Antigravitation

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Introduction: Objections

- The equivalence principle would be violated.
- If there were negative energies, the vacuum would be unstable.
- A geodesic is independent of the particle's mass.
- Leads to unphysical solutions.
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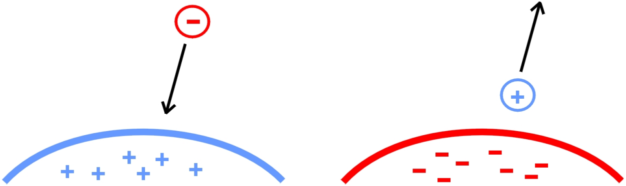

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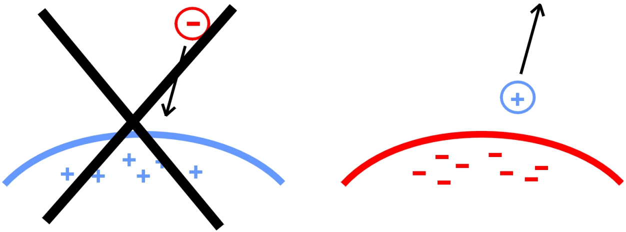

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- And if you have argued it away - why should I care?
- Everybody knows there is no such thing as anti-gravitation.

Motivation: Problems

- Dark Matter
- Dark Energy
- Inflation
- Singularities
- Coincidence problem
- Horizon problem
- Homogeneity problem
- Axis of evil
- ...

$\longrightarrow$ Something is missing in our understanding of the universe.

What do I mean with Anti-gravitating matter

- A second copy of the standard model, identical to the one we know, except for its graviational interaction.
- Both sorts of particles interact only graviationally.
- In particular, anti-matter has completely normal graviational properties.
- Disclaimer: This talk is unquantized.

How do we get the funny stuff to move differently?

- Covariant curves are defined via a connection. Yet this connection is unique only after requiring it to be torsion-free and metric-compatible.
- Need second derivative, throw out metric-compatibility.
- Introduce instead second metric $\underline{\mathbf{h}}$ with which the second connection ${}^{(h)}\underline{\nabla}$ is compatible.

$$\begin{aligned} {}^{(g)}\nabla, {}^{(g)}\mathcal{R} \quad \text{with} \quad {}^{(g)}\nabla\mathbf{g} &= 0 \\ {}^{(\underline{h})}\underline{\nabla}, {}^{(\underline{h})}\mathcal{R} \quad \text{with} \quad {}^{(\underline{h})}\underline{\nabla}\underline{\mathbf{h}} &= 0 \end{aligned}$$

The Pullovers

- 2nd metric provides another interpretation of the manifold (same manifold, different distance measures) and results in different curves for particles. Its local coordinate basis doesn't normally coincide with ours.
- Two sorts of indices: ν raised/lowered with $\mathbf{g}$, $\underline{\nu}$ raised/lowered with $\underline{\mathbf{h}}$
- Pullovers to identify h-fields with observables for g-observer and vice versa: $P_g, P_{\underline{h}}$. Locally linear maps on tensor algebras.
- $\underline{\mathbf{h}}$ is then related to a two-tensor $\mathbf{h} = P_{\underline{h}}(\underline{\mathbf{h}})$ and vice versa $\underline{\mathbf{g}} = P_g(\mathbf{g})$.
- Induces pulled-over derivatives by metric-compatibility:

$$\begin{aligned} P_{\underline{h}}({}^{(\underline{h})}\underline{\nabla}\mathbf{A}) &= {}^{(h)}\nabla P_{\underline{h}}(\mathbf{A}) \\ P_g({}^{(g)}\nabla\mathbf{A}) &= {}^{(\underline{g})}\underline{\nabla} P_g(\mathbf{A}) \end{aligned}$$

Equations of Motion for Matter Fields

- Action for field

$$S = \int d^4x \sqrt{-h}\; P_{\underline{h}}\left(h^{\underline{\nu\kappa}}\; {}^{(\underline{h})}\underline{\nabla}_{\underline{\kappa}}\underline{\phi}\, {}^{(\underline{h})}\underline{\nabla}_{\underline{\nu}}\underline{\phi} \right)$$

- Leads to

$$P_{\underline{h}}\left({}^{(\underline{h})}\underline{\nabla}^{\underline{\alpha}}\, {}^{(\underline{h})}\underline{\nabla}_{\underline{\alpha}}\underline{\phi} \right) = 0$$

Same as

$${}^{(h)}\nabla^{\underline{\alpha}}\, {}^{(h)}\nabla_{\underline{\alpha}} P_{\underline{h}}(\underline{\phi}) = 0$$

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How to determine the second metric?

- For convenience

$$\begin{aligned} g_{\varepsilon\lambda} &= a_{\varepsilon}{}^{\nu} a_{\lambda}{}^{\kappa}\, h_{\nu\kappa} \quad , \quad a_{\underline{\varepsilon}}{}^{\underline{\nu}} = [P_g]^{\varepsilon}{}_{\underline{\varepsilon}}\, a_{\varepsilon}{}^{\nu}\, [P_{\underline{h}}]^{\underline{\nu}}{}_{\nu} \\ g_{\underline{\varepsilon\lambda}} &= a_{\underline{\varepsilon}}{}^{\underline{\nu}} a_{\underline{\lambda}}{}^{\underline{\kappa}}\, h_{\underline{\nu\kappa}} \quad , \quad g_{\varepsilon\lambda} = a_{\varepsilon}{}^{\underline{\nu}} a_{\lambda\underline{\nu}} \;,\; h_{\underline{\nu\kappa}} = a^{\varepsilon}{}_{\underline{\nu}}\, a_{\varepsilon\underline{\kappa}} \end{aligned}$$

- The a's are not independent: $\delta a^{\nu\kappa} = \delta a^{\underline{\nu\kappa}} = 0$.
- Now use symmetry principle

$$\begin{aligned} {}^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu}{}^{(g)}R &= T_{\kappa\nu} - |P_{\underline{h}}| \sqrt{\frac{\underline{h}}{g}}\, a_{\nu}{}^{\underline{\nu}} a_{\kappa}{}^{\underline{\kappa}}\, \underline{T}_{\underline{\nu\kappa}} \\ {}^{(\underline{h})}R_{\underline{\nu\kappa}} - \frac{1}{2} h_{\underline{\nu\kappa}}{}^{(\underline{h})}R &= \underline{T}_{\underline{\nu\kappa}} - |P_g| \sqrt{\frac{g}{\underline{h}}}\, a^{\kappa}_{\underline{\kappa}} a^{\nu}_{\underline{\nu}}\, T_{\kappa\nu} \end{aligned}$$

with

$$T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} + \frac{1}{2} g_{\mu\nu} \mathcal{L} \quad , \quad \underline{T}_{\underline{\nu\kappa}} = -\frac{1}{\sqrt{-\underline{h}}} \frac{\delta \underline{\mathcal{L}}}{\delta h^{\underline{\nu\kappa}}} + \frac{1}{2} h_{\underline{\nu\kappa}} \underline{\mathcal{L}}$$

- Degrees of freedom in pull-overs needed to fulfill Bianchi identities.

Action

- Full action

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left({}^{(g)}R/8\pi G + \mathcal{L}(\psi)\right) + \sqrt{-h}P_{\underline{h}}(\underline{\mathcal{L}}(\underline{\phi})) \\ &+ \int d^4x\sqrt{-\underline{h}}\left({}^{(\underline{h})}R/8\pi G + \underline{\mathcal{L}}(\underline{\phi})\right) + \sqrt{-\underline{g}}P_g(\mathcal{L}(\psi)) \quad , \end{aligned}$$

- Dynamical variables $\mathbf{g}$ and $\underline{\mathbf{h}}$, ψ and $\underline{\phi}$.
- Variation of $g_{\varepsilon\lambda}h_{\kappa\nu}a^{\varepsilon\kappa}a^{\mu\nu} = \delta^{\mu}{}_{\lambda}$ with $\delta a^{\varepsilon\kappa} = 0$ yields

$$\delta h_{\kappa\lambda} = -[a^{-1}]^{\mu}{}_{\kappa}[a^{-1}]^{\nu}{}_{\lambda}\delta g_{\mu\nu}$$

- Note: There is no negative kinetic energy in the action. Variation over fields (for 'inertial' stress-energy) has no change of sign. Sign change only for source term of field equations.
- Both graviational AND inertial mass (energy) are conserved, thus no vacuum decay possible.

Schwarzschild Metric

- Spherical symmetry with regular particle of mass M in center. Outside solution

$$
\begin{aligned}
g_{tt} &= -\left(1-\frac{2M}{r}\right) \;,\; g_{rr} = -1/g_{tt} \;,\; g_{\theta\theta} = r^2 \;,\; g_{\phi\phi} = r^2 \sin^2\theta \\
h_{\underline{tt}} &= -\left(1+\frac{2M}{r}\right) \;,\; h_{\underline{rr}} = -1/h_{\underline{tt}} \;,\; h_{\underline{\theta\theta}} = r^2 \;,\; h_{\underline{\phi\phi}} = r^2 \sin^2\theta
\end{aligned}
$$

  and $h_{\kappa\nu} = h_{\underline{\kappa\nu}}$.

- Does what expected.

Friedmann-Robertson-Walker Metric

- Ansatz for **g**: $ds^2 = -dt^2 + \frac{a^2}{1-k_a r}(dr^2 + d\Omega^2)$
- Ansatz for $\underline{\mathbf{h}}$: $ds^2 = -dt^2 + \frac{b^2}{1-k_b r}(dr^2 + d\Omega^2)$
- Yields Friedmann-equations (w/o CC term)

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 &= \frac{8}{3}\pi G\left(\rho - |P_{\underline{h}}|\left(\frac{b}{a}\right)^3 \underline{\rho} + \frac{k_a}{a^2}\right)\\
\frac{\ddot{a}}{a} &= \frac{4}{3}\pi G\left(-(\rho + 3p) + |P_{\underline{h}}|\left(\frac{b}{a}\right)^3 (\underline{\rho} + 3\underline{p})\right)\\
\left(\frac{\dot{b}}{b}\right)^2 &= \frac{8}{3}\pi G\left(\underline{\rho} - |P_g|\left(\frac{a}{b}\right)^3 \rho + \frac{k_b}{b^2}\right)\\
\frac{\ddot{b}}{b} &= \frac{4}{3}\pi G\left(-(\underline{\rho} + 3\underline{p}) + |P_g|\left(\frac{a}{b}\right)^3 (\rho + 3p)\right)
\end{aligned}$$

- Note: anti-graviating matter does not create accelerated expansion.

Relevance?

- Gravitational Lensing?
- Structure of voids?
- Quantize: what about vacuum energy?

Revisited: Objections

- If there were negative energies, the vacuum would be unstable.

$\longrightarrow$ Inertial masses (energies) remain positive.

- A geodesic is independent of the particle's mass.

$\longrightarrow$ A particle's curve depends on the connection used.

- One could construct a perpetuum mobile.

$\longrightarrow$ Like charges attract, unlike charges repel: no self-acceleration.

- If there is anti-gravitating matter - where is it?

$\longrightarrow$ As far away as possible.

- And if you have argued it away - why should I care?

$\longrightarrow$ Possibly important at large distances, strong curvature, high densities i.e. astrophysics and cosmology.

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