Higher moments characteristics from QGP flow

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Outline

- Introduction – Why higher moments?
- Basic definitions
- Results and discussions
- Summary

Introduction

1. Relativistic Fluid dynamics model

Relativistic fluid dynamics (FD) is based on the **conservation laws** and the assumption of **local equilibrium (→ EoS)**

$$\boxed{\begin{aligned} N^{\mu}{}_{,\mu} &= 0 \\ T^{\mu\nu}{}_{,\mu} &= 0 \end{aligned}} \Rightarrow \boxed{\begin{aligned} [N^{\mu} d\hat{\sigma}_{\mu}] &= 0 \\ [T^{\mu\nu} d\hat{\sigma}_{\mu}] &= 0 \end{aligned}}$$

FD needs an initial state, and a final “freeze-out” stage also.
The intermediate stage is described with a **CFD** model, we use the **PIC** method.

N fuid cells, i = 1; 2; :::;N, with time and expansion the number of fluid cells increases.

Equation of state (EOS): MIT Bag Model

$$e_q(T,\mu) = e_{SB}(T,\mu) + B$$

$$P_q(T,\mu) = P_{SB}(T,\mu) - B$$

It can describe QGP without a phase transition.

note:

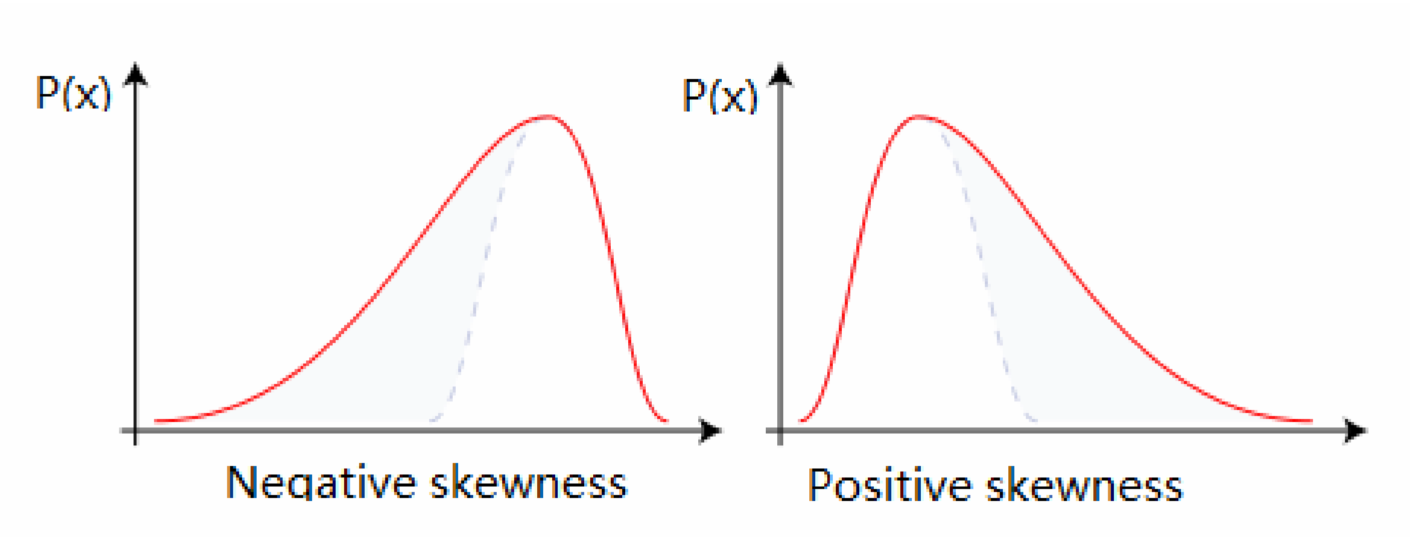
we consider two flavors and massless quarks and gluons;

the bag constant is $B = 0.397\ \mathrm{GeV/fm^3}$

2. Skewness and Kurtosis

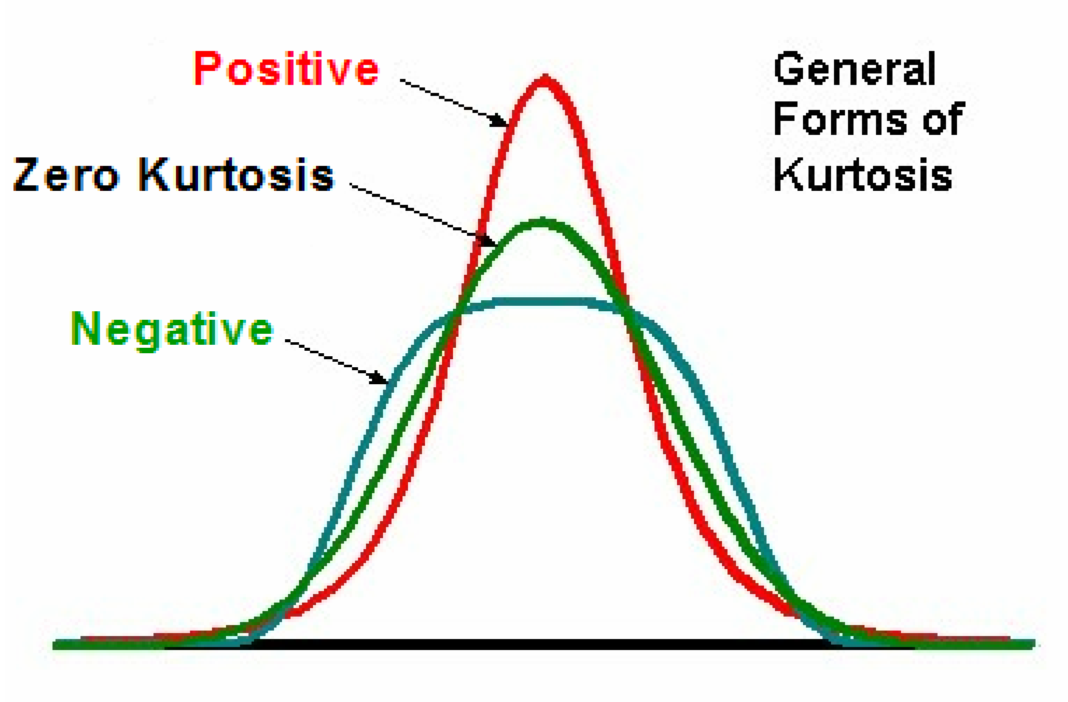
Skewness

A measure of asymmetry of the probability distribution of a real-valued random variable



Kurtosis

A description of the peakedness of a probability distribution



Kurtosis is commonly defined as the forth cumulant divided by the square of the second cumulant.

Near the critical point of a phase transition, the fluctuations will become stronger, the distribution is wider, this leads to negative kurtosis.

Assuming **critical fluctuations** in the QGP to hadronic matter and a first-order transition in a box (similar to the textbook example shown by Prof. I. Mishustin):

$$f(e) = f_1 + \frac{K_1}{e} + K_2(e-e_0) + K_3(e-e_0)^2 + K_4(e-e_0)^3$$

Laurent series instead of 4th order polynomial.

$$P(e) \propto \exp[-\beta F(e)]$$

$$F(e) = \Omega f(e)$$

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Fluctuations in hadronizing quark gluon plasma

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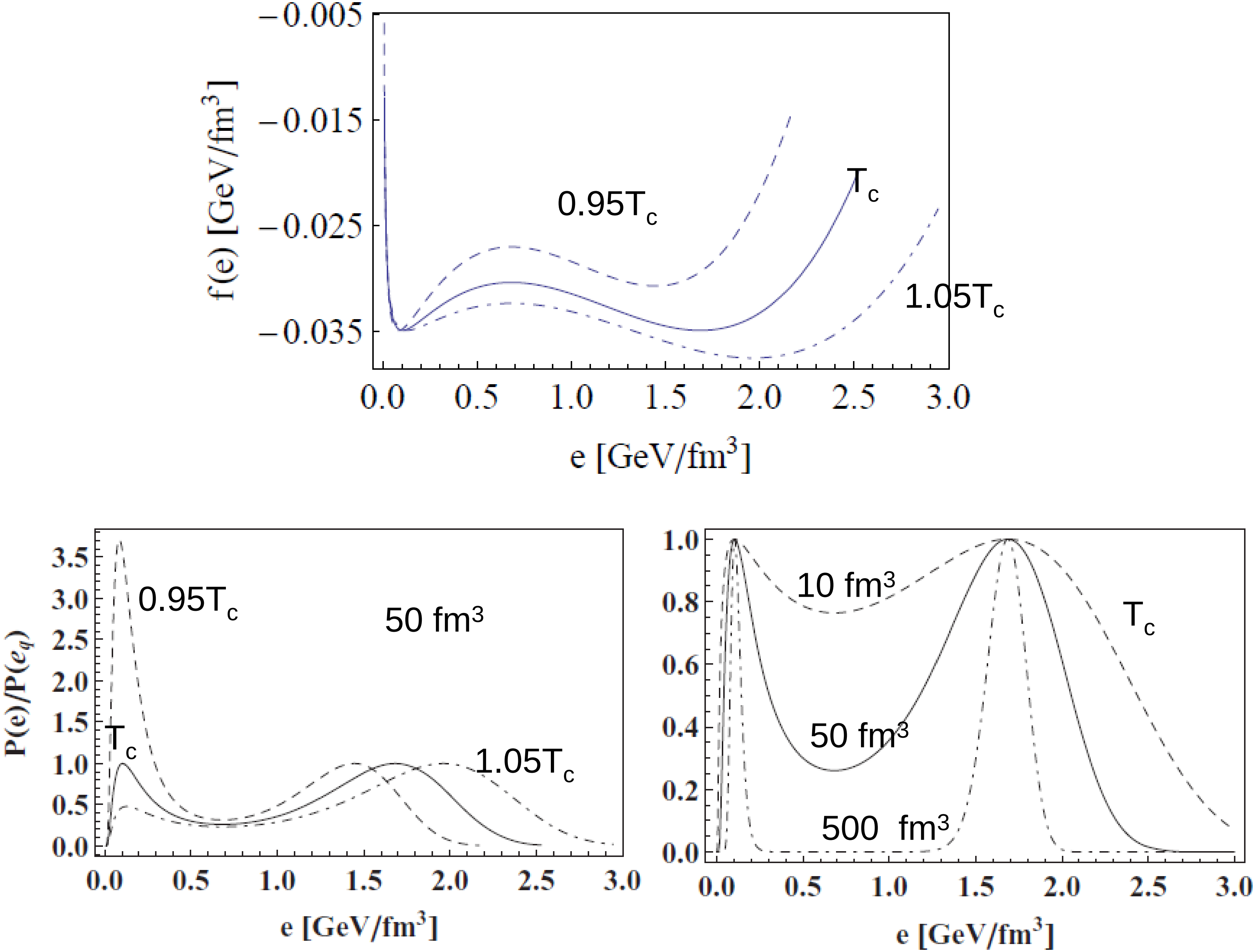
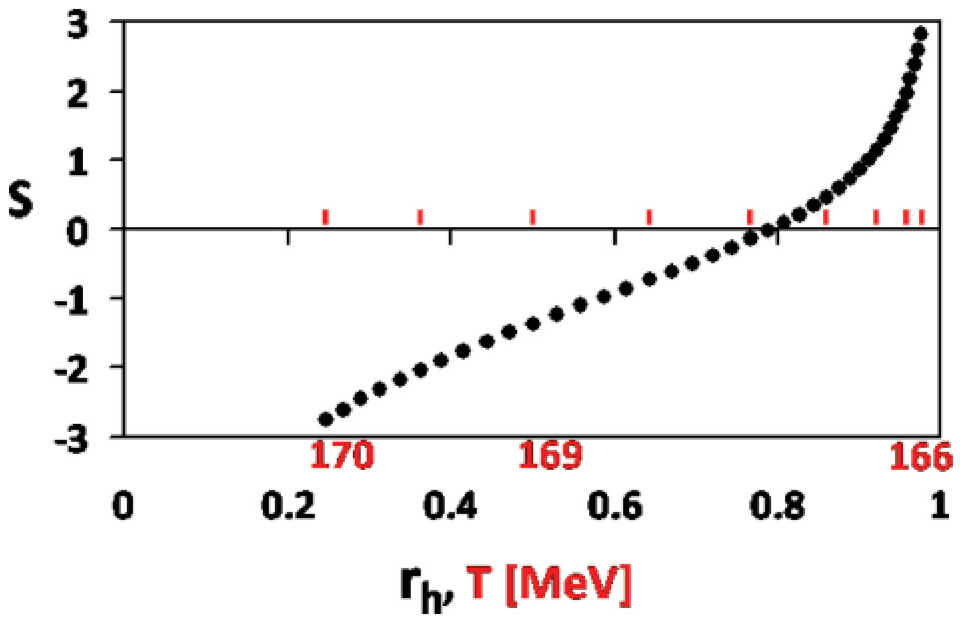
$$r_h = \frac{P(e_h)}{P(e_q) + P(e_h)}$$

volume abundance of the hadronic matter

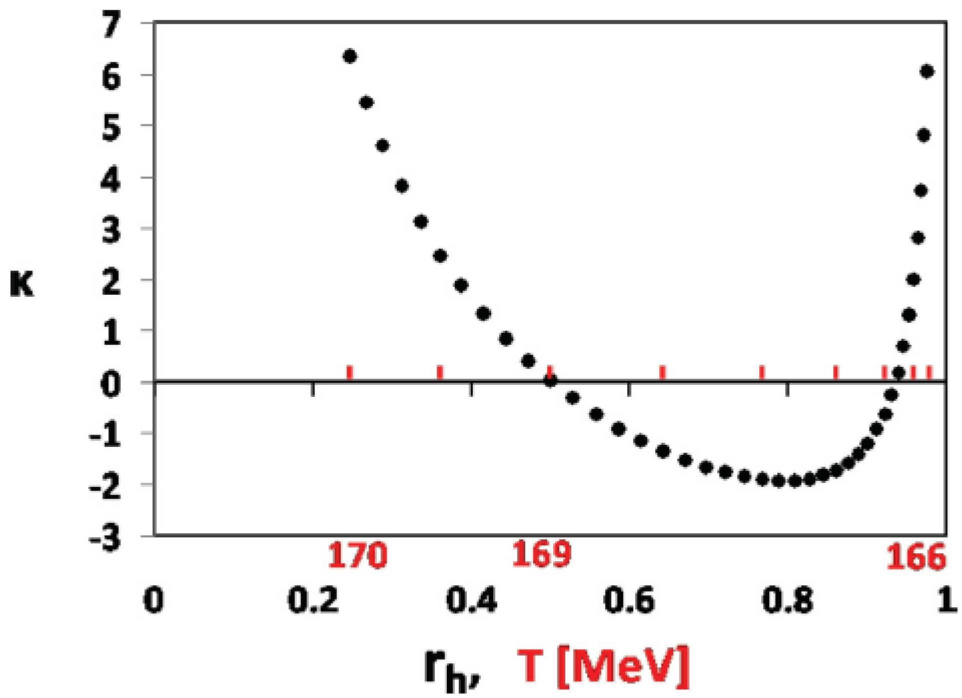


−0.005
−0.015
−0.025
−0.035
f(e) [GeV/fm³]
0.95T$_c$
T$_c$
1.05T$_c$
0.0 0.5 1.0 1.5 2.0 2.5 3.0
e [GeV/fm³]
P(e)/P(e_q)
0.95T$_c$
50 fm³
T$_c$
1.05T$_c$
e [GeV/fm³]
10 fm³
T$_c$
50 fm³
500 fm³
e [GeV/fm³]



If there is a phase transition the skewness and kurtosis change!

What will happen if we avoid a Phase Transition?



We minimize the fluctuations from the initial state, avoid viscosity →

Central collisions

Supercooled QGP

Basic definitions

1. Weighted average

The weighted average of a quantity x is defined as

$$< x > \equiv \sum_i x_i \cdot w_i$$

where $$w_i = \frac{\mathrm{n}_i^{CF} \cdot V_i}{N_{tot}} \qquad \sum_i w_i = 1$$

During the expansion stage, each cell carries less baryon charge with time.

2. Specific energy density and baryon density

$$\boxed{\mathrm{n}_i^{CF} = n_i^{LR}\gamma_i}$$

In Calculation Frame (CF)

$$< \varepsilon^{CF} >= \sum_i \frac{e_i^{CF}}{\mathrm{n}_i^{CF}} \underbrace{\frac{\mathrm{n}_i^{CF} V_i}{N_{tot}}}_{w_i} = \sum_i \frac{e_i^{CF} V_i}{N_{tot}}$$

$$< \mathrm{n}^{CF} >= \sum_i \mathrm{n}_i^{CF} \frac{\mathrm{n}_i^{CF} V_i}{N_{tot}} = \sum_i (\mathrm{n}_i^{CF})^2 \frac{V_i}{N_{tot}}$$

In Local Rest Frame (LR)

$$< \varepsilon^{LR} >= \sum_i \frac{e_i^{LR}}{n_i^{LR}} \frac{\mathrm{n}_i^{CF} V_i}{N_{tot}} = \sum_i \frac{e_i^{LR} \gamma_i V_i}{N_{tot}}$$

$$< n^{LR} >= \sum_i n_i^{LR} \frac{\mathrm{n}_i^{CF} V_i}{N_{tot}} = \sum_i (n_i^{LR})^2 \frac{\gamma_i V_i}{N_{tot}}$$

3. Moments

$$M^{(n)} = < (x - < x >)^n > = \int (x - < x >)^n P(x) dx$$

$$= \sum_i (x - < x >)^n \ w_i$$

$$< x^n > = \int x^n P(x) dx = \sum_i x_i^n \ w_i$$

where P(x) is the spatial distribution weighted by the baryon charge density in the CF frame.

Mean value $<x> = M^{(1)}$

Variance $$\sigma^2 =< (x- < x >)^2 = M^{(2)}$$

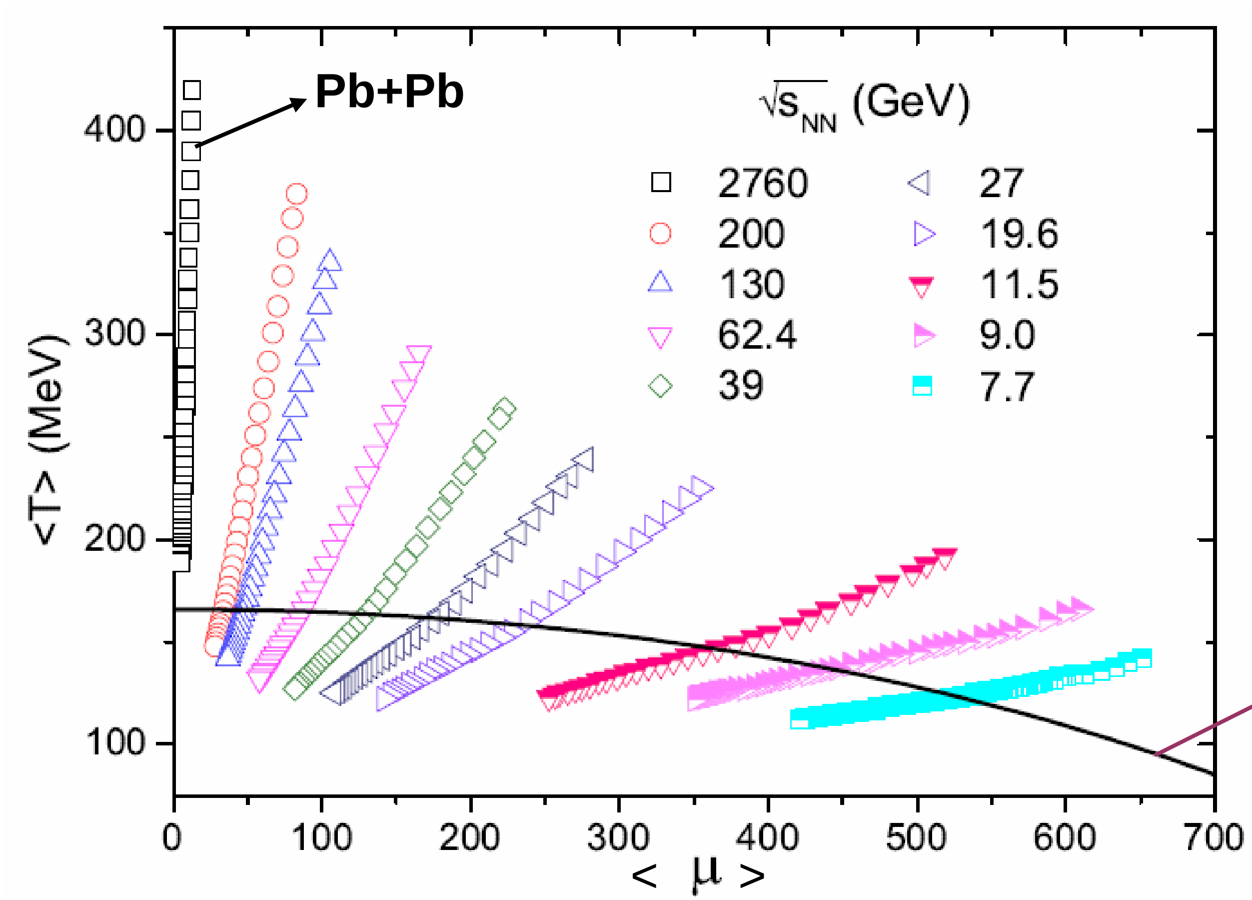
Skewness $$S = \frac{< (x- < x >)^3 >}{\sigma^3} = \frac{M^{(3)}}{(M^{(2)})^{3/2}}$$

Kurtosis $$\kappa =< \frac{(x- < x >)^4}{\sigma^4} > -3 = \frac{M^{(4)}}{(M^{(2)})^2} - 3$$

Results and Discussions

- Trajectory of FD development of QGP
- Average specific energy time development
- Average baryon density time development
- Skewness and Kurtosis

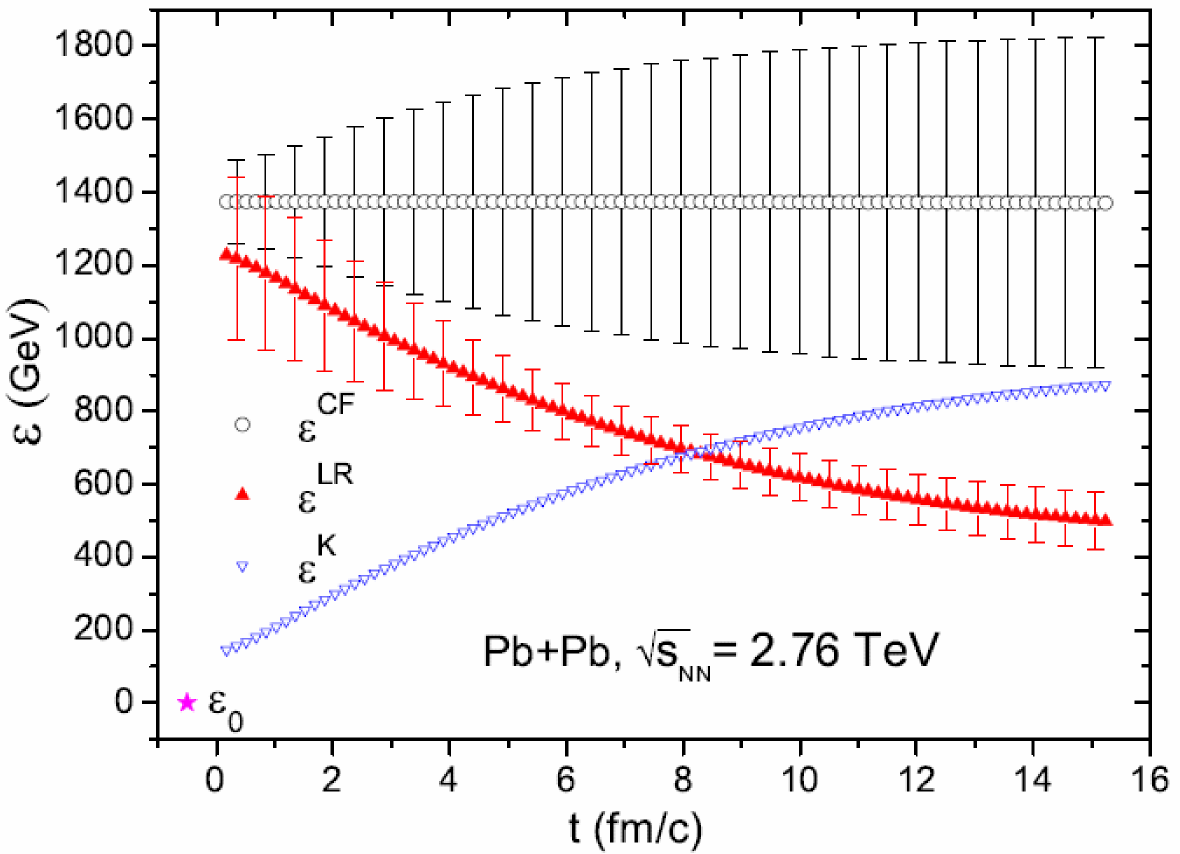
1. Trajectory of FD development of QGP



Hadronic freeze out curve, Cleymans et al.,

Cell size: $dx=dy=dz=0.575\ fm$

2. Average specific energy - time development

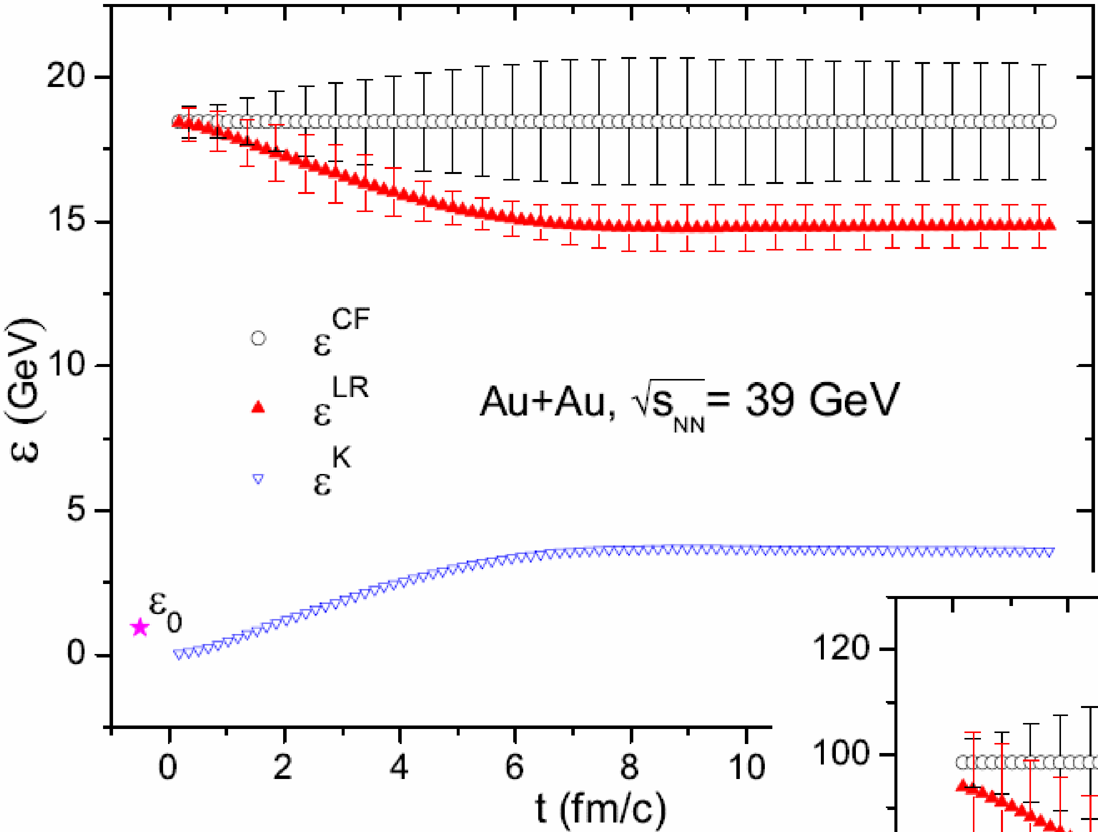


$$\varepsilon^{K} \equiv \varepsilon^{CF} - \varepsilon^{LR}.$$

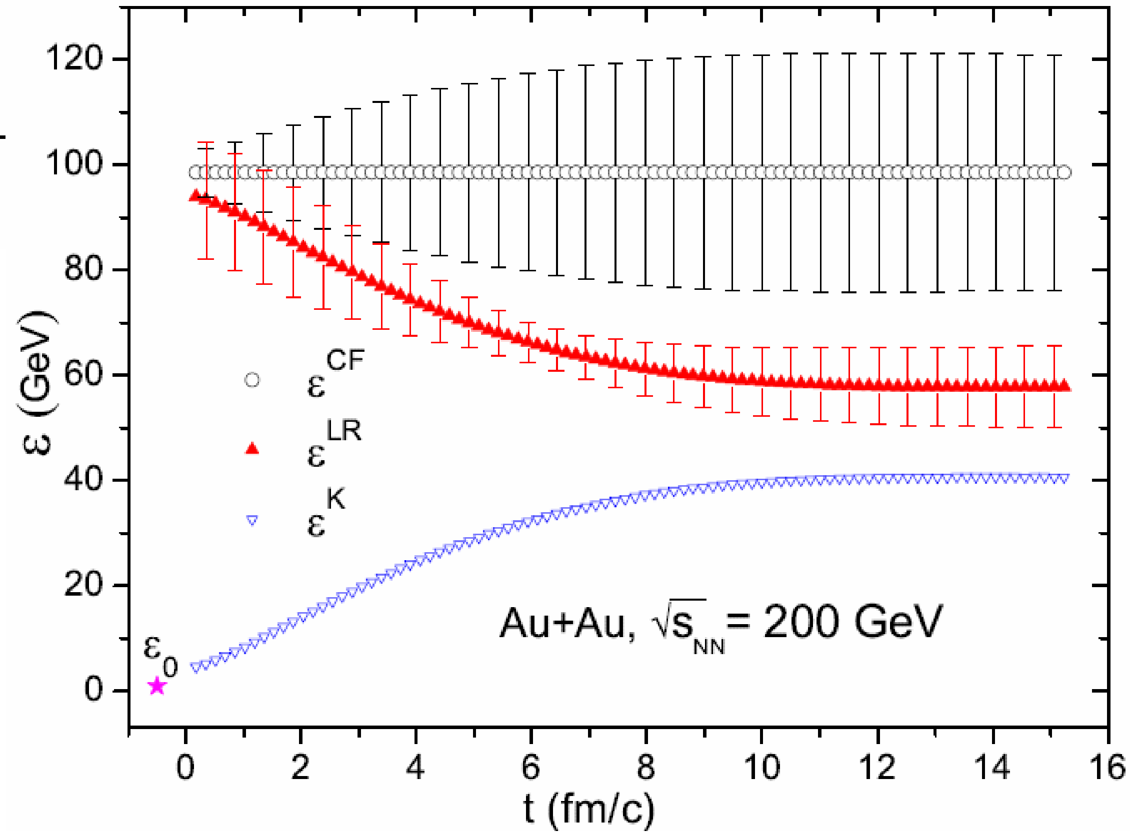
ε_0 =0.938 GeV is the initial specific internal energy before collision.

Central collisions: to minimize the initial state fluctuations.

Spatial variance is increasing due to the expansion.

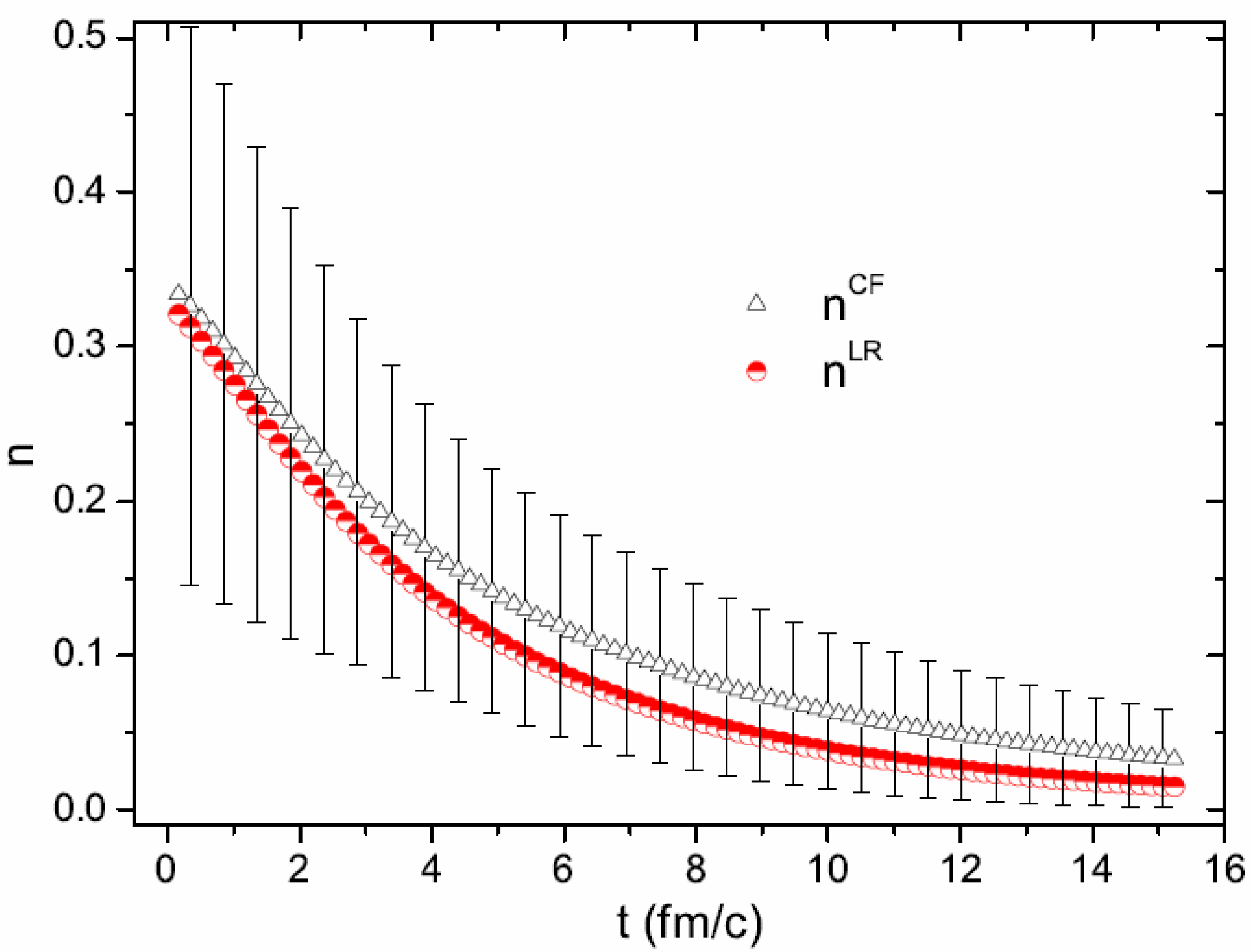


For the lower energy collisions, the freeze out time is at an earlier stage and the fluid dynamic cells have zero pressure very soon.



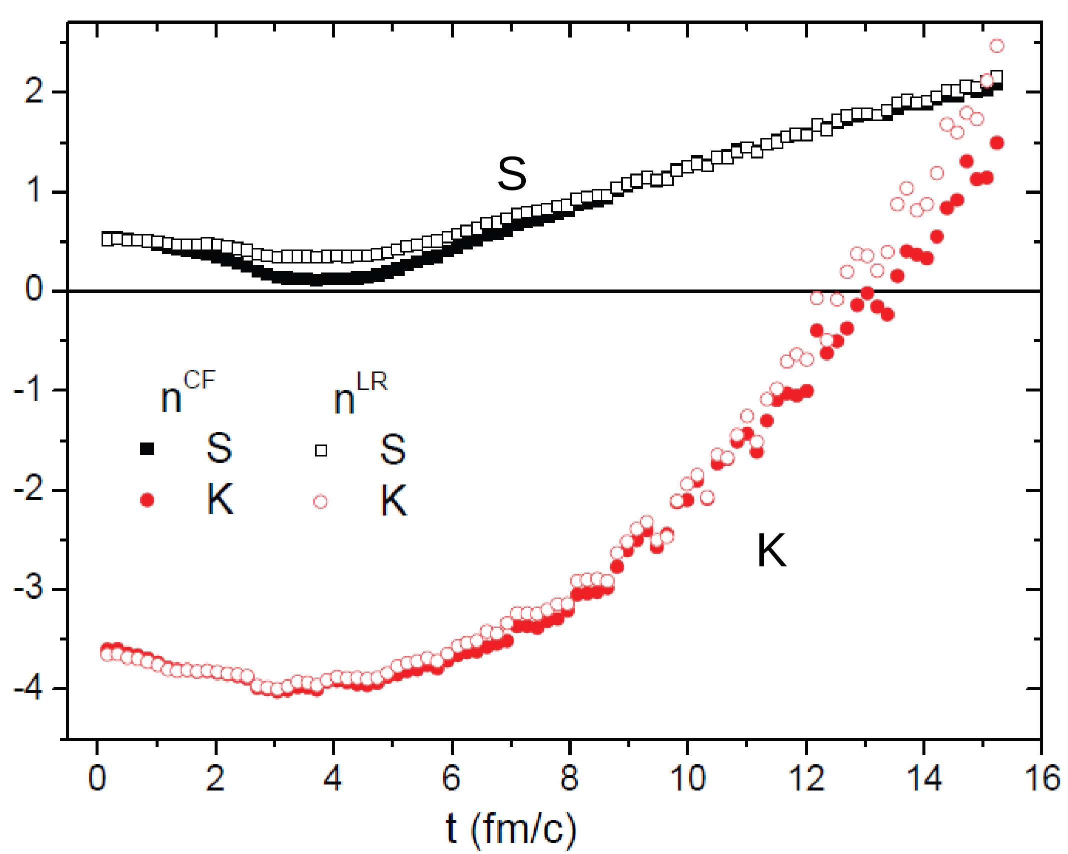
In the following we study the central **Pb+Pb** collisions at **2.76TeV**.

3. Average baryon density - time development



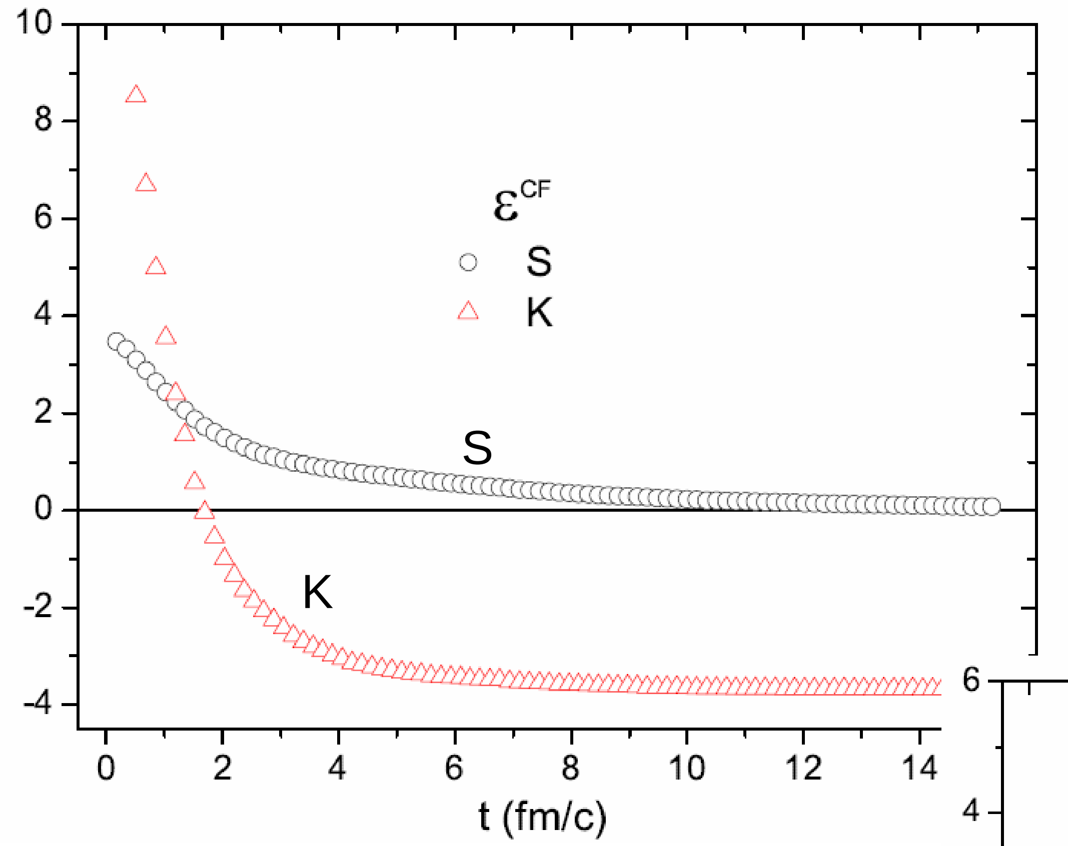
Both the average baryon density
and the variance are decreasing!

4. Skewness and Kurtosis



The skewness calculated in CF is twice larger than that in LR around t=4 fm/c, indicating that the apparent density is more uniform than the invariant scalar density.

The kurtosis changes sign at a very late stage (after usual FO).



In CF,

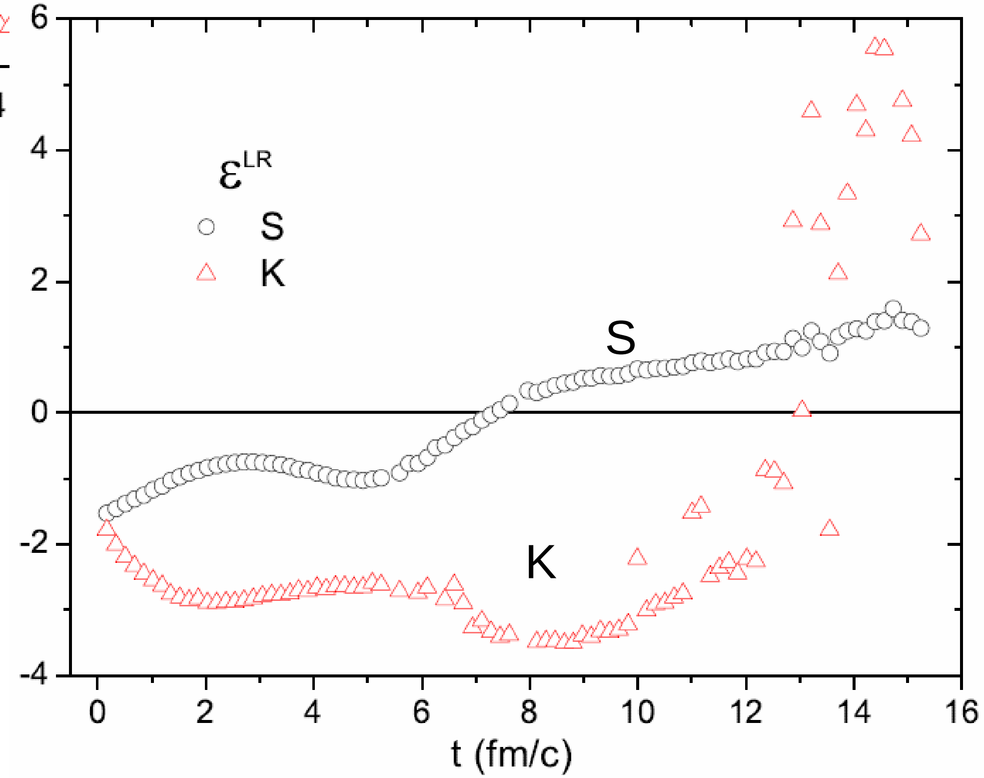
skewness always keeps positive

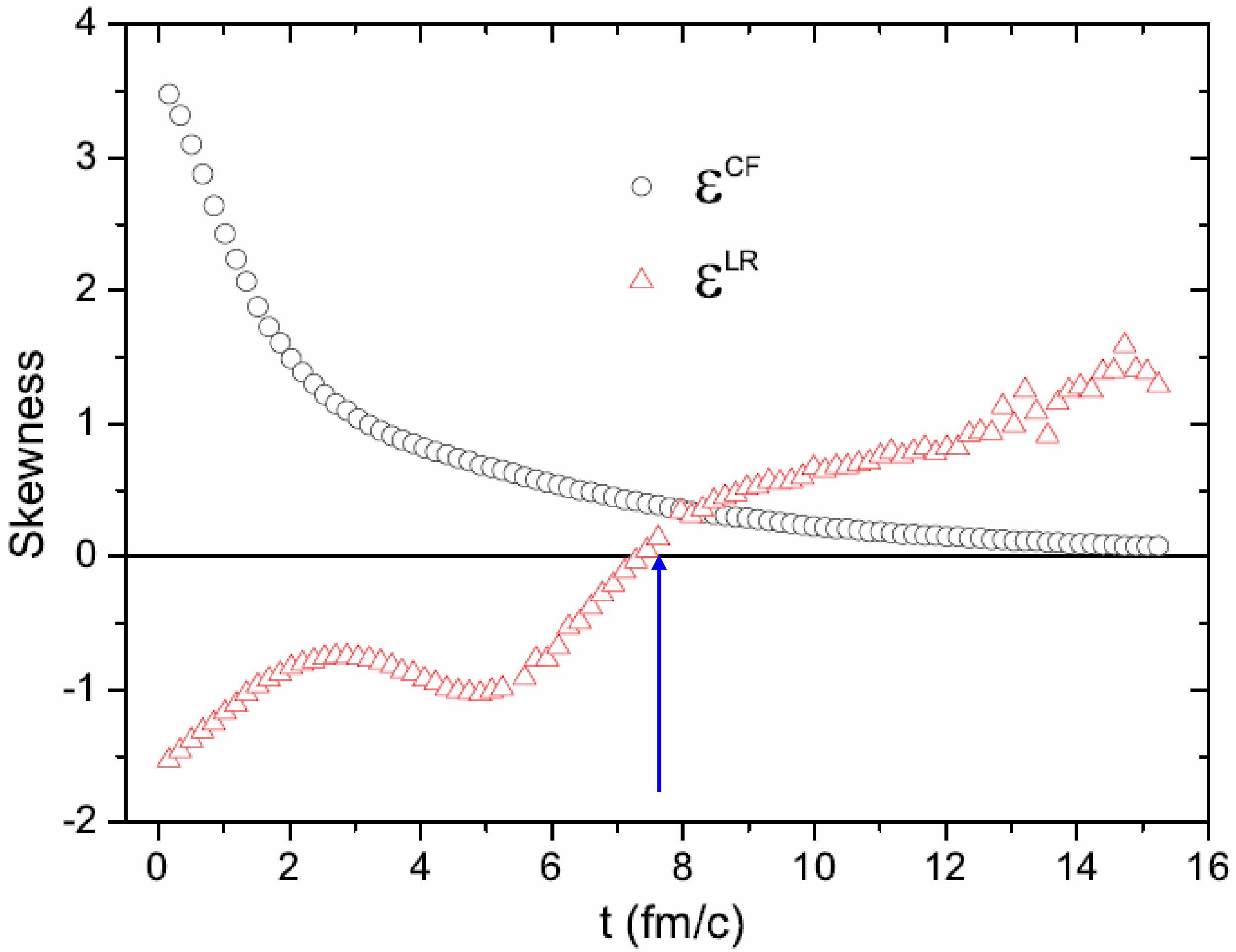
Kurtosis changes sign from + to –

In LR,

Skewness and Kurtosis both

change sign from – to + (*)





Changes sign around the typical FO moment: t=8 fm/c

Summary

1. FD model can describe the late stages of collisions, expanding up to the empirical FO boundary and beyond

2. Skewness and kurtosis of specific energy density change signs during expansion, thus even with least possible initial variation, strongly varying statistical higher moments may develop.

Thank you for your attention!