New NSE equations of state for core collapse supernova simulations

Shun Furusawa (Waseda Univ.)
Kohsuke Sumiyoshi (Numazu college of Tech.)
Shoichi Yamada (Waseda Univ.)
Hideyuki Suzuki (Tokyo Univ. of Science)
1, Introduction
Core-collapse supernova

1, Gravitational Collapse
- Fe core
- $\rho_c = 10^{9-10} \text{ g/cm}^3$

2, Neutrino Sphere
- Neutrinosphere
- $\rho_c > 10^{11} \text{ g/cm}^3$

3, Bounce & Shock Wave Formation
- Bounce
- $\rho_c > 10^{14} \text{ g/cm}^3$
- Shock wave

4, Neutroization Burst
- $\nu_e$ neutronization burst

5, Neutrino Heating
- $\nu(\text{all})$

6, Explosion
- $\tau(\text{collapse}) \sim \Omega(10-100)\text{ms}$
- $\tau(\text{neutronization burst}) < \Omega(10)\text{ms}$
- $t(\text{stall}) = \Omega(100)\text{ms}$
Role of nuclear abundance before bounce

Abundances of nuclear species in cores

⇒ weak interaction rates

1. electron capture
   \[ N(A, Z) + e^- \rightarrow N(A, Z - 1) + \nu_e \]

2. neutrino coherent scatterings
   \[ N(A, Z) + \nu_e \rightarrow N(A, Z) + \nu_e \]

⇒ lepton fractions \( Y_L \)

⇒ core mass at bounce \( M_{\text{core}}(t_B) \)

⇒ Explosion energies
   • Neutrino temperature from \( \nu \) sphere
Role of nuclear abundance after bounce

In Shocked matter, deuteron, triton, He3, He4 are abundant

⇒ Heating and Cooling rates

- Deuterons have \( \frac{1}{3} \text{ to } \frac{1}{2} \) absorption rates of nucleons
  \[
  \sigma(\text{①}) \sim \frac{1}{3} \text{ to } \frac{1}{2} \times \sigma(\text{②})
  \]

  ① \( \nu_e + d \rightarrow e^- + p + p \)  
  ② \( \nu + n \rightarrow e^- + p \)

- \( \nu \) inelastic scatterings on He4 have a few \% increase in heating rate (Ohnishi et al. 2007)
Previous works

**Standard EOSs**

1. Lattimer et al. (1991)
   - Skyrme Type Interaction & Compressible Liquid Drop model

2. Shen et al. (1998)
   - Relativistic Mean Field (RMF) & Thomas Fermi approximation

**Multi-Nuclei EOS**

4. Hempel et al. (2009) RMF + excluded volume
5. G. Shen et al. (2010) Virial expansion + Hartree

- Only one representative heavy nucleus (Single Nucleus Approximation (SNA))
- Only He4 of light nuclei
2, EOS model
Baryonic EOS with Multi Species of Nuclei (Z<1000)

• Formulations

NSE (Nuclear Statistical equilibrium)
\[(A, Z) \leftrightarrow (A - 1, Z) + n\]
\[(A, Z) \leftrightarrow (A - 1, Z - 1) + p\]

\[T > \sim 5 \times 10^9 \text{ [K]}\]

[time scale of all nuclear interactions] < [dynamical time scale]

the minimized Free Energy \( F(X_p, X_n, \{X_i\}) \)

for Abundances \( X_p, X_n, \{X_i\} \) on given \( \rho, T \& Y_p \).

⇒ Thermodynamics Quantities & Abundances \( X_p, X_n, \{X_i\} \)

Base of Free energy of our model

uniform matter : RMF ⇒ Free nucleons + bulk energies

nuclear mass : LDM (Z<1000)
WB Mass formula (one of LDMs) and experimental mass data

\[ \Delta B(A, Z) = B(A, Z)^{Data} - B(A, Z)^{WB} \]

Shell Effects

Light nuclei \( A < 10 \)
The Free Energy density

\[ f = f_{p,n} + \sum n_i (F_i^{\text{trans}} + A_i M_B + E_i^{\text{bulk}} + E_i^{\text{surf}} + E_i^{\text{Coul}} + E_i^{\text{shell}}) \]

The free nucleons

Translational Energy

Nuclear Mass

nuclei

i : index of a nucleus (1<Z<1000, 1<N<1000)

Free Nucleons: RMF theory (TM1 parameter set) and excluded volume

Nuclei: • translational energy with approximate excluded volume

• heavy nuclei: Liquid Drop Model
  • approximate nuclear pasta phase
  • Nuclear Shell term

• Light nuclei (d, t, He3, He4): quantum approach
Approximate pasta phases

Volume fraction in Wigner-Seiz Cell

\[ u_i = \frac{V_i^N}{V_i} \]

Volume of a cell \( V_i \)

Volume of a nucleus \( V_i^N \)

The shapes depend on the volume fractions

- **Surface**: Surface tensions \( \sigma_i \) (Lattimer 1991) + high density correction
  
  \[ E^s_i = \sigma_i \times (\text{surface area}) \times (1 - n_{\text{nucleon}}/n_{s_i})^2 \]

- **Coulomb**: Integration of Coulomb force in WS cell
**Bulk Energy** : RMF Calculation (TM1)

\[ E_{i}^{bulk} = A_i F_i^{RMF}(T) \]

**Shell Energies** : Mass Data – LDM in vacuum

\[
E_{i}^{S_{h}} = \begin{cases} 
M_{i}^{data} - \left[ E_{i}^{B} + E_{i}^{C} + E_{i}^{S_{u}} \right]_{\text{vacuum}} & (\rho \leq 10^{12}\text{g/cm}^3) \\
(M_{i}^{data} - \left[ E_{i}^{B} + E_{i}^{C} + E_{i}^{S_{u}} \right]_{\text{vacuum}}) \times (\rho_{0} - \rho) / (\rho_{0} - 10^{12}) & (\rho > 10^{12}\text{g/cm}^3) 
\end{cases}
\]

\[ E_{i}^{S_{k\ldots}} = 0 \quad \text{for the nuclei with no mass data available} \]

**Light Nuclei** : Quantum Approach (Röpke et al(2009), Typel et al. (2010))

- d, t, He3, He4 in uniform nuclear matter

\[ M_j = M_j^{data} + \Delta E_j^{Pa} + \Delta E_j^{SE} + \Delta E_j^{C} \quad (j = d, t, h \& \alpha) \]

**Pauli energy shifts** : repulsive force from other baryons

empirical quadratic form from Typel (2010)
fitted to the result of a quantum theory Röpke (2009)

**Self energy shifts** : energy shifts of nucleons composing light nuclei

\[
\Delta E_j^{SE}(n'_{p}, n'_{n}, T) = (A_j - Z_j) \Delta E_n^{SE} + Z_j \Delta E_p^{SE} + \Delta E_j^{\text{eff.mass}} \\
\Delta E_{n/p}^{SE} = \Sigma_{n/p}^{0}(T, n'_{p}, n'_{n}) - \Sigma_{n/p}(T, n'_{p}, n'_{n}) \quad \Delta E_j^{\text{eff.mass}} \quad \text{is calculated by a fitting formula (Typel 2010)}
\]
Different parts from Furusawa et al. (2011)

① Mass Data: addition of theoretical mass data
- Audi 2003 (experimental mass data)
- Koura 2005 (theoretical mass data)
- LDM with no shell: \( E_i^{shell} = 0 \)

② Temperature dependence of Bulk Energy

Previous model

Nuclei with mass data: \( E_i^{bulk + shell} = M_i^{data} - [E_i^{Coulomb} + E_i^{Surface}]_{vacuum} \)

(no temperature dependence)

Other nuclei: \( E_i^{bulk} = E_i^{RMF}(T) \)  \( E_i^{shell} = 0 \)

\( E_i^{bulk + shell} = E_i^{RMF}(T) + M_i^{data} - [E_i^{RMF}(T = 0) + E_i^{Coulomb} + E_i^{Surface}]_{vacuum} \)

All nuclei have temperature dependence.

③ Light nuclei: LDM \( \Rightarrow \) quantum approach
The Free Energy density

\[ f = f_{p,n} + \sum_i n_i (F^\text{trans}_i + A_i M_B + E_i^\text{bulk} + E_i^\text{surf} + E_i^\text{Coul} + E_i^\text{shell}) \]

**free nucleons**

**Translational Energy**

**Nuclear Mass**

\( i \) : index of a nucleus (1<Z<1000,1<N<1000)

- Boltzmann gases with approximate excluded volume
- mass of heavy nuclei: Liquid Drop Model with RMF bulk
  + approximate nuclear pasta phase
  + Nuclear Shell term
- mass of Light nuclei \((d, t, \text{He3, He4})\): quantum approach

@Low densities: Boltzmann gases with mass data

⇒ ordinary NSE thanks to Shell energies

@ \( \rho = \rho_0 \): continuous transition to the EOS for RMF \( \rho \geq \rho_0 \)

\( F^\text{trans}_i = E_i^\text{surf} = E_i^\text{Coul} = E_i^\text{shell} = 0 \) and RMF parts only remain)
3 Results

① Mass fractions of nuclei in the (N, Z) plane

Mass fraction $Y_p=0.3$  $T=1.0$ MeV  $\rho_B = 10^{12}$ g/cm$^3$

Previous model (Audi)

Current Model (Audi+KTUY)

- KTUY extend the region of the nuclide with shell effects.
② Isotope abundance of Z=26

A Previous EOS (Audi data only) = Furusawa (2011)
B A New EOS (Audi data & temperature dependent bulk)
C Current EOS (Audi + KTUY data & temperature dependent bulk)

- The bulk energies with temperature dependence avoids the large discreteness at high T.
- KTUY extend the region of the nuclide with shell effects.
③ Average Mass Numbers

A Previous EOS (Audi data only) = Furusawa (2011)
B New EOS (Audi data & temperature dependent bulk)
C New EOS (Audi + KTUY data & temperature dependent bulk)

We can see the effect of neutron magic number (N=126, 184) in KTUY.
light nuclei abundance of $d, t, h (=He3), \alpha$

$Y_e = 0.3, T = 10\text{MeV}$

C, LDM  $M_i = E_i^B + E_i^{Sh} + E_i^C + E_i^{Su}$

same mass formula for heavy nuclei

D, Quantum Approach  

$$M_j = M_j^{\text{data}} + \Delta E_j^{Pa} + \Delta E_j^{SE} + \Delta E_j^C$$

E, $\Delta E_j^C$  Only  

$$M_j = M_j^{\text{data}} + \Delta E_j^C$$

※Models of heavy nuclei are same.

The LDM overestimates the binding energies of light nuclei.

(high bulk energies depending on $T$ & Surface energies diminish due to Pasta phase)
Comparizon with Shen EOS

Our EOS (quantum approach)
- d (red)
- t (green)
- h (light green)
- α (pink)

Shen (α Only)
- ...........

Deuteron and triton are more abundant than alpha at high T.

T=5MeV, Yp=0.3

T=10MeV, Yp=0.3
4, Summary

Purpose
Construction of multi-nuclei EOS to calculate the weak interaction rates in core-collapse supernovae.

Model  NSE EOS
RMF (free nucleons & Bulk)
LDM +Shell Energy +Pasta phase
Quantum approach for light nuclei (d,t,h,α)

Improvements & Results
- Temperatures dependence of bulk ⇒ avoidance of discrete abundances at high T
- KTUY mass data ⇒ more wide range of nuclei with Shell effects in N,Z plane
- Quantum approach ⇒ more realistic abundance of light nuclei

Future works
- Temperature dependence of shell energies
- Improvement of Quantum approach (Momentum dependence)
- Application to hydro-dynamical simulations