Fluctuation and Flow Probes of Early-Time Correlations

NeD & TURIC
Hersonissos, 30 June 2012

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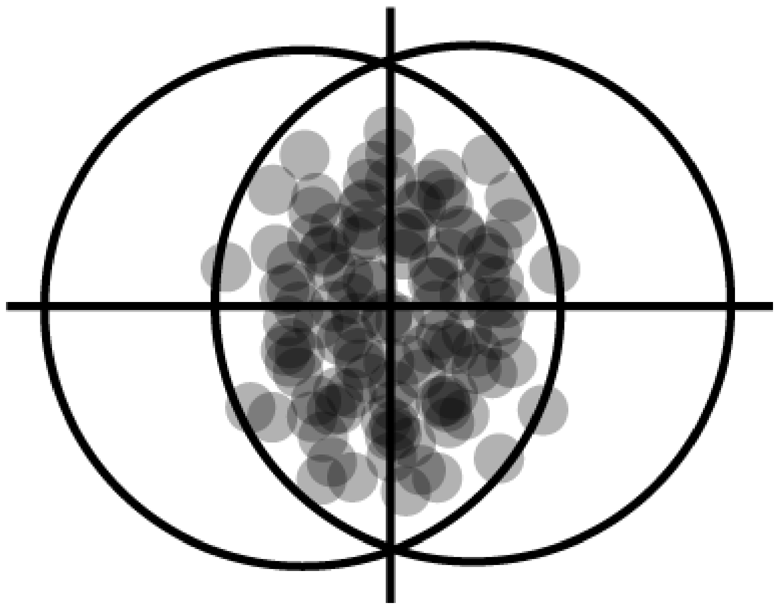
Motivation



Can correlations distinguish lumpy vs. smooth initial conditions?

nucl-th/1107.3317

nucl-th/1205.1218

Two Particle Correlations:

$$\rho_2 = \frac{d^2N}{d\boldsymbol{p}_1 d\boldsymbol{p}_2}$$

Pair Distribution

pairs = singles^2 + correlations

Borghini, Dinh, Ollitrault

Sources of Correlation:

Space $\Rightarrow$ Momentum

- Geometry (global, long range)
- Same source (local, long range)

Other + “non-floiw”

- Momentum conservation (long range)
- Jets (short range)
- Resonance decays (short range)
- …

Correlations and Fluctuations

correlations = pairs - singles^2

$$r(\boldsymbol{p}_1, \boldsymbol{p}_2) = \rho_2(\boldsymbol{p}_1, \boldsymbol{p}_2) - \rho_1(\boldsymbol{p}_1)\ \rho_1(\boldsymbol{p}_2)$$

$$\iint r(\boldsymbol{p}_1, \boldsymbol{p}_2) d\boldsymbol{p}_1 d\boldsymbol{p}_2 = \langle N(N-1)\rangle - \langle N\rangle^2 = Var(N) - \langle N\rangle$$

Influence of "lumps": $r\ (\mathbf{p}_1,\mathbf{p}_2) \neq 0$

Non-zero values indicate non-Poissonian behavior

Multiplicity Fluctuations:

$$\mathcal{R} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

Independent of geometry.
Independent of flow.

GM, Gavin nucl-th/1107.3317

Transverse Momentum Fluctuations

STAR nucl-ex/0504031; Gavin nucl-th/0308067

p_t covariance:

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle \qquad \delta p_t \equiv p_t - \langle p_t \rangle$$

Fluctuations ⇔ Correlations

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \iint \delta p_{t1} \delta p_{t2} \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\langle N(N-1) \rangle} d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

Influence of "lumps": Correlations, r ($\mathbf{p}_1$,$\mathbf{p}_2$), modified by transverse expansion based on origin..

Independent of geometry and anisotropic flow,
but not average expansion.

Momentum Correlation Function

Local equilibrium hydro evolution + Cooper-Frye freeze out

$$r(p_1, p_2) = \iint\limits_{\substack{\text{freeze-out}\\ \text{surface}}} c(x_1, x_2) f(x_1, p_1) f(x_2, p_2)$$

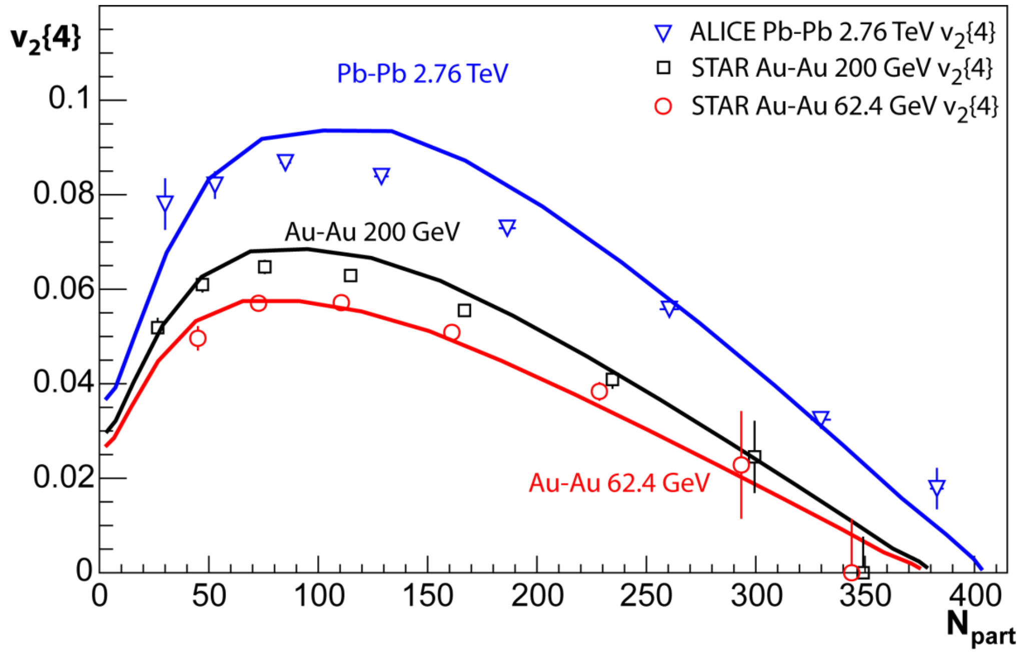
Blast wave expansion:

- Normalized Boltzmann distribution

$$f(\vec{x}, \vec{p}) = n^{-1} e^{-u^{\mu} p_{\mu}/T}$$

- Eccentricity **ε** characterizes elliptic geometry.
- **v** and **T** Average values from spectra

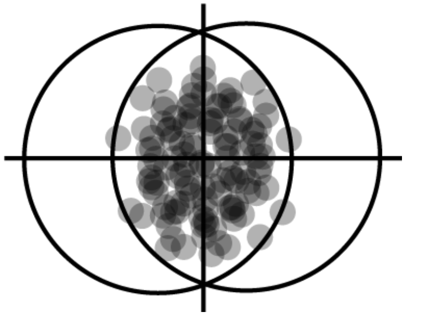
$$\gamma_t \vec{v}_t = \varepsilon_x x \hat{x} + \varepsilon_y y \hat{y}$$



Flux Tubes and Correlations

Spatial correlations:

$$c(\boldsymbol{x}_1,\boldsymbol{x}_2)=\Big\langle\big[n(\boldsymbol{x}_1)-\langle n(\boldsymbol{x}_1)\rangle\big]\big[n(\boldsymbol{x}_2)-\langle n(\boldsymbol{x}_2)\rangle\big]\Big\rangle$$



- Density "lumps" emerge from flux tubes.
- Correlated partons from same flux tube
- Flux tube size << $\mathrm{R_A} \Rightarrow \delta(\vec{r}_t)$
- Average all flux tube distributions $\rho_{FT}(\vec{R}_t)\approx\frac{2}{Area}\left(1-\frac{R_t^2}{R_A^2}\right)$

$$c(\boldsymbol{x}_1,\boldsymbol{x}_2)\approx\mathcal{R}\langle N\rangle^2\delta(\boldsymbol{r}_t)\rho_{FT}(\boldsymbol{R}_t)$$

$$\boldsymbol{r}_t=\boldsymbol{r}_{t1}-\boldsymbol{r}_{t2}$$

$$\boldsymbol{R}_t=(\boldsymbol{r}_{t1}+\boldsymbol{r}_{t2})/2$$

$$\iint r(\boldsymbol{p}_1,\boldsymbol{p}_2)d\boldsymbol{p}_1d\boldsymbol{p}_2=\iint c(\boldsymbol{x}_1,\boldsymbol{x}_2)d\boldsymbol{x}_1d\boldsymbol{x}_2=\langle N\rangle^2\mathcal{R}$$

Flux Tubes in Glasma

Correlations of N_{FT} Flux Tubes :
Fluctuations in tube number

$$\mathcal{R} = \frac{Var(N) - \langle N \rangle}{\langle N \rangle^2} \propto \frac{1}{\langle N_{FT} \rangle}$$

Gluon Rapidity Density

Kharzeev & Nardi

$$\frac{dN}{dy} = \frac{gluons}{tube} \times \langle N_{FT} \rangle \propto \langle N_{FT} \rangle \alpha_s^{-1}\left(Q_s^2\right)$$

Long range Glasma fluctuations:
Depends only on the saturation scale, Q_s.

Dumitru, Gelis, McLerran & Venugopalan;
Gavin, McLerran & GM

$$\mathcal{R}\frac{dN}{dy} \propto \alpha_s^{-1}\left(Q_s^2\right)$$

Multiplicity Fluctuations in Glasma

Glasma prediction:

$$\mathcal{R}\frac{dN}{dy} = \kappa\alpha_s^{-1}\left(Q_s^2\right)$$

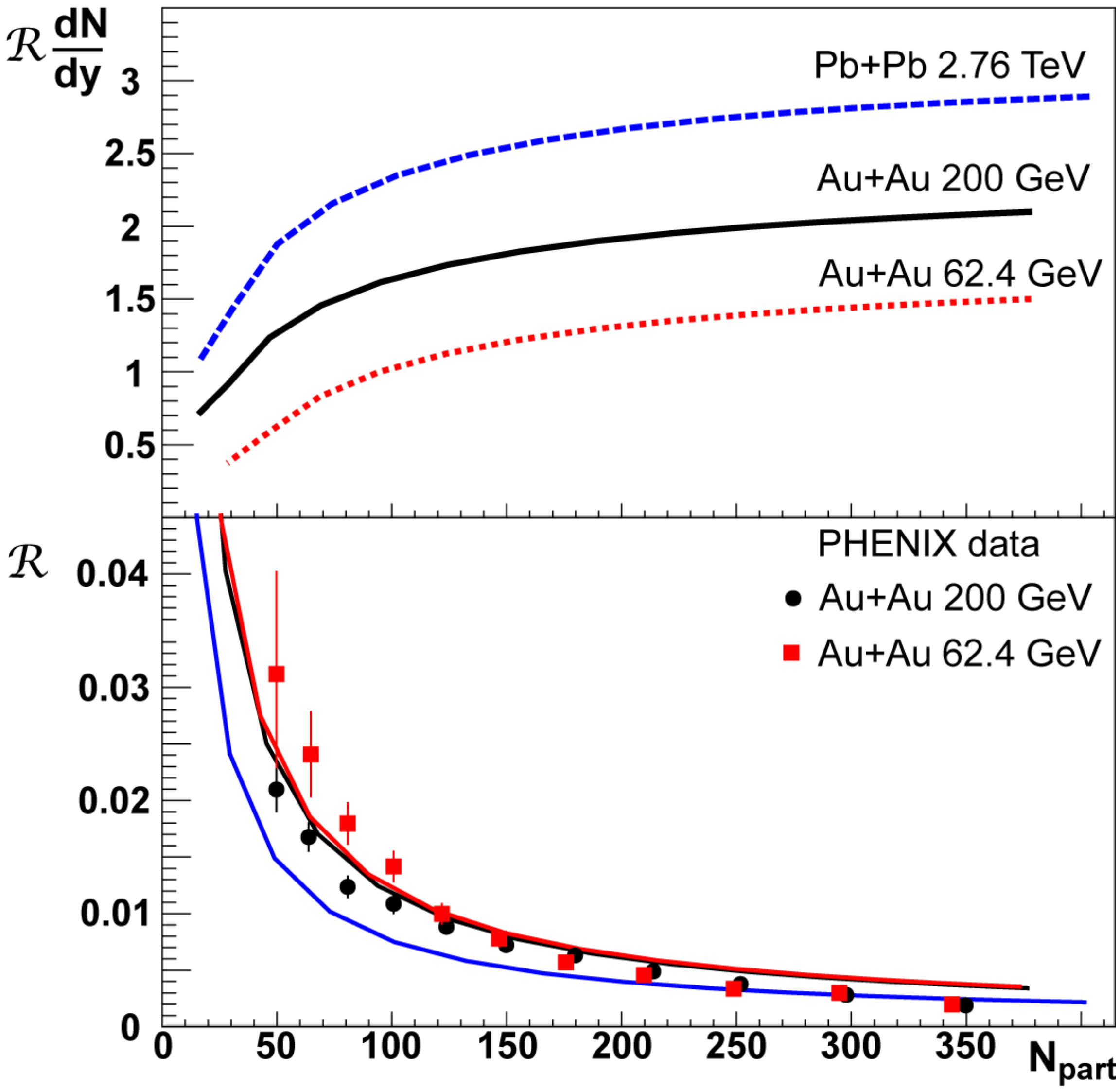
Dumitru, Gelis, McLerran & Venugopalan;
Gavin, McLerran & GM

Fix κ using ridge analysis in 200 GeV Au+Au

Negative binomial distribution

$$\mathcal{R} = k_{NBD}^{-1}$$

Gelis, Lappi & McLerran



GM, Gavin nucl-th/1107.3317

p_t Fluctuations in Glasma

Momentum fluctuations

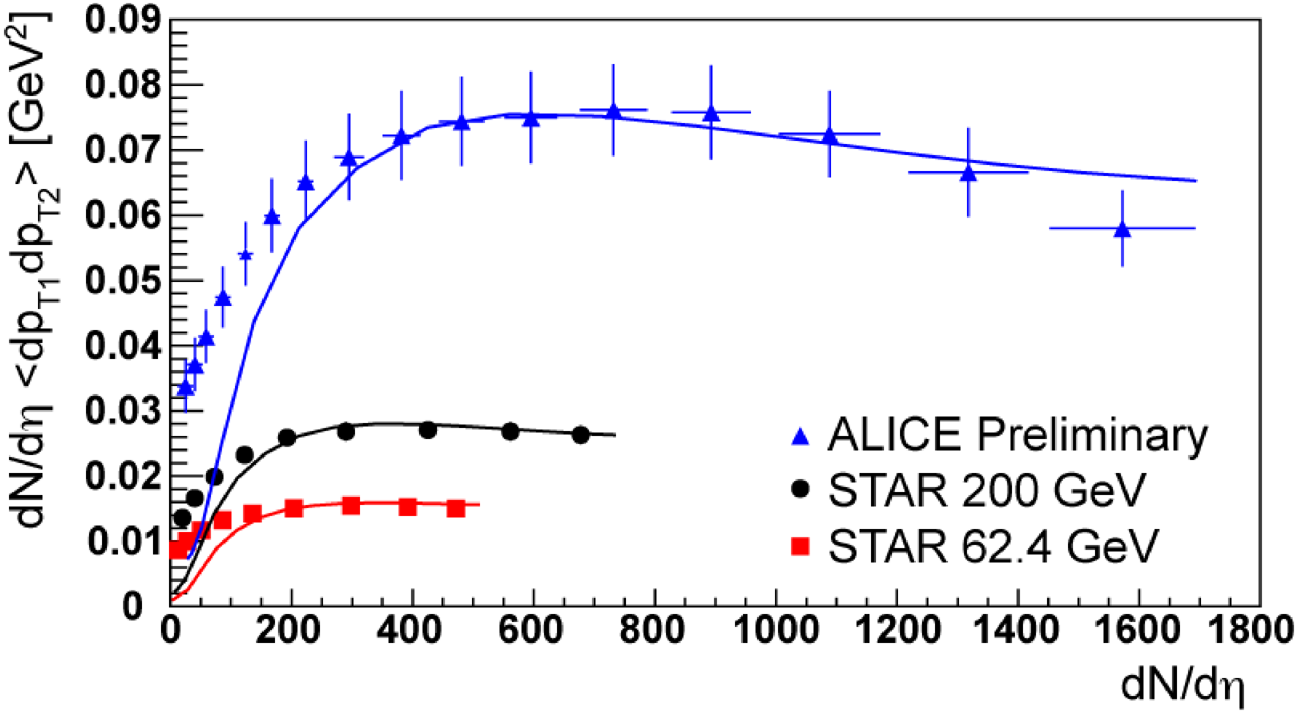
$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \iint \delta p_{t1} \delta p_{t2} \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\langle N(N-1) \rangle} d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

Local momentum excess **averaged over spatial geometries.**

Gavin, GM. nucl-th/1107-3317

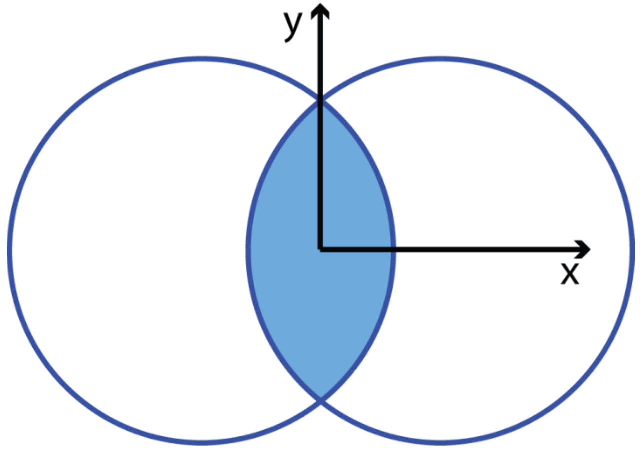
- Blast wave $f(p,x)$
- Same scale factor
- Glasma energy dependence

$$\mathcal{R} \frac{dN}{dy} \propto \alpha_s^{-1}(Q_s^2)$$

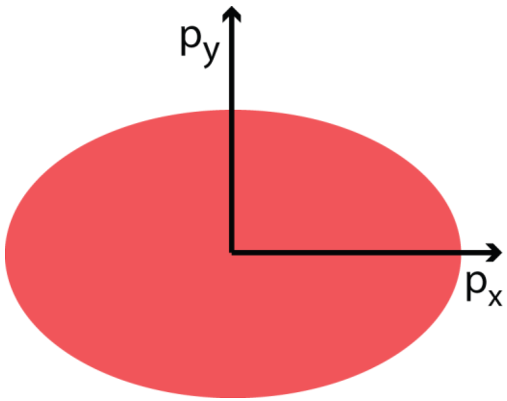


Anisotropic Flow

Initial State Configuration



Final State Momentum



Fourier Flow Coefficient:

$$\langle v_n \rangle = \langle \cos n(\phi - \psi_{RP}) \rangle$$

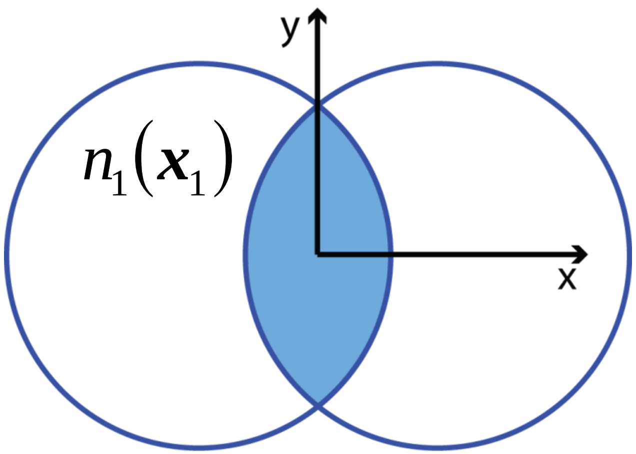
Two-particle coefficient:
no reaction plane needed

$$v_n\{2\}^2 = \frac{\left\langle \sum_{i \neq j} \cos n(\phi_i - \phi_j) \right\rangle}{\langle N(N-1) \rangle}$$

$$\phi = \tan^{-1}(p_y / p_x)$$

Anisotropic Flow

Initial State Configuration



$n_1(\boldsymbol{x}_1)$

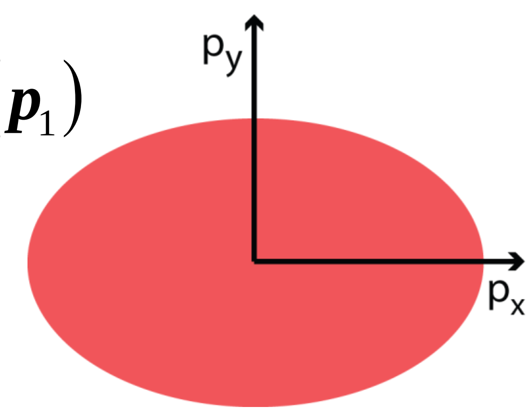
Fourier Flow Coefficient:

$$\langle v_n \rangle = \int \frac{\rho_1(\boldsymbol{p}_1)}{\langle N \rangle} \cos n(\phi_1 - \psi_{RP}) \, d\boldsymbol{p}$$

Two-particle coefficient:
no reaction plane needed

Final State Momentum

$\rho_1(\boldsymbol{p}_1)$



$$v_n\{2\}^2 = \int \frac{\rho_2(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\langle N(N-1) \rangle} \cos(n\Delta\phi) \, d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

$$\Delta\phi = \phi_1 - \phi_2$$

Cumulant Expansion

Pair Distribution:

$$\rho_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = \rho_1(\boldsymbol{p}_1)\ \rho_1(\boldsymbol{p}_2) + r(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

Borghini, Dinh, Ollitrault

Two-particle coefficient:

$$v_n\{2\}^2 \approx \langle v_n \rangle^2 + 2\sigma_n^2$$

- $<v_n>^2$ = reaction plane correlations
- σ^2_n = other correlations
- $v_n\{4\} \approx <v_n>$

Borghini, Dinh, Ollitrault; Voloshin, Poskanzer, Tang, Wang

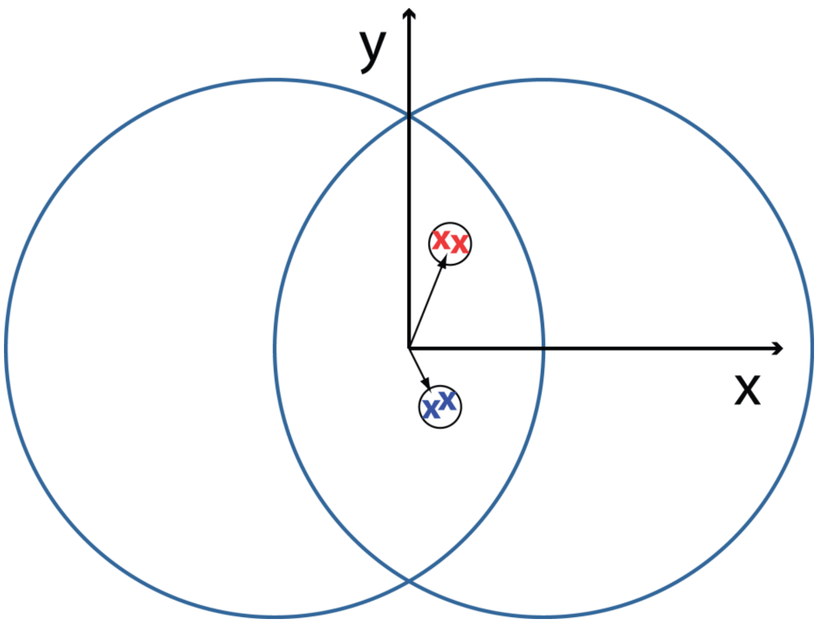
Correlated Part:

$$\sigma_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2} = \int \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{2\langle N(N-1)\rangle} \cos(n\Delta\phi)\ d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

v_n factorization is a signature of flow if $\sigma_n = 0$

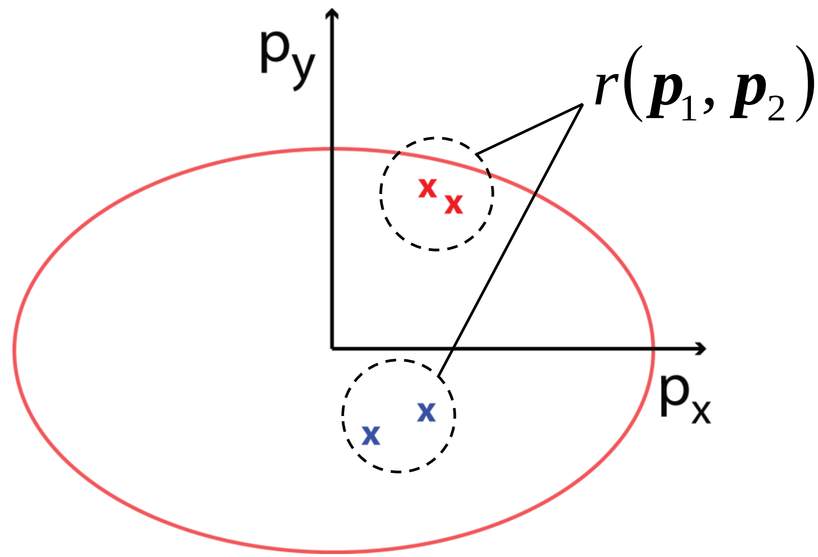
Correlation Mechanism

Initial State Configuration



$$n_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

Final State Momentum



$$\rho_2(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

Final state momenta are correlated to initial position.

- Reaction plane
- Common origin
- Neglect short range correlations

Influence of “lumps”:

- Arbitrary event shapes.
- Transverse expansion modifies correlations based on origin..

Elliptic Flow

Flow and fluctuations:

- Geometry – model input
  Dashed line: Blast Wave $\Rightarrow v_n\{4\}$

- Neglect "non-flow"

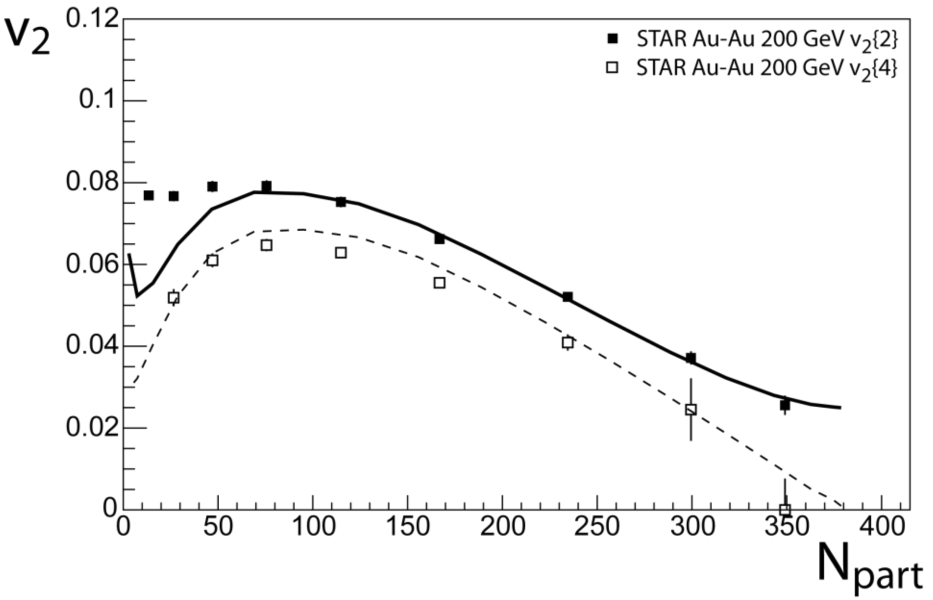
- Calculated fluctuations

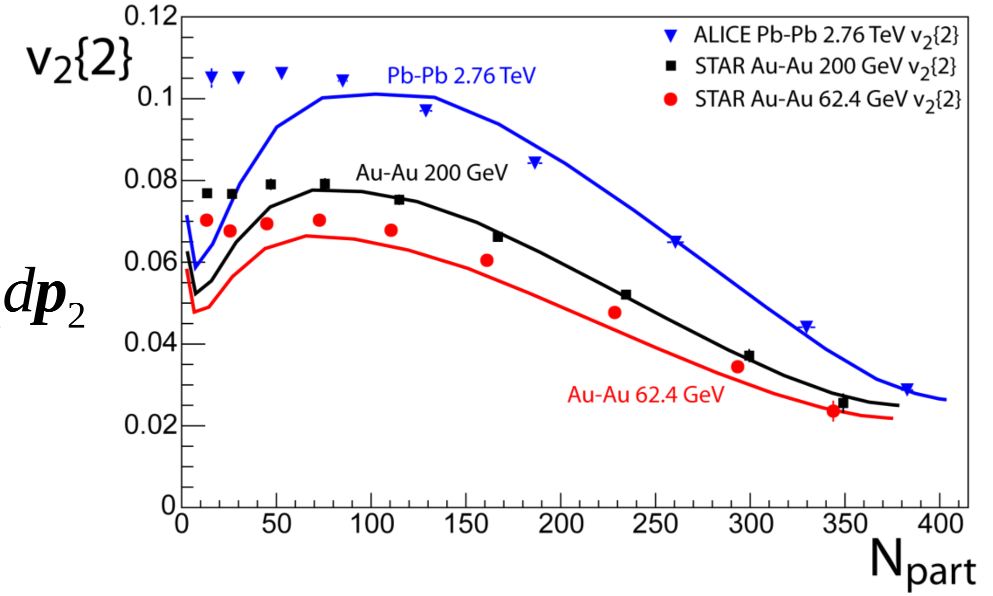
$$\sigma_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2}$$

$$\sigma_n^2 = \int \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{2\langle N(N-1)\rangle} \cos(n\Delta\phi)\ d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

- Energy dependence from $\mathcal{R}$

GM, Gavin nucl-th/1205.1218





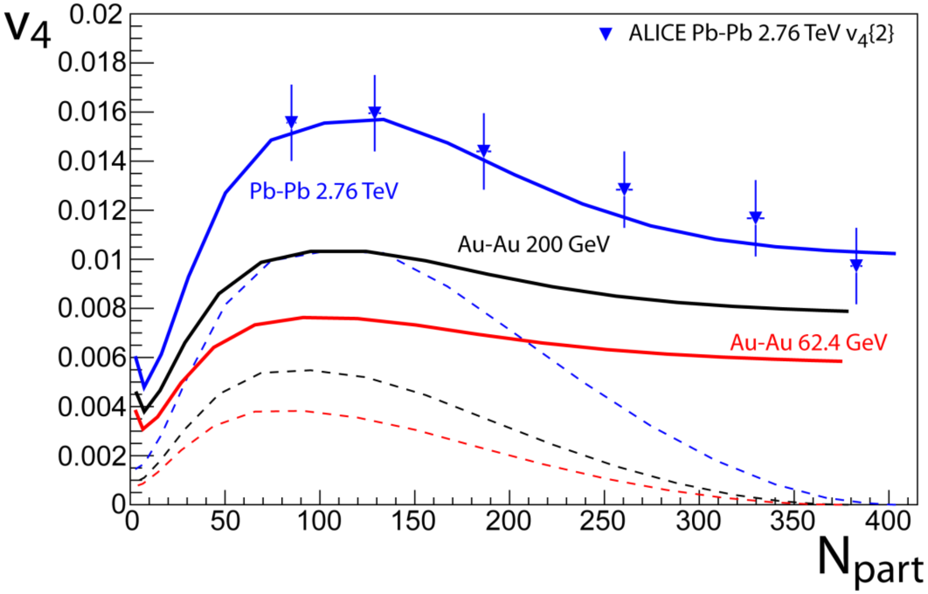
v_4

Flow and fluctuations:

- Geometry – model input
  Dashed line: Blast Wave $\Rightarrow v_n\{4\}$

- Neglect "non-flow"

- Calculated fluctuations



$$\sigma_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2}$$

$$\sigma_n^2 = \int \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{2\langle N(N-1)\rangle} \cos(n\Delta\phi)\ d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

- Energy dependence from $\mathcal{R}$

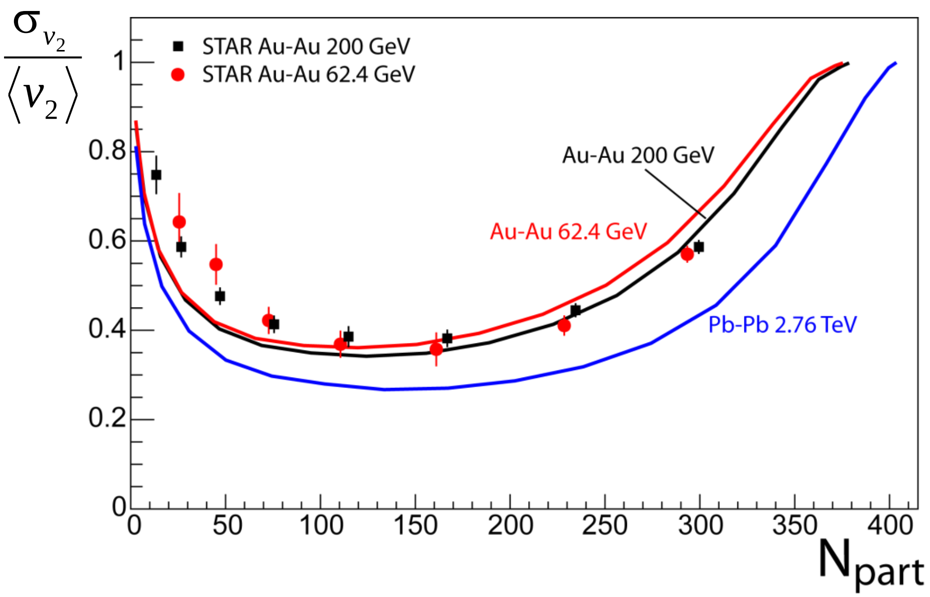
Elliptic Flow Fluctuations

Coefficient of variation: $\dfrac{\sigma_{v_2}}{\langle v_2 \rangle}$

$$\sigma_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2}$$

$$\langle v_n \rangle^2 = \frac{v_n\{2\}^2 + v_n\{4\}^2}{2}$$

Voloshin, Poskanzer, Tang, Wang



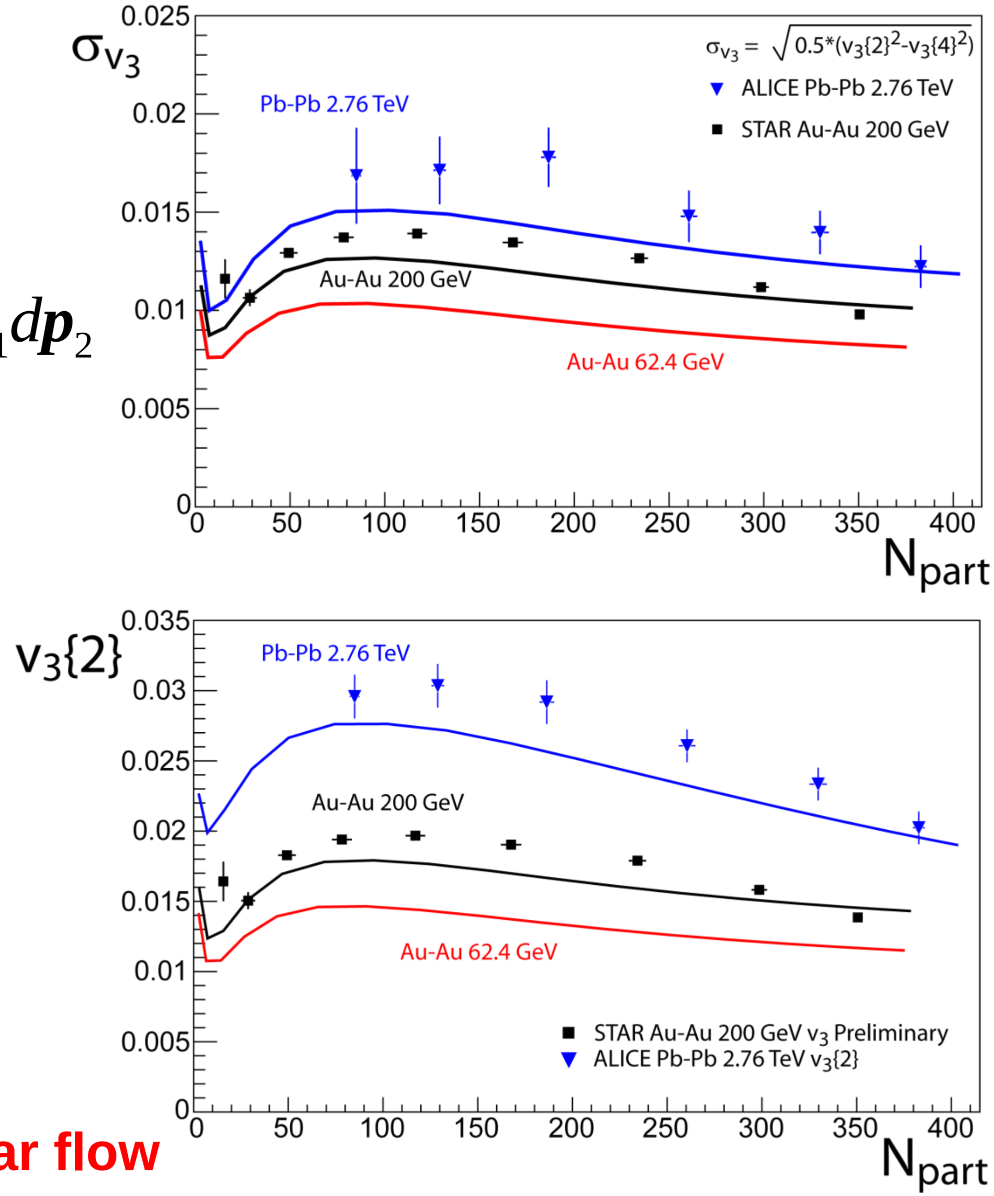
Caution: σ is technically not the variance

Flow Fluctuations and v_3

Flow Fluctuations:

$$\sigma_n^2 = \int \frac{r(\boldsymbol{p}_1, \boldsymbol{p}_2)}{2\langle N(N-1)\rangle} \cos(n\Delta\phi)\; d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

- STAR $v_3\{4\} = 0$
- Energy dependence from $\mathcal{R}$
- Parameterized ALICE $v_3\{4\}$
- A more realistic ρ_{FT} is needed.



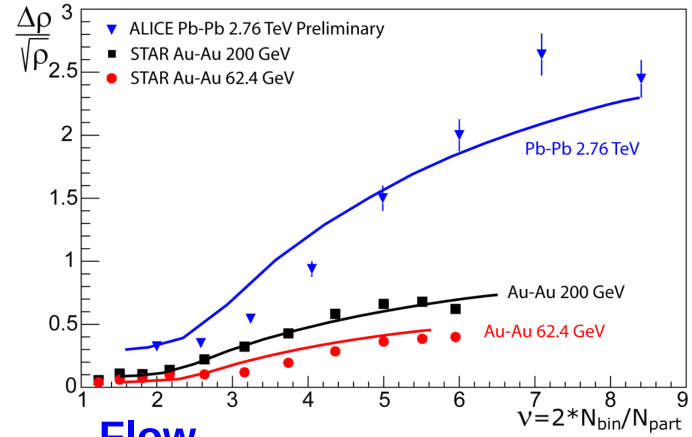
v_3 contributions without triangular flow

Summary:

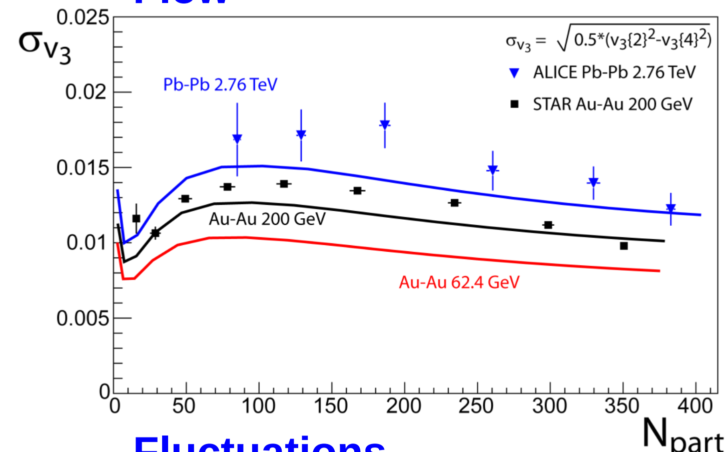
- **Multiplicity fluctuations $\mathcal{R}$** – only depends on the existence of density lumps.
- **Momentum fluctuations** $<\delta p_T \delta p_T>$ - from density lumps and average transverse expansion but not anisotropic flow.
- **Flow fluctuations σ_n** - from density lumps, geometry, and anisotropic flow.
- **The ridge** – the same as flow fluctuations.
- **All depend on the number and size of density lumps ⇒ system, energy, and centrality dependencies.**

Look at them together!

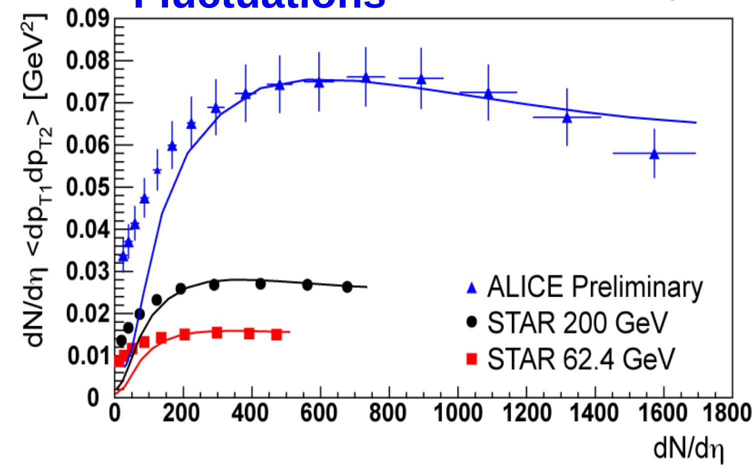
Correlations



Flow



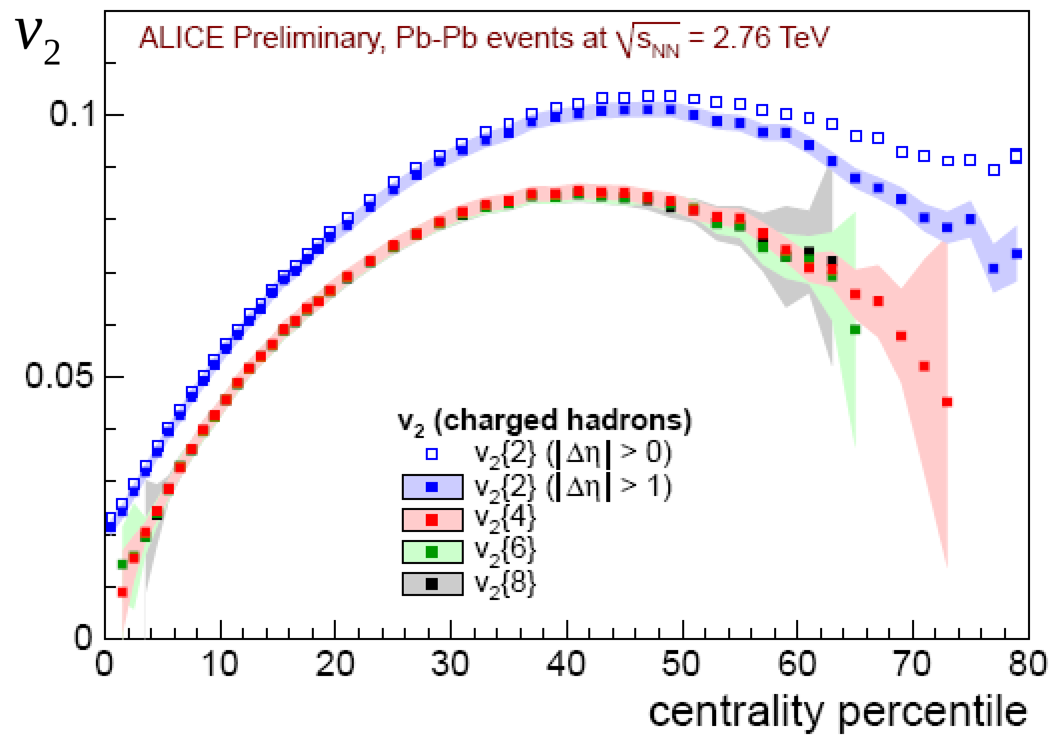
Fluctuations



Long Range Correlations

A. Bilandzic thesis

- Measurements with rapidity gaps do not explain $v_2\{2\}$ and $v_2\{4\}$ differences
- Jets, resonance decays, and other short range effects should be removed
- Long range correlations suggest initial state effects.
- Collision energy dependence of the ridge should mimic v_n



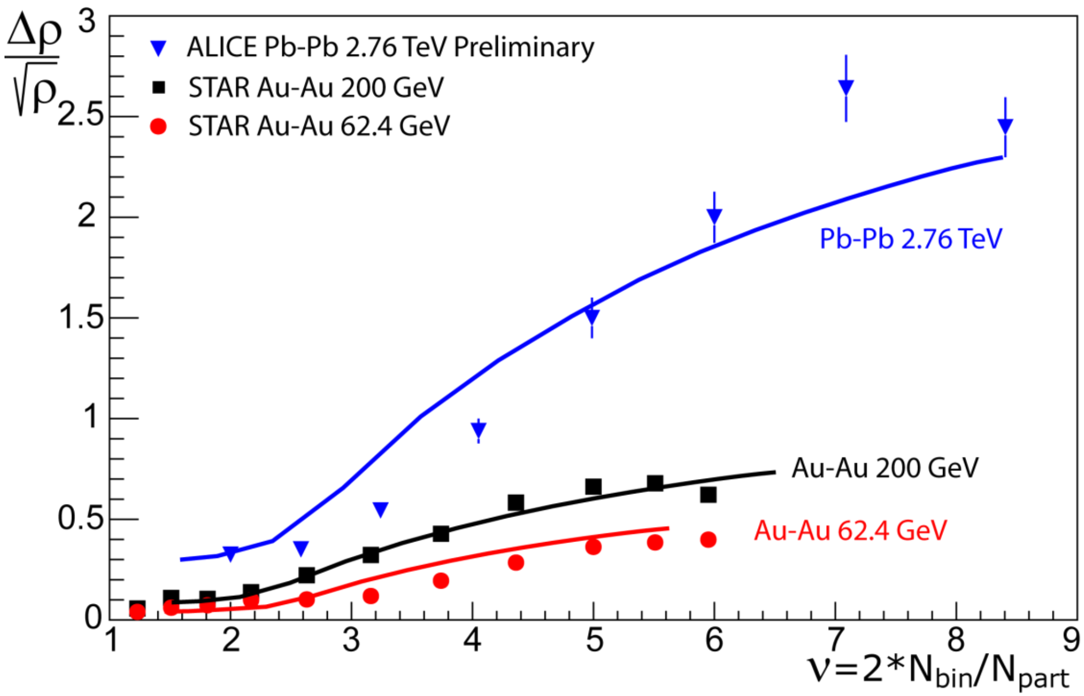
The Soft Ridge

$$\frac{\Delta\rho(\Delta\phi)}{\sqrt{\rho_{ref}}} = \frac{2}{2\pi}\frac{dN}{dy}\sum_{n=1}\langle v_n\rangle^2 \cos n\Delta\phi + \frac{1}{2\pi}\frac{dN}{dy}\frac{r(\Delta\phi)}{\rho_{ref}}$$

Flow subtracted ridge

$$2\frac{dN}{dy}\sigma_n^2 \approx \int \frac{\Delta\rho(\Delta\phi)}{\sqrt{\rho_{ref}}}\cos n\Delta\phi$$

- **Only cos $\Delta\phi$ and cos $2\Delta\phi$ terms subtracted**
- **These terms also contain fluctuations**
- Glasma energy dependence
- $\mathcal{R}$ scale factor set in Au-Au 200 GeV
- Blast wave $f(p,x)$
- Difference in peripheral STAR→ALICE



Four-Particle Coefficients

Four-Particle Distribution: keep only two-particle correlations

$$\rho_4(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4) = \rho_1(\boldsymbol{p}_1)\ \rho_1(\boldsymbol{p}_2)\ \rho_1(\boldsymbol{p}_3)\ \rho_1(\boldsymbol{p}_4)$$
$$+ \rho_1(\boldsymbol{p}_1)\ \rho_1(\boldsymbol{p}_2)\ r(\boldsymbol{p}_3, \boldsymbol{p}_4) + \cdots$$
$$+ r(\boldsymbol{p}_1, \boldsymbol{p}_2)\ r(\boldsymbol{p}_3, \boldsymbol{p}_4) + \cdots$$

Borghini, Dinh, and Ollitrault
Voloshin, Poskanzer, Tang, Wang

Four-particle coefficient:

$$\left\langle \cos n\ (\phi_1 + \phi_2 - \phi_3 - \phi_4) \right\rangle \approx \langle v_n \rangle^4 + 4\langle v_n \rangle^2 \sigma_n^2 + 2\sigma_n^4$$

$$v_n\{4\}^4 = 2v_n\{2\}^4 - \left\langle \cos n\ (\phi_1 + \phi_2 - \phi_3 - \phi_4) \right\rangle \approx \langle v_n \rangle^4$$

$v_n\{4\}$ corrections

Four-particle coefficient:

$$\left\langle \cos n\left(\phi_1+\phi_2-\phi_3-\phi_4\right)\right\rangle$$

$$\approx \left\langle v_n \right\rangle^4 + 4\left\langle v_n \right\rangle^2 \sigma_n^2 + 2\sigma_n^4 - 2\left\langle v_n \right\rangle^2 \mathrm{Re}\left\{\Sigma_n^2\right\} - \left|\Sigma_n^2\right|^2$$

Will cancel with $v_n\{2\}$ terms

$$v_n\{4\}^4 \approx \left\langle v_n \right\rangle^4 - 2\left\langle v_n \right\rangle^2 \mathrm{Re}\left\{\Sigma_n^2\right\} - \left|\Sigma_n^2\right|^2$$

Corrections of order ~1.2%

$$\mathrm{Re}\left\{\Sigma_n^2\right\} = \int \frac{r\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right)}{\left\langle N\left(N-1\right)\right\rangle} \cos 2n\left(\Phi - \psi_{RP}\right) d\boldsymbol{p}_1 d\boldsymbol{p}_2$$

$$\Phi = \left(\phi_1+\phi_2\right)/2$$

$\mathcal{R}$

- K flux tubes, assume

$$\langle N \rangle_K = \mu K$$

$$\langle N^2 \rangle_K - \langle N \rangle_K^2 = \sigma^2 K$$

- K varies event-by-event

$$\langle N \rangle = \mu \langle K \rangle$$

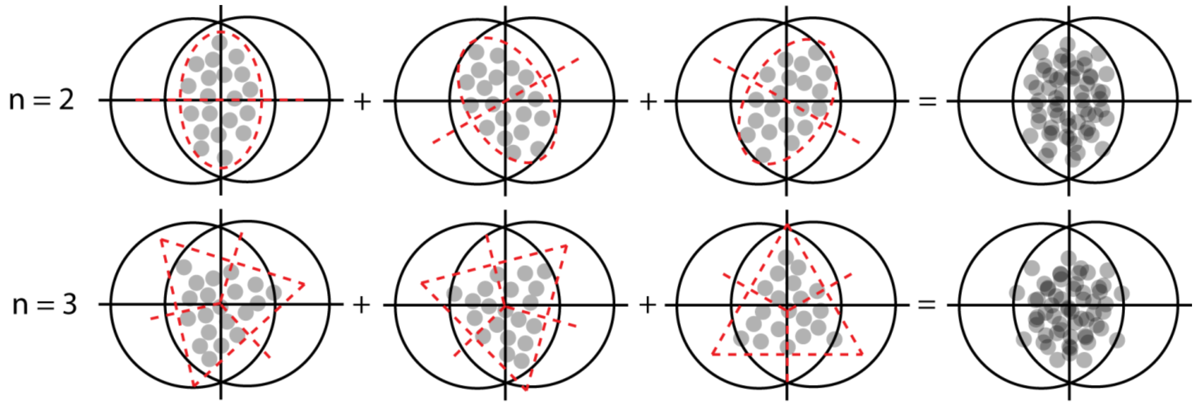
$$\langle N^2 \rangle - \langle N \rangle^2 = \sigma^2 K + \mu^2 \left(\langle K^2 \rangle - \langle K \rangle^2 \right)$$

For K sources that fluctuate per event

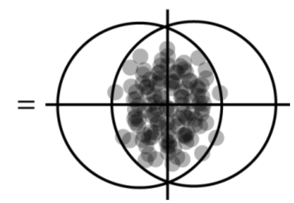
$$\mathcal{R} = \frac{\sigma^2 - \mu}{\mu^2} \frac{1}{\langle K \rangle} + \frac{\langle K^2 \rangle - \langle K \rangle^2}{\langle K \rangle^2} \approx \frac{1}{\langle K \rangle}$$

Fluctuations per source + Fluctuations in the number of sources

Source Distributions



Average all flux tube distributions =



$\Rightarrow \rho_{FT}$

- The v_2 event plane is correlated with real reaction plane.
- The v_3 event plane is arbitrary.
- Event averages represent both event shape and event plane fluctuations.
- Blast wave: Currently captures v_3 event plane fluctuations but not triangular flow.

Geometry: probability distribution of flux tubes ~ nuclear thickness

$$\rho_{FT}\left(\vec{R}_t\right) \approx \frac{2}{Area}\left(1 - \frac{R_t^2}{R_A^2}\right)$$