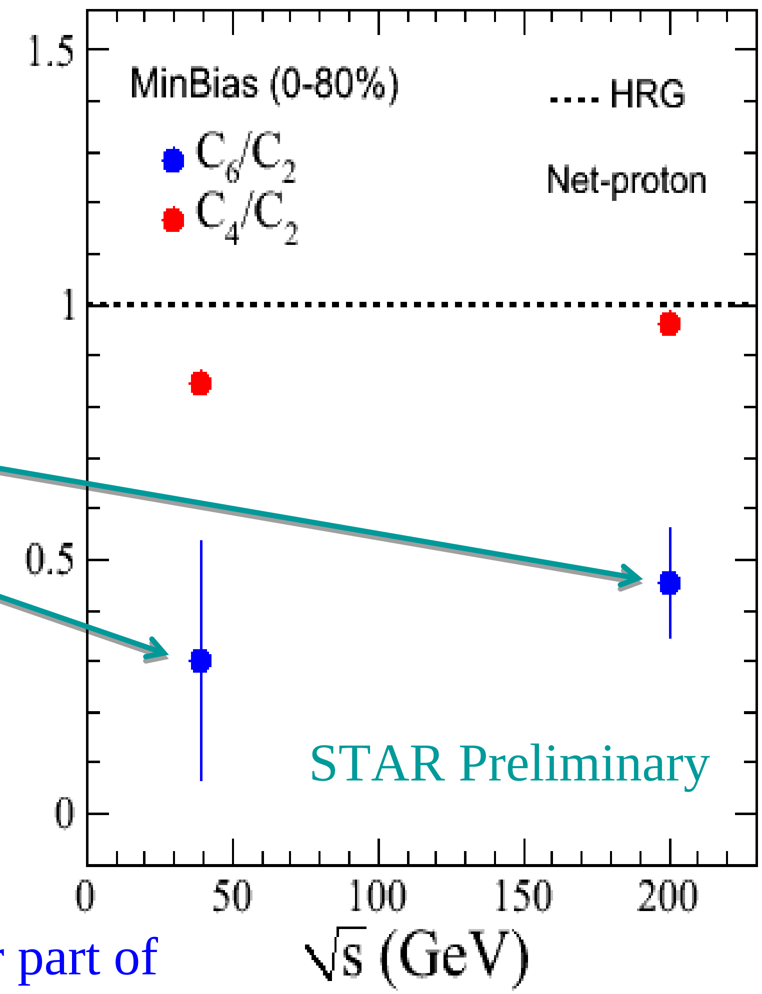
L. Chen, BNL workshop, CPOD 2011
STAR Data

Lattice QCD



Polyakov loop extended Quark Meson Model





Strong deviations of data from the HRG model (regular part of QCD partition function) results: Remnant of O(4) criticality!!??

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k > = \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y + \mu) - p(T, \mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

Probability distribution of the net baryon number

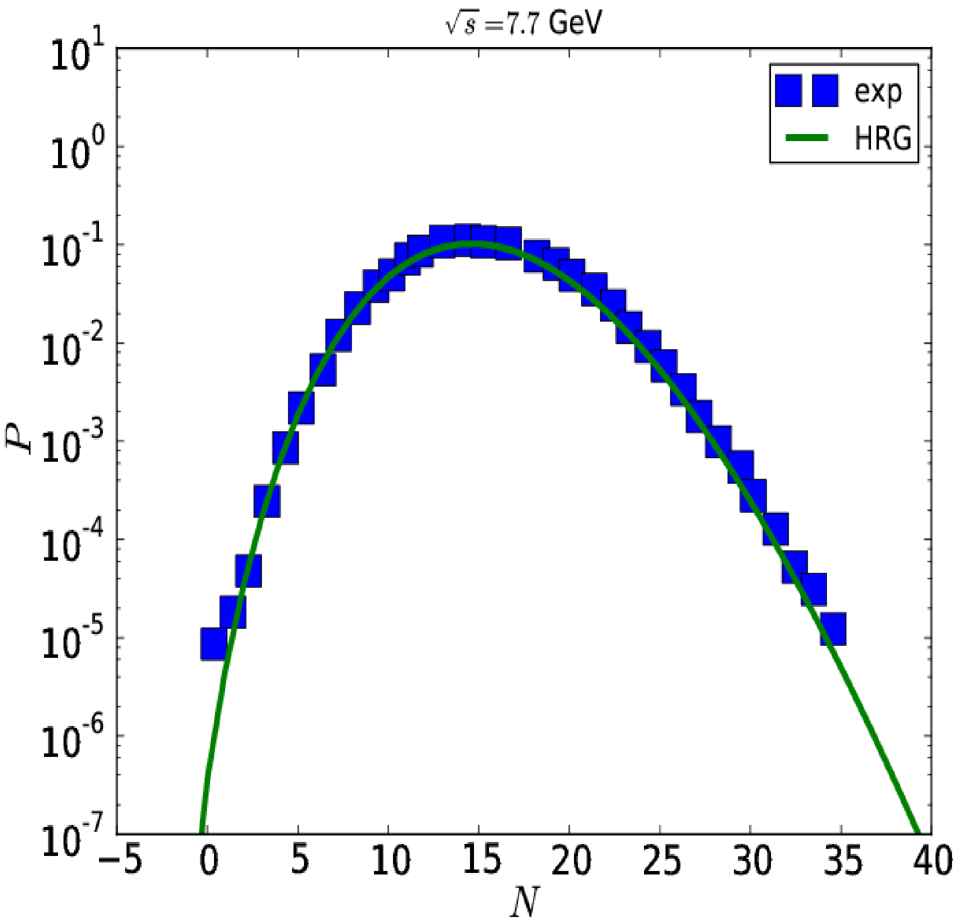
- For the net baryon number P(N) is described as Skellam distribution

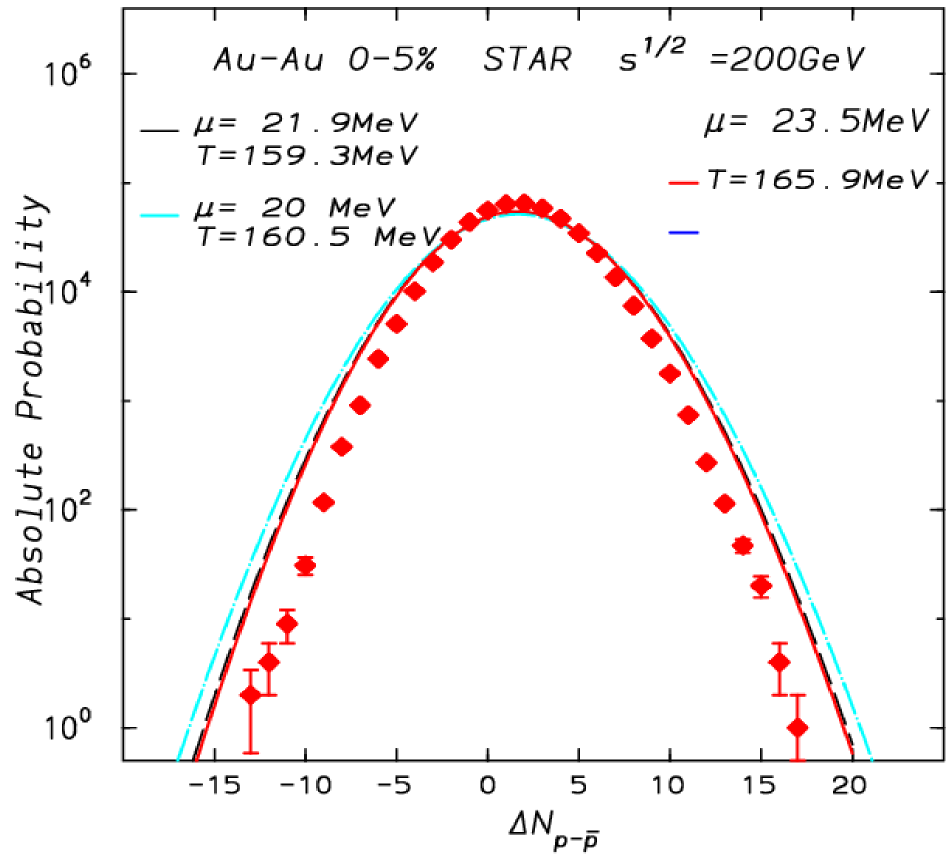
$$P(N) = \left(\frac{B}{\bar{B}} \right)^{N/2} I_N(2\sqrt{B\bar{B}}) \exp[-(B+\bar{B})]$$

- P(N) for net baryon number N entirely given by measured mean number of baryons B and antibaryons $\bar{B}$

In HRG the means $B, \bar{B}$ □ $F(T_f, \mu_f, V)$ are functions of thermal parameters at Chemical Freezeout

Comparing HRG Model with preliminary STAR data: efficiency uncorrected





Data described by Skellam distribution: No sign for criticality

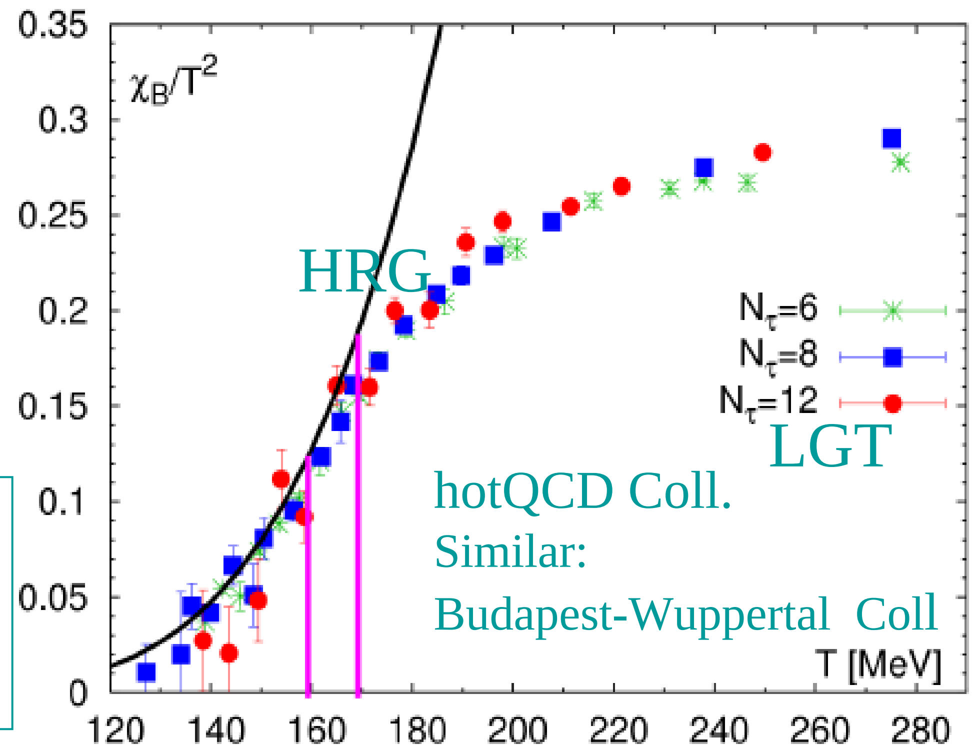
Shrinking of the distribution related with criticality?

Influence of criticality on the shape of P(N)

In the first approximation the χ_B quantifies the width of P(N)

$$P(N) \approx \exp(-N^2 / 2VT^3 \chi_B)$$

Shrinking of the probability distribution already expected due to deconfinement properties of QCD



At the critical point (CP) the width of P(N) should be larger than that expected in the HRG due to divergence of χ_B in 3d Ising model universality class

Influence of O(4) criticality on P(N)

Consider Landau model: $$\Omega = \Omega_{bg} + \frac{1}{2} t_\mu^{\ 2} \sigma^2 + \frac{1}{4}\sigma^4$$

$$\boxed{\begin{aligned} \Omega_0 &\equiv \Omega(T > T_c(\mu), \mu) = \Omega_{\rm bg} \\ \Omega_1 &\equiv \Omega(T \le T_c(\mu), \mu) = \Omega_{\rm bg} - \frac{1}{4} |t_\mu|^{2-\alpha}(T,\mu) \end{aligned}}$$

$$\Omega_{\rm bg} = 2d \cosh(\mu/T)$$

$$t_\mu(T,\mu) \approx A(T - T_c) + B\mu^2$$

Scaling properties:

Mean Field $\alpha = 0$ $\Rightarrow$ $$\boxed{\begin{aligned} c_1^{\rm sing} &= -B\mu|t_\mu|, & c_2^{\rm sing} &= -B|t_\mu| + 2B^2\mu^2 \\ c_3^{\rm sing} &= 6B\mu^2, & c_4^{\rm sing} &= 6B^2. \end{aligned}}$$

O(4) scaling $\alpha \approx -0.21$ $\Rightarrow$ $\boxed{c_n^{\rm sing} \sim -\mu^n |t_\mu|^{2-\alpha-n}}$ $n \ge 3 \Rightarrow c_n^{\sin g} \to \infty$

Contribution of a singular part to P(N)

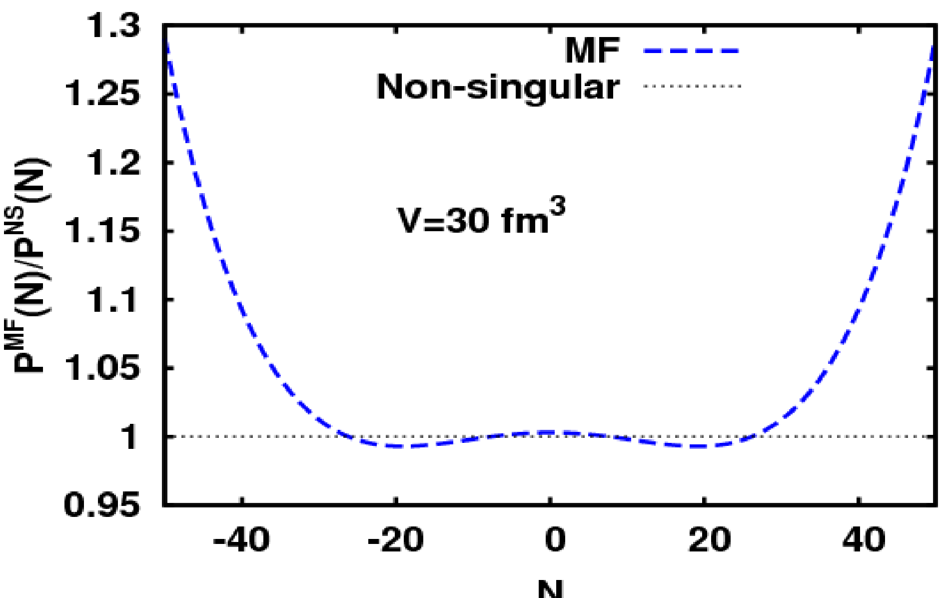
$$P(N) = \frac{Z\ (N,T,V)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

$$P^{NS}(N;T,V,\mu) = I_N(2dVT^3)e^{(\mu N+\Omega_0)/T}$$

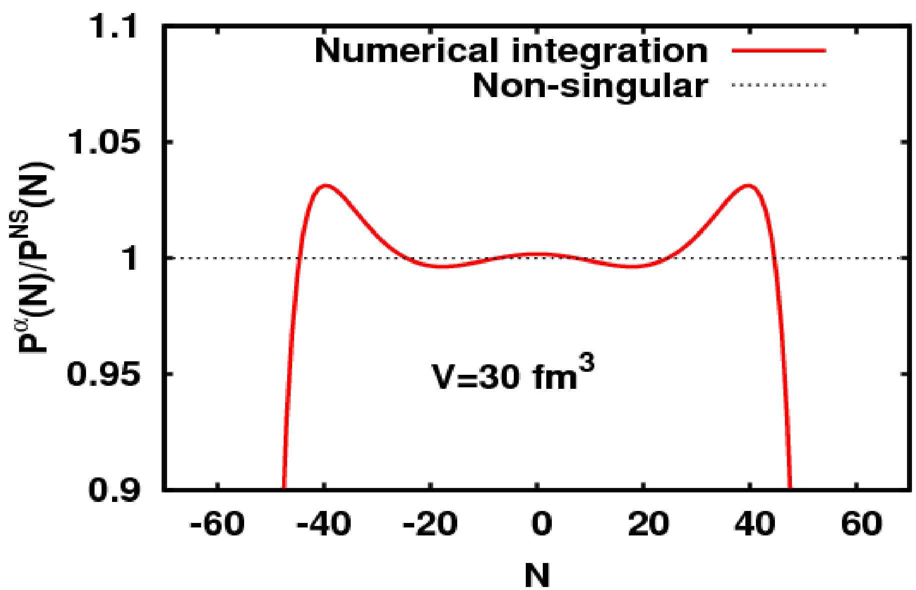
$$Z_c^{\mathrm{MF}}(T,V,N) = e^{\frac{VT^3}{4}[(t+2)^2+2]} \times \sum_{\ell=-\infty}^{\infty} I_{N-2\ell}[(2d-t-2)VT^3] I_\ell\left(\frac{VT^3}{2}\right)$$

Got numerically from:

$$Z(T,V,N) = \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{-i\theta N} \mathcal{Z}(T,V,\theta)$$

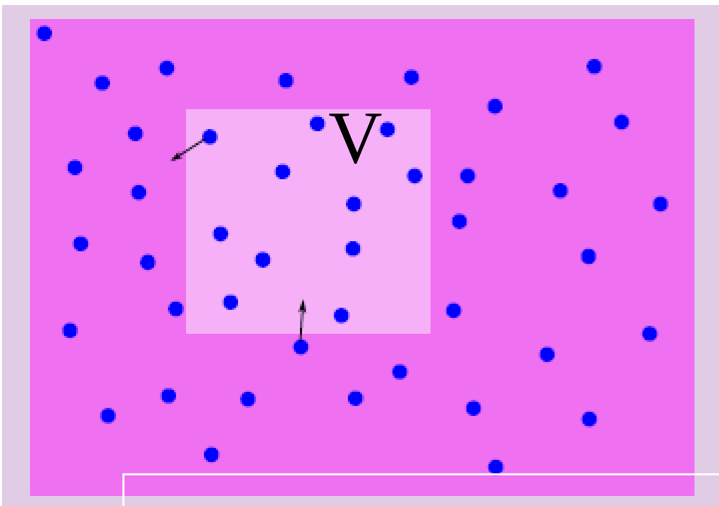


For MF broadening of P(N)



For O(4) narrower P(N)

Influence of volume fluctuations on cumulants



Probability distribution at fixed V

$$P(N,V)=\left(\frac{B}{\overline{B}}\right)^{N/2} I_N(2V\sqrt{n_B n_{\overline{B}}})\exp[-V(n_B+n_{\overline{B}})]$$

Moments of net charge

$$<N^k>=\sum_{N=-\infty}^{N=\infty} N^k P(N)$$

Cumulants

central moment

$$\delta N = N - <N>$$

$$\kappa_1=\frac{1}{V}<N> \qquad \kappa_2=\frac{1}{V}<(\delta N)^2>$$

$$\kappa_3=\frac{1}{V}<(\delta N)^3> \qquad \kappa_4=\frac{1}{V}(<(\delta N)^4>-3<(\delta N)^2>^2)$$

Net charge Moments with Volume fluctuations

Denote P(V) the probability distribution of volume V then

Moments of volume fluctuations: $$<V^n>=\int V^n P(V)dV$$

Cumulants of volume fluctuations: $$v_2=\frac{1}{V}<(\delta V)^2>=<V^2>-<V>^2,\ldots v_n=\ldots$$

Moments of net charge for fluctuating volume:

$$<N^k>_V=\int dV P(V)\sum_{N=-\infty}^{N=\infty} N^k P(N,V)$$

Corresponding Cumulants: $$c_2=<(\delta N)^2>_V,\ldots c_n=\ldots$$

Volume fluctuations and the higher order cumulants for Skellam distribution

apply generating function for $I_n(x)$: $e^{\frac{z}{2}(t+\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n I_n(z)$;

apply the relation : $\left(t\frac{\partial}{\partial t}\right)^k e^{\frac{z}{2}(t+\frac{1}{t})} = \sum_{n=-\infty}^{\infty} n^k t^n I_k(z)$

Get cumulants that include volume fluctuations:

$$c_2 = \kappa_2 + \kappa_1^2 v_2$$

$$c_3 = \kappa_3 + \kappa_1^3 v_3 + 3\kappa_2\kappa_1 v_2$$

$$c_4 = \kappa_4 + \kappa_1^4 v_4 + 6\kappa_2\kappa_1^2 v_3 + 4\kappa_3\kappa_1 v_2 + 3\kappa_2^2 v_2$$

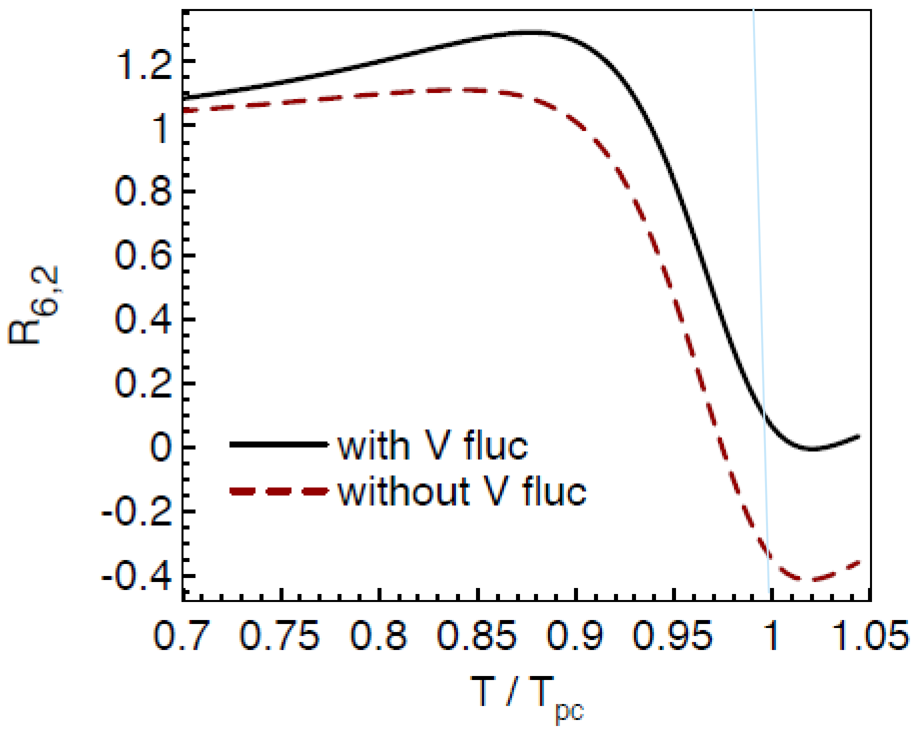
For $v_n = 0$ one recovers cumulants for the fixed volume

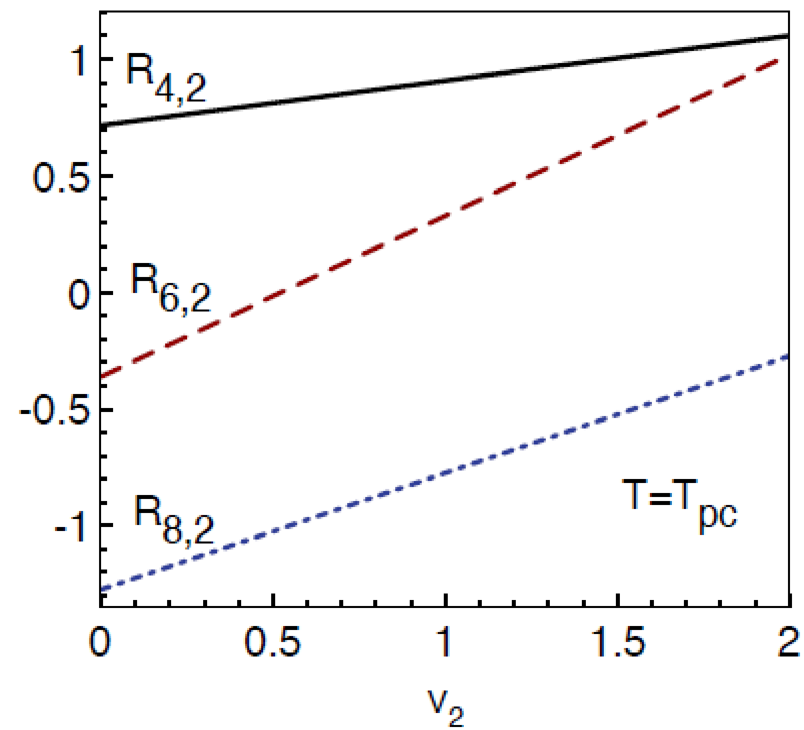
Volume fluctuations and higher order cumulants in PQM model in FRG approach

General expression for any P(N): $B_{n,i}$ - Bell polynomials

$$c_n = \sum_{i=1}^{n} v_n B_{n,i}(\kappa_1, \kappa_2, \cdots, \kappa_{n-i+1});$$

V. Skokov, B. Friman & K.R.



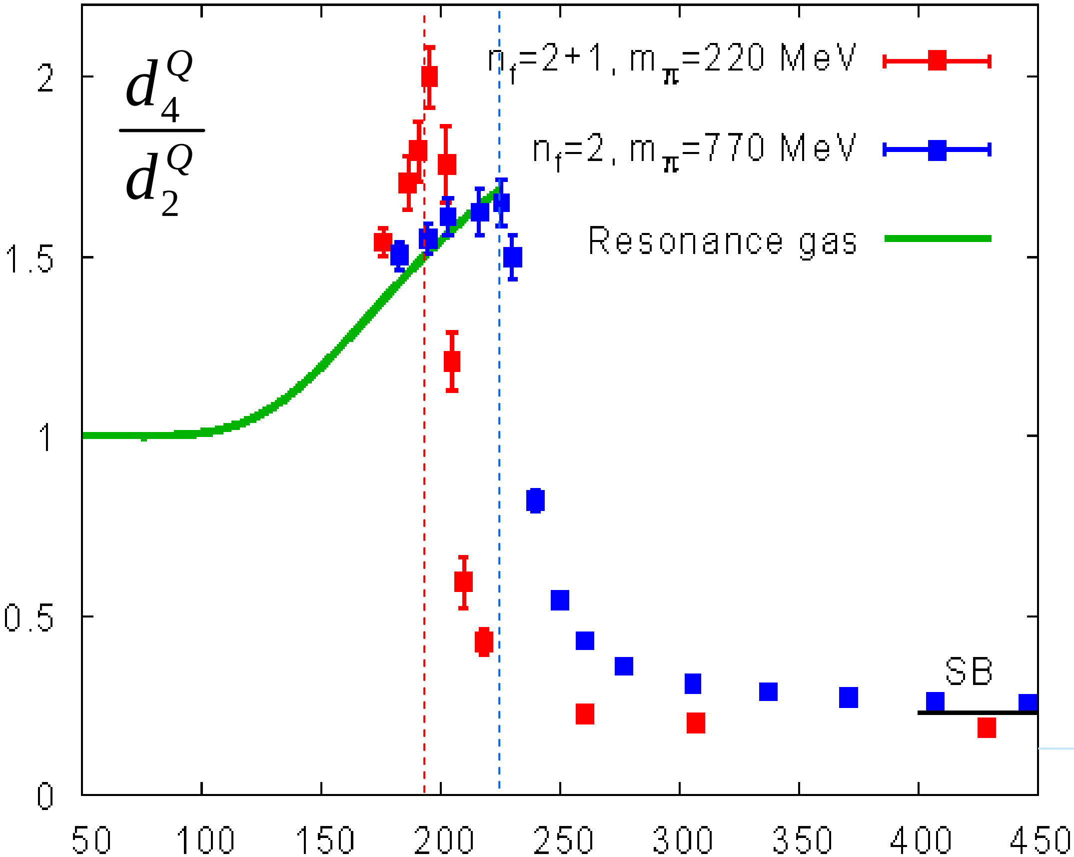


Conclusions:

- Hadron resonance gas provides reference for O(4) critical behavior in HIC and LGT results
- Probability distributions and higher order cumulants are excellent probes of O(4) criticality in HIC
- Observed deviations of the χ_6 / χ_2 from the HRG as expected at the O(4) pseudo-critical line
- Shrinking of P(N) from the HRG follows expectations of the O(4) criticality

Ratios of cumulants d_4^Q / d_2^Q *with* $d_n^Q = \frac{\partial^n \ln Z}{\partial(\mu_Q / T)^n}\Big|_{\mu=0}$

reflect carriers of the net-quark and charge



$$R_{4,2}^Q = \frac{4G^{(3)} + 3F^{(2)} + 27F^{(4)}}{4G^{(3)} + 3F^{(2)} + 9F^{(4)}}$$

$$G, F = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2(m_i / T)$$

$$R_{4,2}^Q(T)|_{\text{pert.th.}} = \frac{34}{15\pi^2} + \mathrm{O}(g^3)$$

$$R_{4,2}^Q \equiv \frac{d_4^Q}{d_2^Q} = \frac{\langle(\delta Q)^4\rangle}{\langle(\delta Q)^2\rangle} - 3\langle(\delta Q)^2\rangle$$