Why should we care about Quantum Gravity

Incompleteness of present theories:

- What is the gravitational field of a superposition state?
- Classical GR predicts singularities (inside black holes, in the beginning of the universe) indicating a breakdown of physics as we know it.
- Black hole evaporation leads to unresolved conflict with standard quantum mechanics.
- Finding a fundamental theory can shed light on currently open questions (CC, parameters in SM, multiverse, etc)

“If we knew what we were doing it wouldn’t be research.”
– Albert Einstein
“According to my hypothesis [...] the two theories are mathematically different and cannot be applied simultaneously. But no inconsistency can arise from using both theories, because any differences between their predictions are physically undetectable.”
Simplify the Complicated

Top-down

Top-down inspired bottom-up approaches
... Extra Dimensions ... Minimal Length ...
... DSR ... Holographic Principle ...

Bottom-up
Top-down inspired bottom-up approaches

- **Extra Dimensions**: KK-excitations, graviton-production, black hole production

- **Deformed Special Relativity**: Shift of reaction-thresholds, energy dependent speed of light (Amelino-Camelia, Magueijo, Smolin, Ellis, SH...)

- **Generalized Uncertainty**: Stagnation of cross-section, modifications of loop contributions (Kempf, Niemeyer, Cavaglia, SH ...)

- **Violation of Lorentz invariance**: Preferred frame effects, higher order operators (Jacobson, Kostolecky, Mattingly, Liberati ...)

- **Quantum Cosmology**: Imprints of QG fluctuations in the CMB/ν background, spectral index (Hofmann, Danielsson, Smolin, Sudarsky ...)

- **Space-time Foaminess**: CPT violation, stochastic deviations from lightcone, accelerated decoherence, noise, (Mavromatos, Farakos, Hogan, ...)

- **Emergent Gravity**: imprints in CMB through modified inflation/phase transition, non-local links, potentially violations of Lorenz-invariance (Konopka, Markopoulou, Prescod-Weinstein, Smolin, Visser ...)
Models with Extra Dimensions

- ADD-model: large extra dimensions $R \gg 1/M_f$
  - Solves Hierarchy-problem, $m_p^2 = R^d M_f^{d+2}$

- RS-model (I and II), extra dimension is curved
  - AdS-CFT Correspondence
  - Allows non-compact extra dimension

- UXD, TeV-scale dimensions
  - Accelerated unification of coupling constants
Gravitation as Effective Theory (ADD)

Philosophy: use naively quantized gravity in perturbative limit

T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D 59 (1999) 105006
T. G. Rizzo, Phys. Rev. D 64, 095010 (2001)

- Perturbation of metric: $g_{AB} = \eta_{AB} + \Psi_{AB}$
- Decompose: spin-2 $h_{\mu\nu}$, vector $V_{\mu i}$, scalar $\phi_{ij}$ (trace $\phi_{ii} = \phi$)
- Coupling to matter $\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_M$
- Energy momentum tensor on brane $T_{AB} = \eta_A^\mu \eta_B^\nu T_{\mu\nu}(x) \delta(y)$
- Yields coupling terms: $\mathcal{L}_{int} = -\frac{1}{2} T \phi - T^{\mu\nu} h_{\mu\nu}$
Massive Gravitons

ADD: Yields tower of massive gravitons with tiny spacing

- Large phase space makes contributions important at $\sqrt{s} \sim M_f$
- # of excitations with energy $E$ is $N(E) \sim (ER)^d$

E.g. $e^+e^- \rightarrow \gamma G: \quad \sigma \sim \frac{\alpha}{m_p^2} N(\sqrt{s}) \sim \frac{\alpha}{s} \left(\frac{\sqrt{s}}{M_f}\right)^{d+2}$

- Brane breaks Poincaré invariance and momentum conservation on brane

RS: Distinctly different signature! Discrete resonances at multiples of TeV.
Signatures of Gravitons

Collider physics (current bounds on $M_f$ in ADD in TeV-range):

- Real gravitons lead to missing energy
- Virtual exchange modifies cross sections

Astrophysics (ADD bounds weak for $d > 4$, strong for $d \leq 4$):

- Enhanced cooling of supernovae/red giants from graviton emission
- Cooling in early universe and contributions to background from decay of bulk excitations
- Anomalous re-heating of neutron stars by decay of gravitationally trapped massive gravitons
Black Holes in Extra Dimensions

In large extra dimensions (ADD)

- Gravity stronger at small distances $\Rightarrow$ horizon radius $R_H$ larger
- For $M \sim 1$ TeV, $R_H$ increases from $\sim 10^{-38}$ fm to $10^{-4}$ fm!
- For these black holes it is $R_H \ll R$ and they have approx higher dimensional spherical symmetry

- At the LHC partons can come closer than their Schwarzschild horizon $\rightarrow$ a black hole can be created!
Production of Black Holes

- Semi-classical cross-section $\sigma \sim \pi R_H^2$
- Can be improved by modelling colliding wave packets
- Yields $\sim 10^8$ black holes per year for LHC pp-collisions
- Numerical tools available for event simulation
Evaporation proceeds in 3 stages:

1. Balding phase: hair loss – the black holes radiates off angular momentum and multipole moments
2. Hawking phase: thermal radiation into all particles of the standard model as well as gravitons
3. Final decay or remaining black hole relic

Black hole thermodynamics: \( T = \kappa/2\pi \) and \( dS/dM = 1/T \)

Numerical investigation:
black hole event generator CHARYBDIS

Harris et al, [arXiv:hep-ph/0411022]
Observables of Black Holes

- Multi-jet like events, spherical, typical temperature \( \sim 200 \text{ GeV} \)
- Momentum cut-off at \( \sim M_f \)
- Thermal spectrum \( \rightarrow \) (ideally) allows to reconstruct \( d \) and \( M_f \)
- Virtual black holes: baryon/flavor non-conservation

![Diagram of multi-jet like events and virtual black holes]
The Minimal Length Scale

- Very general expectation for quantum gravity: fluctuations of spacetime itself disable resolution of small distances
- Can be found e.g. in string theory, Loop Gravity, NCG, etc.
- Minimal length scales acts as UV cutoff
- Lowering the Planck mass means raising the Planck length

...is there a fundamental limit to the resolution of structures?

Discreteness $\Rightarrow$ finite resolution, but finite resolution $\neq$ discretenesss!
A Model for the Minimal Length

- For large momenta, $p$, Compton-wavelength $\lambda = 1/k$ can not get arbitrarily small $\lambda > L_f = 1/M_f$

- Model by modifying relation between wave-vector $k$ and momentum $p$. Results in modified commutation relations
  
  $k = k(p) = \hbar p + a_1 p^3 + a_2 p^5 \ldots \Rightarrow [p_i, x_j] = i \partial p_i / \partial k_j$

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Consequences of the Minimal Length

- Implies a **generalized uncertainty principle**, first correction
  \[ \Delta x \Delta p \geq \frac{1}{2} \hbar \left( 1 + b_1 \frac{\Delta p^2}{M_f^2} \right), \]

- A squeezed phase space at high energies
  \[ \langle p|p'\rangle = \frac{\partial p}{\partial k} \delta(p - p') \Rightarrow dk \rightarrow \frac{dp \partial k}{\hbar \partial p} = \frac{dk}{\hbar} e^{-\frac{p^2}{2M_f^2}} , \]

- And a **modified dispersion relation**
  \[ \omega^2 - k^2 - \mu^2 = \Pi(k, \omega) \]

- Can but need not have an **energy dependent speed of light**
  \[ d\omega/dk \neq 1. \]
Quantisation with a Minimal Length

- Lagrangian for free fermions

\[ \mathcal{L}_f = i \bar{\Psi}(p(k) - m)\Psi \]

- Coupling of the gauge field via \( \partial_\nu \rightarrow D_\nu := \partial_\nu - ieA_\nu \) yields the gauge- and Lorentz-invariant higher order derivative interaction

\[ \mathcal{L} = \bar{\Psi}p(D)\Psi \]

- To first order one finds the usual

\[ \mathcal{L} = \mathcal{L}_f - e\bar{\Psi}\eta^{\nu\kappa}\gamma_\kappa A_\nu \Psi + O(eL^2_{\text{min}}) \]

and the dominant modification comes from the propagators

\[ (p(k) - m)^{-1} \quad (g^{\nu\kappa}(k)\gamma_\nu k_\kappa - m)^{-1} \]

\[ (p^\nu(k)p_\nu(k) - m^2)^{-1} \quad (g^{\nu\kappa}(k)k_\nu k_\kappa - m^2)^{-1} \]

- **Recipe:** replace \( p \) with \( p(k) \rightarrow \) higher order derivative Lagrangian
From the commutator

\[ [a_p, a_p^\dagger] = \delta(p - p') \left| \frac{\partial p}{\partial k} \right| \]

And the field expansion

\[ \phi(x) = \int d^3p \left| \frac{\partial k}{\partial p} \right| \left[ v_p(x)a_p + v_p^*(x)a_p^\dagger \right] \]

One finds the equal time commutator for \( x = (x, t), y = (y, t) \).

\[ [\phi(x), \pi(y)] = i \int \frac{d^3p}{(2\pi)^3} \left| \frac{\partial k}{\partial p} \right| e^{ik(x-y)} \rightarrow i \int \frac{d^3p}{(2\pi)^3} e^{ik(x-y)-\varepsilon p^2} \]

where \( \varepsilon \sim L_{\text{min}}^2 \). I.e.

\[ [\phi(x), \pi(y)] \neq \delta(x - y) \]

The Propagator

\[ \frac{1}{p^\nu(k)p_\nu(k) - m^2} \]

- Since \( p(k) \) has exactly one zero, there are no additional poles on the real axis.

This goes wrong in the first order approx (signs of coefficients are not fixed).

- For the same reason, the characteristic polynomial of the wave-equation has only one (real) zero.
Applications of the Model

The model is useful to examine effects of a minimal length scale

- Modified quantum mechanics:
  - Schrödinger’s equation, levels in hydrogen atom, g-2, Casimir-effect
- Derivation of modified Feynman-rules:
  - General prescription for calculations
- Tree-level cross-sections (e.g. $e^+ e^- \rightarrow f^+ f^-$):
  - Show overall suppression relative to SM-result
- Loop-contributions (e.g. running coupling):
  - Finite, minimal length acts as UV-regulator
Deformed Special Relativity

- Minimal length $L_{\text{min}}$ requires new Lorentz-transformations
- New transformations have 2 invariants: $c$ and $L_{\text{min}}$
- Generalized Uncertainty $\iff$ Deformed Special Relativity
  * When relation $k(p)$ is known and $p$'s (usual) transformation, then also the transformation of $k$ is known.
  * When the new transformation on $k$ is known, then one gets $k(p)$ by boosting in and out of the restframe where $k = p$.

SH, Class. Quantum Grav. 23 (2006) 1815.
Deformed, Non-linear Action on Momentum Space

- Lorentz-algebra remains unmodified

\[ [J^i, K^j] = \varepsilon^{ijk} K_k, \quad [K^i, K^j] = \varepsilon^{ijk} K_k, \quad [J^i, J^j] = \varepsilon^{ijk} J_k \]

- But it acts non-linearly on momentum space, e.g.*

\[ e^{-iL_{ab}\omega^{ab}} \rightarrow U^{-1}(p_0)e^{-iL_{ab}\omega^{ab}} U(p_0) \text{ with } \quad U(p_0) = e^{L_{\min}p_0 p_a \partial p^a} \]

- Leads to Lorentz-boost (z-direction)

\[
\begin{align*}
    p'_0 &= \frac{\gamma(p_0 - vp_z)}{1 + L_{\min}(\gamma - 1)p_0 - L_{\min}\gamma vp_z} \\
    p'_z &= \frac{\gamma(p_z - vp_0)}{1 + L_{\min}(\gamma - 1)p_0 - L_{\min}\gamma vp_z}
\end{align*}
\]

which transforms \((1/L_{\min}, 1/L_{\min}) \rightarrow (1/L_{\min}, 1/L_{\min})\)

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Interpretation of an Invariant Minimal Length

Besides $c$ there is a second invariant $L_{\text{min}}$ for all observers

- DSR approach (from SR)
  - Deformed transformation applies to free particles
  - Physical momentum is subject to deformed transformation
  - If caused by quantum gravity effects what sets the scale?

- GUP approach (from particle physics)
  - Two observers cannot compare lengths without interaction
  - The strength of gravitational effects sets the scale for the importance of quantum gravity
  - Free particles do not experience any quantum gravity or DSR
  - Effects apply for virtual particles in the interaction region only
  - Physical momentum transforms under standard Lorentz transformation

→ Propagator of exchange particles is modified
Features and Observables of DSR

- Non-linear transformation of physical momenta results in unusual addition law

\[ \tilde{\Lambda}(p_1 + p_2) \neq \tilde{\Lambda}(p_1) + \tilde{\Lambda}(p_2) \]

\[ p_1 \oplus p_2 = p(k_1 + k_2) \neq p(k_1) + p(k_2) \]


- Modified dispersion relation for free particles

\[ m^2 \approx E^2 - \vec{p}^2 + \eta \left( \frac{E}{m_p} \right)^n \]

- Energy dependent speed of light: Modifications in the time of flight for \( \gamma \)-ray bursts (Amelino-Camelia, Magueijo & Smolin, Judes & Visser), No-no (SH, Phys. Rev. D 75:105005 (2007))
Fermi Observations of High-Energy Gamma-Ray Emission from GRB 080916C

Science 27 March 2009, Vol. 323. no. 5922, pp. 1688 - 1693

z ≈ 4.35, highest energetic photon ≈ 13 GeV, arriving ≈ 16.5 seconds after the onset of the burst

lower limit on the scale of quantum gravity $M_{QG} > 1.3 \times 10^{18}$ GeV.
At 11:05:15 UT on 2 Sep 2009, the Fermi Large Area Telescope (LAT) detected gamma rays from the long GRB 090902B, which was triggered and located by the Fermi Gamma-ray Burst Monitor (GBM) (trigger 273582310/090902462, GCN9866). The angle of the GBM best position (RA, Dec= 264.5, 26.5) with respect to the LAT boresight was 51 degrees at the time of the trigger, which is close the edge of our field of view. [...] 

More than 200 photons above 100 MeV and more than 30 photons above 1 GeV are observed within 100 seconds. The highest energy photon is a 33.4 GeV event which is observed 82 seconds after the GBM trigger [...] 

Further analysis is ongoing.

The point of contact for this burst is Francesco de Palma (francesco.depalma@ba.infn.it) 

→ A puzzle!
Fluctuations of background metric

Leading to deviations from lightcone (Ford, Phys. Rev. D 51, 1692 (1995)), modified dispersion relations or CPT violation

Halos in images of quasars, decoherence (e.g. in neutrino oscillations), novel CPT violating effects in entangled states of neutral kaons (Alexandre, Farakos, Mavromatos and Pasipoularides, Phys. Rev. D 77, 105001 (2008))

Noise in gravitational wave interferometers...
GEO600 noise with and without holographic noise as in Hogan, arXiv:0806.0665
Plot: Stefan Hild

A mystery? News this Monday: with different readout method most of the noise can be explained.
If space-time is not fundamental, then the early phases of the universe might not be describable by a geometry at all. Konopka, Markopoulou, Severini, Phys. Rev. D 77, 104029 (2008)

Phase transition can have consequences for CMB/structure formation. Magueijo, Smolin, Contaldi, Class. Quant. Grav. 24, 3691 (2007)

Remaining distortions of locality constitute deviations from QFT and be responsible for the cosmological constant

Prescod-Weinstein, Smolin [arXiv:0903.5303]

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A challenge!
The Inverse Problem

Quantum

Semiclassical

Gravity

Quantum Gravity

Modifications of GR

Effective Models

Extra Dimensions / DSR / MDR / Space Time Foams /
Lorentz-Violation / Quintessence / etc.

Earth based collider / high precision

Astrophysics cosmic rays/ γ ray bursts...

Cosmology CMB/ structure / etc

Phenomenology New+Old
The Inverse Problem

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Phenomenology New+Old
Summary

- Various effective models that incorporate quantum gravitational features, some of which make predictions that will be testable soon.

- The “conservative” ones are typically very hard to test. The not-so-conservative ones are frequently weak on the side of consistency.

- The connection between these models and a possibly underlying fundamental theory of quantum gravity is currently unsatisfactory.

→ Develop models that can be applied to various effects, and combine predictions to solve inverse problem.