Non-equilibrium phenomena with Boltzmann equation in ultracold atomic Fermi gases

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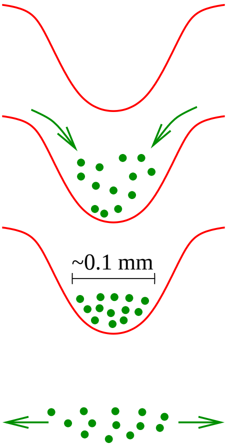
Outline

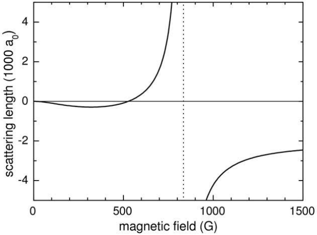
- Introduction
  - Cold atomic gases
  - Applications
- Non-equilibrium phenomena
  - Spontaneous breaking of a symmetry
  - Boltzmann equation :
    - test particle method
    - moments method
- Conclusion

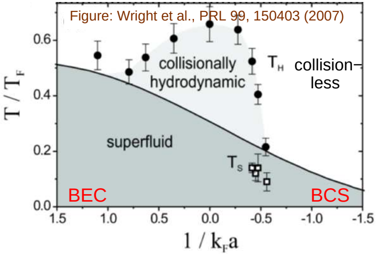
Cold atomic gases

- Create trap potential (combining lasers and/or magnetic fields)
- Load the atoms into the trap: $N \sim 10^5 - 10^6$
- Cool them down : $\simeq$ 10-100 nK
- Trap size : 10-100 μm
- Dilute : typical density $< 10^{15}$ cm^{-3} (air $\simeq 10^{19}$ cm^{-3})
- Measure density profile (switch off the trap)

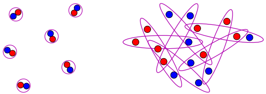


Cold atoms





Novelty : interaction strength and sign can be tuned!



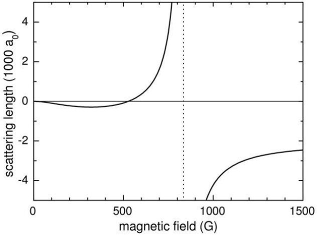
Cross-over BEC-BCS

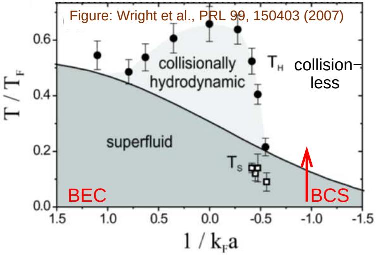
Dynamical regimes : superfluid/hydrodynamics/collisionless

Cold atoms at unitarity $\equiv$ neutron matter at low density

$$(k_F R < 1 < k_F a_{nn})$$

Cold atoms





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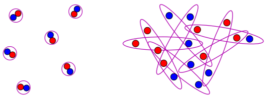

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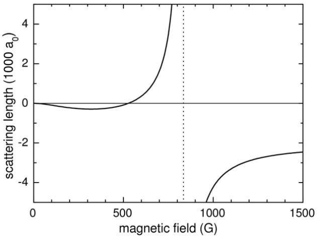

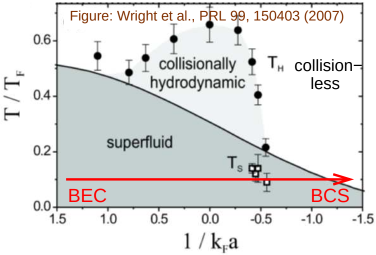

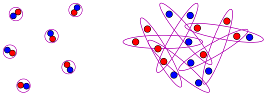

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Possible applications

- BEC-BCS cross-over : deuterons $\rightarrow$ nuclear matter
- Pairing among fermions : nuclear physics (some differences : isospin, finite size effects, small number of pairs)
- Color superconductivity of quark matter :
  pairing of quarks of different masses $\rightarrow$ pairing between different atoms in a trap
- Superfluid hydrodynamics :
  - quasiparticle approach
  - hadronic phase

Pion hydrodynamics

Modification of hydrodynamic theory itself :

$$\partial_\mu (n_0 u^\mu - V^2 \partial^\mu \phi) = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$u^\mu \partial_\mu \phi = -\mu_0$$

with :

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} + V^2 \partial^\mu \phi \partial^\nu \phi$$

For pions : $SU(2)$-matrix $\Sigma \equiv e^{i\vec{\tau}.\vec{\pi}/f_\pi}$
and :

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} + V^2 \mathrm{tr}(\partial^\mu \Sigma \partial^\nu \Sigma^\dagger + \partial^\nu \Sigma \partial^\mu \Sigma^\dagger)$$

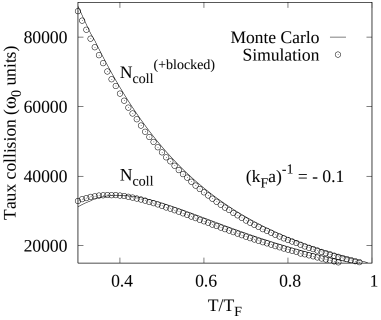
Y. Lallouet, D. D., C. Pujol Phys.Rev. C67(2003) 0149010 ; Phys.Rev. C67 (2003) 057901

Boltzmann equation : test particles method

General framework :

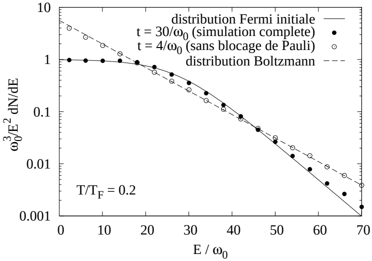
test particles method

and Pauli blocking



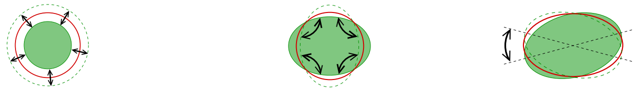
To be checked :

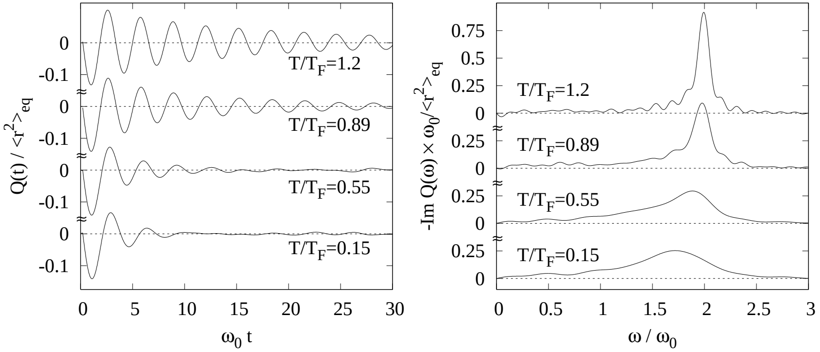
- Energy conservation
- Time step
- Collision rate
- Equilibrium distribution
- ...



Boltzmann equation : test particles method

Collective modes :





S. Chiacchiera, T. Lepers, D.D. , M. Urban Phys. Rev. A79 (2009) 033613

T. Lepers, D.D. , S. Chiacchiera , M. Urban , Phys.Rev. A82 (2010) 023609

Boltzmann equation : moments method

Number of particles : 6.10^5!!!
$\rightarrow$ other method required : moments method

$$f = f_{eq} + f_{eq}(1 - f_{eq})\Phi \tag{1}$$

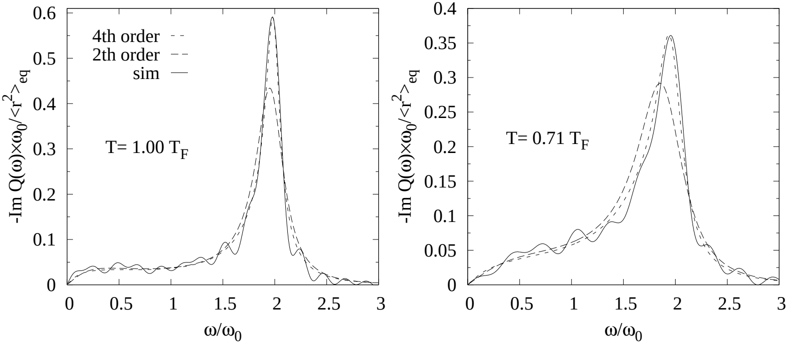
$$f_{eq}(1 - f_{eq})\left(\dot{\Phi} + \frac{\mathbf{p}}{m}\cdot\nabla_r\Phi - \nabla_r(V_T + U_{eq})\cdot\nabla_p\Phi + \beta\frac{\mathbf{p}}{m}\cdot\nabla_r\delta U\right) = -I[\Phi]\,. \tag{2}$$

with linearized collision term :

$$I[\Phi] = \int \frac{d\mathbf{p_1}}{(2\pi)^3}\int d\Omega\frac{d\sigma}{d\Omega}|\mathbf{v} - \mathbf{v}_1|f_{eq}f_{eq\,1} \times (1 - f'_{eq})(1 - f'_{eq\,1})(\Phi + \Phi_1 - \Phi' - \Phi'_1)$$

Choice of ϕ : physical considerations (next to leading order)

Boltzmann equation : moments method



- a good substitute to a complete resolution!
- medium effects : easily incorporated (cross-section + left-hand side)
- cpu time : 1 min instead of several hours/days!

T. Lepers, D.D. , S. Chiacchiera , M. Urban Phys.Rev. A84 (2011) 043634

Principle : Kohn mode

$$\Phi(\vec{r}, \vec{v}, t) = \sum_{i=1}^{2} c_i(t)\phi_i(\vec{r}, \vec{p}) \,,$$

where:

$$\phi_1 = x \,,\ \phi_2 = p_x$$

- multiplication of Boltzmann equation by each ϕ_i
- integration on phase space
- closed system of equations for c_i

First order $\rightarrow$ next order!

$$\Phi(\vec{r}, \vec{p}, t) = \sum_{i=1}^{18} c_i(t)\phi_i(\vec{r}, \vec{p}) \,,$$

where:

$$\phi_1 = x \,,\ \phi_2 = p_x \,, \phi_3 = x^3 \,,\ \phi_4 = x^2 p_x \,,\ \phi_5 = x p_x^2 \,,\ \phi_6 = p_x^3 \,,$$
$$\phi_7 = x y^2 \,, \phi_8 = y^2 p_x \,,\ \phi_9 = x y p_y \,,\ \phi_{10} = y p_x p_y\ \phi_{11} = x p_y^2 \,,\ \phi_{12} = p_x p_y^2 \,,$$
$$\phi_{13} = x z^2 \,,\ \phi_{14} = z^2 p_x \,, \phi_{15} = x z p_z \,,\ \phi_{16} = z p_x p_z \,,\ \phi_{17} = x p_z^2 \,,\ \phi_{18} = p_x p_z^2$$

Conclusion

Trapped atomic gases :

a laboratory for non-equilibrium processes

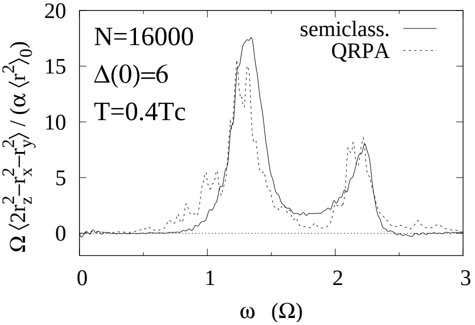
for strongly correlated particles and with a lot of available data!

THANK YOU!

Boltzmann equation and superfluidity : quasiparticle method

- Semi-classical approach for $T < T_c$
- Hydrodynamical equation for phase $\phi(\vec{r}, t)$ of the order parameter coupled to a Vlasov-type equation for the quasiparticles distribution function $\nu(\vec{r}, \vec{p}, t)$

- Numerical solution using the test-particle method
- Example: quadrupole mode
- Transport theory vs. QRPA: reasonable agreement
- Two peaks corresponding to the superfluid and normal parts, respectively



M. Urban, Phys. Rev. A 75 (2007)