Shock wave modelling via Kinetic Theory

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Work together with:

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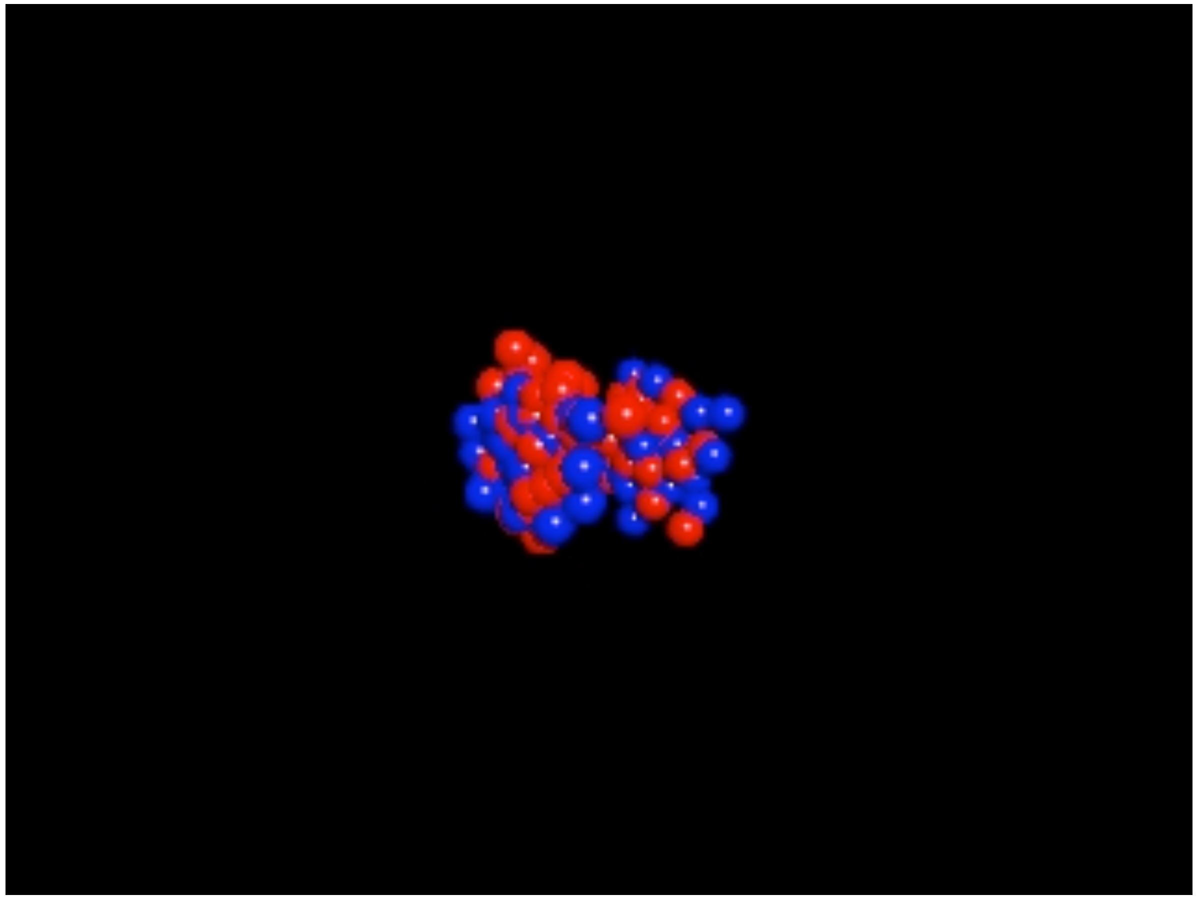
- Dirk Colbry
- Rodney Pickett
- Terrance Strother

Kinetic theory



Ca+Ca collisions with GiBUU code

- Study macroscopic properties of matter in terms of the microscopic particles of which it is composed
- E.g. rarified gas flows, heavy ion collisions
- Numerical tools: Molecular Dynamics, Direct Simulation Monte Carlo, ...
- Solve transport equations (e.g. Boltzmann Eq.)
- $(\partial_t + \mathbf{v} \cdot \nabla_r + \mathbf{F}/m \cdot \nabla_v) f(\mathbf{r},\mathbf{v},t)$

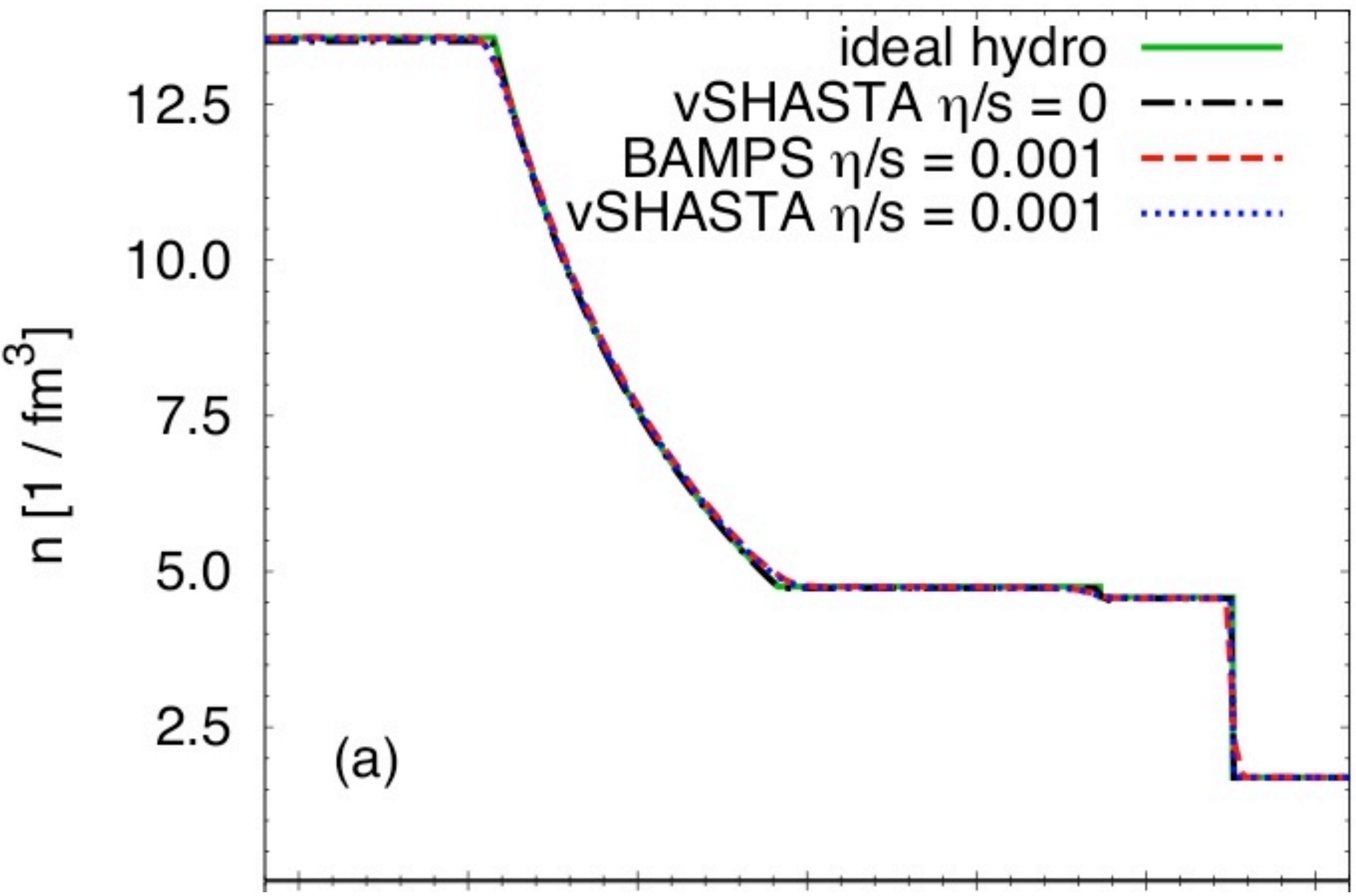
  $= [\partial_t f(\mathbf{r},\mathbf{v},t)]_{Coll}$
- Test particle method:

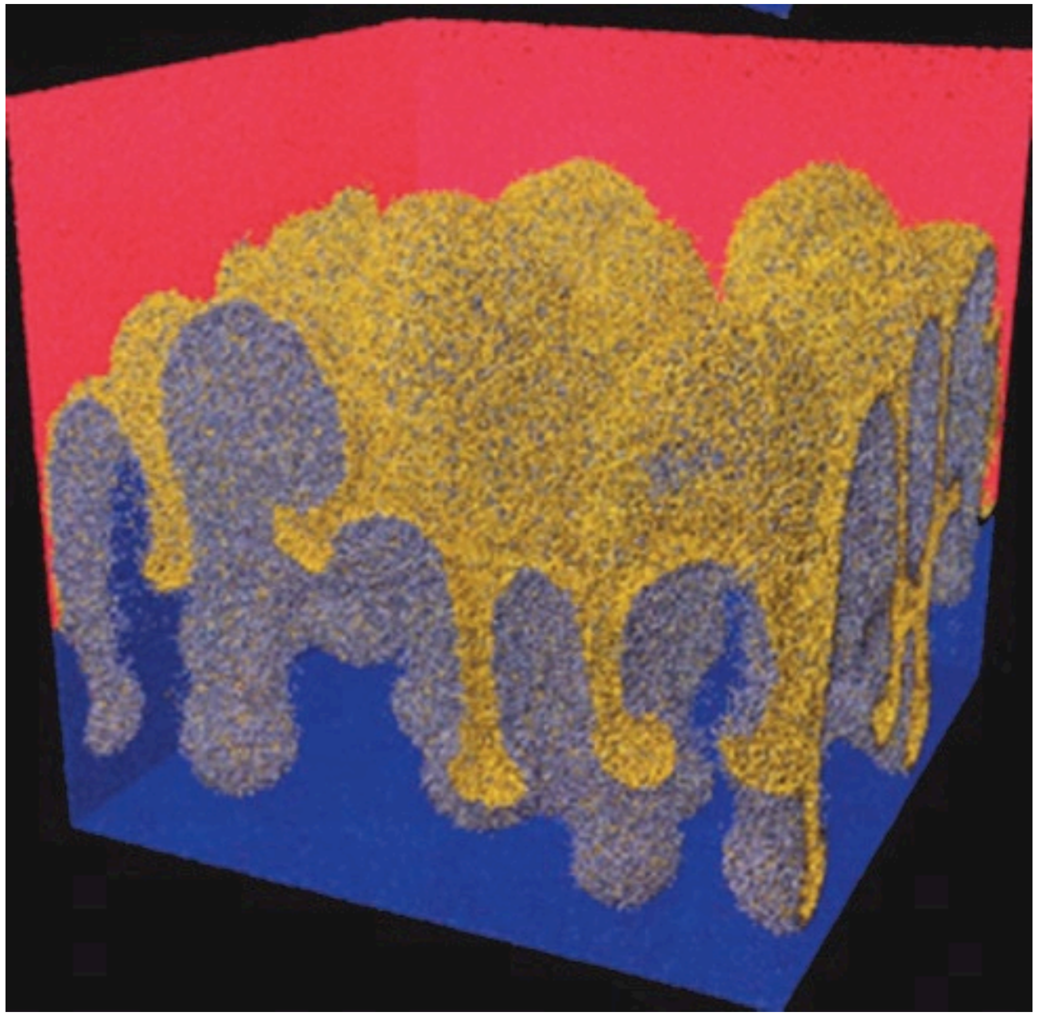
  $f(\mathbf{r},\mathbf{v},t) = \sum \delta^3(\mathbf{r} - \mathbf{r}_i(t))\, \delta^3(\mathbf{v} - \mathbf{v}_i(t))$

$$\frac{\partial f_b(xp)}{\partial t} + \frac{\Pi^i}{E_b^*(p)} \nabla_i^x f_b(xp) - \frac{\Pi^\mu}{E_b^*(p)} \nabla_i^x U_\mu(x) \nabla_p^i f_b(xp) + \frac{M_b^*}{E_b^*(p)} \nabla_i^x U_s \nabla_p^i f_b(xp) = I_{bb}^b(xp) + I_{b\pi}^b(xp)$$

$$\frac{\partial f_\pi(xk)}{\partial t} + \frac{\vec{k} \cdot \vec{\nabla}^x}{E_\pi(k)} = I_{b\pi}^\pi(xk)$$

Hydrodynamic limit

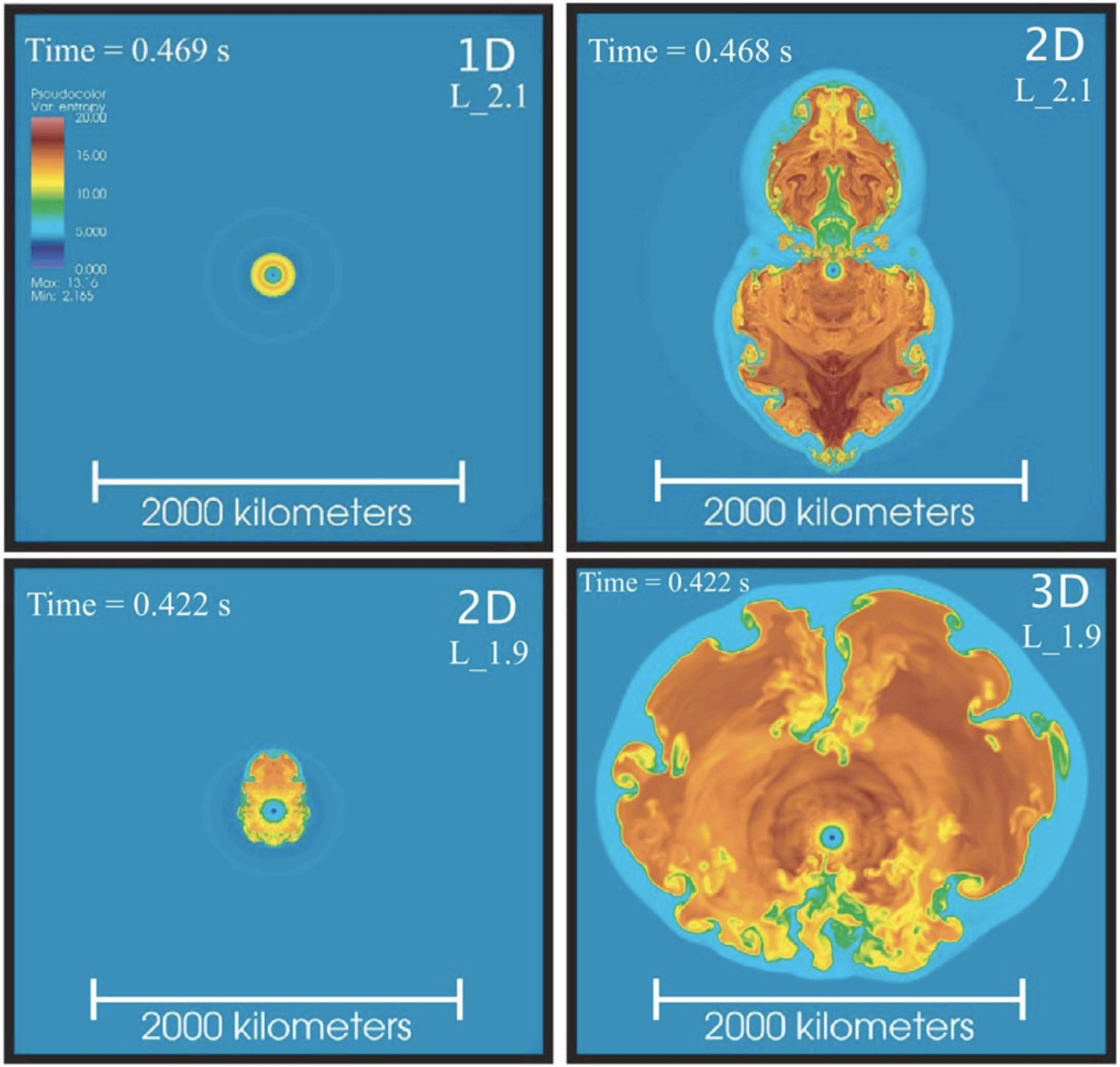




- In the limit of small Knudsen number: K = λ/L
- Transport models can reproduce hydrodynamic behavior, e.g.:
  - Hydrodynamic shocks
  - Fluid instabilities

Figures: Bouras et al., PRC 82, 2 (2010), Kadau et al. PNAS 101, 16 (2004)

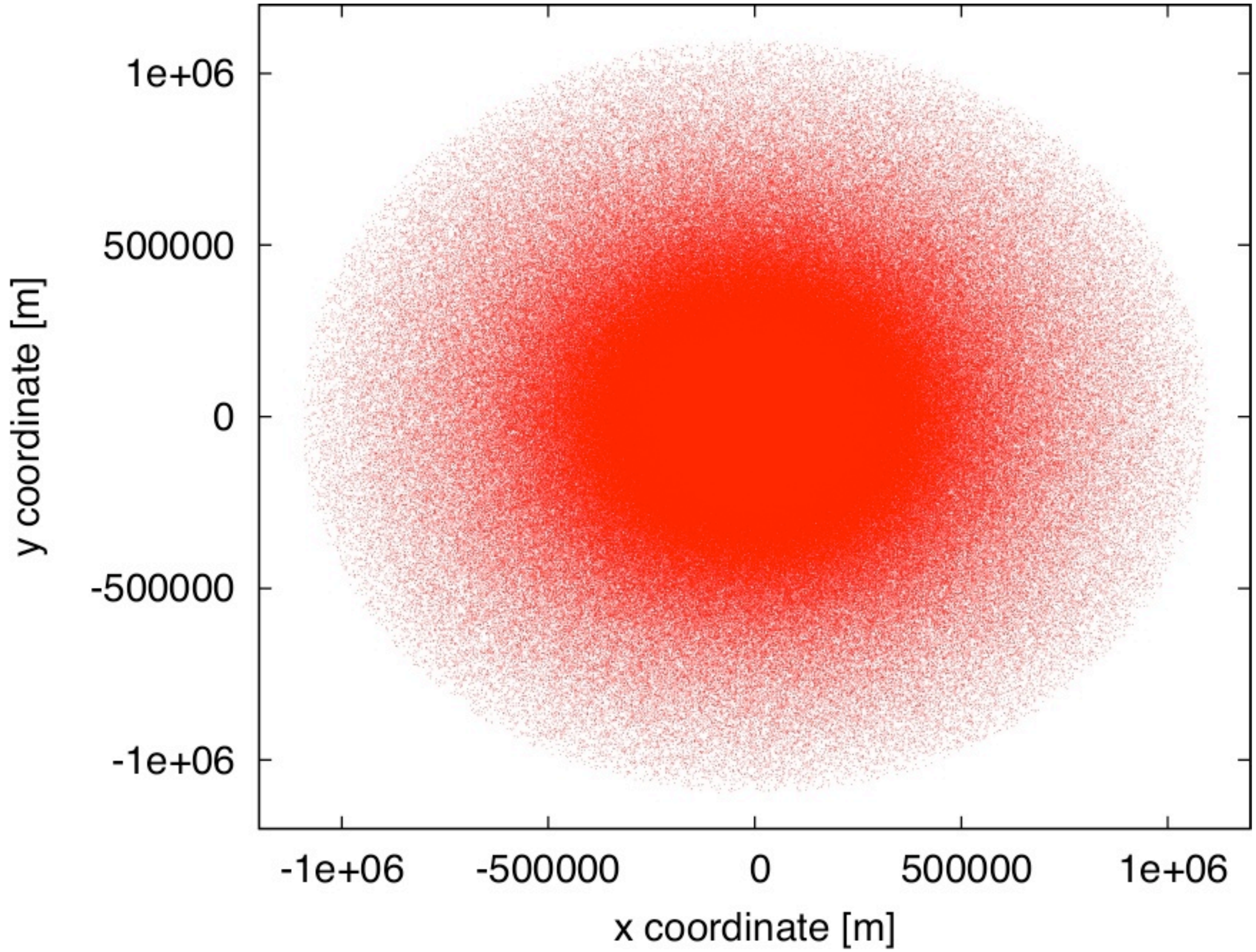
Core collapse supernova



Nordhaus et al., ApJ 720, (2010)

Kinetic Approach

- Describe collapse and explosion of star‘s iron core with transport model
- $\sim 10^6$ matter/baryon test particles & neutrino test particles
- $\sim 10^{51}$ baryons/test particle
- Transport equations similar to baryons and pions in heavy-ion collisions



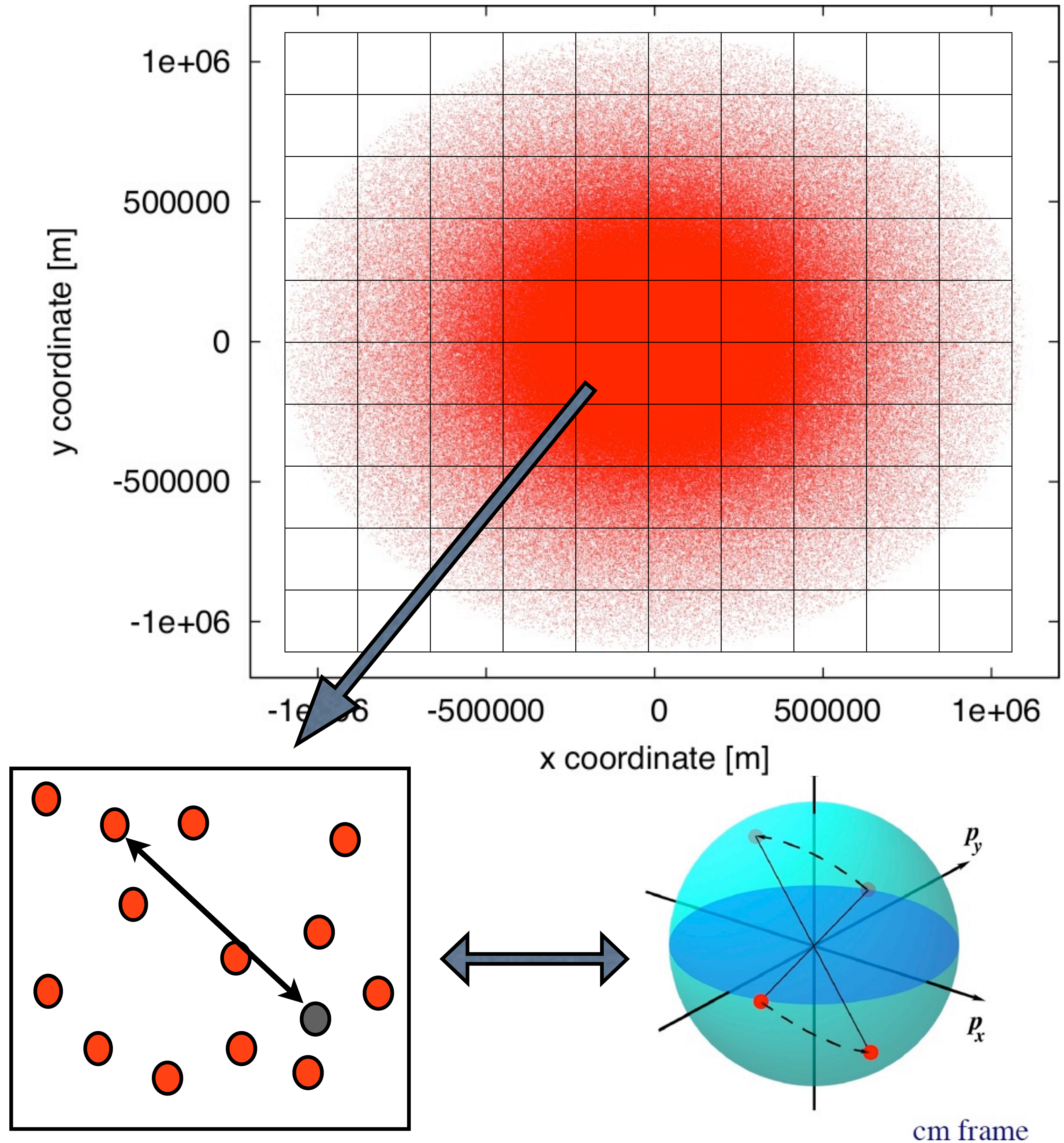
$$\frac{d}{dt}\mathbf{p}_j = -\nabla U_{EoS,e^-}(\vec{r}_j) + \mathbf{F}_{Grav(j)} + \mathbf{F}_{Coll}(\mathbf{p}_j)$$

$$\frac{d}{dt}\mathbf{r}_j = \mathbf{p}_j / \sqrt{m^2 + p_j^2}$$

Strother & Bauer, Int. Journal Mod. Phys. D (2009),
Strother & Bauer, Journal of Phys. 230 (2010)

Scattering of matter test particles

- Collisions by Direct Simulation Monte Carlo
- Randon choice of scattering partners in a cell
- Collision is performed in the Center-of-Mass frame
- Random choice for orientation of outgoing velocity vector



Strother & Bauer, Int. Journal Mod. Phys. D (2009),
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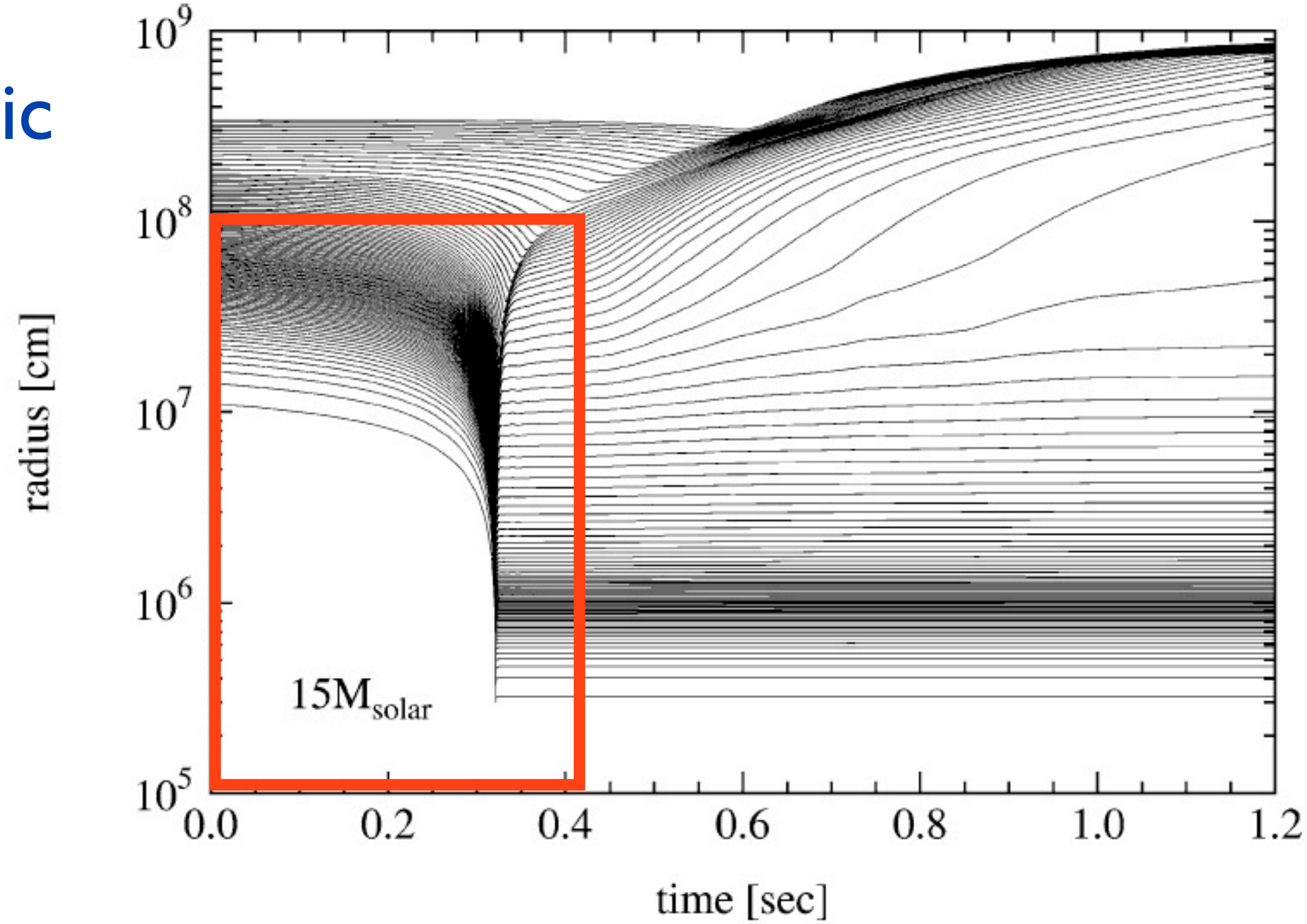
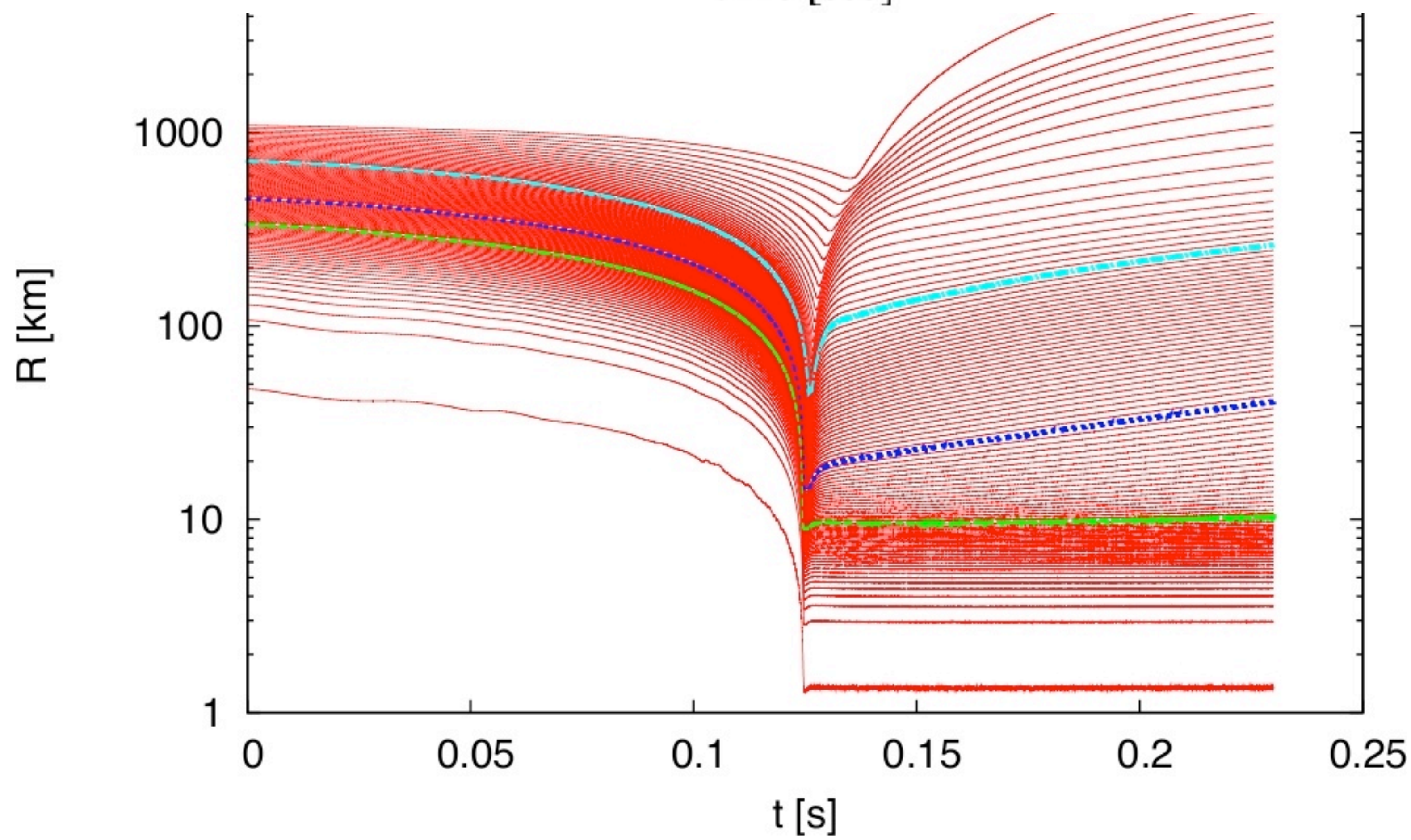
Comparison to hydrodynamic simulations

- Collapse of the iron core of a 15 solar mass star
- General relativistic hydrodynamics in spherical symmetry
- Relativistic mean field equation of state
- No inclusion of neutrinos ➔ Only hydrodynamic evolution
- Similarities in collapse phase and shock formation

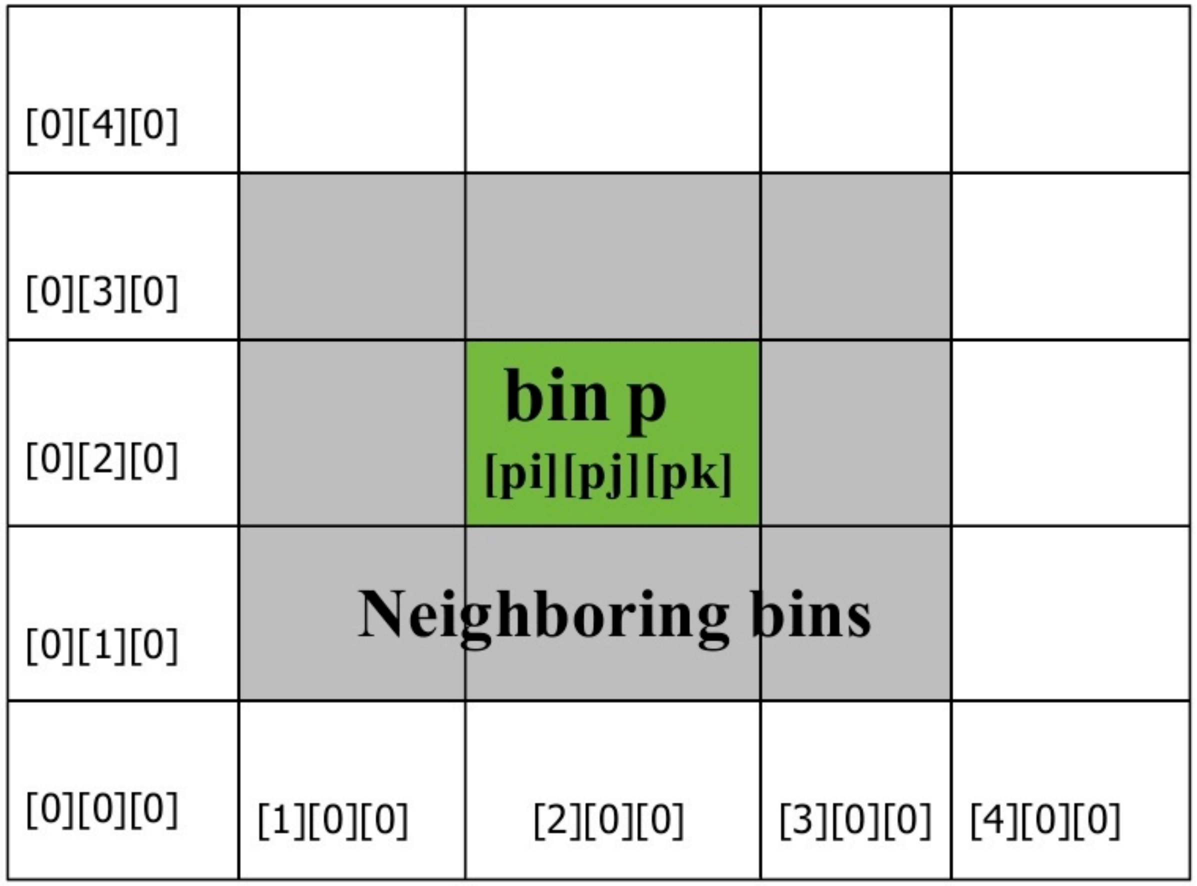


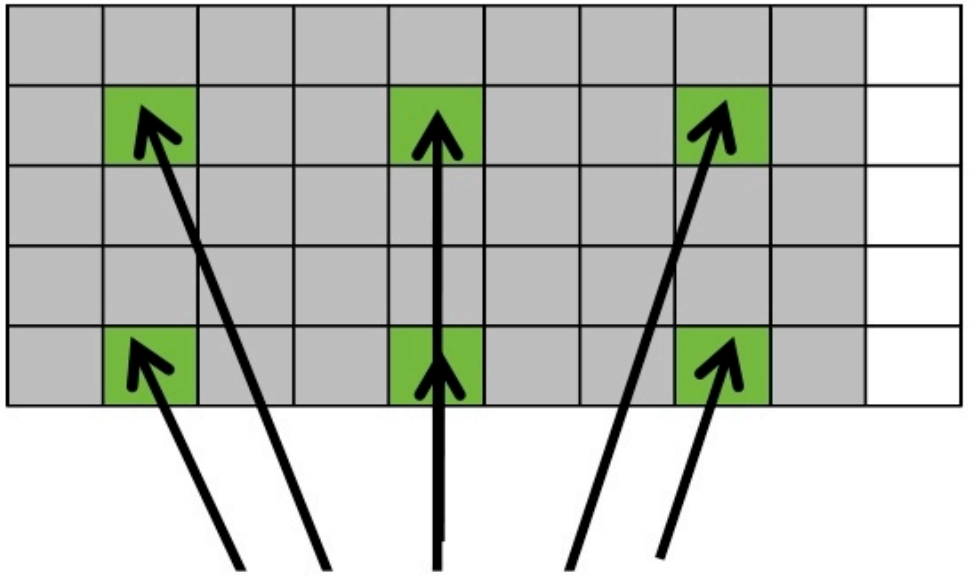


Top figure: Sumiyoshi et al. NPA 730 (2004)

New transport code setup

- Aim: Transport code that can handle >>10^6 test particles in a computationally efficient way
- Simulation space is divided into bins
- Scattering partner search is performed over the neighbouring cells (8 in 2D, 26 in 3D)
- Particle interaction range < bin size
- Scattering partner search can be performed in parallel
- Neighbouring bins should not overlap





Parallel neighbor search by 6 CPUs

Collision detection

1. Relative position change:

   $\mathbf{r}_{rel}(t) = \mathbf{r}_A(t) - \mathbf{r}_B(t)$,

   $\mathbf{v}_{rel}(t) = \mathbf{v}_A(t) - \mathbf{v}_B(t)$

   $C = (\mathbf{r}_{rel}(t) \cdot \mathbf{v}_{rel}(t))\,(\mathbf{r}_{rel}(t + \Delta t) \cdot \mathbf{v}_{rel}(t))$

   $C < 0$: Collision, $C > 0$: No collision

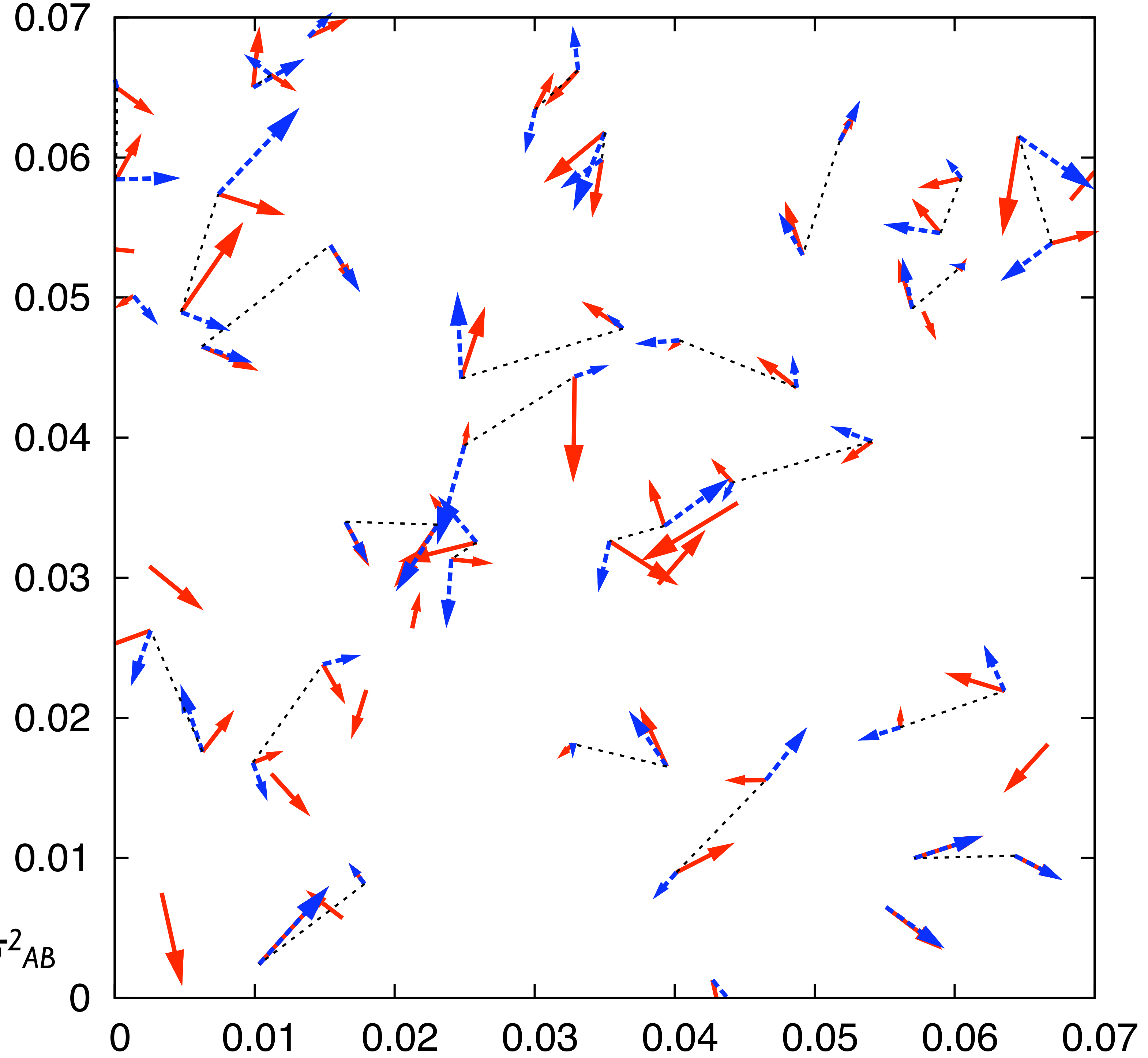
2. Real collision time:

   $|\mathbf{v}_{rel}|^2 t_c^2 + 2(\mathbf{v}_{rel} \cdot \mathbf{r}_{rel})\, t_c + |\mathbf{r}_{rel}|^2 = \sigma^2_{AB}$

   $\sigma_{AB} = r_A(\text{eff}) + r_B(\text{eff})$

   $r(\text{eff})_{3D} = (V_{bin}/(\pi \lambda N_{bin}))^{1/2}$, $r(\text{eff})_{2D} = A_{bin}/(2 \lambda N_{bin})$

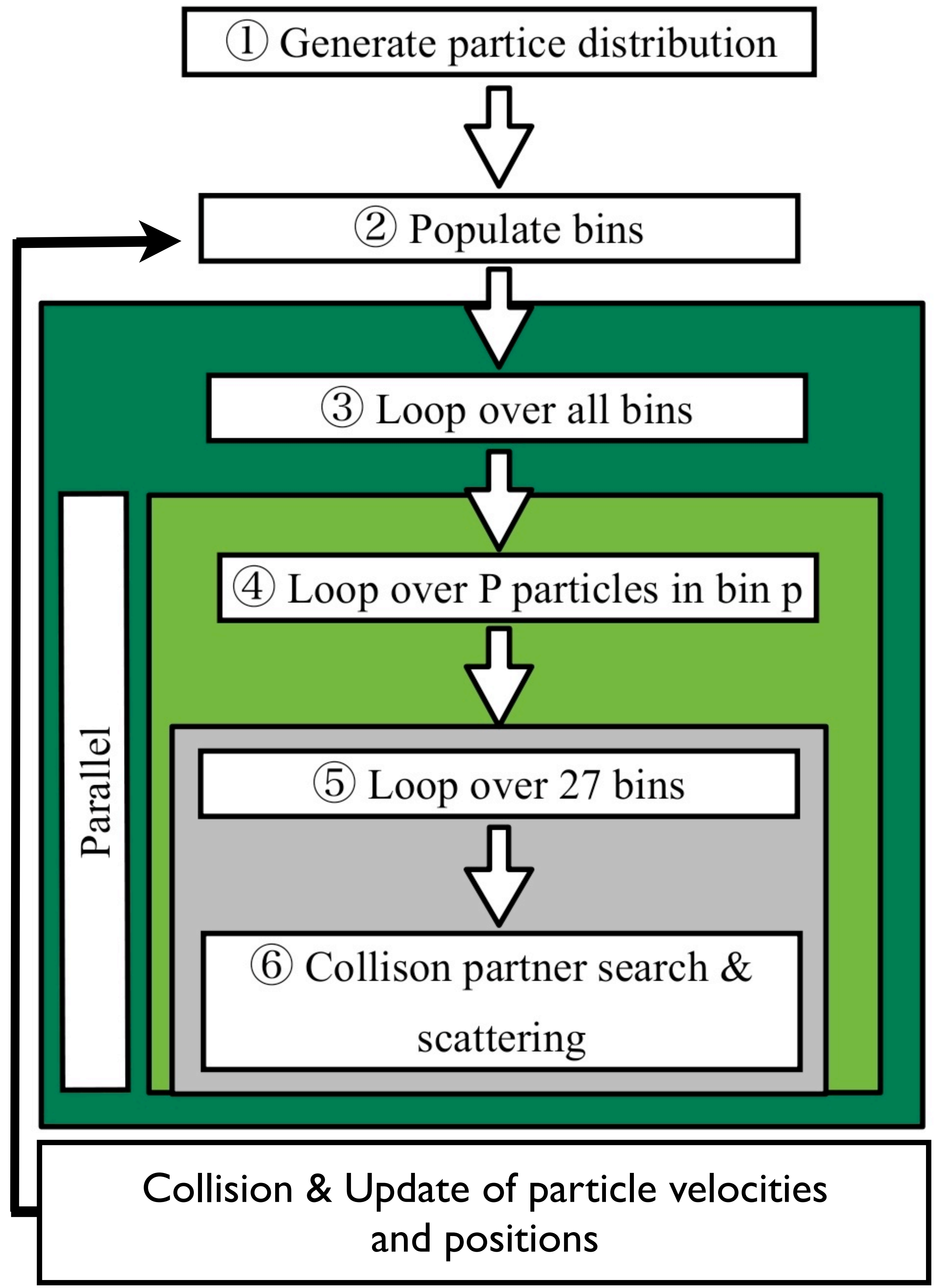
3. Final collision partner: Shortest collision time or smallest distance



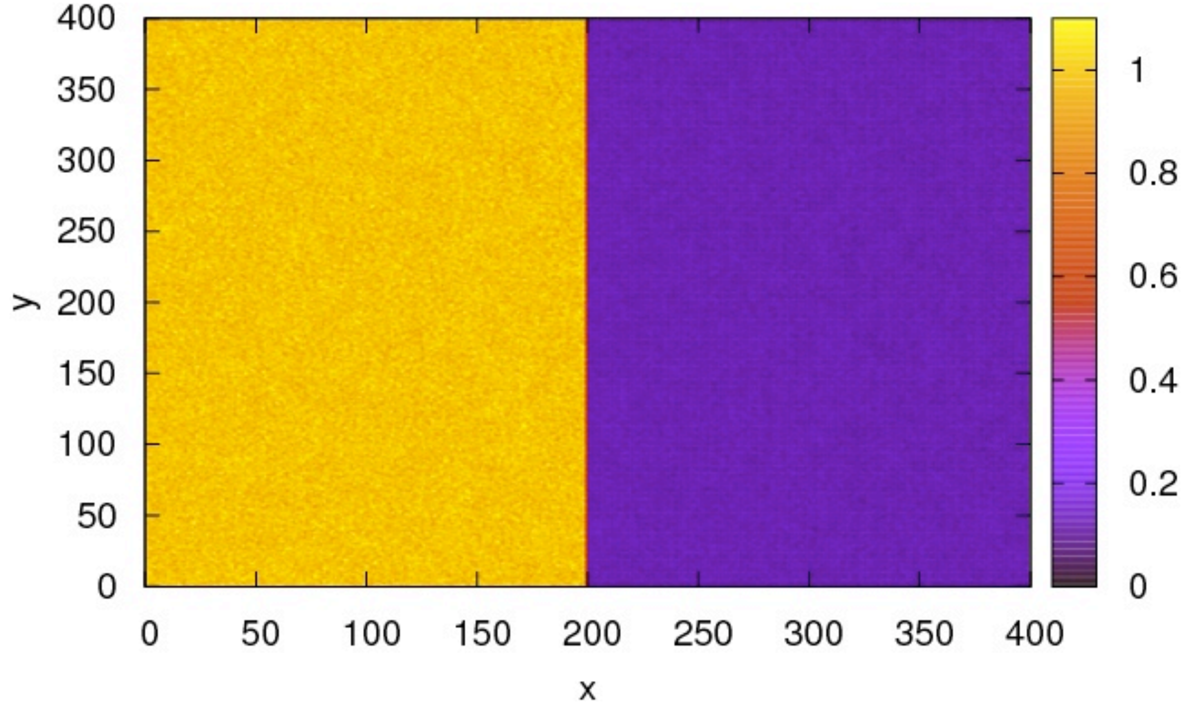
Simulation overview

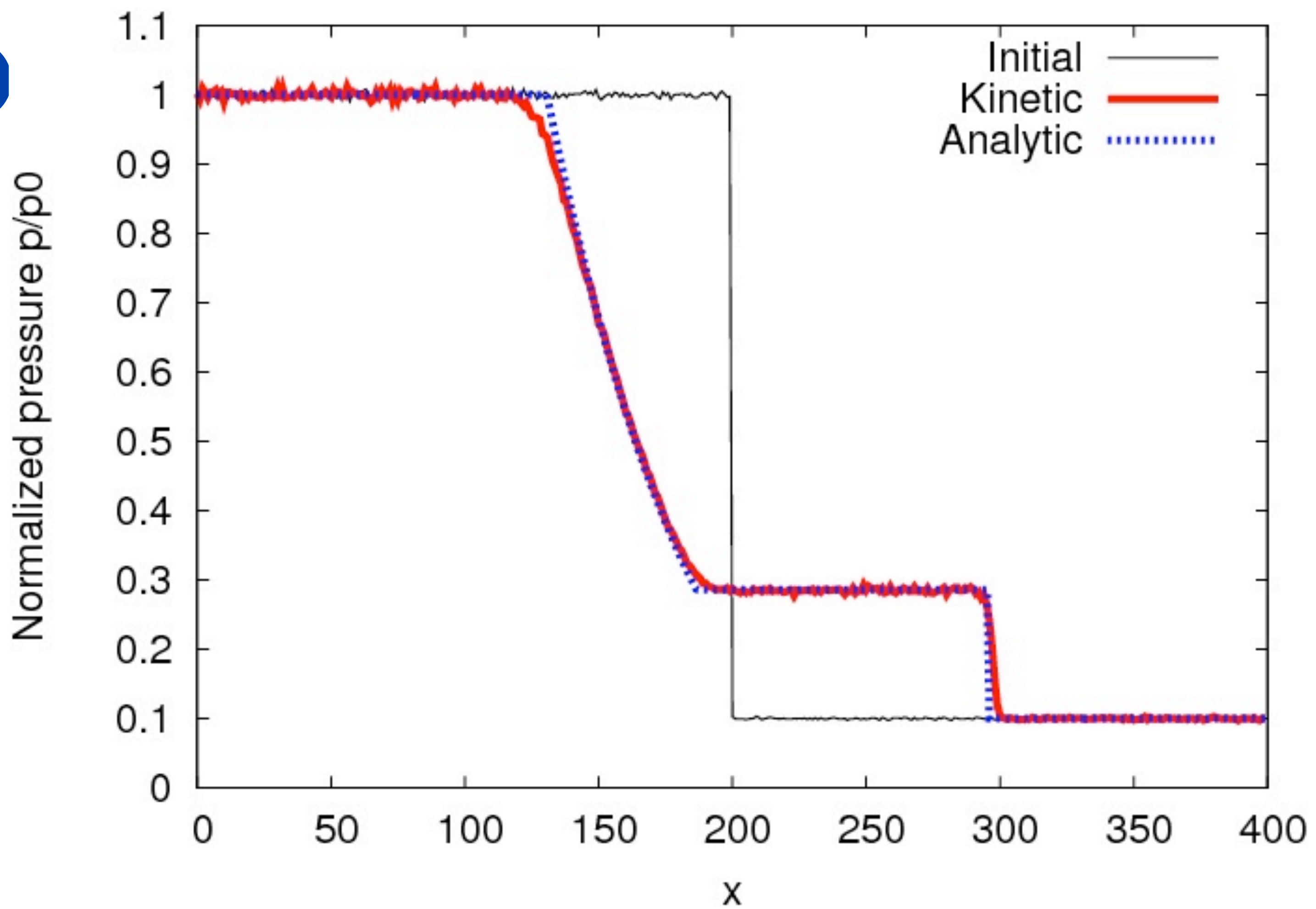
First tests:

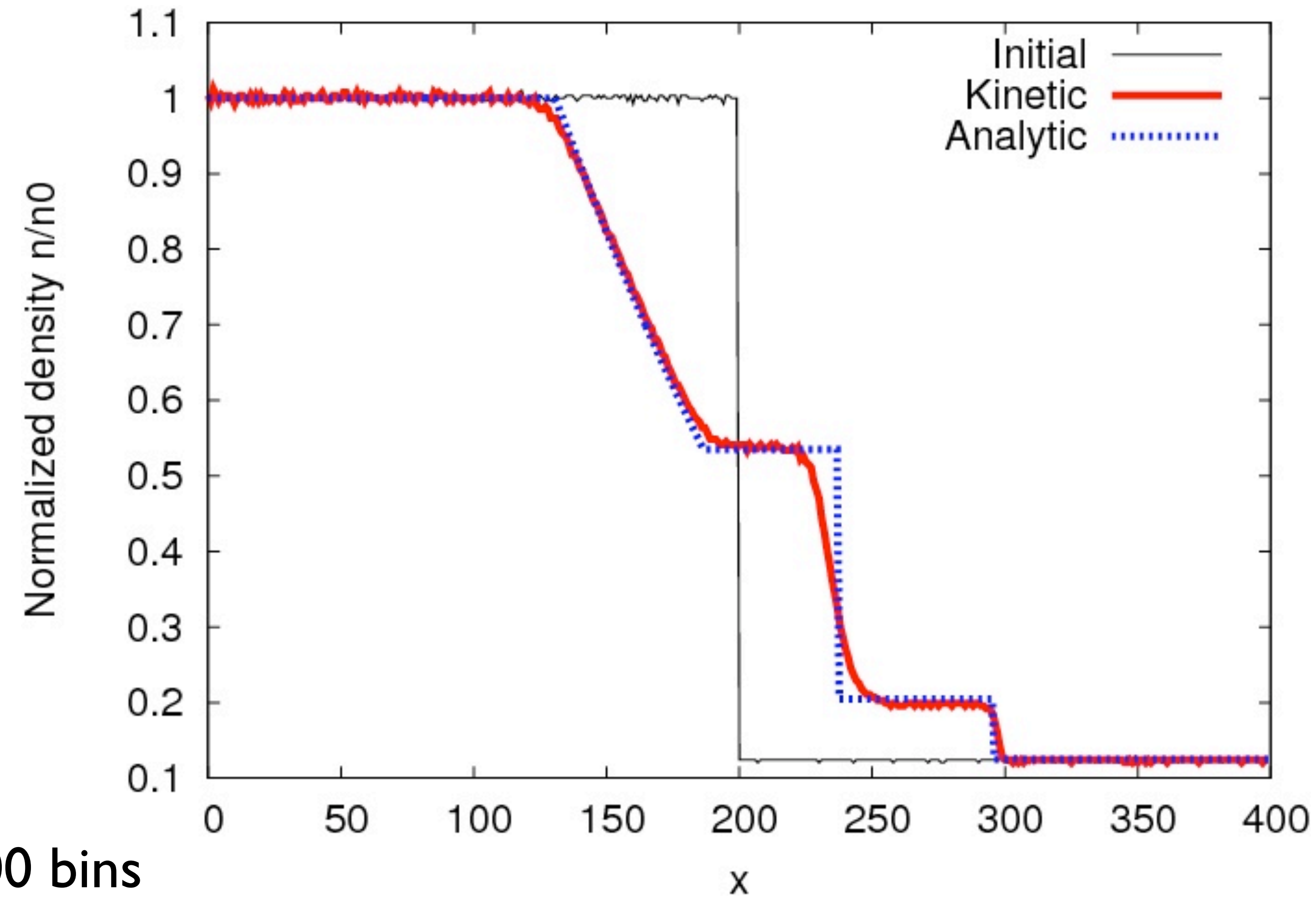
- Box with 10^6 - 10^8 test particles
- Reflective boundary conditions
- Interaction: Elastic scattering
- Degrees of freedom: Number of dimensions in the system



Sod shock tube test, 2D







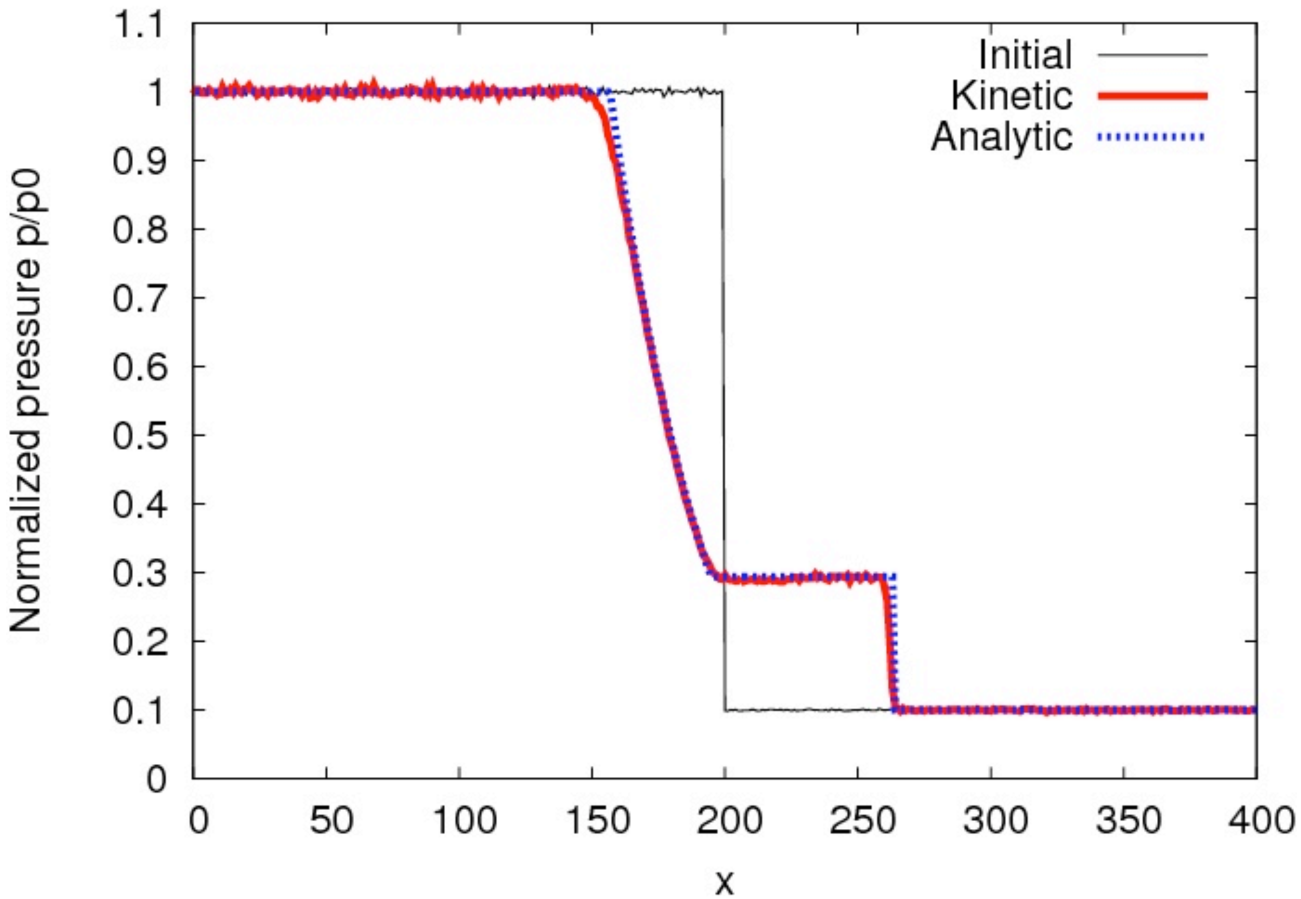
- Riemann problem
- Initial conditions:

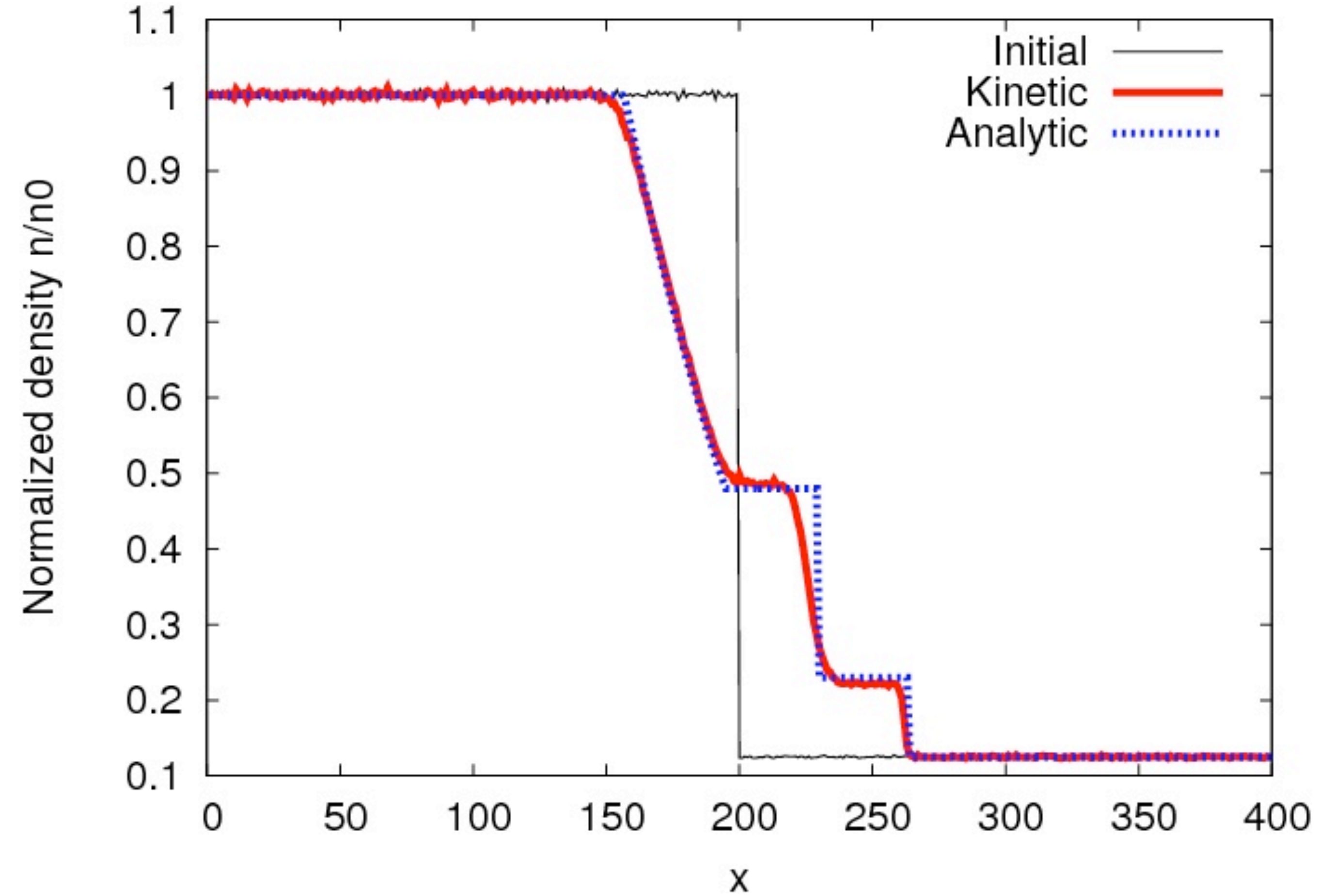
  $n_1 = 1, n_2 = 0.125, P_1 = 1, P_2 = 0.1,$
  $v_1=0, v_2 = 0$

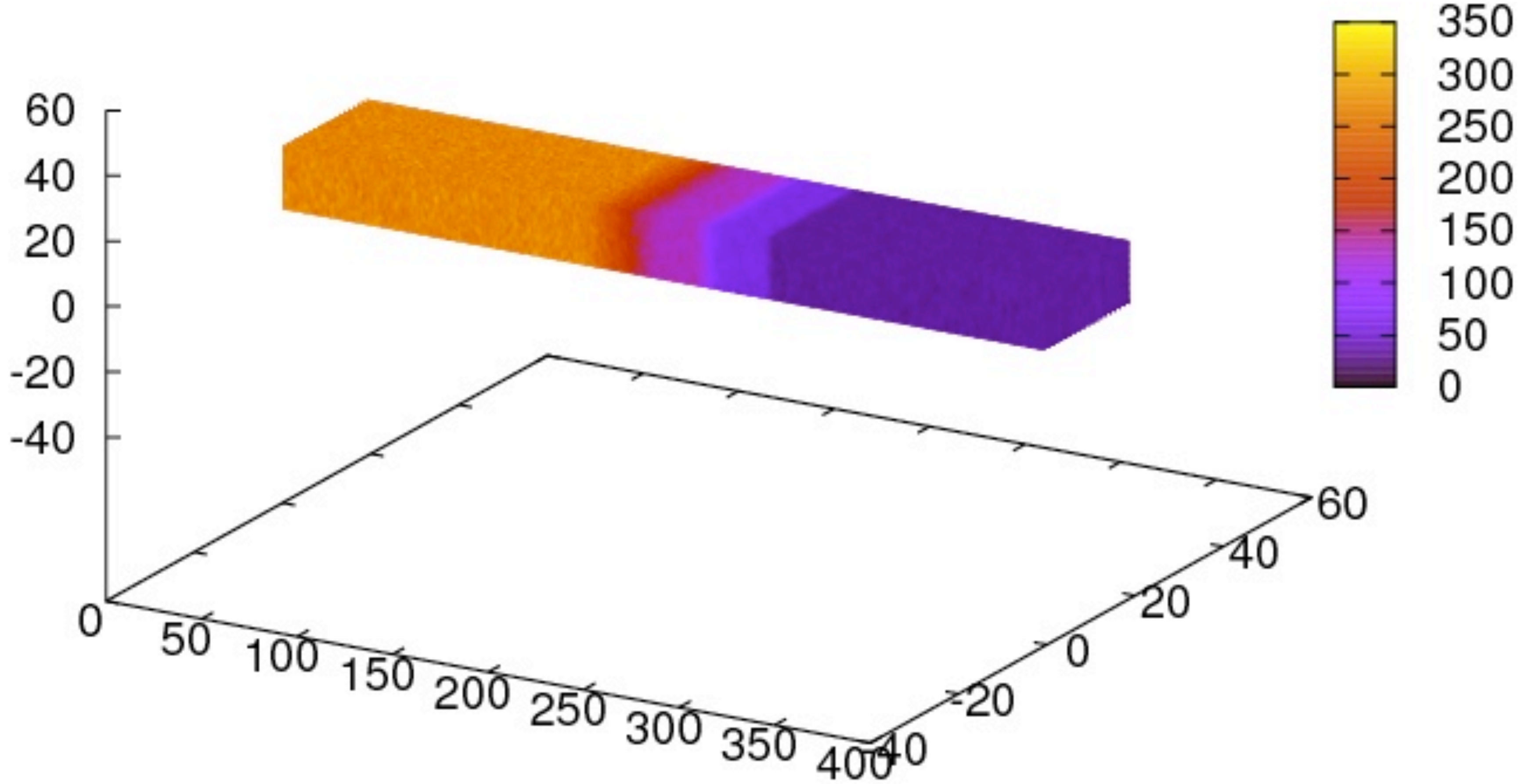
- Analytic solution: Shock front, contact discontinuity and rarefaction wave

N_{TP} =24 000 000, mfp = 0.01 bin-width, 400x400 bins

Sod shock tube test, 3D

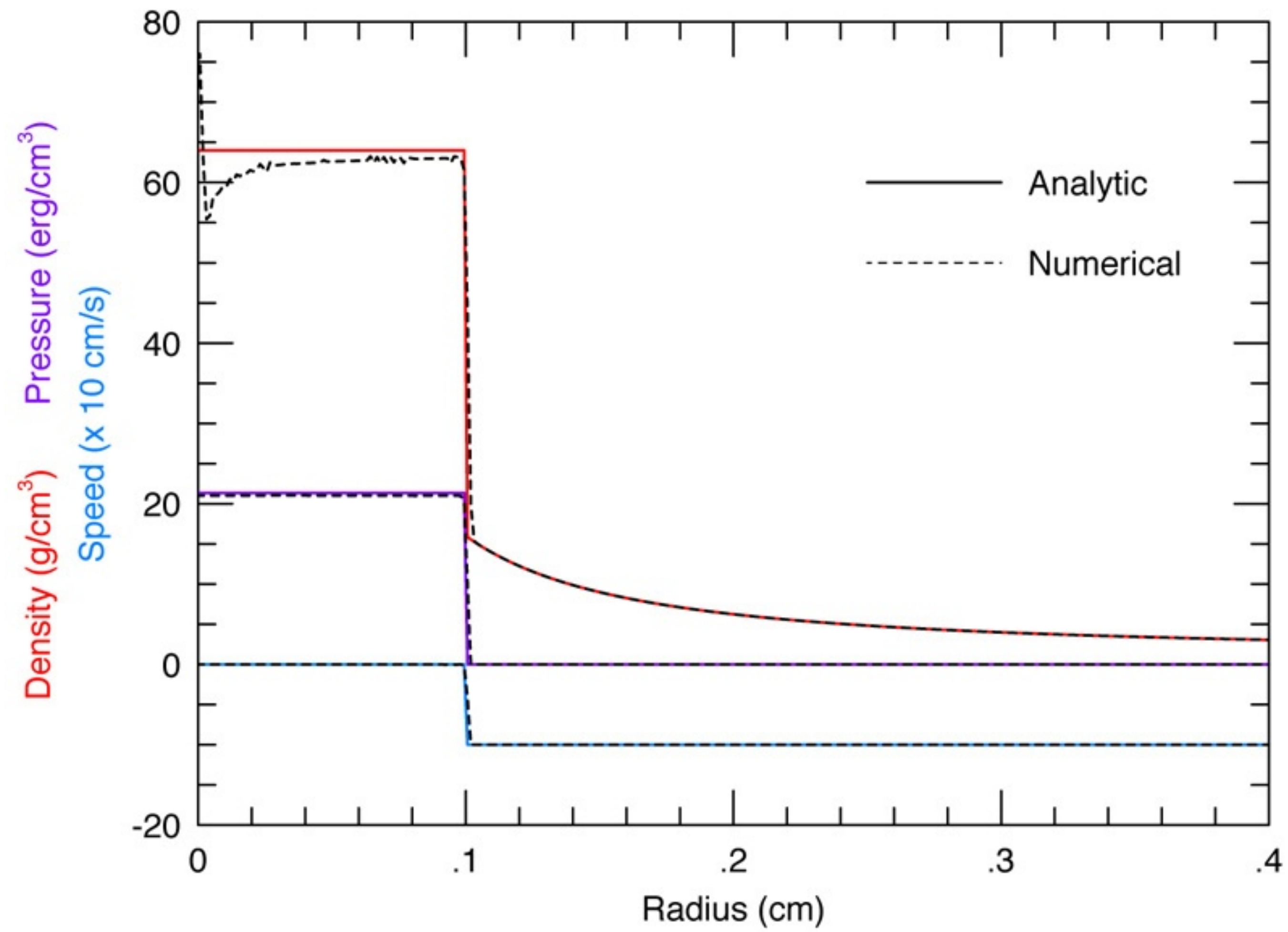


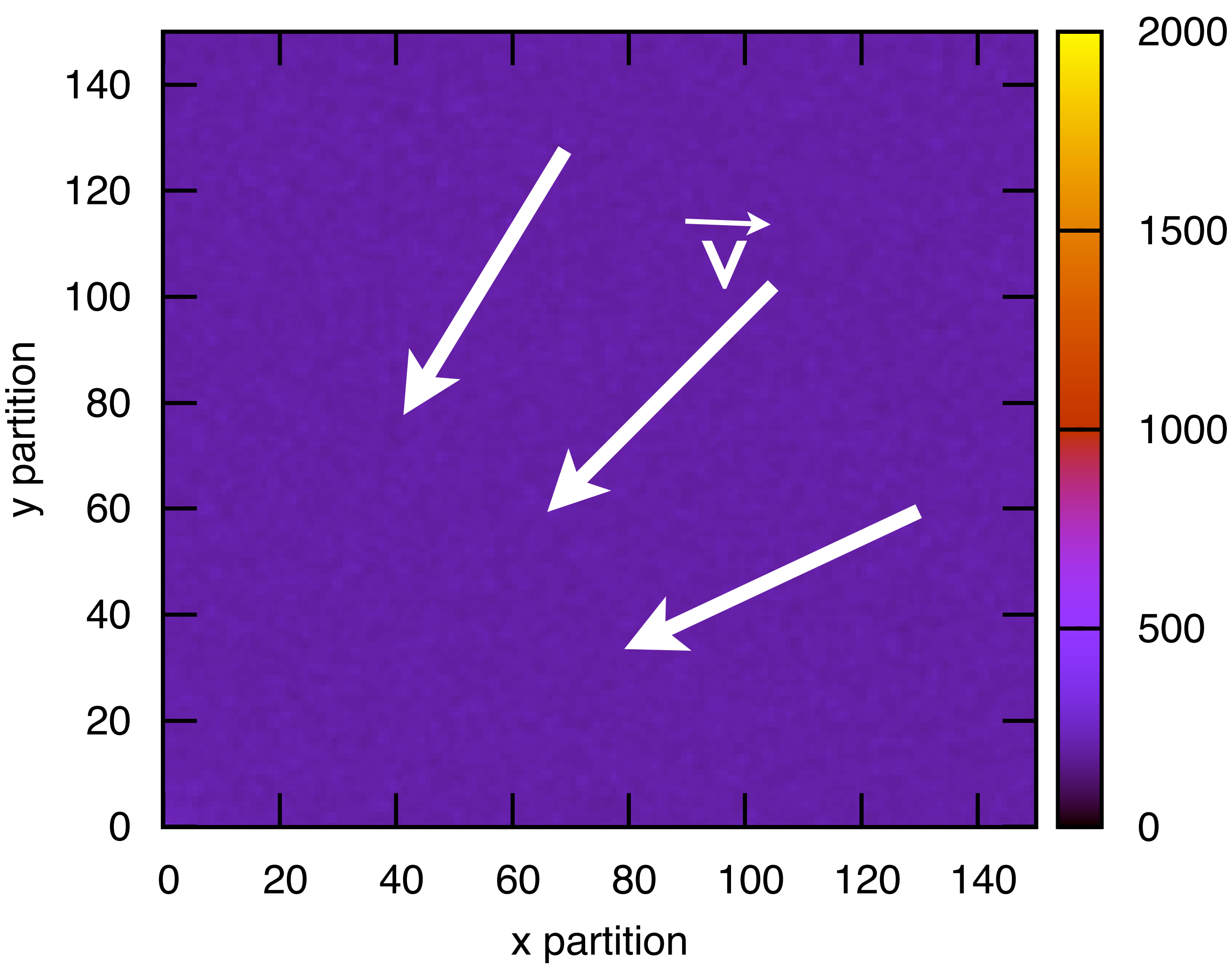


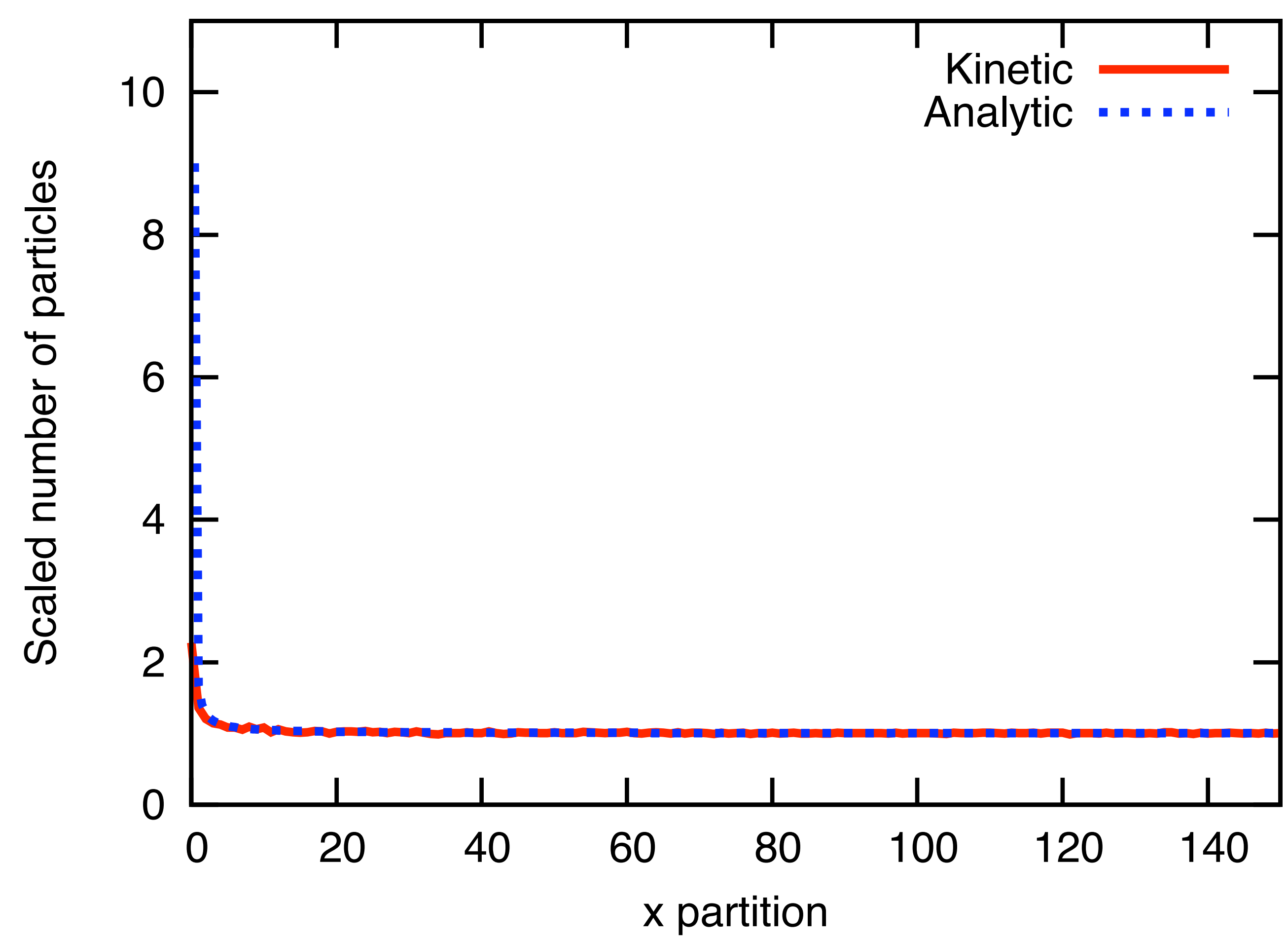


Noh test

- Cold, ideal gas with uniform, radially inward speed
- Shock form at the origin and propagates outward as gas stagnates
- Many hydro codes show anomalous wall-heating near the origin







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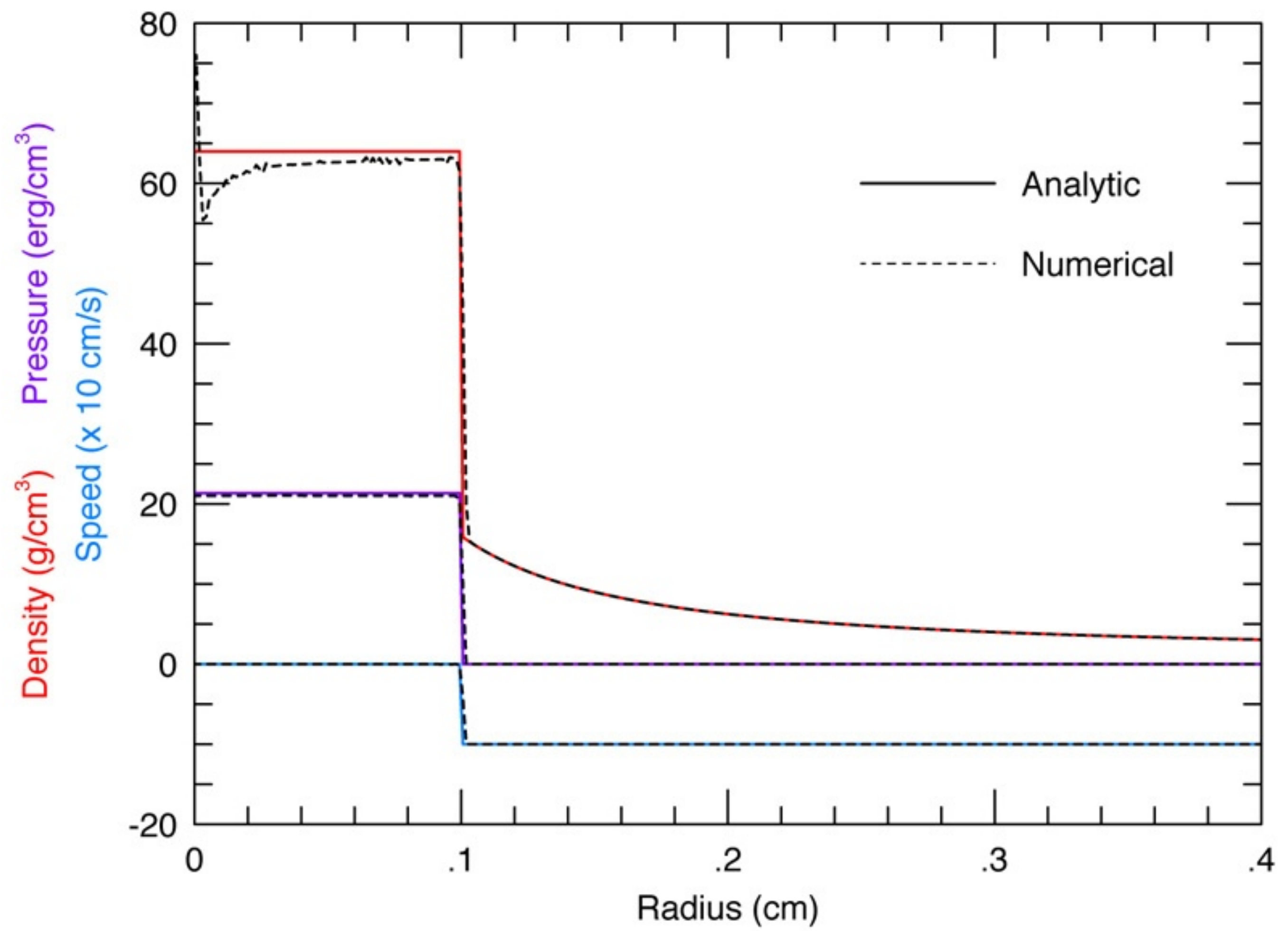
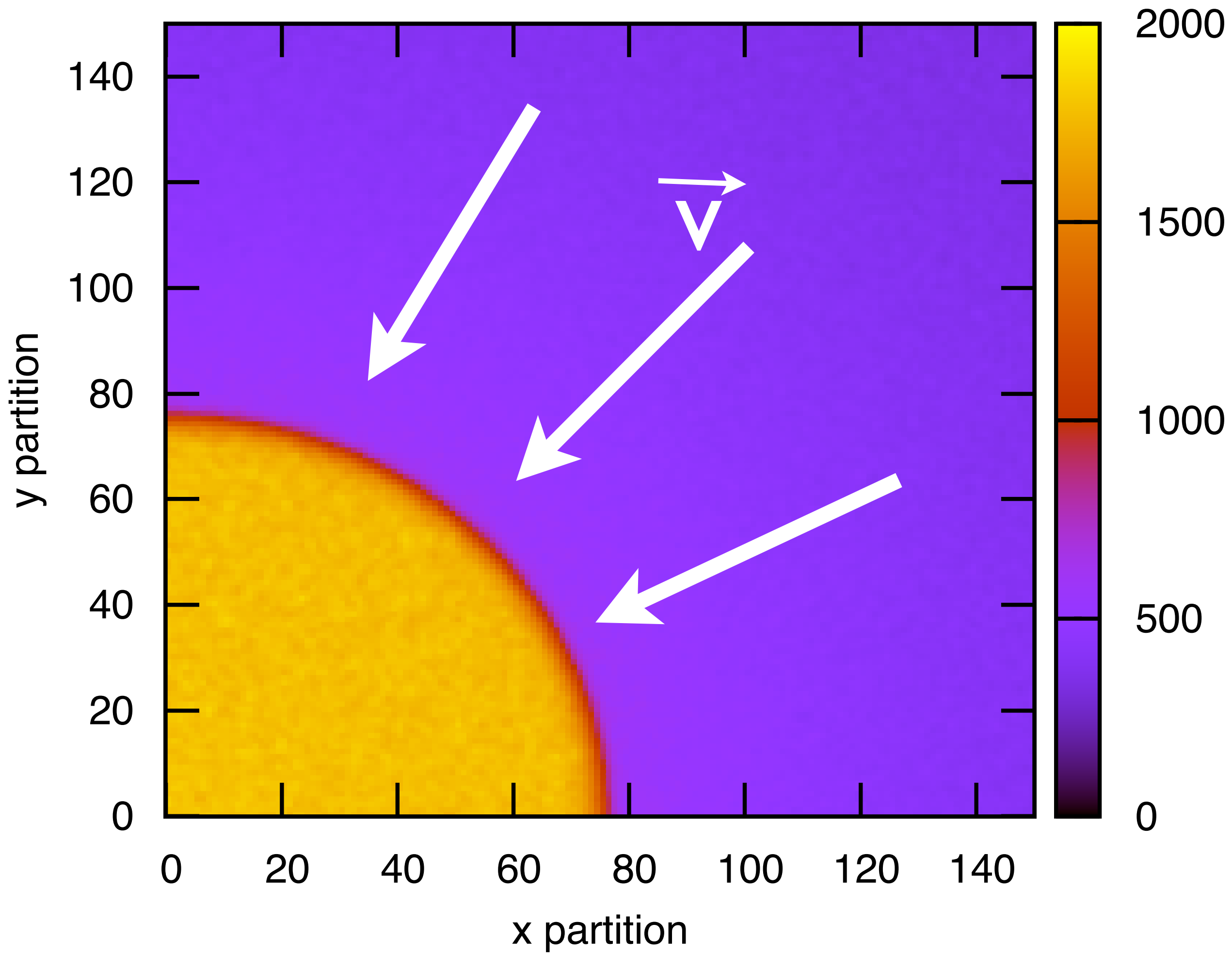
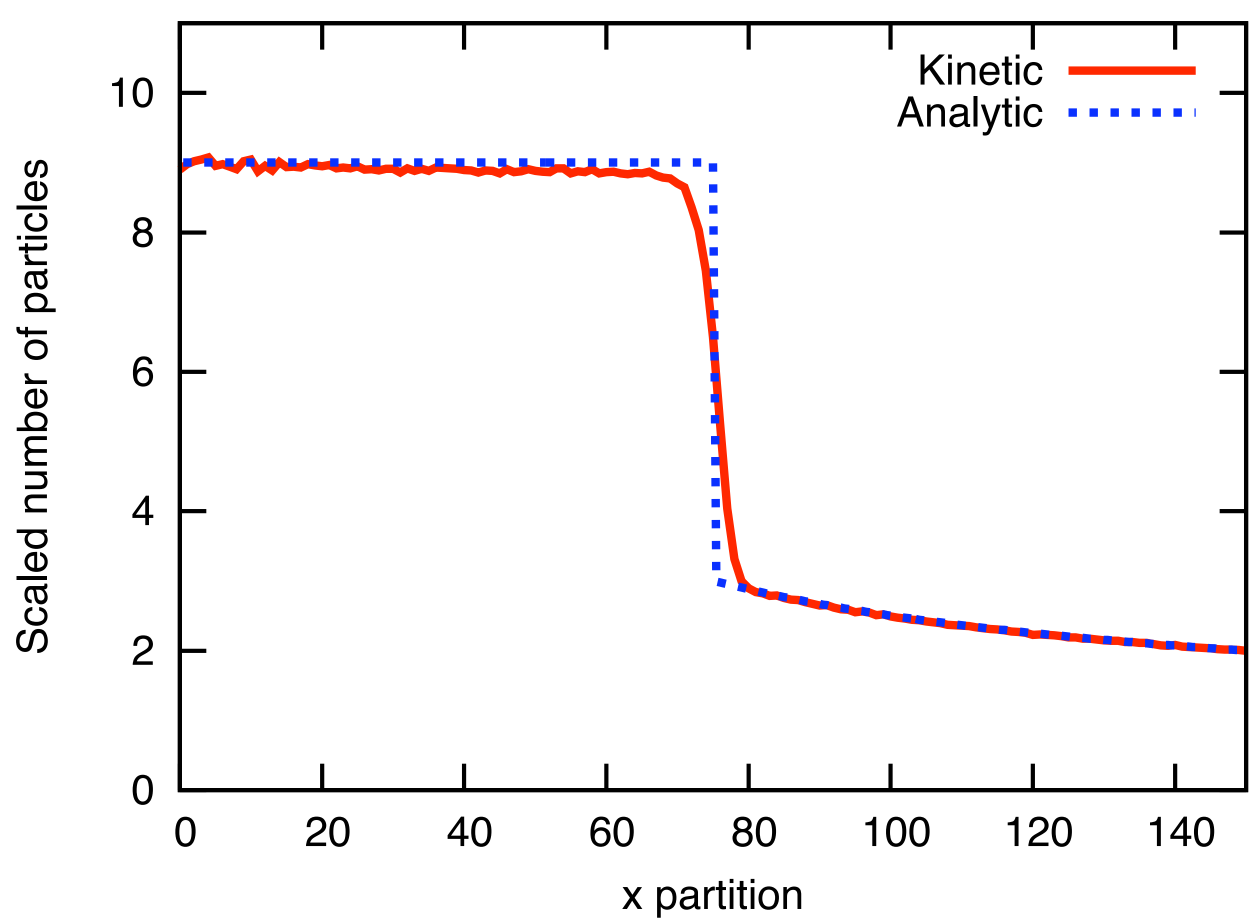
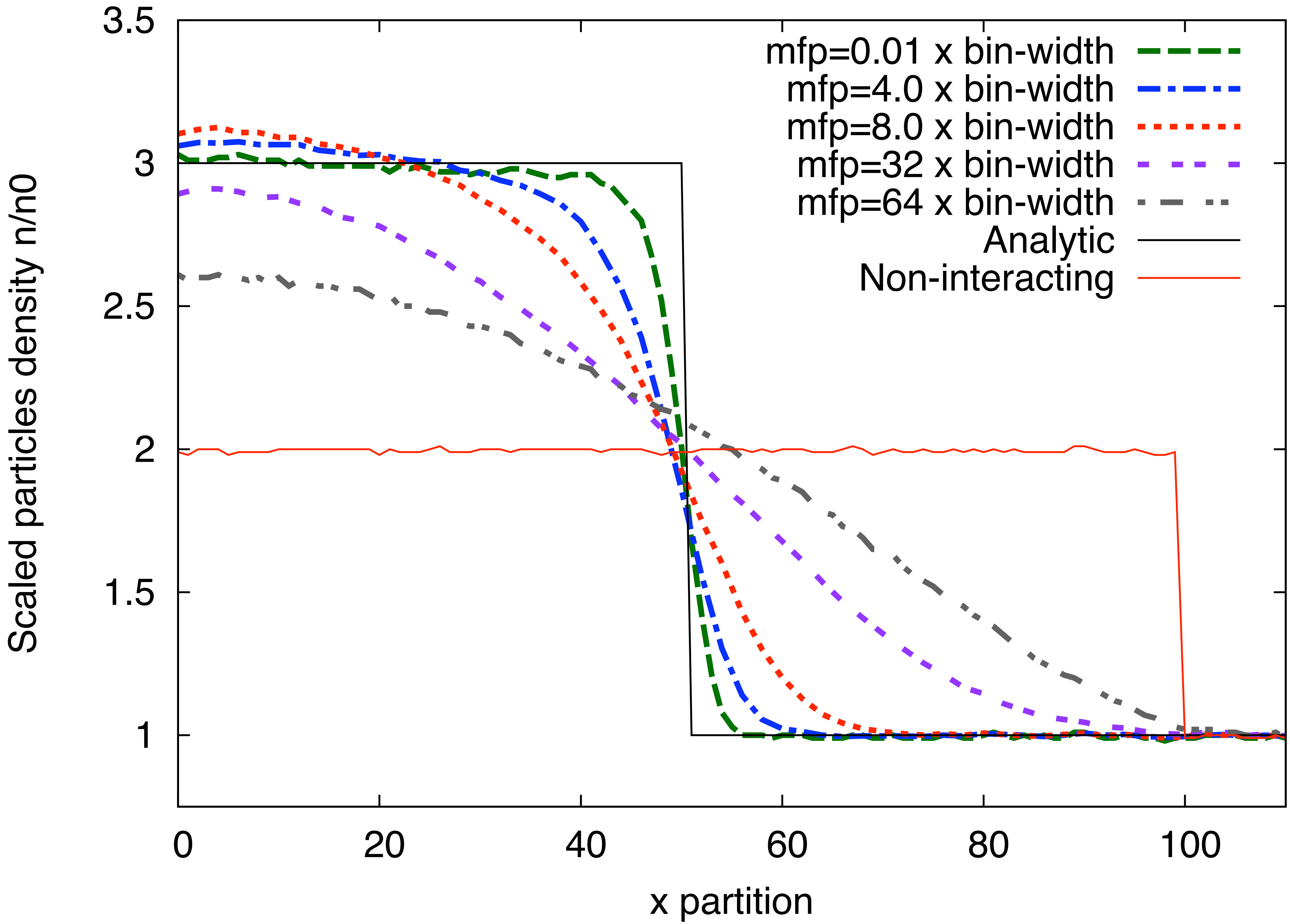






Noh test & mean free path

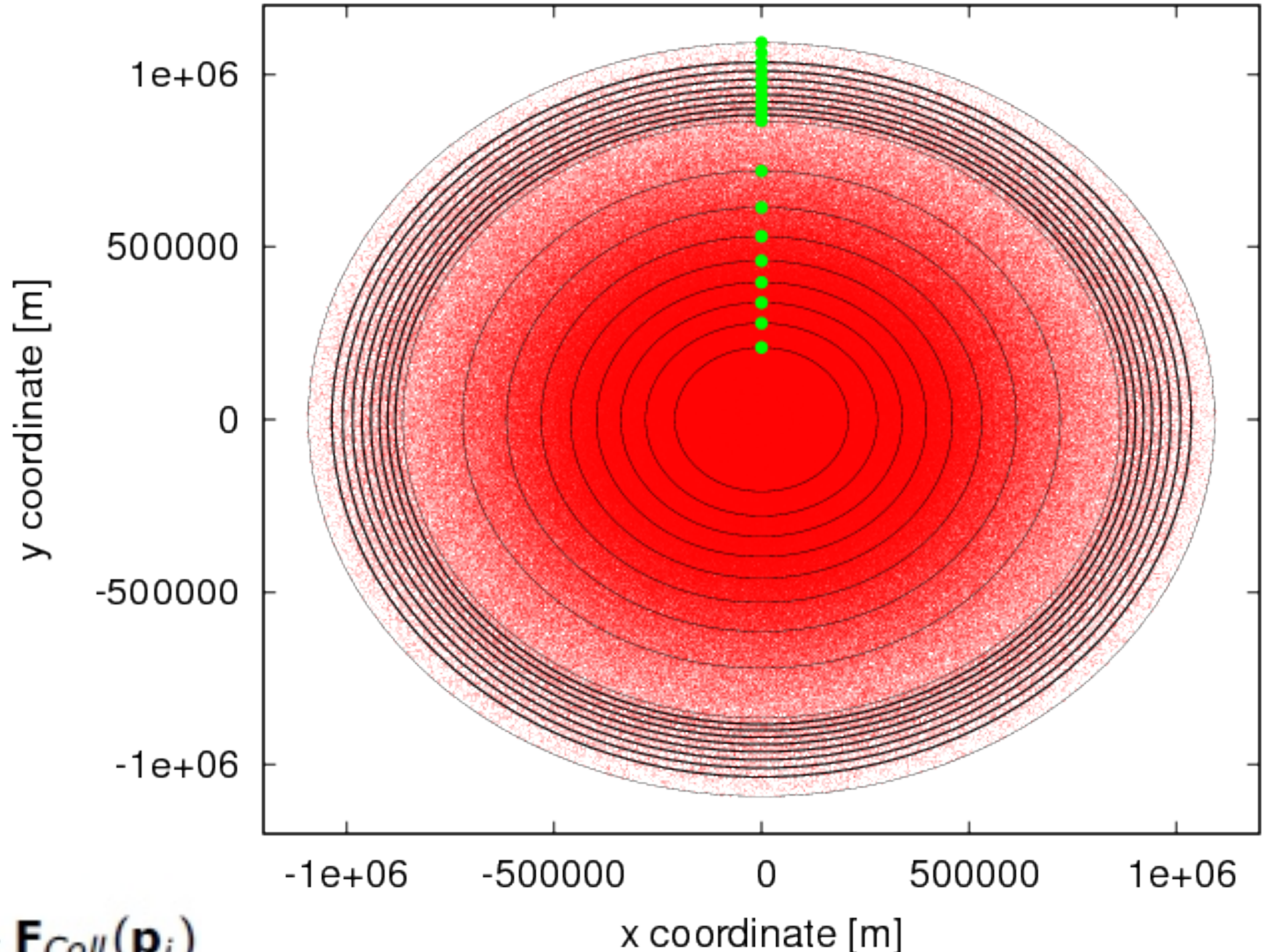


Summary & Outlook

- Transport models can describe matter in and out of equilibrium, shock wave phenomena, fluid instabilities ...
- Idea: Supernova simulation via Kinetic Theory
- Comparisons of first results to hydrodynamic codes look promising
- Current development: Transport code that can handle >> 10^6 test particles in a computationally efficient way
- First hydrodynamic tests to reproduce shock phenomena

- Extension of test suite (maybe also with fluid instabilities)
- Implement in supernova simulation code

Kinetic Approach



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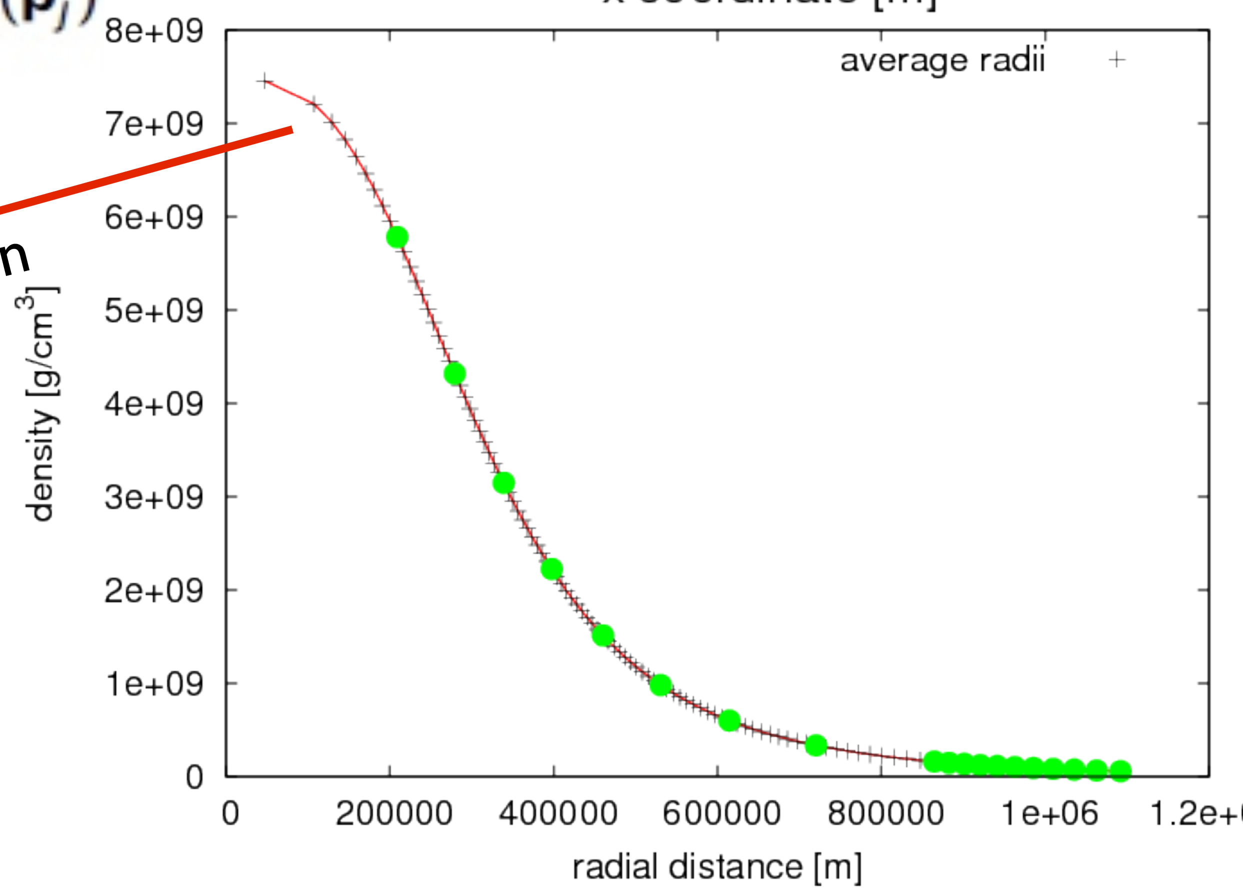
Interpolation

$$\nabla U_{eos} = \frac{\partial U_{eos}}{\partial \varrho} \nabla \varrho + \frac{\partial U_{eos}}{\partial \delta} \nabla \delta$$

tabulated



Strother & Bauer, Int. Journal Mod. Phys. D (2009),
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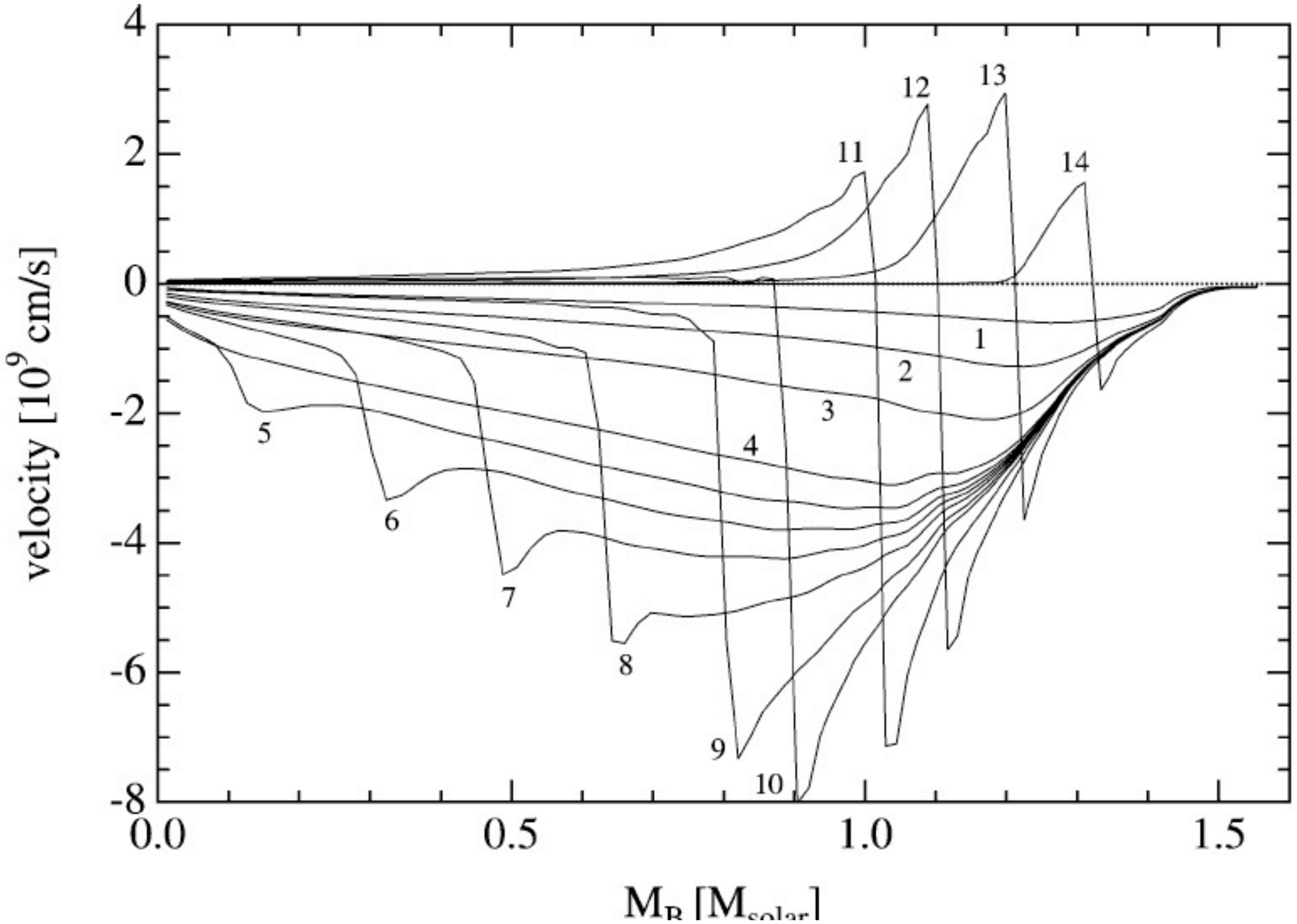

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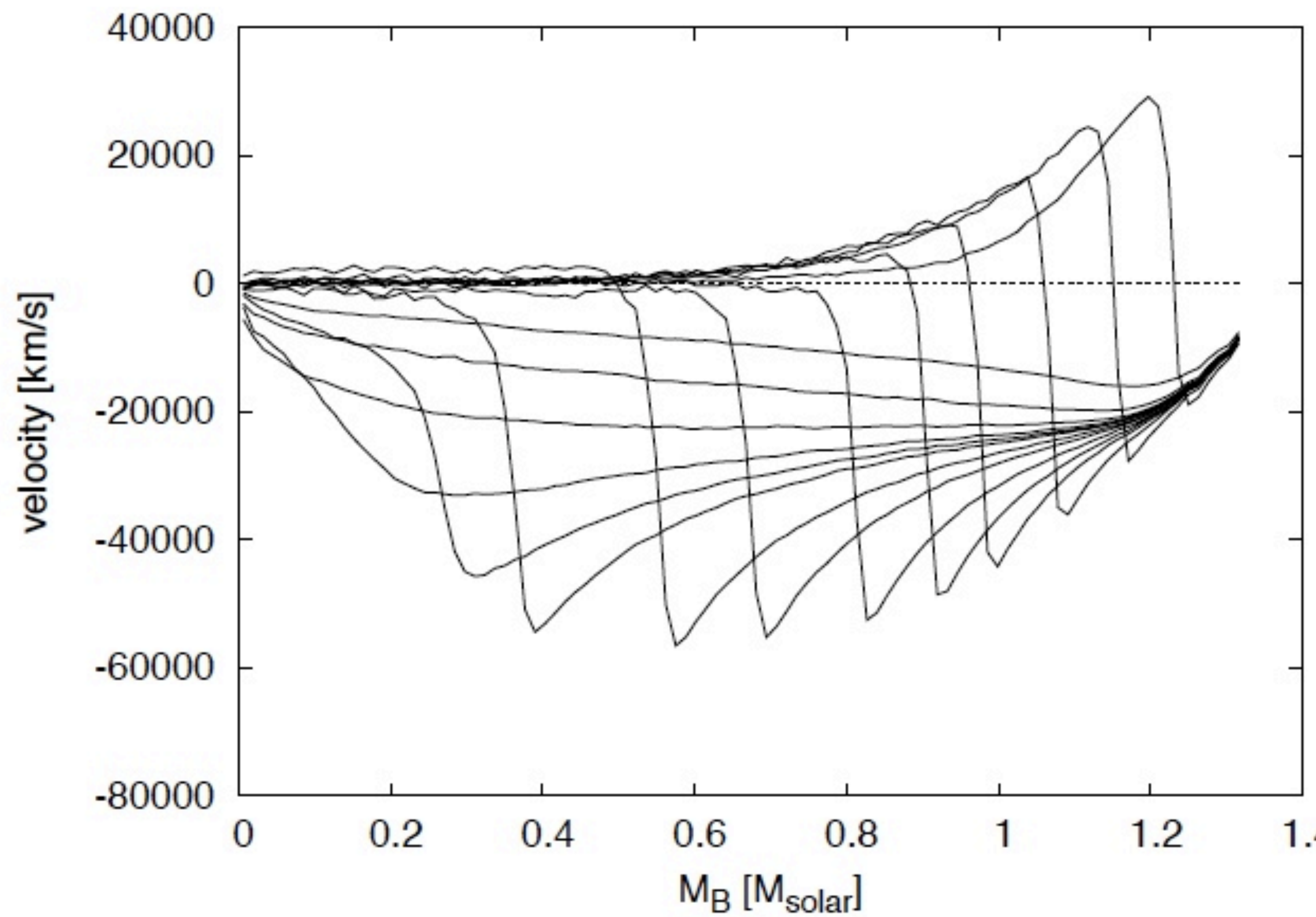

Top figure: Sumiyoshi et al. NPA 730 (2004)