Comparative study of statistical models for supernova matter

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CONTENT

1. Introduction
2. SMSM, HS & FYSS models
3. Comparison of results
4. Summary, discussion and conclusions

fragmentation of nuclear matter in nuclear reactions and astrophysical processes

• Thermal multifragmentation of nuclei:
  Production of hot fragments at $T \approx 3-8$ MeV $\rho = 0.1 \rho_0$
  $\rho_0 = 0.15$ fm$^{-3} = 2.5 \times 10^{14}$ g cm$^{-3}$

• Interpretation: liquid-gas type phase transition in finite nuclei. A chance to investigate properties of hot fragments in dense environment of other nuclei and nucleons, which can be different from their ground state properties.

• Collapse of massive stars leading to Supernova II explosions:
  • We expect production of hot fragments in nuclear matter at $T \approx 1-10$ MeV $\rho \leq 0.3 \rho_0$
  • Characteristic times of the processes are very large (milliseconds), nuclear equilibrium is expected. Properties of hot fragments influence processes during the collapse and explosions.

$$T = 0.5 - 10 \text{ MeV} \quad \rho / \rho_0 = 10^{-1}, 10^{-2} \text{ and } 10^{-3} \quad Y_e = 0.2 - 0.4$$
Production of nuclear fragments in multifragmentation and supernova explosions

Variation of mass distribution with $T$ and $E^*$
U-shape
Power law
Exponential

Statistical models for supernova matter
SMSM, HS, FYSS:
Nuclear Statistical Equilibrium, Wigner-Seitz approximation


Generalized form of SMM for astrophysical conditions,

Liquid drop model,
Grand canonical approximation,
Monte carlo method,
$1 \leq A \leq 1000$, $0 \leq Z \leq A$.


RMF theory,
experimentally measured masses: mass table 2003 Audi, Wapstra and Thibault, (same FYSS),
theoretical nuclear structure calculation of Geng et al. 2005 mass table for experimentally unknown nuclei: $6969$ even-even, even-odd, odd-odd nuclei: _8O^16 - 100Fm^{331}. NO very heavy fragments, $1 \leq A \leq 331$, $0 \leq Z \leq 100$.


RMF theory,
experimentally measured masses: mass table 2003 Audi, Wapstra and Thibault, (same HS)
liquid drop approximation for heavy nuclei $Z>100$, $1 \leq A \leq 1000$, $0 \leq Z \leq A$. 
Statistical ensemble with fix $T, \rho_B, Y_e$

Calculations in a box containing 1000 baryons, density of fragments is fixed at $\rho_0=0.15\text{ fm}^{-3}$

All system is divided to Wigner-Seitz cells containing one nucleus, neutrons and electrons.

$1 \leq A \leq 1000 \quad 0 \leq Z \leq A$


\[ \mu_i = B_i \mu_B + Q_i \mu_Q + L_i \mu_L \]

nuclear species $(A,Z)$:
\[ \mu_{AZ} = A \mu_B + Z \mu_Q \]

electrons $e^-$:
\[ \mu_e^- = -\mu_Q + \mu_L \]

neutrinos $\nu$:
\[ \mu_\nu = \mu_L = -\mu_\nu \]

Protons:
\[ \mu_p = \mu_B + \mu_Q \]

Neutrons:
\[ \mu_n = \mu_B \]

Baryon number conservation:
\[ \rho_B = \frac{B}{V} = \sum_{A,Z} AN_{AZ} \rightarrow \mu_B \]

Electrical neutrality:
\[ \rho_Q = \frac{Q}{V} = \sum_{A,Z} ZN_{AZ} - \rho_e = 0 \rightarrow \mu_Q \]

Lepton number conservation:
\[ \text{trapped } \nu: \quad Y_L = \frac{L}{B} = \frac{\rho_e + \rho_\nu}{\rho_B} \]
\[ \text{free } \nu: \quad Y_e = \frac{\rho_e}{\rho_B} \]
The total free energy density: 

\[ f = \frac{1}{V} \sum_{AZ} N_{AZ} \left[ -T \left( \ln \left( \frac{g_{AZ}^0 V_f A^{3/2}}{\rho_{AZ} V^{3/2}_T} \right) + 1 \right) \right] + F_{AZ} \]

The nucleon thermal wavelength:

\[ \lambda_T = \left( \frac{2\pi \hbar^2}{m_N T} \right)^{1/2} \quad m_N = 939 \text{ MeV} \]

Excluded volume correction:

\[ V_f / V = (1 - \rho_B / \rho_0) \quad \rho_B < 0.1 \rho_0 \]

\[ f = \frac{1}{V} \sum_{AZ} N_{AZ} \left\{ F_{AZ}^{t0} + \ln \left( \frac{V_f}{V} \right) + F_{AZ} \right\} \quad F_{AZ}^{t0} = -T \left[ \ln \left( \frac{g_{AZ}^0 V_f A^{3/2}}{N_{AZ} V^{3/2}_T} \right) + 1 \right] \]

\[ \langle N_{AZ} \rangle = \frac{g_{AZ}^0 V_f A^{3/2}}{\lambda^3_T} \exp \left[ \frac{1}{T} (F_{AZ} - \mu_{AZ}) \right] \]

Internal free energy of species \((A,Z)\) (\(A>4\)):

\[ F_{AZ} (T, \rho) = F_{AZ}^B + F_{AZ}^S + F_{AZ}^C + F_{AZ}^{sym} \]

**Bulk energy:**

\[ F_{AZ}^B(T) = \left( -w_0 - \frac{T^2}{\varepsilon_0} \right) A, \quad \omega_0 = 16 \text{ MeV}, \quad \varepsilon_0 = 16 \text{ MeV} \]

**Surface energy:**

\[ F_{AZ}^S(T) = \beta_0 \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3} \quad \beta_0 = 18 \text{ MeV}, \quad T_c = 18 \text{ MeV} \]

**Coulomb energy:**

\[ F_{AZ}^C(\rho) = \frac{3}{5} c(\rho) \left( \frac{e Z}{r_0 A^{1/3}} \right)^2 \quad c(\rho) = \left[ 1 - \frac{3}{2} \left( \frac{\rho_c}{\rho_0} \right)^{1/3} + \frac{1}{2} \left( \frac{\rho_c}{\rho_0} \right) \right] \]

**Symmetry energy:**

\[ F_{AZ}^{sym} = \gamma \frac{(A - 2Z)^2}{A} \quad \gamma = 25 \text{ MeV} \]

**Nucleons and light fragments** \(A \leq 4\)

\[ F_{AZ} = -B_{AZ} + F_{AZ}^C \]
| **SMSM**  
| (1 ≤ A ≤ 1000, 0 ≤ Z ≤ A) | **HS**  
| (1 ≤ A ≤ 331, 0 ≤ Z ≤ 100) | **FYSS**  
| (1 ≤ A ≤ 1000, 0 ≤ Z ≤ A) |
| \( f = \frac{1}{V} \sum_{AZ} N_{AZ} \left\{ F_{AZ}^{t0} + \ln (V_f / V) + F_{AZ} \right\} \) | \( f = \eta f^{RMF} (T, \rho_n', \rho_p') \) + \( \sum_{A>1,Z} \rho_{AZ} \left\{ F_{AZ}^{t0} - T \ln (V_f / V) + F_{AZ} \right\} \) | \( f = \eta f^{RMF} (T, \rho_n', \rho_p') \) + \( \sum_{AZ} \rho_{AZ} \left\{ F_{AZ}^{t0} V_f / V + F_{AZ} \right\} \) |
| \( F_{AZ}^{t0} = -T \left[ \ln \left( \frac{g_{AZ}^0 V_f A^{3/2}}{N_{AZ} V \lambda_T^3} \right) + 1 \right] \) | \( F_{AZ}^{t0} = -T \left[ \ln \left( \frac{g_{AZ}(T)}{\rho_{AZ} \lambda_{AZ}^3} \right) + 1 \right] \) | \( F_{AZ}^{t0} V_f / V \) |
| \( g_{AZ}^0 = \left\{ \begin{array}{ll} 0.2 & \text{if } A < 1000 \\ 0.6 & \text{if } A \geq 1000 \end{array} \right. \) | \( \lambda_{AZ} = (2\pi h^2 / M_{AZ} T)^{1/2} \) | \( \lambda_{AZ} \sim \text{HS, } M_{AZ} \text{ slightly different} \) |
| \( \lambda_{T} = (2\pi h^2 / m_N T)^{1/2} \) m\(_N\) = 939 MeV | \( E_{AZ}^{max} = B_{AZ} \) | \( E_{AZ}^{max} = \infty \) |
| \( \text{nuclei (A > 4):} \) \( F_{AZ}(T, \rho) = F_{AZ}^B + F_{AZ}^S + F_{AZ}^C + F_{AZ}^{sym} \) | \( F_{AZ} = -B_{AZ} + F_{AZ}^C \) | \( F_{AZ} = F_{AZ}^B + F_{AZ}^S + F_{AZ}^C \) |
| \( \text{nucleons and light particles (A \leq 4):} \) \( F_{AZ} = -B_{AZ} + F_{AZ}^C \) | \( M_{AZ} = M_{AZ}^0 - B_{AZ} \) | \( F_{AZ} = M_{AZ} - [F_{AZ}^C]_{\text{vacuum}} - [F_{AZ}^S]_{\text{vacuum}} \) |
| \( F_{AZ}^S(T) = \beta_0 \left( \frac{T_c - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3} \) \( \beta_0 = 18 \text{ MeV, } T_c = 25 \text{ MeV} \) | | \( F_{AZ}^S(\rho) = 4\pi \left( \frac{3}{4\pi} \right) \frac{V_{AZ}^N}{4\pi R_{AZ}^2} \frac{1}{\rho_0} \) |
| \( \sigma_{AZ} = \frac{A^{2/3}}{4\pi R_{AZ}^2} S_s [1 - 2(Z/A)]^2 \) | \( \sigma_0 = 1.15 \text{ MeV/fm}^3 S_s = 45.8 \text{ MeV} \) |
mass distributions: SMSM, FYSS and HS

\[ \frac{\rho}{\rho_0} = 10^{-3} \]
mass distributions: SMSM, FYSS and HS

\[ \rho/\rho_0 = 10^{-2} \]
mass distributions: SMSM, FYSS and HS

\[ \rho/\rho_0 = 10^{-1} \]
mass fractions of n and p: SMSM, FYSS and HS

![Graphs showing mass fractions of n and p for different values of Ye and T (MeV) for different density ratios (ρ/ρ₀). The graphs compare SMSM, FYSS, and HS models.]
mass fractions of alphas and heavy fragments (A>4): SMSM, FYSS and HS
Average mass and charge numbers of heavier fragments $A_h (A > 4)$ and dispersion of $A_h$.
Comparison of isotopic distributions: $^8$O

- $T=1$ MeV, $Y_e=0.2$
- $T=2$ MeV, $Y_e=0.2$
- $T=3$ MeV, $Y_e=0.2$
- $T=2$ MeV, $Y_e=0.4$
- $T=3$ MeV, $Y_e=0.4$
- $T=5$ MeV, $Y_e=0.2$
- $T=5$ MeV, $Y_e=0.4$

Isotope yield vs. $A$ for different temperatures and electron fractions, with predictions from SMSM, HS, and FYSS models.
Comparison of isotopic distributions: $^{26}$Fe

- $T=0.5$ MeV, $Y_e=0.2$
- $T=0.5$ MeV, $Y_e=0.4$, $\rho/\rho_0=10^{-3}$
- $T=1$ MeV, $Y_e=0.2$
- $T=1$ MeV, $Y_e=0.4$
- $T=2$ MeV, $Y_e=0.2$
- $T=2$ MeV, $Y_e=0.4$
- $T=3$ MeV, $Y_e=0.2$
- $T=3$ MeV, $Y_e=0.4$

Isotope yield as a function of $A$ for different temperatures and neutron fractions.
Comparison of isotopic distributions: \(^{26}\text{Fe}\) and \(^{50}\text{Sn}\)

- \(T=1\text{ MeV} \), \(Y_e=0.2\)
- \(T=2\text{ MeV} \), \(Y_e=0.2\)
- \(T=3\text{ MeV} \), \(Y_e=0.2\)
- \(T=5\text{ MeV} \), \(Y_e=0.2\)
- \(\rho/\rho_0=10^{-2}\)
- \(\rho/\rho_0=10^{-1}\)

Graphs show isotope yield vs. mass number (A). Z values are 26 and 50.
Thermodynamical properties
entropy

\[ S/A \]

\[ T (\text{MeV}) \]

\[ \rho/\rho_0 = 10^{-1}, 10^{-2}, 10^{-3} \]

\[ Ye = 0.2, 0.4 \]
Thermodynamical properties

Pressure

\[
P_C = \rho_B \sum_{AZ} \rho_{AZ} \frac{\partial F_{AZ}^C}{\partial \rho_B},
\]
Thermodynamical properties
chemical potentials of protons and neutrons
Conclusions

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• We have compared SMSM, HS and FYSS model results for nuclear composition and EOS.

• The basic thermodynamical properties of stellar matter for $T \approx 0.5-10$ MeV, $Y_e \approx 0.2-0.4$ and $\rho/\rho_0 \approx 10^{-1}-10^{-3}$ have more or less similar behavior except higher $\rho$ and lower $T$.

• We demonstrate that mass and isotopic distributions have considerable differences related to the different assumptions of the models on properties of nuclei at these stellar conditions.

• Three models give similar trends, but the details reflect the uncertainties related to the modeling of medium effects, such as the temperature and density dependence of surface and bulk energies of heavy nuclei, and the nuclear shell structure effects.

Our future plan: to construct unified EOS for supernova conditions…

Note: SMSM EOS Table and manual will be available soon!
See http://fias.uni-frankfurt.de/physics/mishus/research/smsm/
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HS group: M. Hempel, M. Liebendörfer, F.K. Thielemann,
FYSS group: S. Furusawa, K. Sumiyoshi, S. Yamada, H. Suzuki

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Thank you for your attention!