

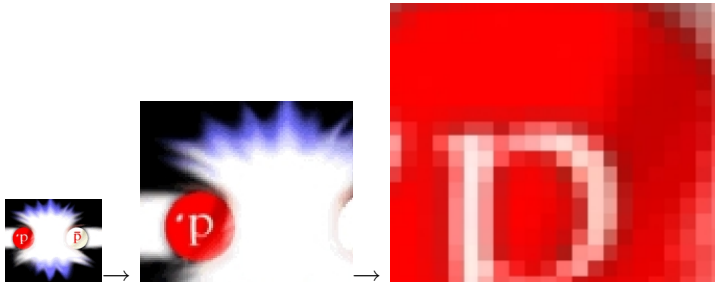
The Minimal Length

Sabine Hossenfelder

Based on

Phys. Lett. B **575**, 85 (2003) [arXiv:hep-th/0305262]
Class. Quantum Grav. 23 (2006) 1815 [arXiv:hep-th/0510245]
Phys. Rev. D **15** ?????? (2006) [arXiv:hep-th/0603032]

The Minimal Length Scale



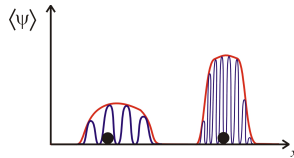
- **Very general expectation** for quantum gravity: fluctuations of spacetime itself disable resolution of small distances
- Can be found e.g. in String Theory, Loop Gravity, NCG, etc.
- Minimal length scale acts as UV cutoff

With Large Extra Dimensions

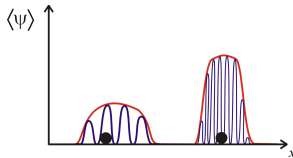
- Lowering the Planck mass means raising the Planck length!
- Effects of a finite resolution become important at the same energy scale as other new signatures.
- Relevance for high energy and high precision experiments.

$$m_{p^2} = M_f^{d+2} R^d$$

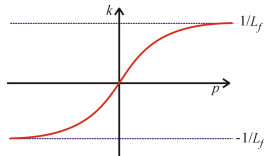
Finite Resolution of Structures



Finite Resolution of Structures



- For large momenta, p , Compton-wavelength $\lambda = 1/k$ can not get arbitrarily small $\lambda > L_{\min} = 1/M_f$



A Model for the Minimal Length

- Modify wave-vector k and commutation relations
 $k = k(p) = \hbar p + a_1 p^3 + a_2 p^5 \dots \Rightarrow [p_i, x_j] = i \hbar \delta_{ij} / \partial k_j$
- Results in a **generalized uncertainty principle**

$$\Delta x \Delta p \geq \frac{1}{2} \hbar \left(1 + b_1 L_{\min}^2 \langle p^2 \rangle \right)$$

- And a **squeezed phase space at high energies**

$$\langle p | p' \rangle = \frac{\partial p}{\partial k} \delta(p - p') \Rightarrow dk \rightarrow \hbar dp \frac{\partial k}{\partial p} = \hbar dp e^{-L_{\min}^2 p^2}$$

- Can but need not have a **varying speed of light** $d\omega/dk \neq 1$.

Quantisation with a Minimal Length

Parametrization in $p_v = f_v(k)$ – input from underlying theory.

- Quantize via $k \rightarrow -i\partial$, $p \rightarrow f(-i\partial) := F(\partial)$
- The Klein-Gordon equation, alias **modified dispersion relation**

$$E^2 - p^2 = m^2 \quad \Rightarrow \quad F^v(\partial)F_v(\partial)\psi = m^2\psi$$

- The Dirac equation

$$(F(\partial) - m)\psi = 0$$

- (Anti)-commutation relations

$$\left[\hat{a}^\dagger(p), \hat{a}(p') \right]_{\pm} = \delta(\vec{p} - \vec{p}') \left| \frac{\partial p}{\partial k} \right|$$

Alternative Description with Energy Dependent Metric*

- Even on a classical level, a particle's energy influences the metric it propagates in

$$R_{\kappa\nu} - \frac{1}{2}g_{\kappa\nu} = \frac{1}{m_p^2} T_{\kappa\nu}$$

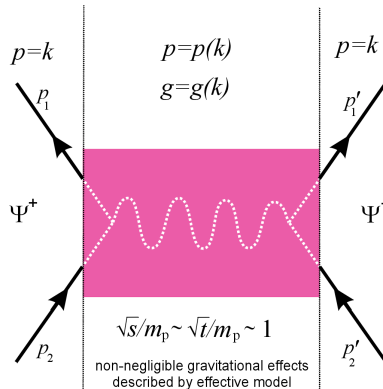
- On a quantum level, expect this dependence to be dominant
- Re-write modified dispersion relation in collision region as

$$g^{\kappa\nu}(k)k_\kappa k_\nu = \mu^2$$

- At small energies $g^{\kappa\nu} \sim \eta^{\kappa\nu}$

*Kimberly, Magueijo and Medeiros, Phys. Rev. D **70**, 084007 (2004).

The Collision Region



Quantisation with a Minimal Length

- Lagrangian for free fermions

$$\mathcal{L}_f = i\bar{\Psi}(\not{F}(k) - m)\Psi \quad \mathcal{L}_f = i\bar{\Psi}(g^{v\kappa}(k)\gamma_v k_\kappa - m)\Psi$$

- Coupling of the gauge field via $\partial_v \rightarrow D_v := \partial_v - ieA_v$ yields the gauge- and Lorentz-invariant higher order derivative interaction

$$\mathcal{L} = \bar{\Psi}\not{F}(D)\Psi \quad \mathcal{L} = \bar{\Psi}\gamma_v g^{v\kappa}(D)D_\kappa\Psi$$

- To first order one finds the usual $\mathcal{L} = \mathcal{L}_f - e\bar{\Psi}\eta^{kv}\gamma_\kappa A_v\Psi + O(eL_{\min}^2)$ and the dominant modification comes from the propagators

$$\begin{array}{ll} (\not{F}(k) - m)^{-1} & (g^{v\kappa}(k)\gamma_v k_\kappa - m)^{-1} \\ (F^v(k)F_v(k) - m^2)^{-1} & (g^{v\kappa}(k)k_v k_\kappa - m^2)^{-1} \end{array}$$

Deformed Special Relativity

- Minimal length L_{\min} requires new Lorentz-transformations
- New transformations have 2 invariants: c and L_{\min}
- Generalized Uncertainty \Longleftrightarrow Deformed Special Relativity
 - * When relation $k(p)$ is known and p 's (usual) transformation, then also the transformation of k is known.
 - * When the new transformation on k is known, then one gets $k(p)$ by boosting in and out of the restframe where $k = p$.

SH, Class. Quantum Grav. 23 (2006) 1815.

Deformed, Non-linear Action on Momentum Space

- Lorentz-algebra remains unmodified

$$[J^i, K^j] = \varepsilon^{ijk} K_k, \quad [K^i, K^j] = \varepsilon^{ijk} K_k, \quad [J^i, J^j] = \varepsilon^{ijk} J_k$$

- But it acts non-linearly on momentum space, e.g.*

$$e^{-iL_{ab}\omega^{ab}} \rightarrow U^{-1}(p_0)e^{-iL_{ab}\omega^{ab}}U(p_0) \quad \text{with} \quad U(p_0) = e^{L_{\min}p_0 p_a \partial p^a}$$

- Leads to Lorentz-boost (z-direction)

$$p'_0 = \frac{\gamma(p_0 - v p_z)}{1 + L_{\min}(\gamma - 1)p_0 - L_{\min}\gamma v p_z}$$

$$p'_z = \frac{\gamma(p_z - v p_0)}{1 + L_{\min}(\gamma - 1)p_0 - L_{\min}\gamma v p_z}$$

which transforms $(1/L_{\min}, 1/L_{\min}) \rightarrow (1/L_{\min}, 1/L_{\min})$

*Magueijo and Smolin, Phys. Rev. Lett. **88**, 190403 (2002).

Interpretation of an Invariant Minimal Length

Besides c there is a second invariant L_{\min} for all observers

- Standard DSR approach
 - * DSR applies for each observer to agree on minimal-ness ... ?
 - * Therefore deformed transformation applies to free particles
 - * If caused by quantum gravity effects what sets the scale?
 - My DSR approach
 - * Two observers can not compare lengths without interaction
 - * The strength of gravitational effects sets the scale for the importance of quantum gravity
 - * Free particles do not experience any quantum gravity or DSR
 - * Effects apply for particles in the interaction region only
- Propagator of exchange particles is modified

Problems of DSR

The Soccer-Ball-Problem



- * Usual DSR: A particle has an upper bound on its energy. There better were no such bound for composite objects. How to get the correct multi-particle limit for DSR-trafos?
- * My DSR: Composite objects don't experience anything funny as long as the gravitational interaction among the components is weak.

The Conservation-Law Problem

- * Usual DSR: the physical momentum p is the one that experiences DSR. But then p is not additive and $f(p_1 + p_2) \neq f(p_1) + f(p_2)$. Which quantity is to be conserved in multi-particle interactions*?
- * My DSR: the physical momenta p_1, p_2 are the asymptotic momenta, they transform, are added, and are conserved in the usual way[†].

* Judes and Visser, Phys. Rev. D **68**, 045001 (2003) [arXiv:gr-qc/0205067].

† SH, Phys. Rev. D **15** ????? (2006) [arXiv:hep-th/0603032]

The Locality Bound*

From the commutator

$$[a_p, a_{p'}^\dagger] = \delta(p - p') \left| \frac{\partial p}{\partial k} \right|$$

And the field expansion

$$\phi(x) = \int d^3p \left| \frac{\partial k}{\partial p} \right| \left[v_p(x) a_p + v_p^*(x) a_p^\dagger \right]$$

One finds the equal time commutator for $x = (\mathbf{x}, t), y = (\mathbf{y}, t)$.

$$[\phi(x), \pi(y)] = i \int \frac{d^3p}{(2\pi)^3} \left| \frac{\partial k}{\partial p} \right| e^{ik(x-y)} \rightarrow i \int \frac{d^3p}{(2\pi)^3} e^{ik(x-y) - \varepsilon p^2}$$

where $\varepsilon \sim L_{\min}^2$.

*Giddings and Lippert, Phys. Rev. D **65**, 024006 (2002), Phys. Rev. D **69**, 124019 (2004).

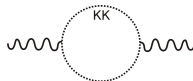
The Minimal Length as UV-Regulator

- Minimal length regularizes loop-integrals

$$\int d^4 p \frac{1}{p^2(p-q)^2} \sim \ln \frac{\Lambda}{\mu_0} \quad \text{for } d=0$$

$$\int d^{4+d} p \frac{1}{p^2(p-q)^2} \sim \left(\frac{\Lambda}{\mu_0} \right)^d \quad \text{for } d > 1?$$

$$\rightarrow \int d^{4+d} p \left| \frac{\partial k}{\partial p} \right| \frac{1}{p^2(p-q)^2} < \infty$$



- Regularizes also higher-dimensional QFT's.
- Natural interpretation of regulator.

SH, Phys. Rev. D **70** (2004) 105003.

The Minimal Length as UV-Regulator

Example: Running Coupling

- Looking close, the propagator exhibits complex structures

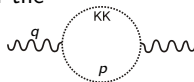
$$D^{\mu\nu}(q) = -g^{\mu\nu}/q^2 \quad \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line} \rightarrow \tilde{D}^{\mu\nu}$$

- Expansion in series of one-particle irreducible contributions

$$\text{wavy line} \text{---} \text{shaded circle} \text{---} \text{wavy line} = \text{wavy line} + \text{wavy line} \text{---} \text{shaded circle} \text{---} \text{wavy line} + \text{wavy line} \text{---} \text{shaded circle} \text{---} \text{shaded circle} \text{---} \text{wavy line} + \dots$$

$$i\tilde{D}^{\mu\nu} = iD^{\mu\nu} + i\tilde{D}^{\mu\lambda}[ie^2\Pi_{\lambda\sigma}]i\tilde{D}^{\sigma\nu} + i\tilde{D}^{\mu\lambda}[ie^2\Pi_{\lambda\sigma}]i\tilde{D}^{\sigma\kappa}[ie^2\Pi_{\kappa\rho}]i\tilde{D}^{\rho\nu} + \dots$$

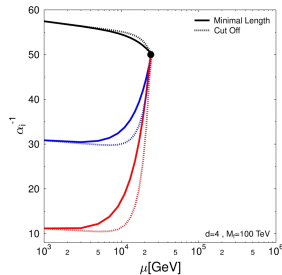
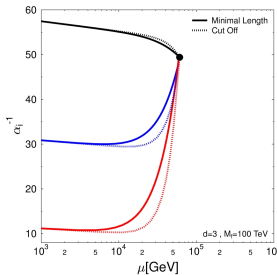
- Can be summarized in $[q^2 g_{\mu\nu} - e^2 \Pi_{\mu\nu}] \tilde{D}_\lambda^\nu = -g_{\mu\lambda}$
- Loop-integral is dampened at high energies from squeezing of the momentum space measure \rightarrow becomes finite



The Minimal Length as UV-Regulator

Example: Running Coupling

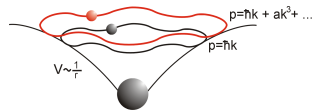
- Interpretation: finite resolution of vacuum structure.
- In extra dimensions, power-law running accelerates unification



Observables of a Minimal Length - High Precision

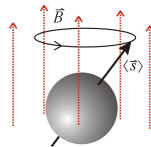
- Hydrogen atom: shift of energy levels

SH, M. Bleicher, S. Hofmann, S. Scherer, J. Ruppert
and H. Stöcker, Phys. Lett. B **598** (2004) 92-98



- g-2

U. Harbach, SH, M. Bleicher and H. Stöcker
Phys. Lett. B **584** (2004) 109-113



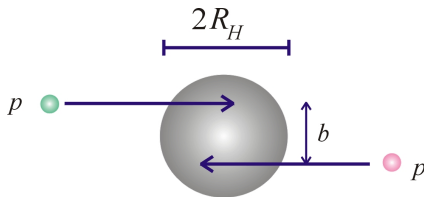
- Varying speed of light! Energy dependent time of flight

Amelino-Camelia, Phys. Rev. D **64** (2001) 036005
Magueijo and Smolin, Phys. Rev. Lett. **88** (2002) 190403
Judes and Visser, Phys. Rev. D **68** (2003) 045001

Advertisement Break

Large Extra Dimensions

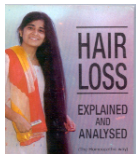
- More volume: the volume factor makes gravity appear weak at large distances \gg radius
- True strength: at small distances ($\sim 1\text{TeV}$) gravity becomes stronger
- 2 in 1: graviton- and black hole production at the LHC becomes possible*



* Restrictions apply

Evaporation proceeds in 3 stages:

- 1 Balding phase: hair loss – the black holes radiates off angular momentum and multipole moments
- 2 Hawking phase: thermal radiation into all particles of the standard model as well as gravitons
- 3 Final decay (remaining black hole relic?)



Black hole thermodynamics: $T = \kappa/2\pi \sim 200 \text{ GeV}$

Numerical investigation: black hole event generator CHARYBDIS

TEILCHENPHYSIK

Angst vor dem großen Knall

Physiker wollen bei New York den Anfang des Universums erforschen und lösen Endzeitstimmung aus

In der „Unendlichen Geschichte“ von Michael Ende breitet sich das Nichts unaufhaltsam aus. Es reißt Tiere und Pflanzen fort, verschlingt Berge und Seen – und lässt von ganz Phantasien nicht mehr als ein Sandkorn übrig. So ein Schicksal steht vielleicht der Erde bevor, fürchten jetzt viele Amerikaner, wenn ein neuer Teilchenbeschleuniger bei New York ab Herbst

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„Eine Kettenreaktion könnte den Planeten verschlingen“, warnte im Juli

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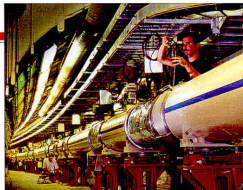
CRASH-TESTS MIT ATOMEN SIMULIEREN URKNALL

Goldatome umrunden den Beschleuniger fast 80 000-mal pro Sekunde. Wenn sie zusammenstoßen, schmelzen ihre Kerne zu einem Quark-Gluon-Plasma. Dieser eigenartige Materiezustand existierte nur einen Sekundenbruchteil nach dem Urknall.



Elektrisch geladene Atome (Ionen) lagern in beiden Richtungen durch den Beschleuniger. An sechs Kreuzungspunkten (blau) können sie zusammenstoßen

Ein Physikerkomitee befasst sich mit mehreren Weltuntergangsszenarien



VOR DEM ERSTEN STOSS Seit Juli flitzen Goldatome durch den unterirdischen Ringtunnel. Ab Herbst gehen sie auf Kollisionskurs

Big Bang Machine: Will it destroy Earth?

The London Times July 18, 1999

Creation of a black hole on Long Island?

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The committee will also consider an alternative, although less likely, possibility that **the colliding particles could achieve such a high density that they would form a mini black hole**. In space, black holes are believed to generate intense gravitational fields that suck in all surrounding matter. The creation of one on Earth could be disastrous. [...]

John Nelson, professor of nuclear physics at Birmingham University who is leading the British scientific team at RHIC, said the chances of an accident were infinitesimally small - but Brookhaven had a duty to assess them. **"The big question is whether the planet will disappear in the twinkling of an eye. It is astonishingly unlikely that there is any risk - but I could not prove it,"** he said.

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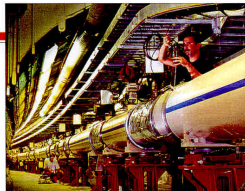
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Beim Aufprall pressen sich die Goldkerne zu einem winzigen Schwarzen Loch zusammen



Das Schwarze Loch saugt alles in seiner Umgebung auf und verschlingt den ganzen Erdball in Minuten

...back to the main feature

Observables of a Minimal Length - High Energy

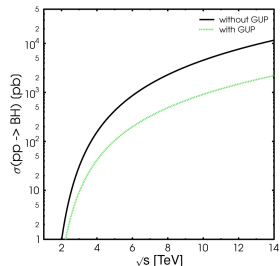
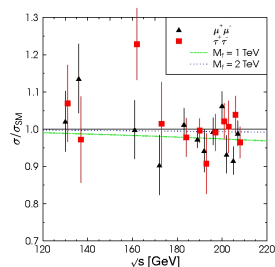
- Suppression of cross-section from $\sqrt{s} \sim M_f$ on.

SH, M. Bleicher, S. Hofmann, S. Scherer, J. Ruppert
and H. Stöcker, Phys. Lett. B598 (2004) 92-98

- Black hole production becomes more difficult

SH, Phys. Lett. B598 (2004) 92-98

- Cosmic ray puzzle? (modified threshold)



Outlook

- Thermodynamics with minimal length
 - * Finite minimal density?
 - * Modified equation of state?
- MDR from mean field approximation
 - * Integrating out the graviton soup
 - * Non-local effects compatible with stringy limits?
- Curved momentum space $\rightarrow \sqrt{g(p)}$
 - * Constrain $p(k)$ from geometrical arguments?
 - * Classify DSR's.

Summary

- Effective theory that allows to examine phenomenology
- Related to DSR but different interpretation of free particles
- Input $p(k)$, rspt. $\sqrt{g(p)}$, should eventually be provided by the underlying theory
- With more investigation, it should be possible to further classify and constrain the model