Nonequilibrium quark-pair and photon production

Hendrik van Hees

Frank Michler, Dennis Dietrich, Carsten Greiner, and Stefan Leupold

Goethe Universität Frankfurt

June 29, 2012

Outline

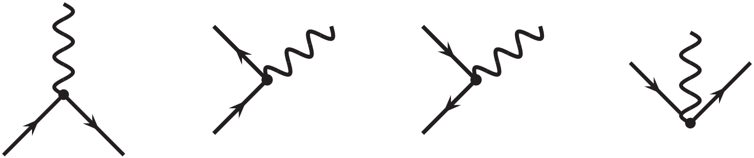
1. Motivation: Chiral symmetry and quark-mass changes
2. Quark-pair creation in an Yukawa-background field
3. Photon production in leading order in α_e
4. Conclusions
5. References

Motivation: Chiral symmetry and quark-mass changes

- approximate chiral symmetry of QCD (light-quark sector)
  - spontaneously broken in the vacuum due to formation of $\langle \bar{q} q \rangle \neq 0$
  - pions (+kaons): pseudo-Nambu-Goldstone bosons (massless in χ limit)
- at high temperatures and/or densities chiral-symmetry restoration
  - chiral-partner hadrons should become mass degenerate
  - expect large in-medium modifications of spectral properties
- electromagnetic probe in heavy-ion collisions
  - photons and dileptons nearly unaffected by FSI
  - provide undisturbed signal from hot and dense fireball
  - $M_{\ell^+ \ell^-}$ spectra $\Leftrightarrow$ medium modifications of light vector mesons
  - p_T spectra of real and virtual photons $\Leftrightarrow$ collective flow/temperature
- time-dependent problem: nonequilibrium production of em. probes

Earlier Work

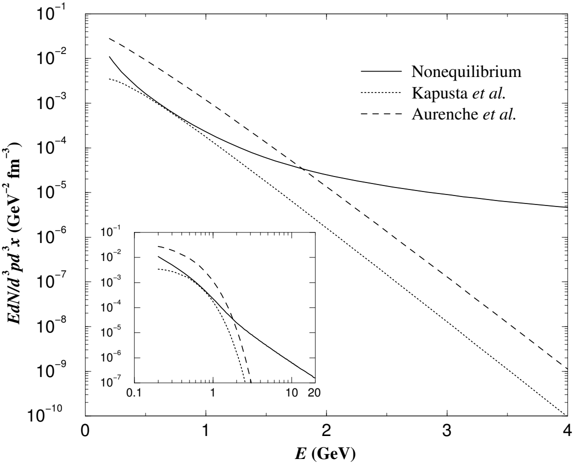
- Wang, Boyanovsky [WB01]: naive definition of transient photon numbers
- first-order processes (kinematically forbidden in equilibrium!)
  - allowed because of violation of energy at finite times
  - spontaneous pair annihilation, particle/antiparticle bremsstrahlung, pair+γ production



- Boyanovsky, Vega [BV03]
  - vacuum contribution to γ self-energy divergent; renormalization?!?
  - other contributions $\cong_{k \to \infty} 1/k^3$: γ number UV divergent
- Fraga, Gelis, Schiff [FGS05]
  - vacuum contribution unphysical; renormalization prescription criticized
  - no alternative ansatz
  - counter arguments by Boyanovsky and Vega [BV05]

Earlier Work

- Wang, Boyanovsky [WB01]: comparison to LO equilibrium processes



- at large k non-equilibrium γ's outshining thermal contributions
- spectra $\cong_{k \to \infty} 1/k^3$ vs. $\propto \exp(-k/T)$ for thermal contrib.
- but total photon number divergent

QED with external Yukawa field

- address toy model
  - start with Dirac (quark) field coupled to a Yukawa-background field
  - couple photon field minimally to quarks
  - keep basic principles: current conservation, em. gauge symmetry
- "matter" Lagrangian

$$\mathscr{L}^{(0)} = \overline{\boldsymbol{\psi}}(\mathrm{i}\not{\partial} - m - g\phi)\boldsymbol{\psi}$$

- gauge phase symmetry $\Leftrightarrow \mathrm{U}(1)_{\mathrm{em}}$
  - add minimal gauge coupling $\partial_\mu \rightarrow \partial_\mu + \mathrm{i}q\mathbf{A}_\mu$
  - kinetic term for photons $\mathscr{L}_\gamma^{(0)} = -\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}/4$

$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}_\gamma^{(0)} \underbrace{-\overline{\psi} q \not{\mathbf{A}} \psi}_{\mathscr{L}_{\text{int}}}$$

- make classical Yukawa field time dependent: $\phi = \phi(t)$
- invariant under $\mathrm{U}(1)_{\mathrm{em}}$ gauge symmetry

$$\boldsymbol{\psi}(x) \rightarrow \exp[\mathrm{i}q\chi(x)]\boldsymbol{\psi}(x), \quad \overline{\boldsymbol{\psi}}(x) \rightarrow \exp[-\mathrm{i}q\chi(x)]\overline{\boldsymbol{\psi}}(x),$$

$$\mathbf{A}_\mu(x) \rightarrow \mathbf{A}_\mu(x) - \partial_\mu\chi, \quad \phi(t) \rightarrow \phi(t)$$

Mode decomposition

- start with
$$\mathscr{L}^{(0)} = \overline{\boldsymbol{\psi}}(x)[\mathrm{i}\not{\partial} - m - g\phi(t)]\boldsymbol{\psi}(x)$$
- equation of motion for field operators: free Dirac eq. with t-dep. mass
$$[\mathrm{i}\not{\partial} - M(t)]\boldsymbol{\psi}(t) = 0, \quad M(t) = m + g\phi(t)$$
- spatial translation invariance ⇒ find momentum-eigenmodes:
$$\boldsymbol{\psi}(x) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^{3/2}} \sum_{\sigma=\pm 1/2} \exp(\mathrm{i}\vec{p}\cdot\vec{x})[\mathbf{b}(\vec{p},\sigma)u_{\vec{p},\sigma}^{(+)}(t) + \mathbf{d}^{\dagger}(-\vec{p},\sigma)u_{\vec{p},\sigma}^{(-)}(t)]$$
- mode functions
$$u_{\vec{p},\sigma}^{(\lambda)}(t) = [\mathrm{i}\gamma^0\partial_t - \vec{\gamma}\cdot\vec{p} + M(t)]\chi_\sigma^{(\lambda)}\varphi_{\vec{p}}^{(\lambda)}(t),$$
$$\gamma^0\chi_\sigma^{(\lambda)} = \lambda\chi_\sigma^{(\lambda)}, \quad \Sigma^3\chi_\sigma^{(\lambda)} = \sigma\chi_\sigma^{(\lambda)}, \quad \Sigma^3 = \frac{\mathrm{i}}{4}[\gamma^1,\gamma^2],$$
$$[-\partial_t^2 - \vec{p}^2 - M^2(t) + \mathrm{i}\lambda\dot{M}(t)]\varphi_{\vec{p}}^{\lambda} = 0.$$

Particle Interpretation?

- investigate time-dependences of mass with

$$M(t) \underset{t \to \pm\infty}{\cong} M_{\pm} = \text{const}$$

- define modes such that they allow particle interpretation for $t \to \pm\infty$

$$\varphi^{(\lambda=1)}_{\text{in},\vec{p}}(t) \underset{t \to -\infty}{\cong} N_{\text{in},\vec{p}} \exp[-\mathrm{i}\omega_-(\vec{p})t], \quad \omega_-(\vec{p}) = +\sqrt{M_-^2 + \vec{p}^2}$$

$$\varphi^{(\lambda=1)}_{\text{out},\vec{p}}(t) \underset{t \to +\infty}{\cong} N_{\text{out},\vec{p}} \exp[-\mathrm{i}\omega_+(\vec{p})t], \quad \omega_+(\vec{p}) = +\sqrt{M_+^2 + \vec{p}^2}$$

$$\varphi^{(\lambda=-1)}_{\text{in/out}}(t) := \varphi^{(\lambda=+1)*}_{\text{in/out},\vec{p}}(t), \quad |\Omega_{\text{in}}\rangle \neq |\Omega_{\text{out}}\rangle!$$

- normalization of mode functions from equal-time anticommutators
- for $M(t) \neq \text{const} \Rightarrow \varphi^{(\lambda)}_{\text{in},\vec{p}} \neq \varphi^{(\lambda)}_{\text{out},\vec{p}} \Rightarrow |\text{vac}_{\text{in}}\rangle \neq |\text{vac}_{\text{in}}\rangle$
- particle interpretation uniquely defined only for asymptotic states!
- in the following: $\mathbf{b}(\vec{p},\sigma) = \mathbf{b}_{\text{in}}(\vec{p},\sigma)$, $\mathbf{d}(\vec{p},\sigma) = \mathbf{d}_{\text{in}}(\vec{p},\sigma)$

Particle Interpretation at finite times?

- diagonalize time-dependent Hamiltonian
- Bogoliubov transformation to new creation and annihilation operators

$$\begin{pmatrix} \tilde{\mathbf{b}}(t,\vec{p},\sigma) \\ \tilde{\mathbf{d}}^{\dagger}(t,\vec{p},\sigma) \end{pmatrix} = \begin{pmatrix} \xi_{\vec{p},\sigma}(t) & \eta_{\vec{p},\sigma}(t) \\ -\eta^{*}_{\vec{p},\sigma}(t) & \xi^{*}_{\vec{p},\sigma}(t) \end{pmatrix} \begin{pmatrix} \mathbf{b}(\vec{p},\sigma) \\ \mathbf{d}^{\dagger}(\vec{p},\sigma) \end{pmatrix},$$

$$|\xi_{\vec{p},\sigma}(t)|^2 + |\eta_{\vec{p},\sigma}(t)|^2 = 1$$

- after normal ordering wrt. instantaneous vacuum

$$\mathbf{H}(t) = \sum_{\sigma} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \omega_{\vec{p}}(t) \left[\tilde{\mathbf{b}}^{\dagger}(t,\vec{p},s)\tilde{\mathbf{b}}(t,\vec{p},s) + \tilde{\mathbf{d}}^{\dagger}(t,\vec{p},s)\tilde{\mathbf{d}}(t,\vec{p},s)\right],$$

$$\omega_{\vec{p}}(t) = \sqrt{M^2(t) + \vec{p}^2}$$

- instantaneous particle/antiparticle number operators

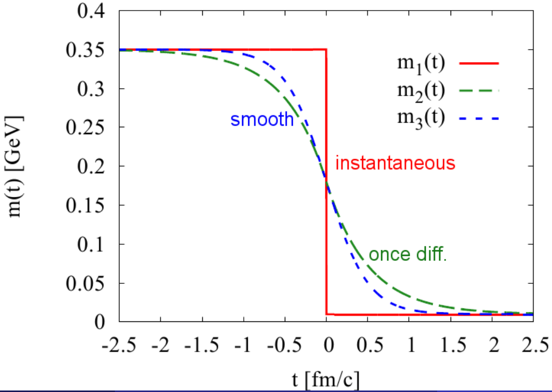
$$\mathbf{N}_{\vec{p},\sigma}(t) = \tilde{\mathbf{b}}(t,\vec{p},\sigma)\tilde{\mathbf{b}}^{\dagger}(t,\vec{p},\sigma), \quad \overline{\mathbf{N}}_{\vec{p},\sigma}(t) = \tilde{\mathbf{d}}^{\dagger}(t,\vec{p},\sigma)\tilde{\mathbf{d}}(t,\vec{p},\sigma)$$

Well-defined initial-value problem!

- give initial state $\mathbf{R}_0$ at $t \to -\infty$; here vacuum: $\mathbf{R}_0 = |\Omega_{\text{in}}\rangle \langle \Omega_{\text{in}}|$
- calculate spectra for particles as

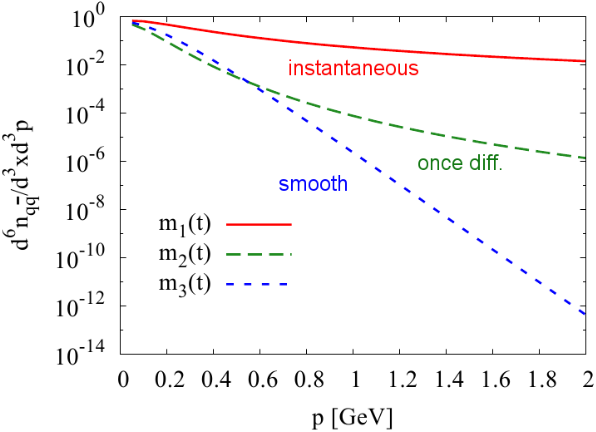
$$\frac{\mathrm{d}N_{\vec{p}}}{\mathrm{d}^3\vec{x}\mathrm{d}^3\vec{p}} = \sum_{\sigma} \left\langle \Omega_{\text{in}} \left| \mathbf{N}_{\vec{p},\sigma}(t) \right| \Omega_{\text{in}} \right\rangle = \frac{\mathrm{d}\overline{N}_{\vec{p}}}{\mathrm{d}^3\vec{x}\mathrm{d}^3\vec{p}} = \sum_{\sigma} \left| \eta_{\vec{p},\sigma}(t) \right|^2$$

- Mass-switching functions



Asymptotic pair spectra

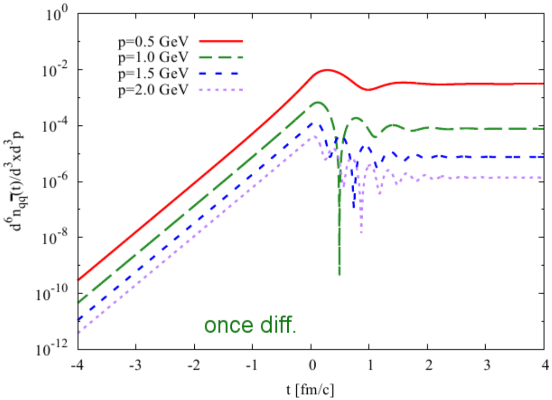
- asymptotic spectra ($t \to \infty$)



- behavior for $p \to \infty$:

$\propto (m_c - m_b)^2/|\vec{p}|^2$, $\propto 1/|\vec{p}|^6$, $\propto \exp(-p/T_{\text{eff}})$

Time dependence of spectra

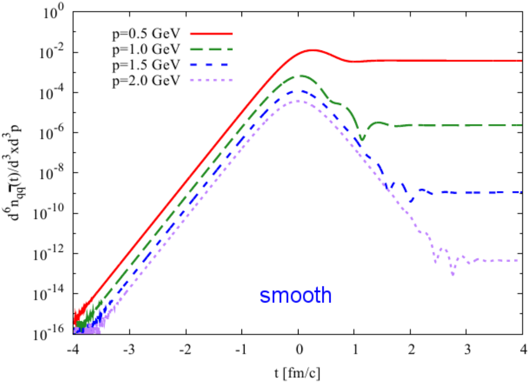


- large overshoot at times with large variations in $M(t)$
- reason:

$$\eta_{\vec{p},\sigma}(t) \underset{|\vec{p}| \gg m(t)}{\cong} \frac{\exp(-\mathrm{i}|\vec{p}|t)}{2|\vec{p}|^2} \int_{-\infty}^{t} \mathrm{d}t' \dot{M}(t') \exp(2\mathrm{i}|\vec{p}|t')$$

- well-behaved UV limit only for asymptotic spectra

Time dependence of spectra



- for finite t: spectra $\cong_{|\vec{p}|\to\infty} 1/|\vec{p}|^4$
- asymptotic spectrum exponential for large $|\vec{p}|$
- total energy density divergent for finite t!
- artifact of Yukawa background field independent of $\vec{x}$?

Probable way out?

- problems with definition of transient particle numbers
  - instantaneous single-particle energy-eigenmodes: no clear definition of "positive frequency solutions"
  - particle interpretation of those modes ambiguous
- possible well-defined "particle models" @ $t \neq \pm\infty$:
  - adiabatic solutions to the mode function

$$\varphi_{\vec{p}}^{+}(t) = \frac{1}{\sqrt{2w(t)}} \exp\left[-\mathrm{i} \int_{t_0}^{t} \mathrm{d}t' w(t')\right]$$

  - $w(t)$ given in WKB approximations of mode equation
  - positive definite $w(t) \Rightarrow \varphi_{\vec{p}}^{(+)} =$ "positive frequency solutions"
  - usually well behaved finite particle densities [Ful79, Ful89]
  - admit proper particle definition (but not unique!)
  - asymptotic particle interpretation unique!
- WKB semiclassical approximations
- "transient" particle interpretations approximately equivalent
- transient particle spectra/yields good under "transport conditions"

Perturbation theory for em. interaction

- treat photon production perturbatively
  - use quark fields from $\mathscr{L}^{(0)}$
  - J-interaction picture includes exact dynamics from $\phi(t)$ (all orders in g)
  - photon field operators: free-field evolution in J-interaction picture
- to get correct asymptotic limit
  - adiabatic switching a la Gell-Mann-Low theorem crucial
  - $\mathbf{H}_{\text{int}} \rightarrow \exp(-\epsilon|t|)\mathbf{H}_{\text{int}}$ $[\mathbf{H}_{\text{int}} = q \int \mathrm{d}^3\vec{x}\overline{\boldsymbol{\psi}}(x)\not{\mathbf{A}}(x)\boldsymbol{\psi}(x)]$
  - $\mathbf{U}_\epsilon(t_1, t_2)$: interaction-picture time evolution for states

$$|\Omega_{\text{out}}\rangle = \lim_{\epsilon\to 0}\lim_{t\to\infty} \frac{\mathbf{U}_\epsilon(-\infty, t)\,|\Omega_{\text{in}}\rangle}{\langle\Omega_{\text{in}}\,|\mathbf{U}_\epsilon(-\infty, t)|\,\Omega_{\text{in}}\rangle}$$

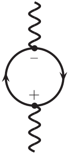
  - order of limits crucial: first $t \to \infty$ and then $\epsilon \to 0$
  - projects to vacuum state
  - damping out transient contributions from higher states
- only applicable for asymptotic observables (S-matrix prescription)
  - advantage: perturbation theory applicable in NEqQFT
  - disadvantage: no transient photon numbers but only asymptotic!
  - definition for γ numbers for interacting γ fields???

Photon production in leading order in α_e

- asymptotic photon-production yield
- photon polarization in terms of Schwinger-Keldysh real-time diagrams
- turns out to be absolute square

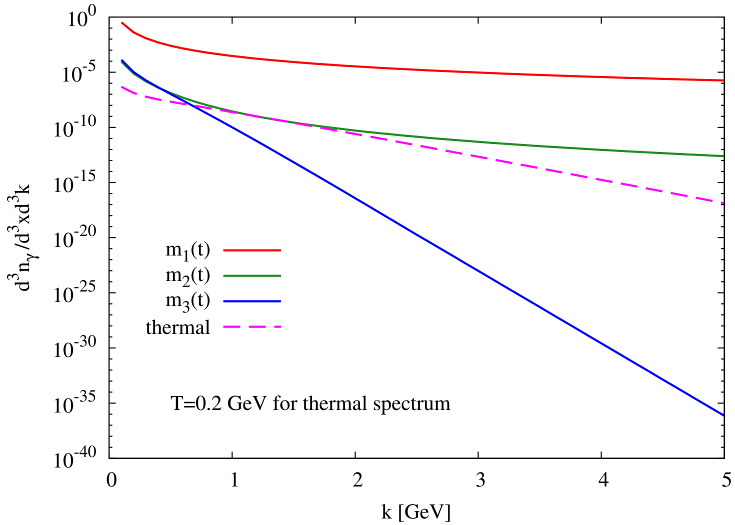
$$(2\pi)^3|\vec{k}|\frac{\mathrm{d}n_\gamma}{\mathrm{d}^4x\mathrm{d}^3\vec{k}} = -\operatorname{Im}\left\{\int_{-\infty}^{\infty}\overline{\mathrm{d}t_1}\int_{-\infty}^{\infty}\overline{\mathrm{d}t_2}\,\Pi_{\perp}^{<}(\vec{k},t_1,t_2) \times \exp\left[\mathrm{i}|\vec{k}|(t_1-t_2)\right]\right\},$$

- $\overline{\mathrm{d}t} := \mathrm{d}t\exp(-\epsilon|t|)$ with limit $\epsilon \to 0^+$ after all time integrations!
- photon-polarization tensor

$$\mathrm{i}\Pi_{\mu\nu}^{<}(\vec{k},t_1,t_2) =$$ 

- quark propagators: full time evolution from external Yukawa field!

Asymptotic photon spectra

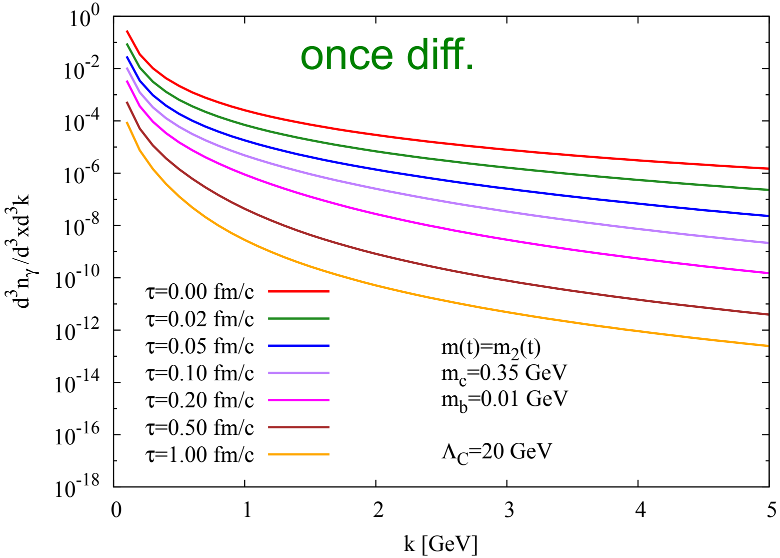


Asymptotic photon spectra

- spectra=modulus squared $\Rightarrow$ positive photon number
- all pure vacuum contributions vanish for asymptotic spectra
- one-loop $\Pi^{<}$ convergent (except for instantaneous switching!)
- instantaneous switching
  - loop integral divergent (here cut-off regulated with $\Lambda = 100\text{ GeV}$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} 1/|\vec{k}|^3$
  - total photon number and energy UV divergent
- once-differentiable switching function
  - loop integral convergent (here: extrapolated spectrum for $\Lambda \to \infty$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} 1/|\vec{k}|^6$
  - total photon number and energy UV convergent
- Smooth switching function
  - loop integral convergent (here: extrapolated spectrum for $\Lambda \to \infty$)
  - asymptotic photon spectrum $\cong_{|\vec{k}| \to \infty} \exp(-|\vec{k}|/T_{\text{eff}})$
  - total photon number and energy UV convergent

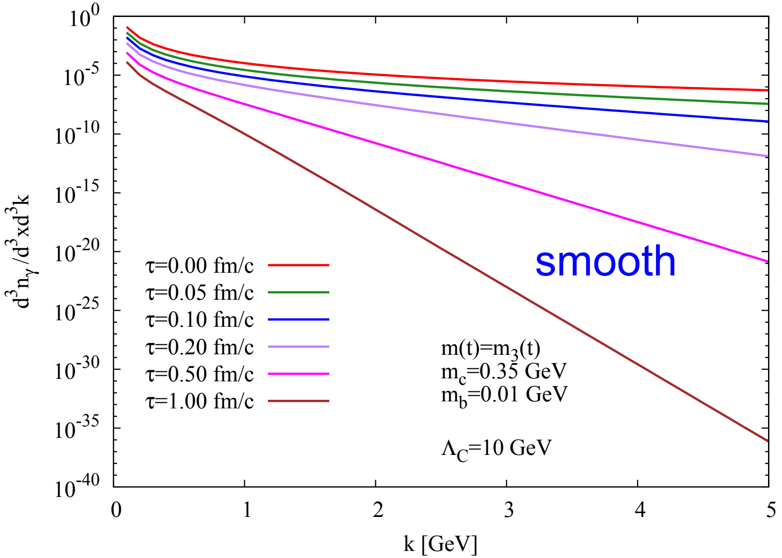
Asymptotic photon spectra

- dependence on switching duration



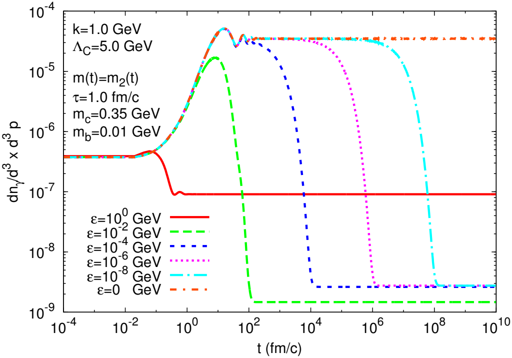
Asymptotic photon spectra

- dependence on switching duration



Transient photon spectra?

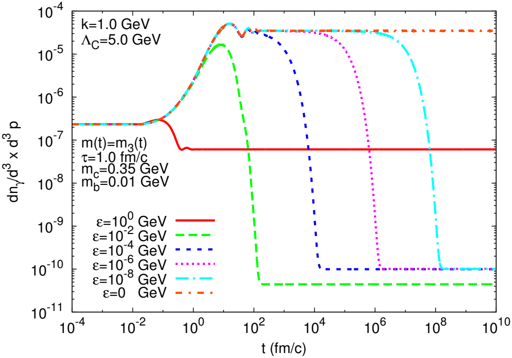
- naive attempt: keep time-integration limit finite
- keep only non-vacuum contributions (ad-hoc renormalization)



- spectra at finite t many orders of magnitude above asymptotic limit
- ill-defined transient photon numbers
- no sensible physical interpretation! of naive rates
- interchanging orders of limits $t \to \infty$ and $\epsilon \to 0 \Rightarrow$ forbidden!

Transient photon spectra?

- naive attempt: keep time-integration limit finite
- keep only non-vacuum contributions (ad-hoc renormalization)



- spectra at finite t many orders of magnitude above asymptotic limit
- ill-defined transient photon numbers
- no sensible physical interpretation! of naive rates
- interchanging orders of limits $t \to \infty$ and $\epsilon \to 0 \Rightarrow$ forbidden!

Conclusions

- spontaneous $q\bar{q}$-pair creation in time-dep. Yukawa background
  - simplified model for transient effect of chiral phase transition $m_{\text{const}} \rightarrow m_{\text{curr}}$
  - definition of transient particle numbers non-trivial
  - naive interpretation wrt. instant energy eigenmodes unphysical
  - probably cured by using particle interpretations wrt. adiabatic modes
  - asymptotic particle yields well defined
- Nonequilibrium photon radiation
  - well-defined perturbation theory for asymptotic photon numbers
  - contributions to $\mathcal{O}\alpha_{\text{em}}$ (kinematically forbidden in equil.)
  - LO neq. asymptotic photon yield convergent (except for instantaneous mass shift)
  - naively defined transient photon spectra ill-defined

Outlook

- straight-forward extension: asymptotic non-eq. dilepton spectra
- Challenges
  - what about IR divergences (soft photons, LPM effect)?
  - physically sensible definition of transient photon numbers?
  - relation to quantum-kinetic/transport picture/equations?
  - self-consistent electromagnetic mean field ("back-reaction problem")?
  - Noneq. photon (dilepton) asymptotic spectra in realistic HIC fireball?

BACKUP SLIDES

Normalization of fermion-mode functions

- equal-time anticommutators

$$\left\{\boldsymbol{\psi}_\alpha(t,\vec{x}),\boldsymbol{\psi}_\beta^\dagger(t,\vec{y})\right\} = \delta_{\alpha\beta}\delta^{(3)}(\vec{x}-\vec{y})$$

- fulfilled if $\mathbf{b}(\vec{p},\sigma)$, $\quad\mathbf{d}(\vec{p},\sigma)$ like usual fermionic annihilation operators

$$u_{\vec{p},\sigma}^{(\lambda)\dagger} u_{\vec{p},\sigma'}^{(\lambda')} = \delta_{\sigma\sigma'}\delta_{\lambda\lambda'}$$

- with ansatz

$$u_{\vec{p},\sigma}^{(\lambda)}(t) = [\mathrm{i}\gamma^0\partial_t - \vec{\gamma}\cdot\vec{p} + M(t)]\varphi_{\vec{p}}^{(\lambda)}(t)\chi_\sigma^{(\lambda)}$$

  fulfilled with $\varphi_{\vec{p}}^{(\lambda=1)} = \varphi_{\vec{p}}^{(\lambda=1)*} =: \varphi_{\vec{p}}(t)$ if

$$|\dot{\varphi}_{\vec{p}}|^2 + \mathrm{i}m\varphi_{\vec{p}}^* \overleftrightarrow{\partial}_t \varphi_{\vec{p}} + \omega_{\vec{p}}^2(t)|\varphi_{\vec{p}}|^2 \equiv 1, \quad \omega_{\vec{p}}^2(t) = M^2(t) + \vec{p}^2$$

- lhs time-independent due to EoM for φ (charge conservation!)
- can express normalization factors in terms of asymptotic frequencies:

$$N_{\text{in/out},\vec{p}}^{(\lambda)} = \frac{1}{\sqrt{2\omega_\pm(\omega_\pm + \lambda M_\pm)}}$$

Bibliography I

[BV03] D. Boyanovsky, H. de Vega, Are direct photons a clean signal of a thermalized quark gluon plasma?, Phys. Rev. D **68** (2003) 065018.

[BV05] D. Boyanovsky, H. J. de Vega, Photon production from a thermalized quark gluon plasma: Quantum kinetics and nonperturbative aspects, Nucl. Phys. A **747** (2005) 564.

[FGS05] E. Fraga, F. Gelis, D. Schiff, Remarks on transient photon production in heavy ion collisions, Phys. Rev. D **71** (2005) 085015.

[Ful79] S. Fulling, Remarks on positive frequency and hamiltonians in expanding universes, Gen. Rel. Grav. **10** (1979) 807.

Bibliography II

[Ful89] S. A. Fulling, Aspects of Quantum Field Theory in Curved Space-Time, Cambridge University Press, Cambridge, Port Chester, Melbourne, Sydney (1989).

[WB01] S.-Y. Wang, D. Boyanovsky, Enhanced photon production from quark - gluon plasma: Finite lifetime effect, Phys. Rev. D **63** (2001) 051702.