Black holes, Gauge/Gravity Duality, and Energetic Probes

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Contents

1. Black holes in anti-de Sitter space

1.1. From dyonic black holes to black three-branes

The duality revolution of the mid-1990's hinged in no small part on generalizations of the Reissner-Nordstrom black hole.

$$\mathcal{L} = \frac{1}{2\kappa^2} R - \frac{1}{4} F_{\mu\nu}^2 \,. \tag{1}$$

$$\begin{aligned} ds^2 &= -\frac{h}{H^2} dt^2 + H^2 \left[\frac{dr^2}{h} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) \right] \\ F_2 &= \frac{Q_E}{r^2 H^2} dt \wedge dr + Q_B \sin\theta \, d\theta \wedge d\phi \end{aligned} \tag{2}$$

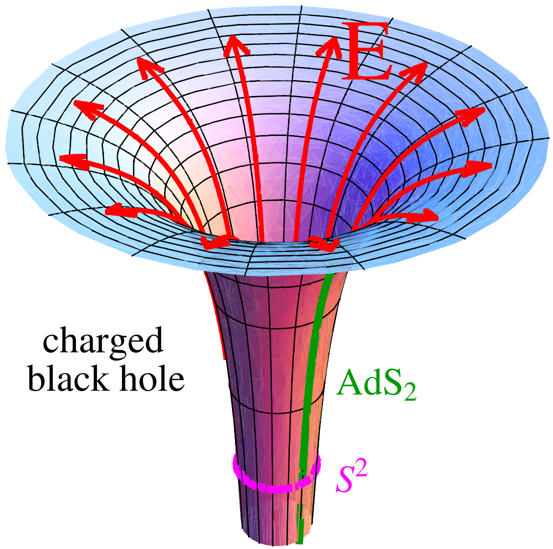
where

$$H = 1 + \frac{L}{r} \qquad\qquad h = 1 - \frac{r_H}{r} \tag{3}$$

$$L\sqrt{1 + \frac{r_H}{L}} = \kappa \sqrt{\frac{Q_E^2 + Q_B^2}{2}} \qquad\qquad M = \frac{4\pi}{\kappa^2}(2L + r_H) \,. \tag{4}$$

Many lovely properties:

- Mass-charge inequality, $M \geq \frac{8\pi}{\kappa}\sqrt{\frac{Q_E^2+Q_B^2}{2}}$.
- No-force condition at extremality between identical RN BH's: electrostatic / magnetostatic repulsion balances gravitational attraction.
- Near-horizon geometry of extremal solution is $AdS_2 \times S^2$.



To get $AdS_2 \times S^2$, set $r_H = 0$ and "drop the 1" from H (i.e. take $r \ll L$):

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2}{r^2}dr^2 + L^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,. \qquad (5)$$

Type IIB string theory in ten dimensions has a close analog: the black three-brane.

Part of the IIB equations of motion read:

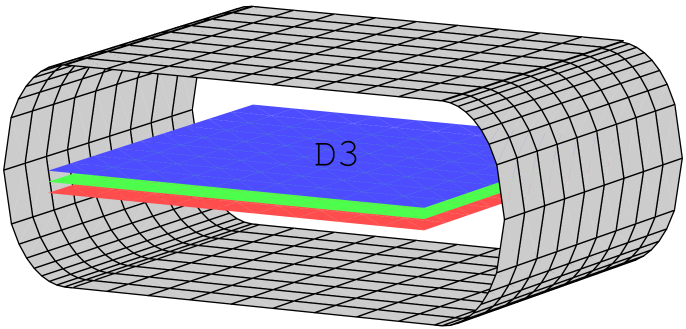
$$\begin{aligned} R_{MN} &= \frac{\kappa^2}{6} F_{MP_1P_2P_3P_4} F_N{}^{P_1P_2P_3P_4} \\ dF_{(5)} &= 0 \qquad F_{(5)} = *F_{(5)} \,. \end{aligned} \tag{6}$$

The black three-brane is

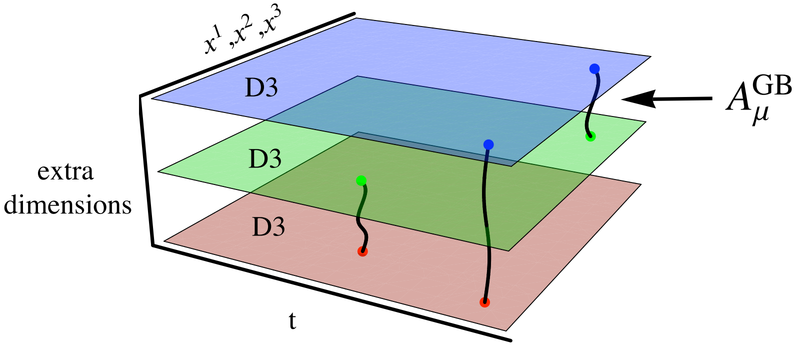
$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H}}\left(-h\,dt^2 + d\vec{x}^2\right) + \sqrt{H}\left(\frac{dr^2}{h} + r^2 d\Omega_5^2\right) \\ F_{(5)} &= \underbrace{\frac{Q}{H^2 r^5} dt \wedge d^3x \wedge dr}_{\text{electric}} + \underbrace{\text{(Hodge dual)}}_{\text{magnetic}} \end{aligned} \tag{7}$$

where

$$\begin{aligned} H &= 1 + \frac{L^4}{r^4} \\ h &= 1 - \frac{r_H^4}{r^4} \\ Q &= \frac{L^4}{\kappa} \quad \text{if } r_H = 0 \,. \end{aligned} \tag{8}$$



Polchinski's picture of D-branes, at small $g_{\rm str}$, is that they are $3+1$-dimensional planes on which strings can end. He showed that such planes generate $F_{(5)}$.



weak coupling

If N D3-branes are on top of one another, the surrounding geometry must be the black three-brane with

$$\frac{L^8}{\kappa^2} = \frac{N^2}{4\pi^5}\,. \tag{9}$$

This also implies

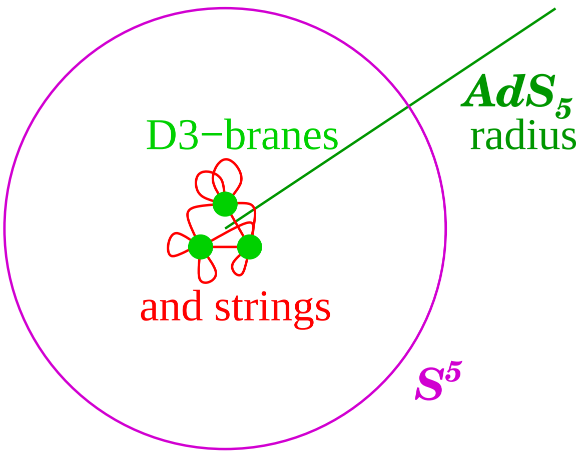
$$\frac{L^4}{\alpha'^2} = 4\pi g_{\rm str} N = g_{\rm YM}^2 N\,. \tag{10}$$

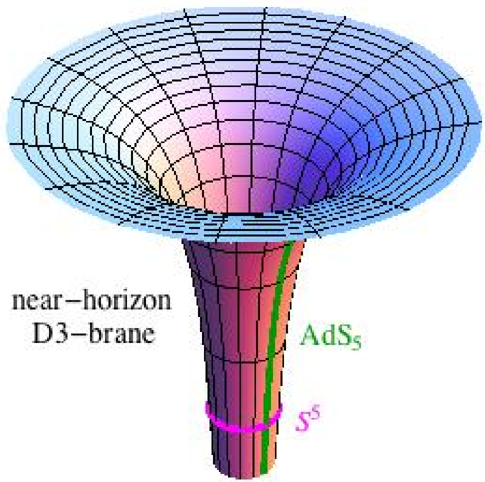
To use the black three-brane metric reliably, we need $N \gg 1$ (suppressing graviton loops) and $g_{\rm YM}^2 N \gg 1$ (suppressing stringy corrections).

1.2. AdS/CFT

The viewpoint of AdS/CFT is that the black three-brane describes the same object as N coincident D3-branes, so there must be a direct equivalence of the physics.

More precisely, the equivalence should be between the near-horizon limit of the black three-brane ($AdS_5 \times S^5$) and the field theory limit of open strings on D3-branes ($\mathcal{N} = 4$ super-Yang-Mills theory) [Maldacena, hep-th/9711200].





D3-branes at weak coupling, viewed end-on

D3-branes at strong coupling are replaced by $AdS_5 \times S^5$ geometry.

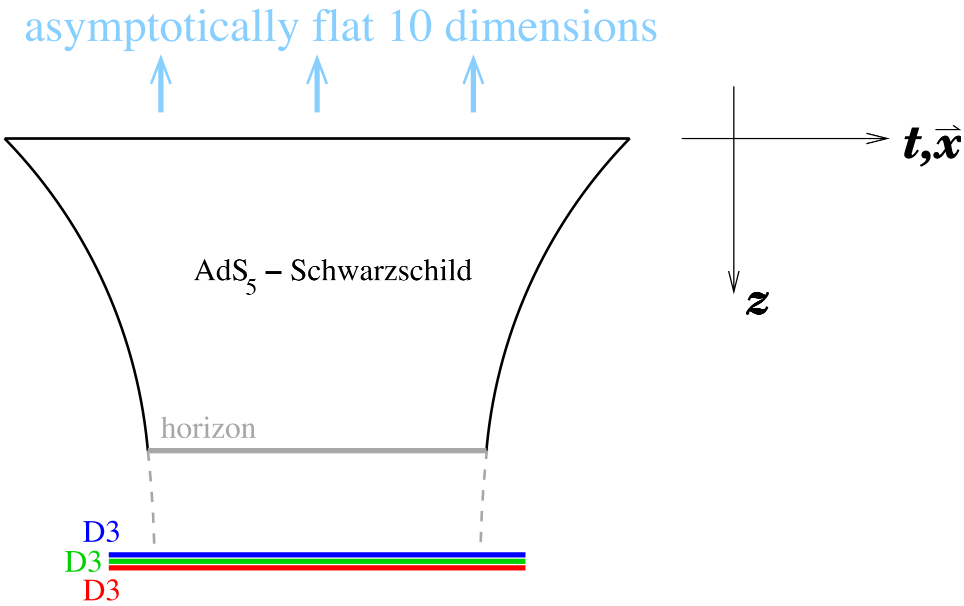
"Drop the 1" in H to get AdS_5-Schwarzschild $\times$ S^5:

$$ds^2 = \frac{r^2}{L^2}\left(-h dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}\frac{dr^2}{h} + L^2 d\Omega_5^2 \,. \tag{11}$$

The entropy and temperature provide a first check:

$$S = \frac{V_3}{4G_5}\frac{r_H^3}{L^3} \qquad T = \frac{r_H}{\pi L^2}\,, \tag{12}$$

so we see [Gubser, Klebanov, and Peet, hep-th/9602135]



$$S = \frac{\pi^2}{2} V_3 N^2 T^3 = \frac{3}{4} S_{\text{free SYM}}\,, \tag{13}$$

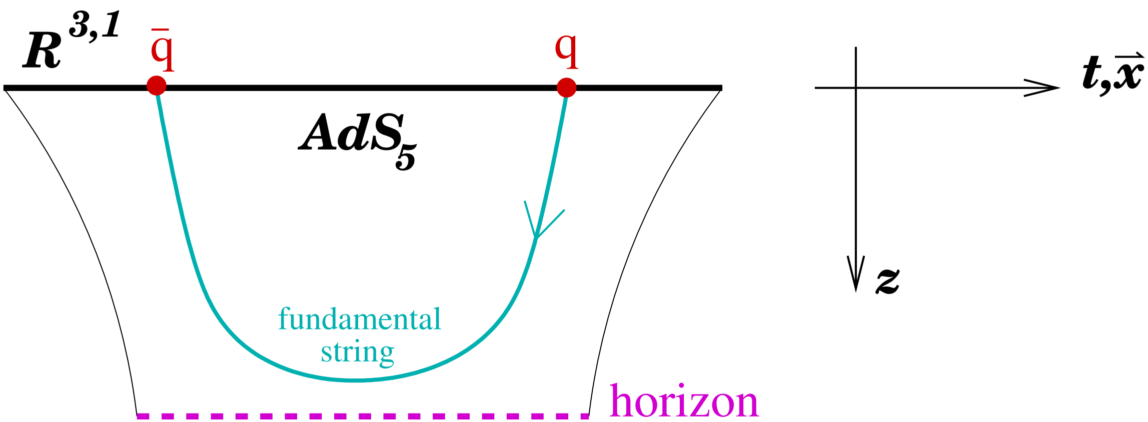
which makes sense because black hole methods work at large $g_{\text{YM}}^2 N$ and the free particle counting of entropy is for $g_{\text{YM}}^2 N \to 0$.

A related statement is that the onshell action of supergravity in AdS_5 generates the connected correlators of the dual gauge theory, at leading order in large N and $g^2_{YM}N$ [Gubser, Klebanov, and Polyakov, hep-th/9802109; Witten, hep-th/9802150]:

$$\text{extremum } e^{-I_{\text{SUGRA}}[\phi]} = \left\langle e^{\int d^4x\, \phi_0 \mathcal{O}} \right\rangle . \tag{14}$$

Another standard observable is the potential between a static, infinitely heavy quark and anti-quark [Rey and Yi, hep-th/9803001; Maldacena, hep-th/9803002]:

$$V_{q\bar{q}}(r) = \frac{4\pi^2}{\Gamma(1/4)^4} \frac{\sqrt{g^2_{\text{YM}}N}}{r} \qquad \text{at } T = 0 \text{ from hanging string} . \tag{15}$$

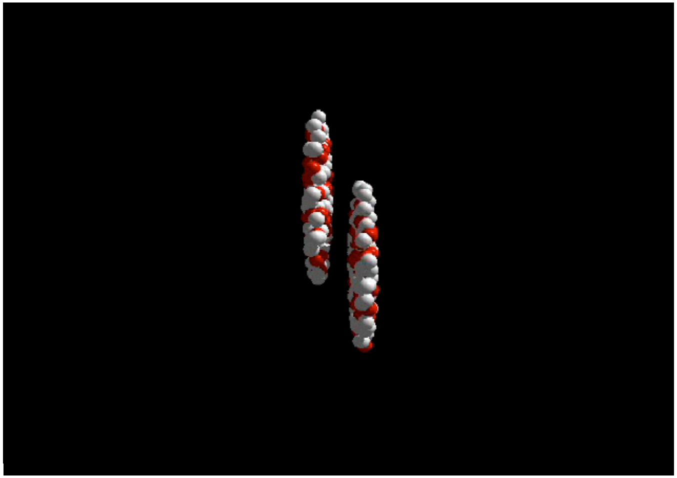


1.3. Relation to the QGP

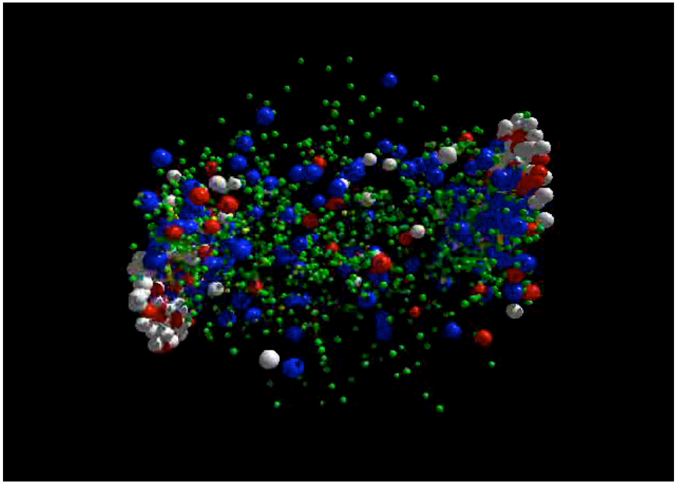
Because $S \propto N^2$ and $V_{q\bar{q}} \propto 1/r$, AdS_5-Schwarzschild describes a deconfined but strongly coupled gauge theory, like QCD somewhat above $T_c \approx 170\,\mathrm{MeV}$.

RHIC, and now LHC, creates such a medium in heavy-ion collisions.
Focus on RHIC here.

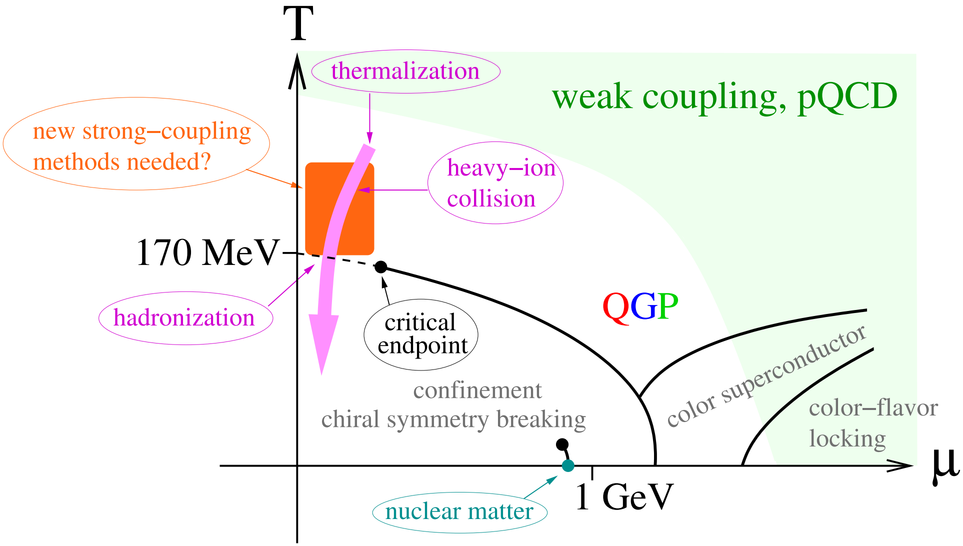
BEFORE collision



AFTER collision (UrQMD)



- 7500 particles come out, and data supports fleeting existence of a thermal medium.
- Peak temperature is believed to be 3 or 4 trillion degrees.



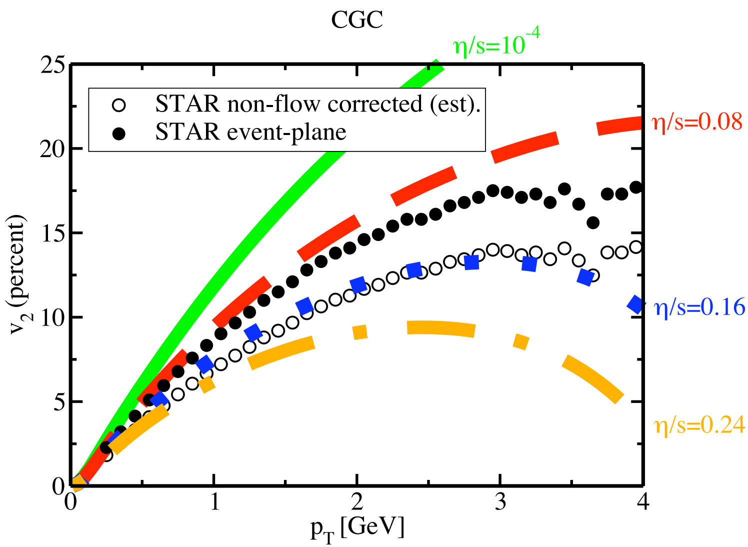
pQCD is useful but can't address all interesting questions.

New strong coupling methods are needed to describe transition fluid QGP medium.

Perturbations of black three-branes lead to a result on shear viscosity [Policastro, Son, and Starinets, hep-th/0104066; Kovtun, Son, and Starinets, hep-th/0405231]

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad (16)$$

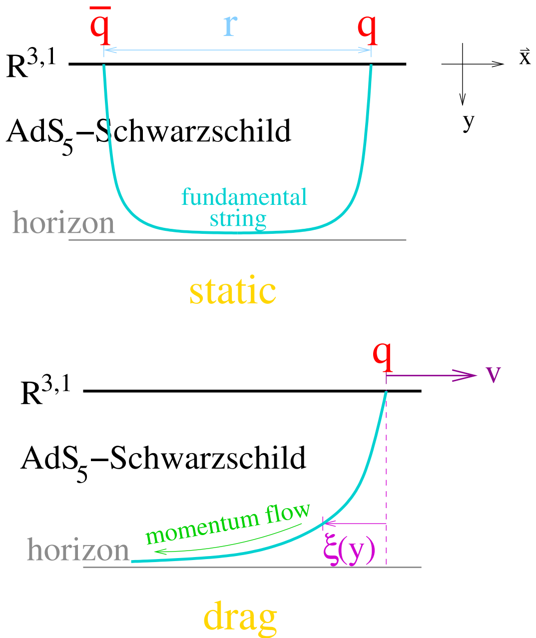
which is in the correct ballpark to agree with data [Luzum and Romatschke, 0804.4015].

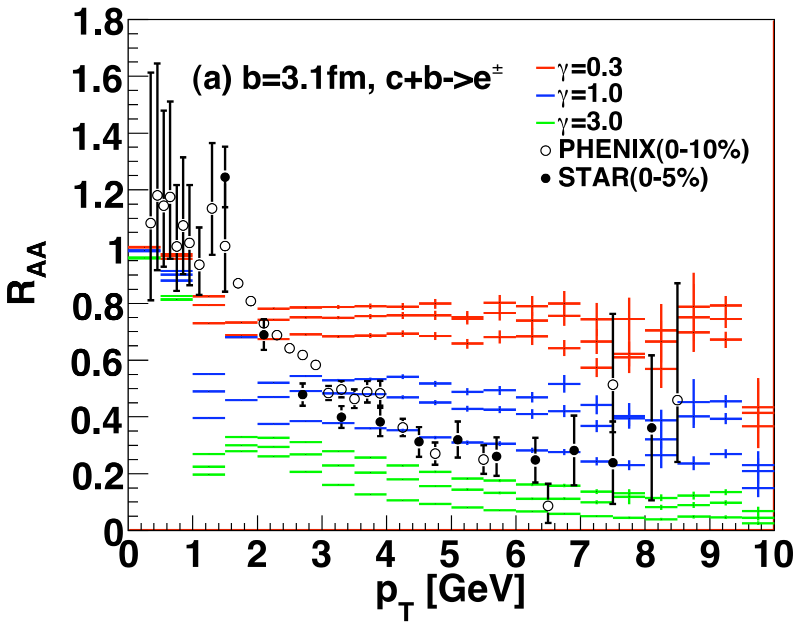


Heavy quarks are described by a string rising up to the boundary, and if such a string moves it experiences a drag force [Herzog et al, hep-th/0605158; Teaney and Casalderrey-Solana, hep-ph/0605199; Gubser, hep-th/0605182]:

$$F_{\rm drag} = \frac{\pi\sqrt{g_{\rm YM}^2 N}}{2} T^2 \frac{v}{\sqrt{1-v^2}} \,. \tag{17}$$

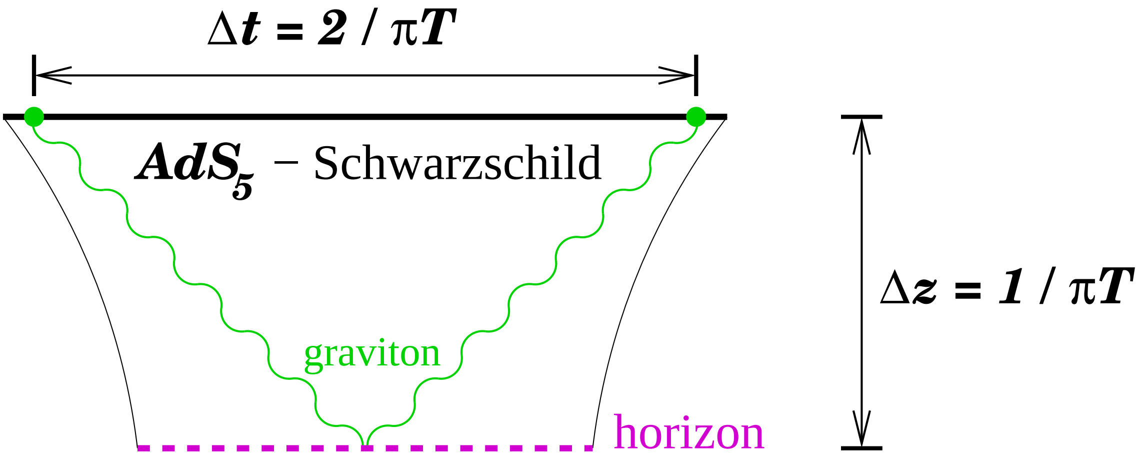
This drag force is also in the right ballpark relative to RHIC data [Akamatsu, Hatsuda, and Hirano, 0809.1499].





AdS-black hole calculations also seem to indicate that thermalization time is short [Kovtun and Starinets, hep-th/0506184; Friess et al, hep-th/0611005; Chesler and Yaffe, 1011.3562; many others], on the order

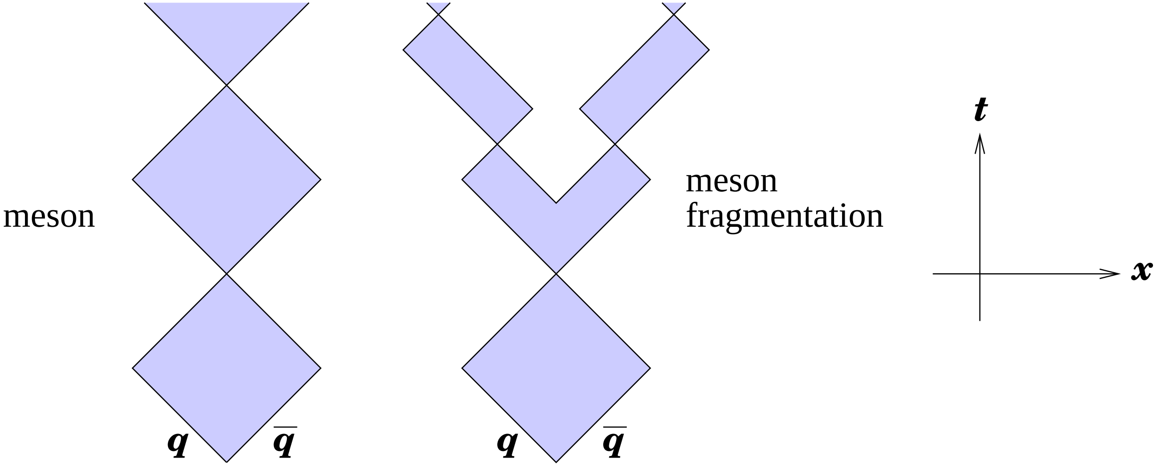
$$t_{\rm therm} \approx \frac{2}{\pi T_{\rm peak}} \approx 0.3\,{\rm fm}/c \qquad \text{if } T_{\rm peak} \approx 400\,{\rm MeV}\,. \tag{18}$$



2. Finite endpoint momentum

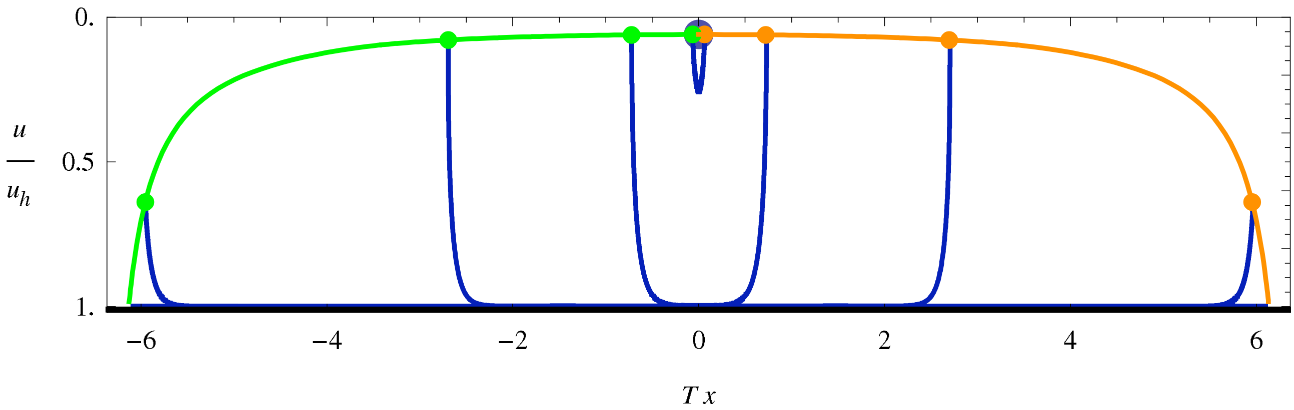
2.1. Why do we need it?

A highly successful phenomenological account of fragmentation (Lund model) starts with energetic quarks moving apart while linked by a string: the "yo-yo" [Andersson et al, 1983; Artru, 1983]. Earlier work goes back to [Bardeen et al, 1976].



- When $g_{\rm str} = 0$, all that can happen is that the massless quark and anti-quark oscillate in a linear potential. $g_{\rm str} \neq 0$ allows for fragmentation events.
- Initial energy is *entirely* in q and $\bar{q}$. Sometime later, it's entirely in the string.

To account for the medium in a heavy ion collision, a related strategy was pursued in AdS_5-Schwarzschild: [Chesler et al, 0804.3110], similar to [Gubser et al, 0803.1470].



Standard boundary conditions were applied: $\partial_\sigma X^\mu = 0$.

Initial state is a short string intended to reflect state of a quark-anti-quark pair produced in an energetic scattering event.

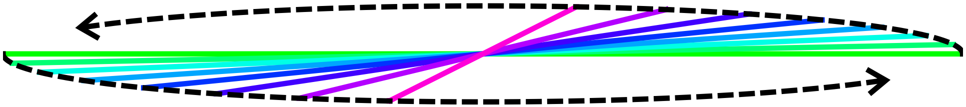
It would be more faithful to the Lund model to have finite momentum at the string endpoints.

To see why finite endpoint momentum makes sense for classical strings, consider an interpolation between Regge and the yo-yo:

$$X^\mu(\tau,\sigma) = \frac{1}{2} Y^\mu(\tau-\sigma) + \frac{1}{2} Y^\mu(\tau+\sigma)\,. \tag{19}$$

where

$$\frac{dY^\mu}{d\xi} = \begin{pmatrix} \sqrt{\ell_1^2 \sin^2\xi + \ell_2^2 \cos^2\xi} \\ \ell_1 \sin\xi \\ \ell_2 \cos\xi \end{pmatrix} \qquad Y^\mu(0) = \begin{pmatrix} 0 \\ -\ell_1 \\ 0 \end{pmatrix}\,. \tag{20}$$



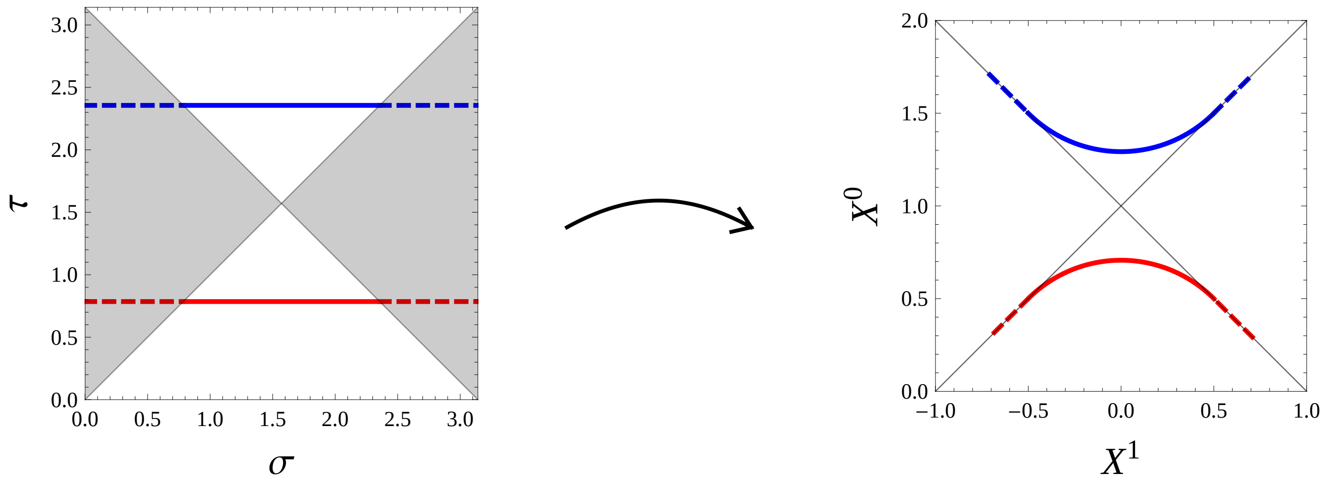
Snapshots at constant τ, with $\ell_2 = \ell_1/10$

Regge case is $\ell_1 = \ell_2$, and then $X^0 = \tau$.

Yo-yo is $\ell_2 = 0$, but now $X^0(\tau,\sigma)$ is complicated because $\dot{Y}^0 = \ell_1 |\sin\xi|$.

Observe $X^\mu(\tau,0) = Y^\mu(\tau)$: endpoint prescribes entire motion of string.

The mapping $(\tau, \sigma) \to (X^0, X^1)$ is partially degenerate when $\ell_2 = 0$: a finite region maps to the edge of the string.



More transparent would be to use a static gauge, $X^0 = t$ and $X^1 = x$, and allow each endpoint to carry $E_{\rm endpoint} = t/(2\pi\alpha')$, so that

$$E_{\rm total} = \frac{2\ell_1 - 2t}{2\pi\alpha'} + 2 \times \frac{t}{2\pi\alpha'} = \frac{2\ell_1}{2\pi\alpha'} \,. \tag{21}$$

2.2. Endpoints follow geodesics

Now I want to argue that endpoint trajectories naturally follow spacetime geodesics when the endpoint momentum is non-vanishing. Argument proceeds in three steps:

Step 1: Formulate an action that includes finite endpoint momentum.

$$S = -\frac{1}{4\pi\alpha'} \int_M d\tau d\sigma \sqrt{-h} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \int_{\partial M} d\xi \, \frac{1}{2\eta} \dot{X}^\mu \dot{X}^\nu G_{\mu\nu} \,, \quad (22)$$

where η is the einbein on the edge of the worldsheet.

Step 2: Formulate eom's in terms of endpoint momenta and bulk momentum density.

$$\begin{aligned} P^a_\mu &= -\frac{1}{2\pi\alpha'} \sqrt{-h} h^{ab} G_{\mu\nu} \partial_b X^\nu && \text{bulk momentum density} \\ p_\mu &= \frac{1}{\eta} G_{\mu\nu} \dot{X}^\nu && \text{endpoint momentum} \end{aligned} \quad (23)$$

$$\begin{aligned} \partial_a P^a_\mu - \Gamma^\kappa_{\mu\lambda} \partial_a X^\lambda P^a_\kappa &= 0 && \text{bulk conservation of momentum} \\ \dot{p}_\mu - \Gamma^\kappa_{\mu\lambda} \dot{X}^\lambda p_\kappa &= \dot{\sigma}^a \epsilon_{ab} P^b_\mu && \text{boundary loses/gains energy from bulk} \end{aligned} \quad (24)$$

Step 3: Manipulate endpoint equations in a conformal gauge.

Use a metric where $\sqrt{-h}h^{ab} = \text{diag}\{-1, 1\}$. Then I claim

$$\dot{\sigma}^a \epsilon_{ab} P^b_\mu \pm \frac{\eta}{2\pi\alpha'} p_\mu = 0\,. \tag{25}$$

or, equivalently,

$$(\epsilon_{ab}\sqrt{-h}h^{bc} \mp \delta^c_a)\dot{\sigma}^a \partial_c X^\nu = 0\,. \tag{26}$$

This is because $M_a{}^c \equiv \epsilon_{ab}\sqrt{-h}h^{bc}$ has eigenvectors $(1, \pm 1)$; and along the world-sheet boundary, we have $\dot{\sigma}^a \propto (1, \pm 1)$.

So

$$\dot{p}_\mu - \Gamma^\kappa_{\mu\lambda}\dot{X}^\lambda p_\kappa = \mp \frac{\eta}{2\pi\alpha'} p_\mu\,, \tag{27}$$

where we take $-$ when the string endpoint is "unrolling."

We can now see that endpoint moves along a geodesic:

$$\dot{\tilde{p}}_\mu - \Gamma^\kappa_{\mu\lambda}\dot{X}^\lambda \tilde{p}_\kappa = 0 \qquad \text{where} \qquad \tilde{p}_\mu = \frac{1}{\tilde{\eta}} G_{\mu\nu}\dot{X}^\nu\,. \tag{28}$$

2.3. Doubled strings in AdS_5

Yo-yo generalizes easily to global AdS_5, most simply as a doubled string.

$$ds_5^2 = L^2 \left(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2\right) \,, \tag{29}$$

and we embed string into an AdS_2 submanifold:

$$ds_2^2 = L^2 \left(-\cosh^2\rho\, d\tau^2 + d\rho^2\right) \tag{30}$$

with endpoint trajectory determined by

$$\tan\frac{\tau}{2} = \tanh\frac{\rho}{2} \,. \tag{31}$$

The endpoint energy is

$$p_\tau = -\frac{EL}{2} + \frac{L^2}{\pi\alpha'}\sinh\rho \,. \tag{32}$$

so snapback occurs at $\rho_* = \sinh^{-1}\left(\frac{\pi\alpha'}{2L}E\right)$.

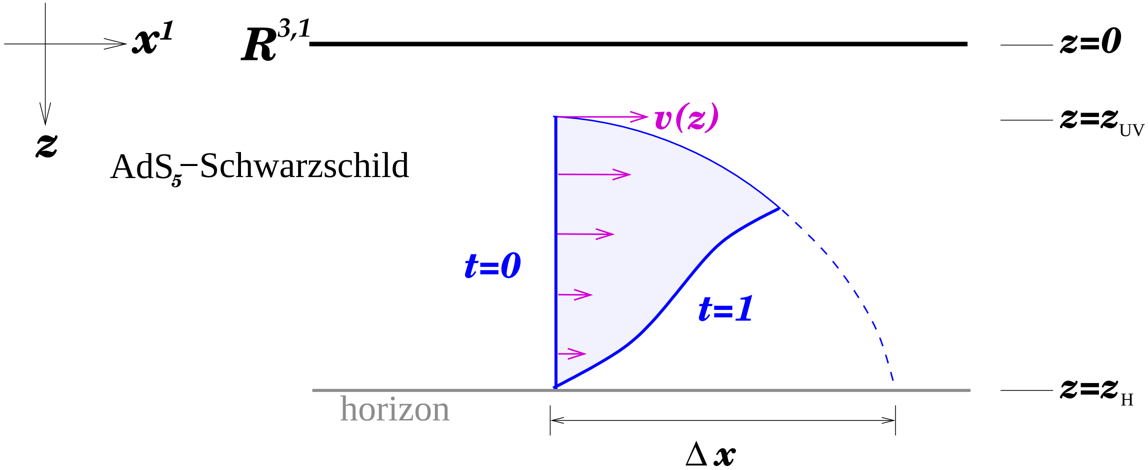
What is dual operator? Propose

$$\mathcal{O} = \mathrm{tr}\, X^I (\nabla_1)^S X^I \,, \tag{33}$$

in same multiplet as the operators $\mathrm{tr}\, X^I(\nabla_2 + i\nabla_3)^S X^I$ dual to Regge strings.

3. Application to light quark energy loss

Single quark setup: [Gubser et al, 0803.1470]

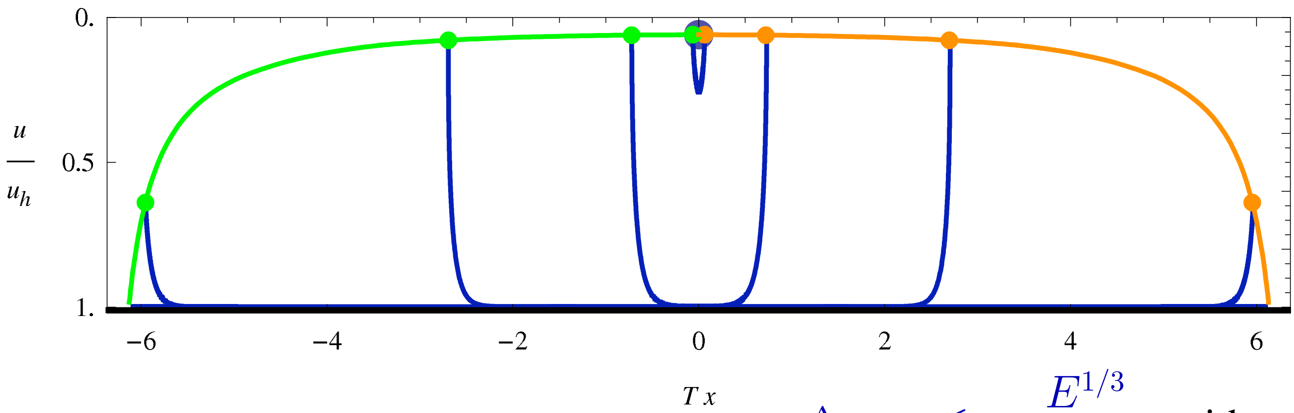


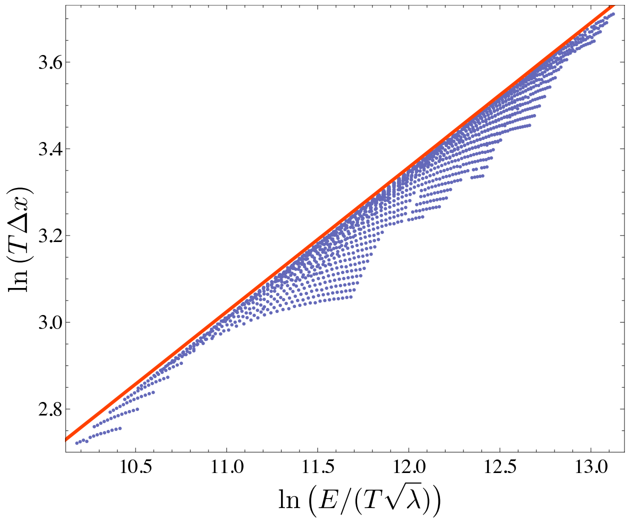
If a string starts at $t = 0$ with one end through the horizon and the other on a flavor brane, how far can it get before it falls through the horizon?

There's a big range of choice of initial conditions.

$\Delta x_{\rm stop} \lesssim \kappa \dfrac{E^{1/3}}{\lambda^{1/6} T^{4/3}}$ with an estimate $\kappa \in (0.35, 0.41)$.

Dissociating meson setup: [Chesler et al, 0804.3110]





- $\Delta x_{\text{stop}} \leq \kappa \dfrac{E^{1/3}}{\lambda^{1/6} T^{4/3}}$ with $\kappa = 0.526$ from extensive numerical study.
- Reminiscent of perturbative BDMPS result (e.g. [Baier et al, hep-ph/9608322]) $\Delta E_{\text{BDMPS}} = \dfrac{1}{4}\alpha_s C_R \hat{q} (\Delta x)^2$.
- Recent PHENIX study [Adare et al, 1208.2254] actually favors $\Delta E \propto \ell^3$ over ℓ^2.

Our plan:

- Show how $\kappa = 0.526 = \frac{2^{1/3}}{\sqrt{\pi}} \frac{\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$ comes out of spacetime geodesics plus a slightly tricky accounting of initial energy.
- Show how finite endpoint momentum gives $\kappa = 0.624$.
- Show how single quark can approach $\kappa = 0.990$.
- Propose a new account of instantaneous energy loss based on endpoint $\dot{p}_\mu$.

3.1. No endpoint momentum

When a string has a lot of momentum in x^1 direction, it quickly settles into a segment of trailing string with velocity $v = \sqrt{f(z_*)}$, where AdS_5-Schwarzschild metric is

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \qquad \text{with} \qquad f(z) = 1 - \frac{z^4}{z_H^4}\,. \tag{34}$$

So we evaluate energy (half the total energy of the meson) as

$$E_* = \frac{L^2}{2\pi\alpha'}\frac{1}{\sqrt{1-v^2}}\left[\frac{1}{z_*} - \frac{1}{z_H}\right] + \frac{1}{v}\frac{dE}{dt}\Delta x(z_*, z_H) \approx \frac{L^2}{2\pi\alpha'}\frac{1}{\sqrt{1-v^2}}\frac{1}{z_*}\,. \tag{35}$$

The endpoint subsequently stays *close* to a null geodesic, which is a solution to

$$\frac{dx_{\rm geo}}{dz} = \frac{1}{\sqrt{f(z_*) - f(z)}} = \frac{z_H^2}{\sqrt{z^4 - z_*^4}} \,. \tag{36}$$

So we find $\Delta x_{\rm stop}$ by intersecting geodesic with horizon:

$$\Delta x_{\rm stop} = \frac{z_H^2}{z_*} \frac{\sqrt{\pi}\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})} - {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{z_*^4}{z_H^4}\right) z_H \,, \tag{37}$$

and in the high-energy limit where $z_* \ll z_H$

$$\Delta x_{\rm stop} = \frac{2^{1/3}}{\sqrt{\pi}} \frac{\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} \frac{1}{T} \left(\frac{E_*}{\sqrt{\lambda} T}\right)^{1/3} \,, \tag{38}$$

3.2. Including endpoint momentum

Following spirit of Lund, assign all energy to the endpoints initially. Also require $E_{\text{endpoint}} \to 0$ just as string crosses horizon.

Calculate evolution of $E_{\text{endpoint}} = -p_t$ using

$$\dot{p}_t = -\frac{\eta}{2\pi\alpha'} p_t = \frac{\sqrt{\lambda}}{2\pi} \frac{f}{z^2} \frac{dt}{dz} \,. \tag{39}$$

Arrive at

$$E_* \approx \frac{\sqrt{\lambda}}{2\sqrt{\pi}} \frac{\sqrt{\pi}\,\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \frac{z_H^2}{z_*^3} \sqrt{f(z_*)} \,. \tag{40}$$

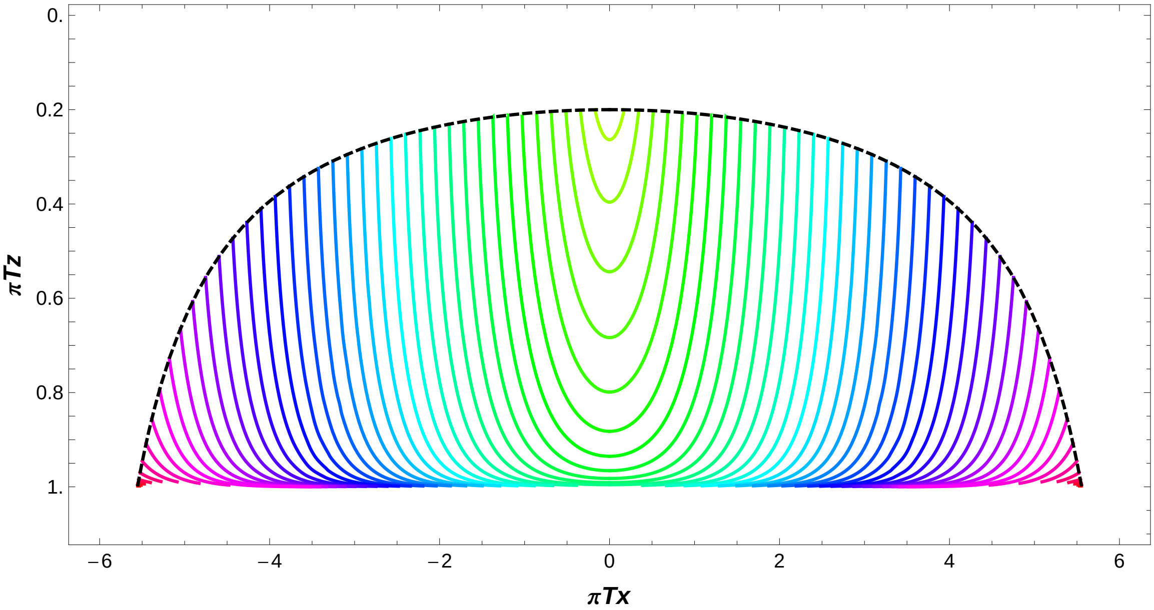
The *same* spacetime geodesic calculation as before now leads to

$$\Delta x_{\text{stop}} = \frac{2^{1/3}}{\pi^{2/3}} \frac{\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right)^{1/3}}{\Gamma\left(\frac{3}{4}\right)^{4/3}} \frac{1}{T} \left(\frac{E_*}{\sqrt{\lambda}T}\right)^{1/3} = \frac{0.624}{T} \left(\frac{E_*}{\sqrt{\lambda}T}\right)^{1/3} , \tag{41}$$

as before with $z_* \ll z_H$.

Only the energy calculation changed.

One can numerically determine the shape of the bulk of the string:



String goes further because we budgeted initial energy differently: no initial downward motion, only longitudinally outward.

3.3. Single quarks and instantaneous energy loss

How far a string can go if one end passes through the horizon and total energy E outside horizon is fixed?

Argument from spacetime geodesics is now familiar: start near the horizon moving upward; require $E_{\rm endpoint} \to 0$ only when we fall completely into the horizon; and use $\dot{p}_\mu$ equation to evolve $E_{\rm endpoint}$ along endpoint geodesic. Answer:

$$\Delta x_{\rm stop} = \frac{2}{\pi^{2/3}} \frac{\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right)^{1/3}}{\Gamma\left(\frac{3}{4}\right)^{4/3}} \frac{1}{T} \left(\frac{E}{\sqrt{\lambda}T}\right)^{1/3} = \frac{0.990}{T} \left(\frac{E}{\sqrt{\lambda}T}\right)^{1/3} \tag{42}$$

To find motion of the bulk of the string, it helps a lot to use Eddington-Finkelstein coordinates:

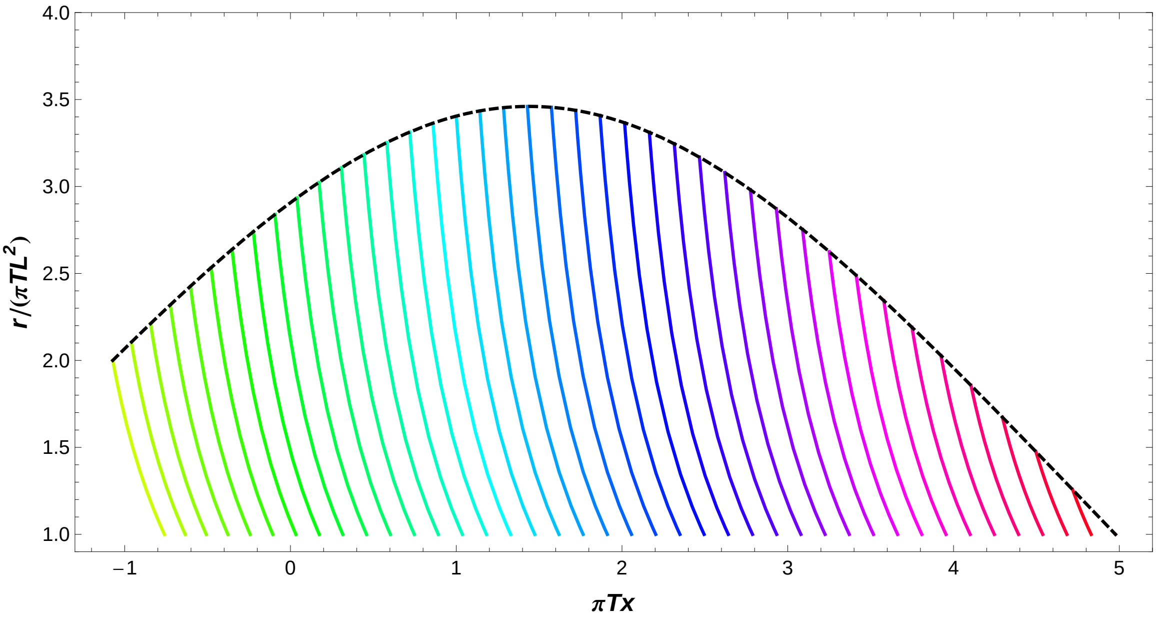
$$ds^2 = -\frac{r^2}{L^2}\left(1 - \frac{r_H^4}{r^4}\right) dv^2 + 2dvdr + \frac{r^2}{L^2} d\vec{x}^2 \,. \tag{43}$$

Initializing with a segment of the trailing string,

$$x_{\rm trailing} = \beta\left(v - \frac{L^2}{r_H}\tan^{-1}\frac{r}{r_H}\right)\,, \tag{44}$$

one finds—qualitatively—a trailing string truncated by the null geodesic.

Amusing feature: at fixed E-F time, "trailing" string *leads* the endpoint (known to [Casalderrey-Solana and Teaney, hep-th/0701123]).

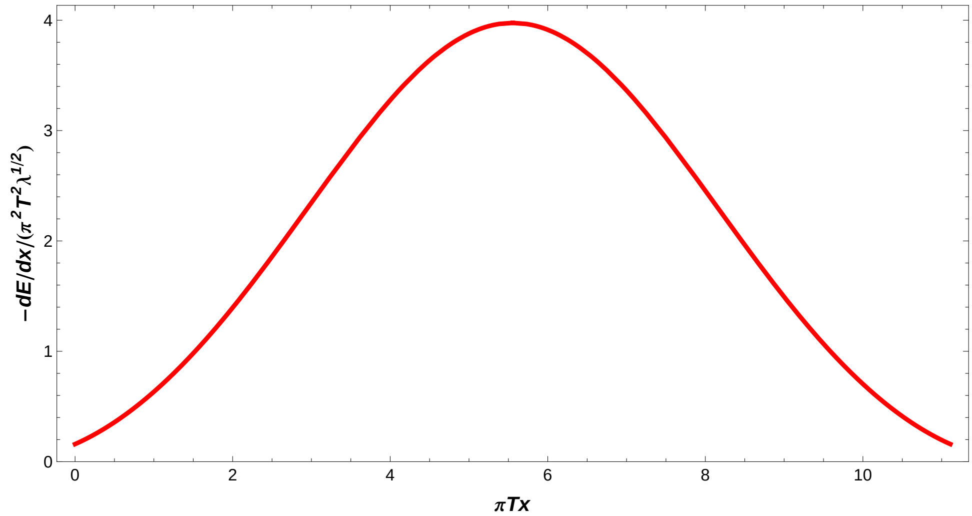


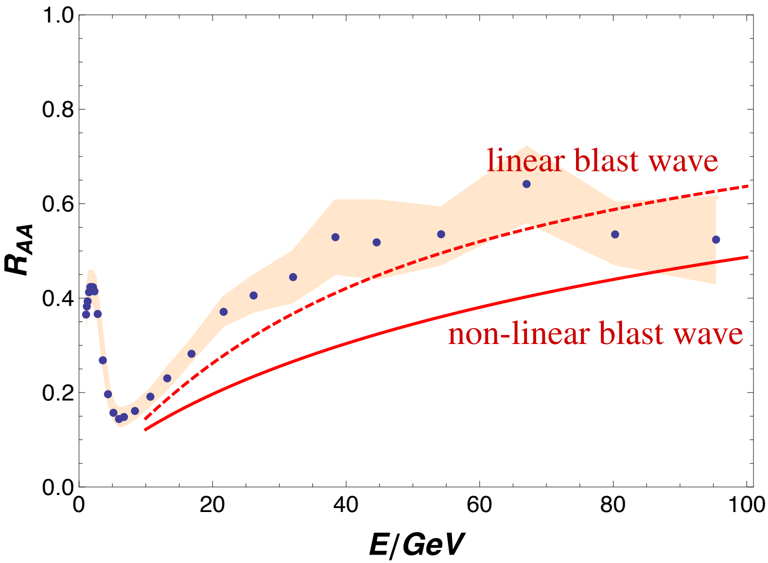
Starting from $\dot{p}_\mu$ for the endpoint, can derive

$$\frac{dE}{dx} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\sqrt{f(z_*)}}{z^2}, \tag{45}$$

where z is determined as the height of the endpoint geodesic at position x.

The result is a bell-shaped dE/dx, different from usual ansatz $dE/dx \sim E^\alpha x^\beta T^\gamma$.





- Preliminary results (red) suggest that R_{AA} at LHC is underpredicted by this model: $\lambda = 1$ here!
- R_{AA} is number of high-energy particles observed divided by expectations from pp.
- Probably need to go beyond conformal models—running coupling is important.

4. Summary

- AdS/CFT develops naturally out of consideration of black three-branes and related objects in 10-dimensional string theory.
- The strongly coupled, deconfined gauge theory described by a black three-brane has some useful similarities to the QGP.
- Finite endpoint momentum is part of classical string theory.
- Endpoints with finite momentum follow spacetime geodesics except for abrupt changes in direction.
- Finite endpoint momentum helps identify trajectories that maximize transverse distance traveled in AdS_5-Schwarzschild with fixed energy.
- Heavy-ion applications of $\Delta x_{\text{stop}} \propto E^{1/3}$ and bell-shaped dE/dx are under consideration.