Dynamical freeze-out

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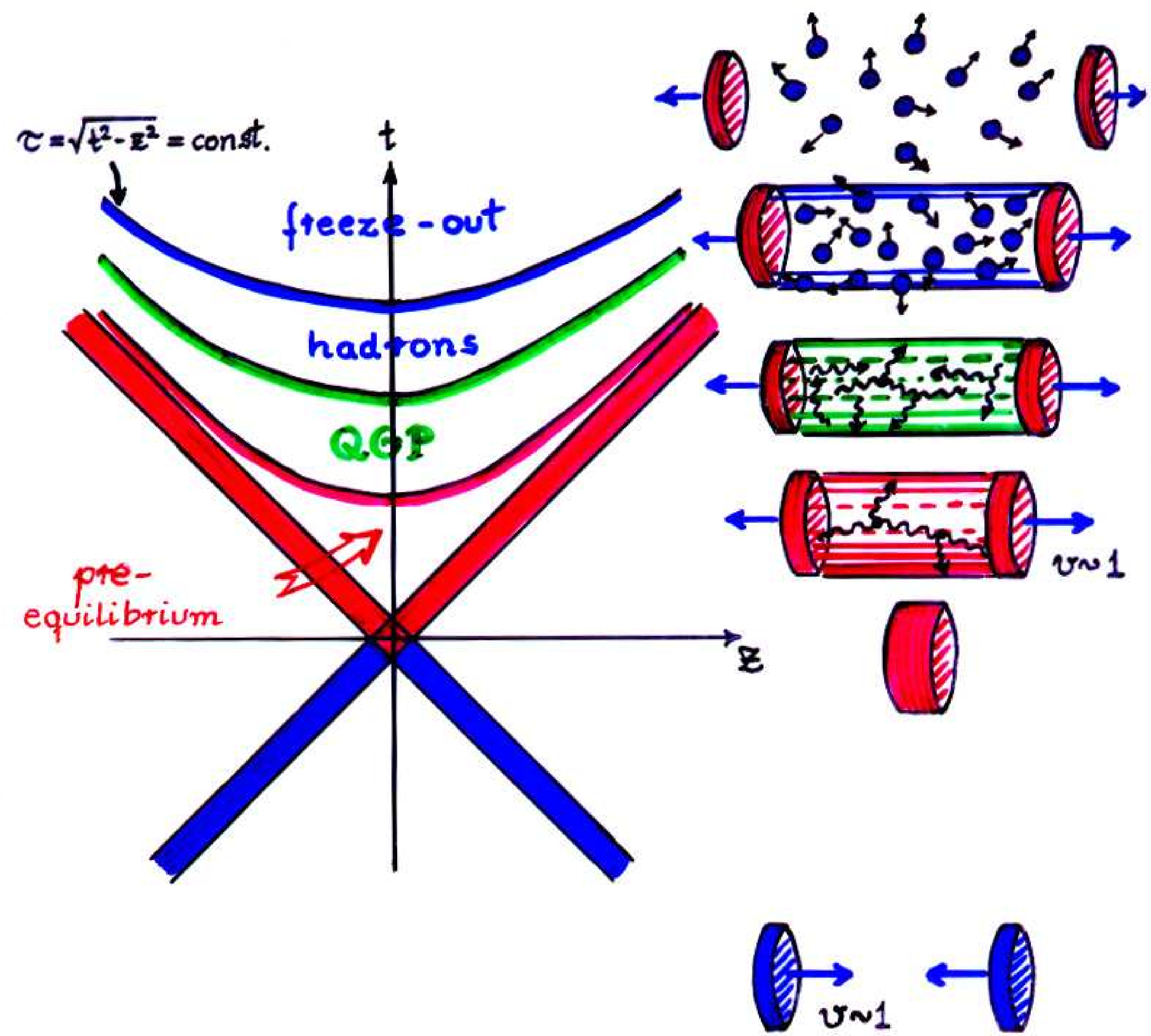
The space-time picture:



Freeze-out

- Kinetic equilibrium requires scattering rate $\gg$ expansion rate
- this not valid $\rightarrow$ system behaves as free streaming particles
- momentum distributions cease to evolve $\rightarrow$ they "freeze-out"
- criterion: expansion rate equal to scattering rate:

$$\frac{1}{K_n} = \frac{\tau_{\mathrm{scat}}^{-1}}{\partial_\mu u^\mu} \approx 1$$

- $\tau_{\mathrm{scat}}^{-1} \propto T^4 \rightarrow$ rapid transition to free streaming
- **Approximation:** decoupling takes place on constant temperature hypersurface $T = T_{\mathrm{fo}}$

Why bother?

- "You cannot describe hadron gas using fluid dynamics?"
  - why not? Prove it!

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    - lack of dissipation?
    - bad freeze-out description?
    - something else?

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  - True, but is the reason
    - lack of dissipation?
    - bad freeze-out description?
    - something else?
- viscous hydro has freeze-out too!

Why bother?

- "Why not to use a hybrid model?"
  - sampling distorts the particle distributions
  - results depend on switching criterion!
    - switch at $K_n = K_{n,\mathrm{sw}}$?

Dynamical criterion

- **need to evaluate**

$$\frac{1}{K_n} = \frac{\tau_{\mathrm{scat}}^{-1}}{\partial_\mu u^\mu}$$

- $\partial_\mu u^\mu$ **known from hydro**
- $\tau_{\mathrm{scat}}^{-1}$**?**

Dynamical criterion

- need to evaluate

$$\frac{1}{K_n} = \frac{\tau_{\rm scat}^{-1}}{\partial_\mu u^\mu}$$

- $\partial_\mu u^\mu$ known from hydro
- $\tau_{\rm scat}^{-1}$?
  - Prakash *et al.*, Phys. Rept. 227, 321 (1993):

    Parametrization: Daghigh & Kapusta, Phys. Rev. D 65, 064028 (2002)

$$\tau_{\pi\pi}^{-1}(T) \approx 16 \left(\frac{T}{100\ \mathrm{MeV}}\right)^4 \mathrm{MeV}$$

- pions only, chemical equilibrium

Scattering rates

- **evaluate scattering rate of pions in thermal hadron gas**
  - **number of scatterings:** $N = F_1 N_2 \sigma_{12} = n_1 |\vec{v}_{12}| N_2 \sigma_{12}$
  - $|\vec{v}_{12}| = \sqrt{(s-s_a)(s-s_b)}/(2E_a E_b)$
    where $s_a = (m_1 + m_2)^2$ **and** $s_b = (m_1 - m_2)^2$
  - **fold over thermal distributions**
  - **sum over all scattering partners**
  - **scatterings per pion** $\rightarrow$ **divide by pion density**

$$\tau_{\mathrm{scat}}^{-1} = \frac{1}{n_\pi(T,\mu_\pi)} \sum_i \int \mathrm{d}^3 p_\pi \, \mathrm{d}^3 p_i \, f_\pi(T,\mu_\pi) f_i(T,\mu_\pi) \frac{\sqrt{(s-s_a)(s-s_b)}}{2E_\pi E_i} \sigma_{\pi i}(s)$$

- **what is $\sigma_{\pi i}$?**

Cross sections

- as in UrQMD:
  - $\sigma_{\pi i}(s)$ for resonance formation using Breit-Wigner

$$\sigma_{\pi i}(s) = \sum_R \sigma_{\pi i \to R}(s)$$

  - estimate $\sigma_{\pi m}(s)$ for elastic π-meson scattering

$\Rightarrow$ check that the result fits the cross section data

Cross sections

$$\sigma_{\pi i\to R}(s) = \frac{2S_R+1}{(2S_\pi+1)(2S_i+1)} \frac{\pi}{{p_{\mathrm{CMS}}}^2} \frac{\Gamma_{R\to\pi i}(\sqrt{s})\,\Gamma_{tot}(\sqrt{s})}{(m_R-\sqrt{s})^2+{\Gamma_{tot}}^2(\sqrt{s})/4}$$

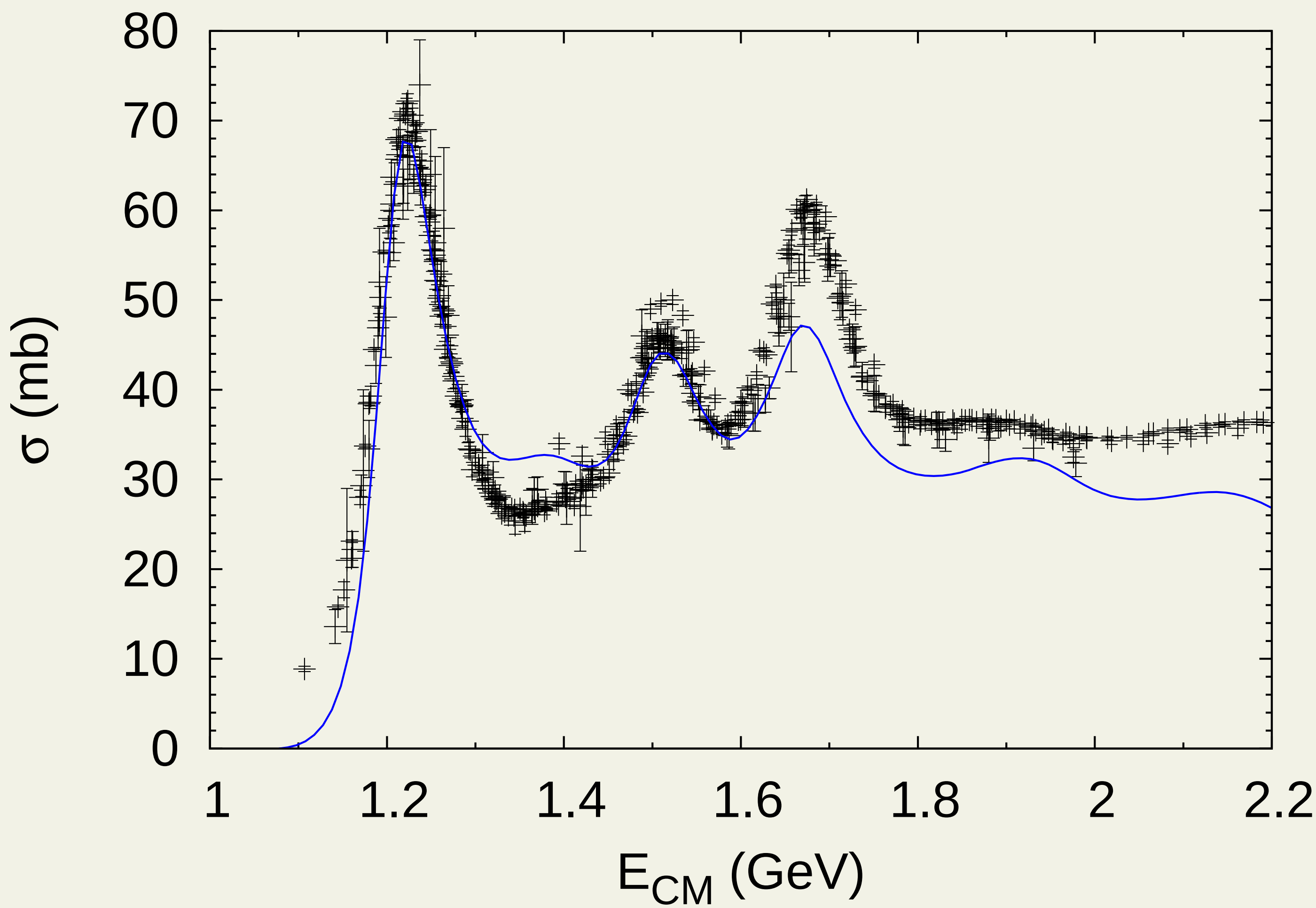
where

- S_j **is spin**
- p_{CMS} **is particle momenta in CMS**
- Γ_{tot} **and** $\Gamma_{R\to\pi i}$ **total and partial decay widths:**

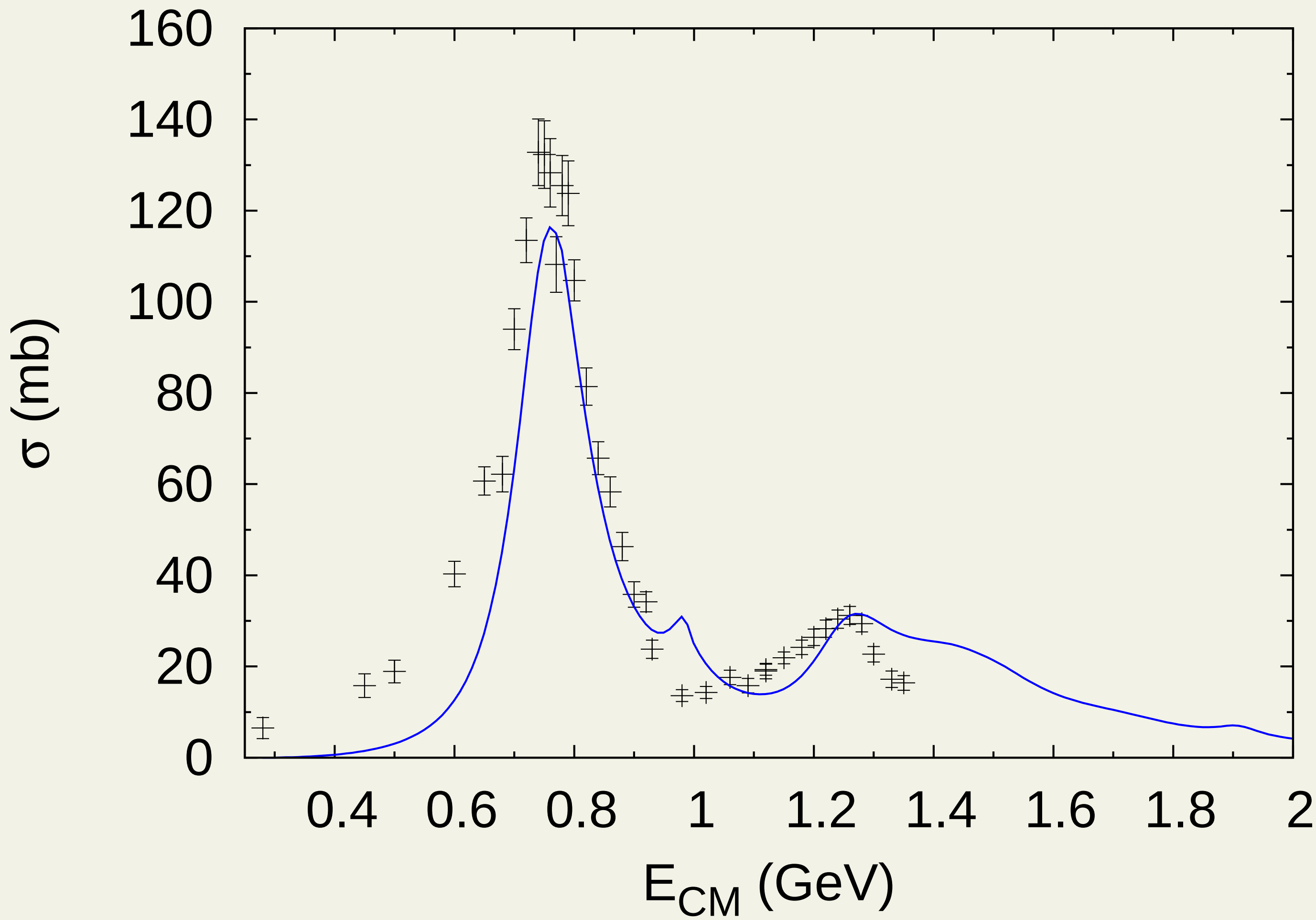
$$\Gamma_{R\to\pi i}(M) = \Gamma_R^{\pi i}\,\frac{m_R}{M}\left(\frac{p_{\mathrm{CMS}}(M)}{p_{\mathrm{CMS}}(m_R)}\right)^{2l+1} \frac{1.2}{1+0.2\left(\frac{p_{\mathrm{CMS}}(M)}{p_{\mathrm{CMS}}(m_R)}\right)^{2l}}$$

- **Note: scattering partner** i **can be a resonance!**

$\sigma_{\pi^- p}$

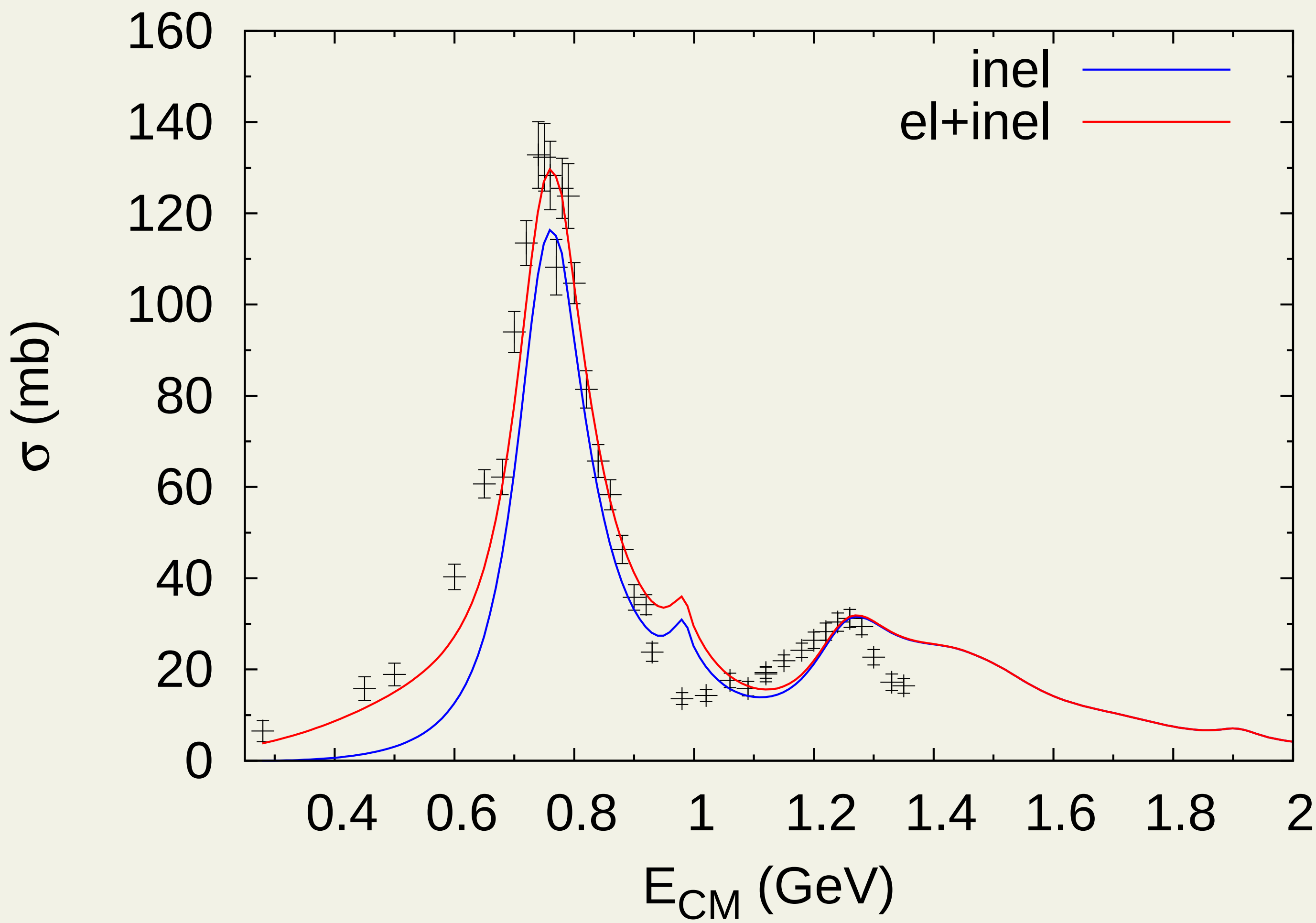


$\sigma_{\pi^+\pi^-}$



160
140
120
100
80
60
40
20
0
σ (mb)
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
E_{CM} (GeV)

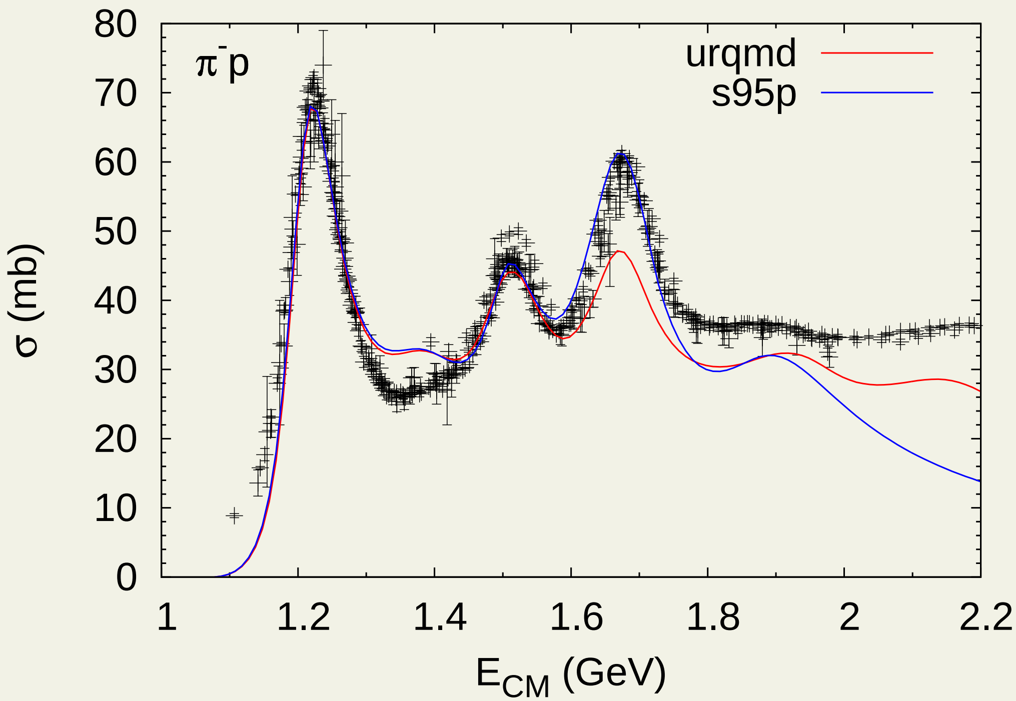
$\sigma_{\pi^+\pi^-}$

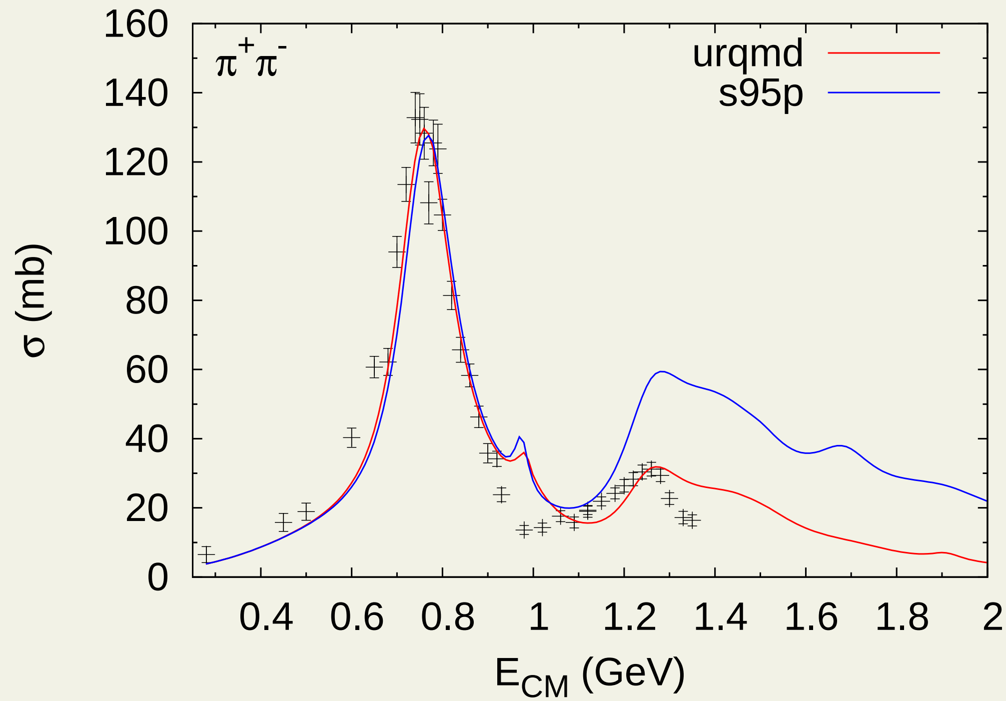


– **elastic meson-meson scattering** $\sigma_{mm} = 5$ **mb**
– **elastic** $\pi\pi$ **scattering** $\sigma_{\pi\pi} = \sigma_0 e^{-(\sqrt{s}-m_0)^2/w}$
$\sigma_0 = 15$ **mb,** $m_0 = 0.65$ **GeV,** $w = 0.1$ **GeV**

Effect of particle properties

- masses, widths, branching ratios not same in UrQMD and s95p





Integrals

$$\tau_{\rm scat}^{-1} = \frac{1}{n_\pi(T,\mu_\pi)} \sum_i \int \frac{\mathrm{d}^3 p_\pi}{(2\pi)^3} \frac{\mathrm{d}^3 p_i}{(2\pi)^3} \frac{1}{e^{(E_\pi-\mu_\pi)/T}-1} \frac{g_i}{e^{(E_i-\mu_i)/T}\pm 1}$$

$$\times \frac{\sqrt{(s-s_a)(s-s_b)}}{2E_\pi E_i} \sigma_{\pi i}(s)$$

$\Downarrow$

. . . some algebra. . .

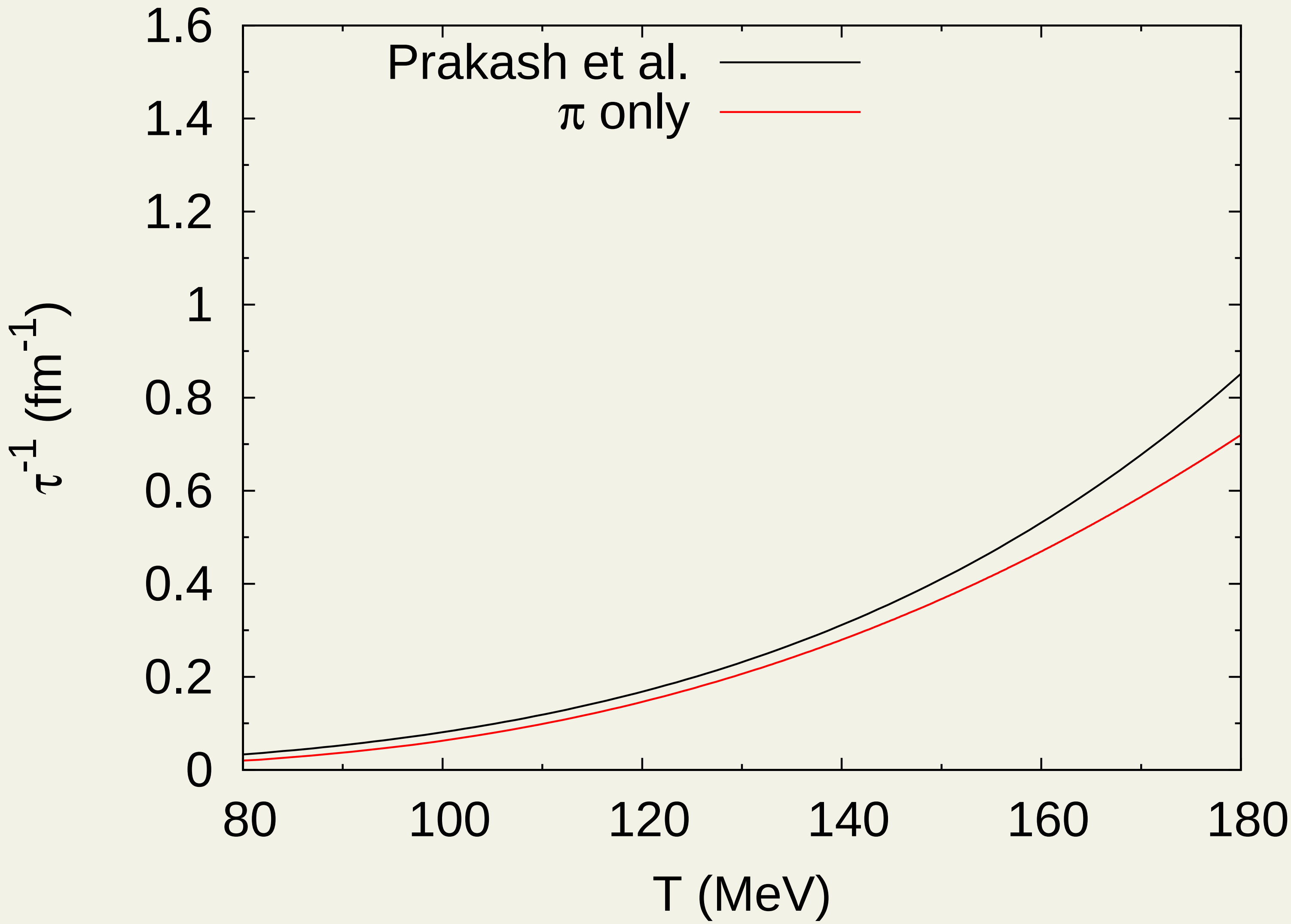
$\Downarrow$

$$\cdots = \frac{T}{n_\pi(T,\mu_\pi)} \sum_i \frac{g_i}{32\,\pi^4} \sum_{n=1}^{\infty} (-1)^{n+1} e^{n\mu_\pi/T} \sum_{m=1}^{\infty} \frac{(\mp 1)^{m+1}}{m} e^{m\mu_i/T}$$

$$\times \int_{s_a}^{\infty} \mathrm{d}s \frac{\sigma_{\pi i}(s)\ (s-s_a)(s-s_b)}{\sqrt{as-(a-1)(m_i^2-am_\pi^2)}} \mathrm{K}_1\!\left(m\sqrt{as-(a-1)(m_i^2-am_\pi^2)}/T\right),$$

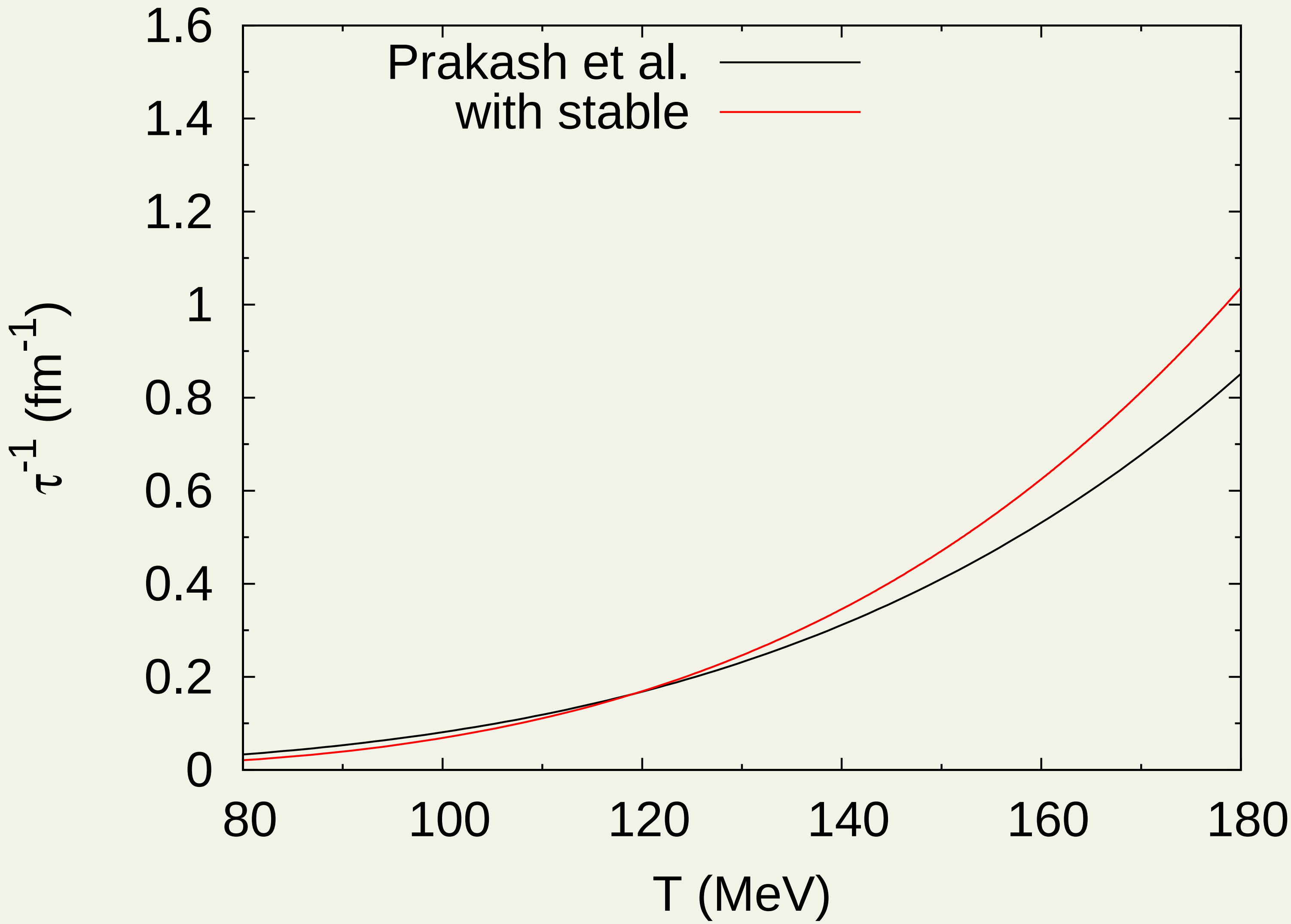
where $a = n/m$

Pions only

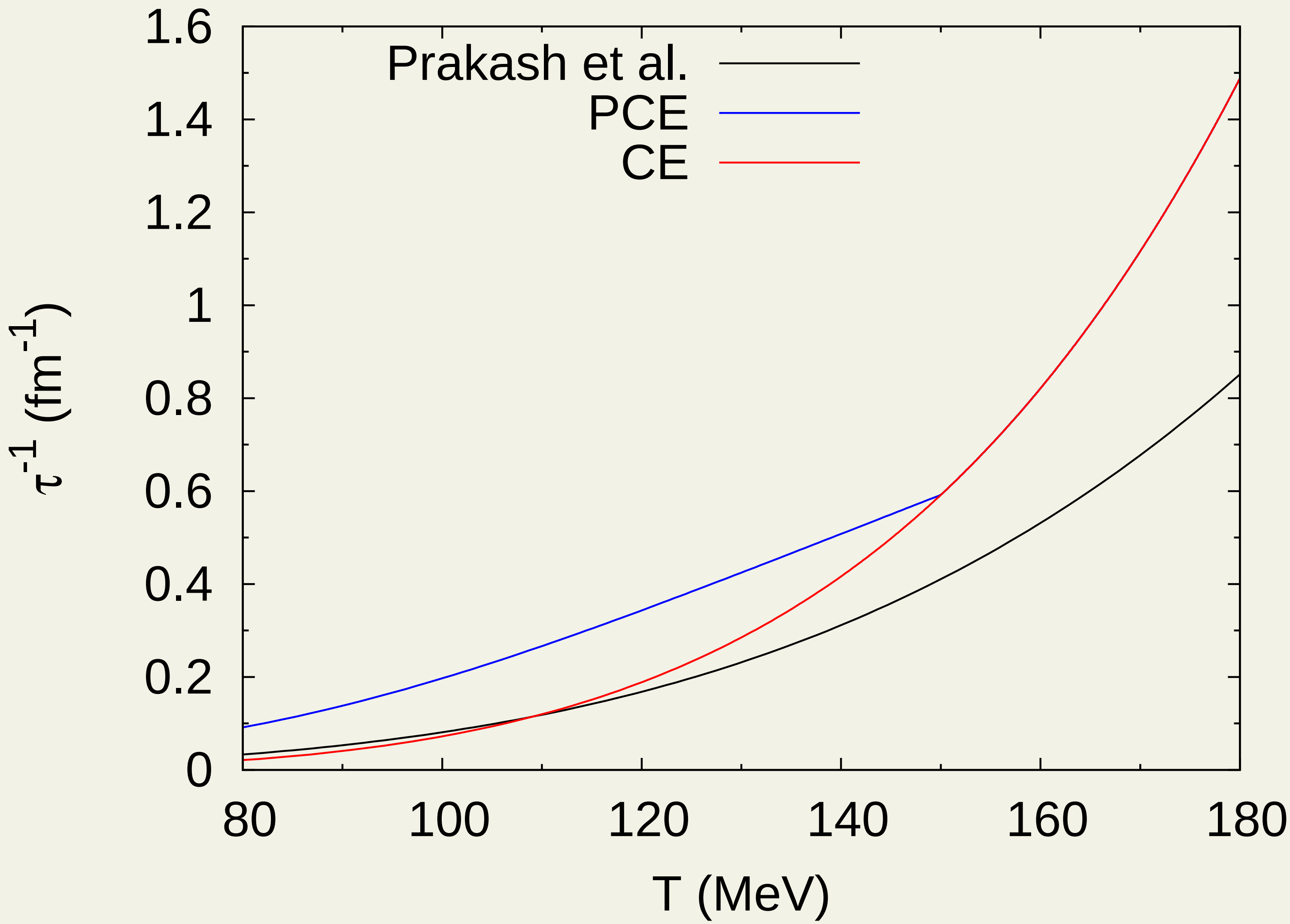


Prakash et al.
π only
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
τ-1 (fm-1)
80
100
120
140
160
180
T (MeV)

Scattering with stable particles



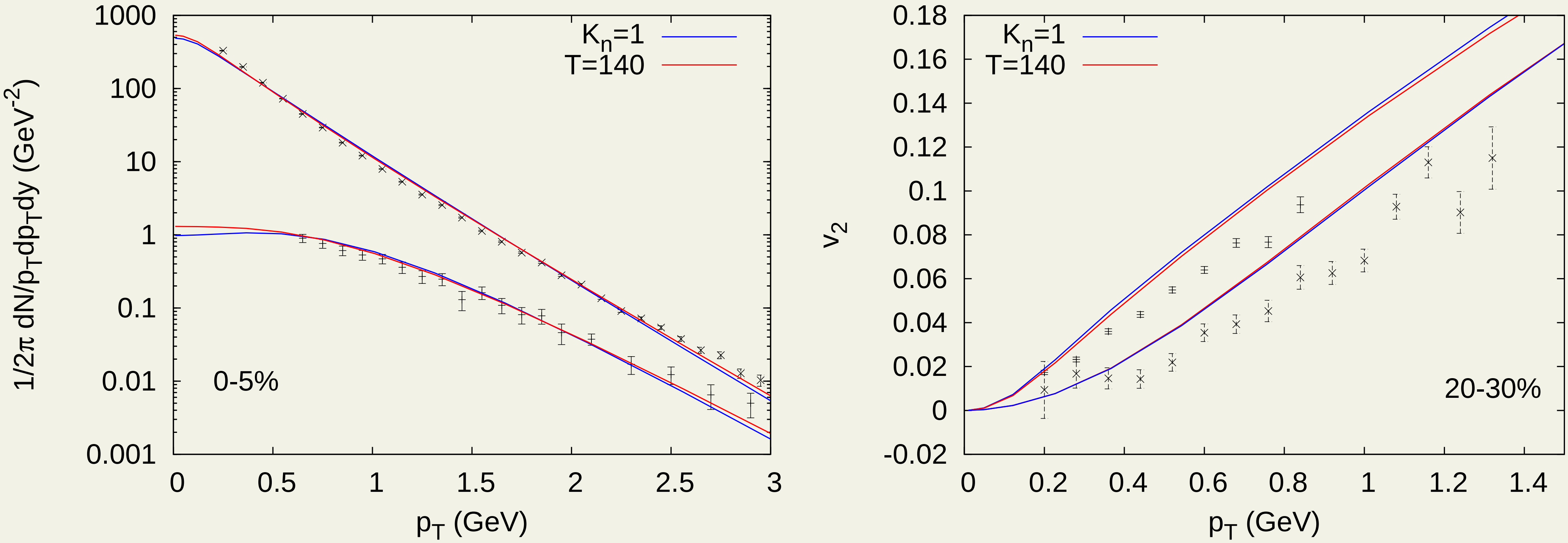
Total rate



Prakash et al.
PCE
CE
τ^{-1} (fm^{-1})
T (MeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
80
100
120
140
160
180

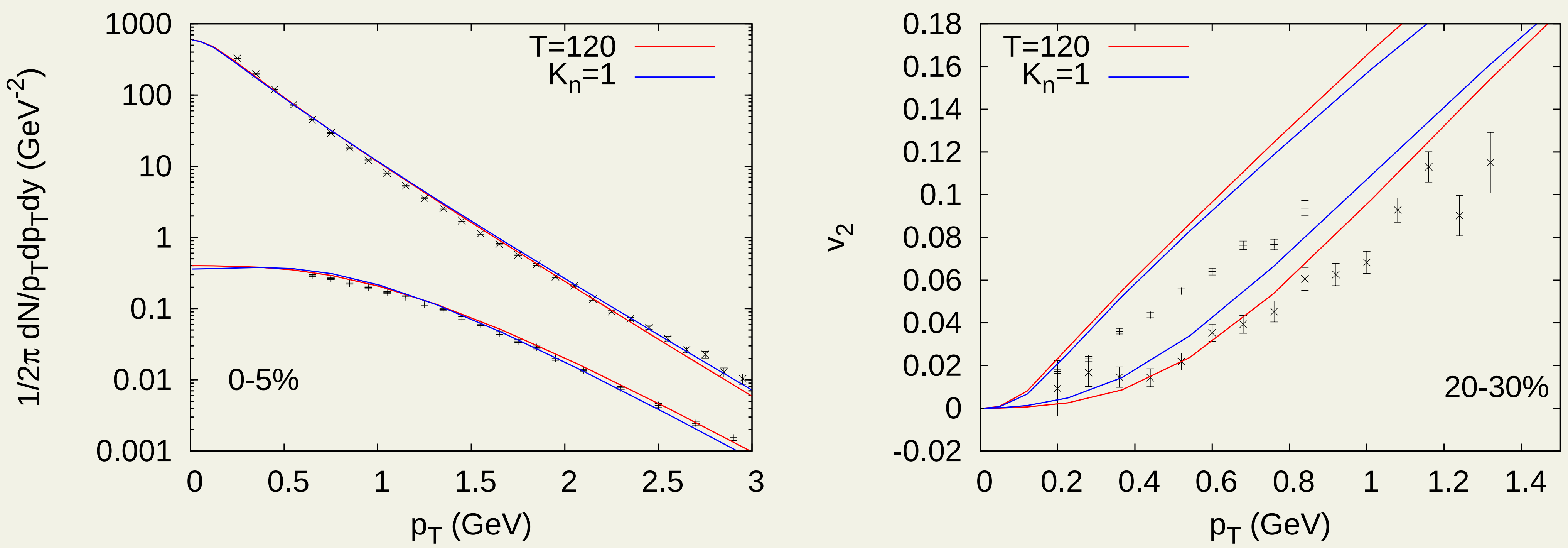
Effect on distributions

- **chemical equilibrium**



Effect on distributions

- chemical freeze-out at $T_{\rm chem} = 150$ MeV



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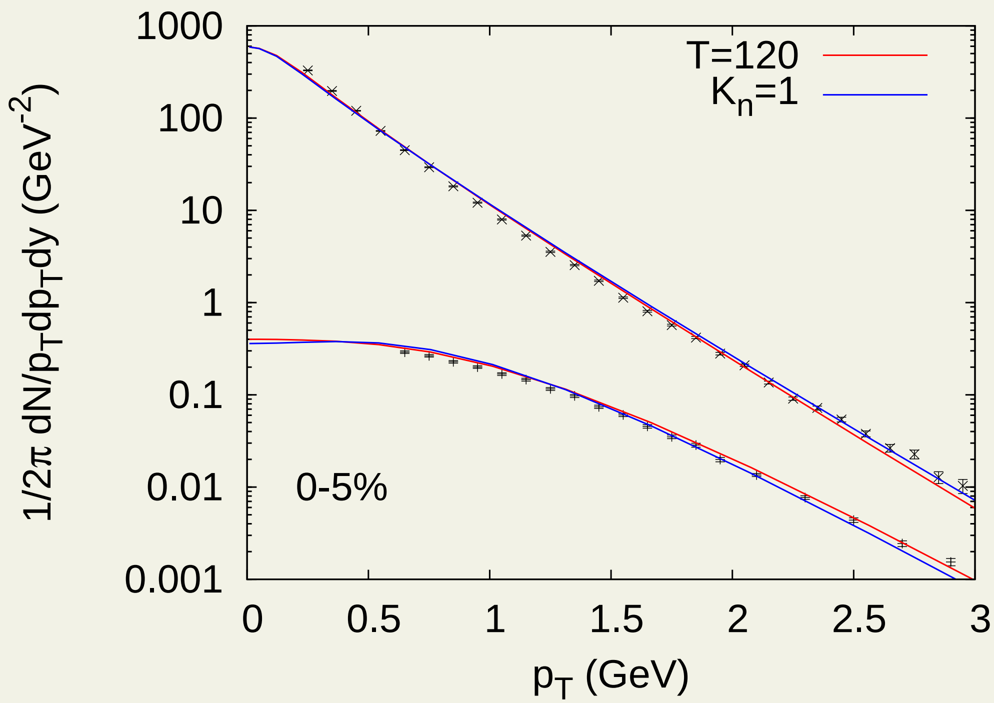

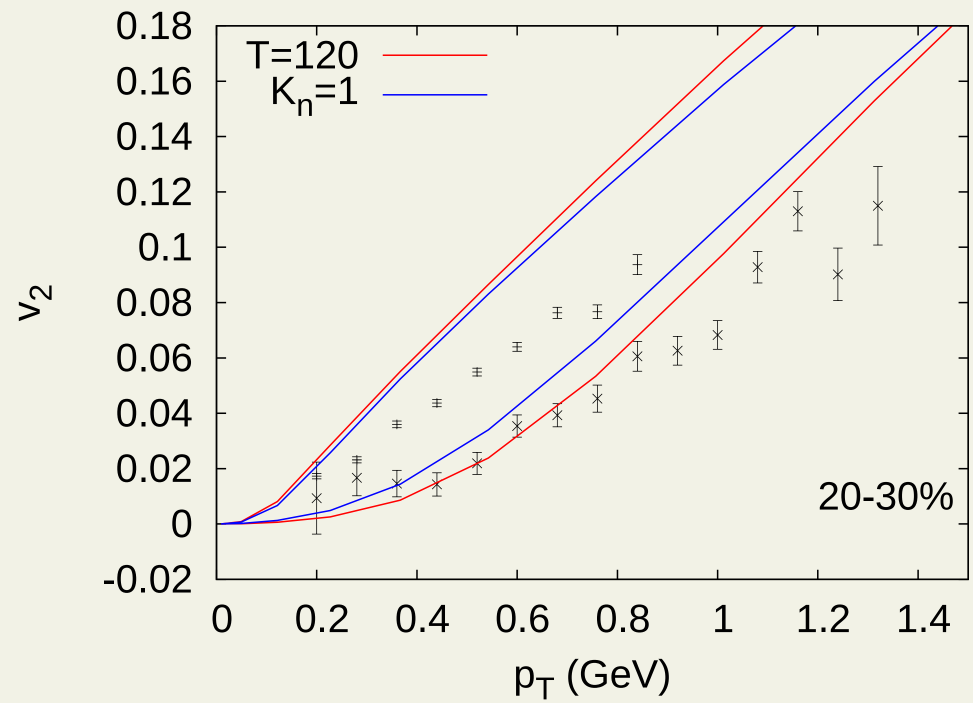

$\tau_0 = 0.6$ **fm, sBC**

Effect on distributions

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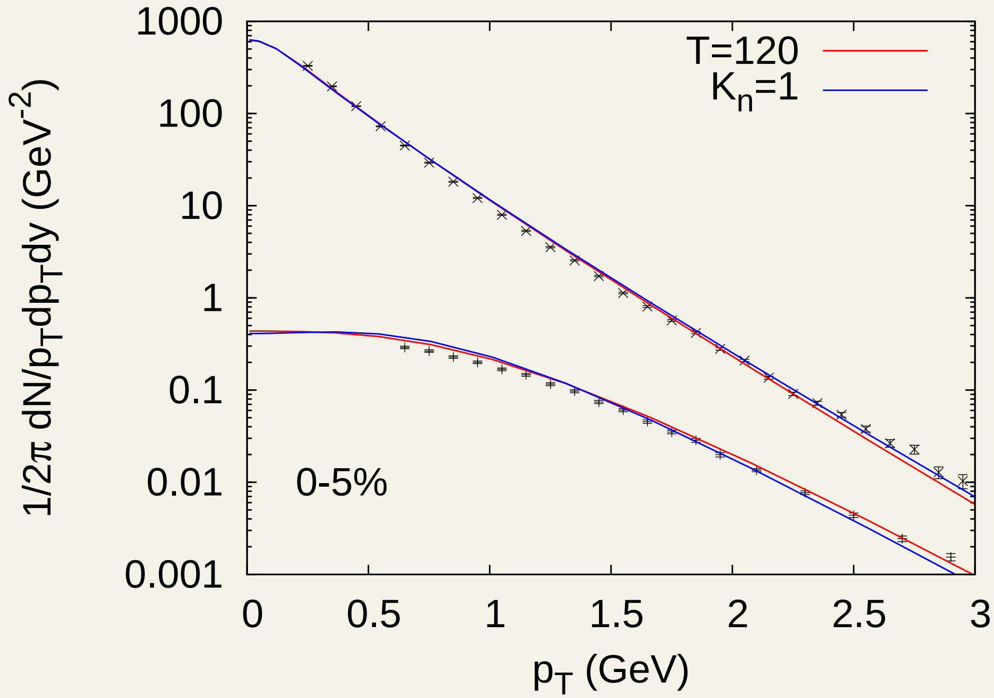

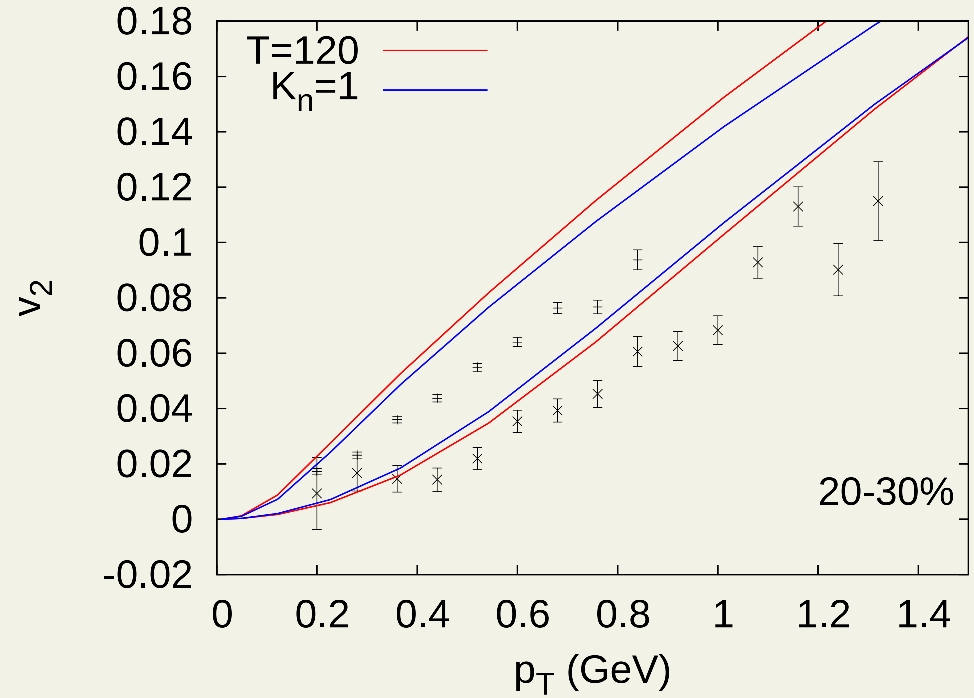

$\tau_0 = 0.2$ **fm, eWN+eBC**

Conclusions

- constant T freeze-out is an oversimplification
- effect is small but non-negligible
- effect on HBT or δf ?

Pressure vs. Budapest-Wuppertal lattice