Statistical thermal model
a.k.a. statistical hadronization model (SHM) or hadron resonance gas (HRG)

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COST Workshop on Interplay of hard and soft QCD probes for collectivity in heavy-ion collisions
Lund, Sweden, February 25 – March 1, 2019
Relativistic heavy-ion collisions

Event display of a Pb-Pb collision in ALICE at LHC

Thousands of particles created in relativistic heavy-ion collisions

Apply concepts of statistical mechanics to describe particle production
Historical perspective

1951-1953: Early applications of statistical concepts to particle production [Fermi; Landau; Pomeranchuk]

1965-1975: Hagedorn’s model (statistical bootstrap), applications to high-energy collisions

\[ \rho(m) = A m^{-\alpha} \exp(m/T_H) \]

1969: S-matrix formulation of statistical mechanics, the basis of the thermal model

Dashen, Ma, Bernstein, PRC 187, 45 (1969)

~1975: QCD as accepted theory of strong interactions

1992-...: Thermal fits to heavy-ion hadron yield data, mapping HIC to the QCD phase diagram

Cleymans, Satz; Braun-Munzinger, Stachel; Rafelski; Redlich; Becattini;...

2003-...: Open-source implementations of the thermal model: SHARE (Rafelski+), THERMUS (Cleymans+), the Thermal-FIST package (V.V.)
Relativistic heavy-ion collisions: Thermal model

Pros:
- Simplest model with very few free parameters ($T, \mu_B, \ldots$)
- Connection to QCD phase diagram
- Easier to test new ideas

Cons:
- No dynamics
- Describes only yields
- Thermal parameters fitted to data at each energy
Hadron resonance gas (HRG) at freeze-out

**HRG:** Equation of state of hadronic matter as a multi-component non-interacting gas of known hadrons and resonances

\[
\ln Z \approx \sum_{i \in M,B} \ln Z_i^d = \sum_{i \in M,B} \frac{d_iV}{2\pi^2} \int_0^\infty \pm p^2 dp \ln \left[ 1 \pm \exp \left( \frac{\mu_i - E_i}{T} \right) \right]
\]

**Grand-canonical ensemble:** \( \mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S \)  *chemical equilibrium*

**Thermal model:**
Equilibrated hadron resonance gas at the chemical freeze-out stage of high-energy collisions

**Model parameters:**
- \( T \) – temperature
- \( \mu_B, \mu_Q, \mu_S \) – chemical potentials
- \( V \) – system volume

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The basis of the HRG model

Dashen, Ma, Bernstein (1969): Inclusion of narrow resonances as free, point-like particles models attractive interactions where they are being formed [S-matrix formulation of statistical mechanics, PRC 187, 345 (1969)]

Example: interacting pion system

Interacting pion gas within the S-matrix approach agrees with the non-interacting $\pi + \rho$ gas

Include all resonances as free, point-like particles

HRG model

Venugopalan, Prakash, NPA (1992)
List of hadrons and resonances

\[
\ln Z^{\text{hrg}} = \sum_{i \in M, B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 \, dp \, \ln \left[ 1 \pm \exp \left( \frac{\mu_i - E_i}{T} \right) \right]
\]

Particle list in the thermal model usually includes all hadrons and resonances listed as established in the PDG listing

\~400 species

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Connecting model to experiment

\[ N_i^{hrg} = \frac{d_iV}{2\pi^2} \int_0^\infty p^2 dp \left[ \exp \left( \frac{E_i-\mu_i}{T} \right) \pm 1 \right]^{-1} \propto e^{-m_i/T} \]

**Particle decays:** Unstable resonances decay before being detected

\[ \Delta \leftrightarrow N \quad \pi \quad K^* \leftrightarrow K \quad \pi \quad \rho \leftrightarrow \pi \quad \pi \quad \text{etc.} \]

Take into account feeddown:

\[ N_i^{\text{fin}} = N_i^{\text{hrg}} + \sum_j BR(j \rightarrow i) N_j^{\text{hrg}} \]

60-70% of \( \pi, p, \text{etc.} \) are from feeddown

**Conservation laws:**

Zero net strangeness \( \rightarrow \mu_S \)

Electric-to-baryon ratio \( Q/B = 0.4-0.5 \) \( \rightarrow \mu_Q \)

** Freeze-out parameters** \( T, \mu_B, V \) extracted through \( \chi^2 \) minimization

\[ \chi^2 = \sum_i \frac{(N_i^{\text{fin}} - N_i^{\text{exp}})^2}{(\sigma_i^{\text{exp}})^2}, \quad i = \pi, K, p, \Lambda, ... \]
Thermal fits at SPS and RHIC energies

- Fair data description across several orders of magnitude
- Evidence for chemical equilibration of matter
Thermal fits at LHC

\[
\begin{array}{cccccccc}
\frac{\pi^+ + \pi^-}{2} & \frac{K^+ + K^-}{2} & K_S^0 & \frac{K^+ + K^-}{2} & \phi & \frac{p + \bar{p}}{2} & \Lambda & \frac{\Xi^- + \Xi^+}{2} & \frac{\Omega^+ + \Omega^0}{2} & d & \frac{\Delta H + \Delta H}{2} & \frac{3H + 3H}{2} & 3\text{He}
\end{array}
\]

\[dN/dy\]

\[10^3\]
\[10^2\]
\[10\]
\[1\]
\[10^{-1}\]
\[10^{-2}\]
\[10^{-3}\]
\[10^{-4}\]

\[\text{Not in fit}\]
\[\text{Extrapolated}\]

\[\text{ALICE Preliminary}\]
\[\text{Pb-Pb } \sqrt{s_{NN}} = 2.76 \text{ TeV, 0-10\%}\]

\[\text{Model} \quad T \ (\text{MeV}) \quad \chi^2/\text{NDF}\]
\[\text{THERMUS 2.3} \quad 155 \pm 2 \quad 24.5/9\]
\[\text{GSI-Heidelberg} \quad 156 \pm 2 \quad 18.4/9\]
\[\text{SHARE 3} \quad 156 \pm 3 \quad 15.1/9\]

\[\text{BR} = 25\%\]

ALICE collaboration (SQM 2015)
Heavy-ion collisions and the QCD phase diagram

Thermal fits for systems at different collision energies map chemical freeze-out stage in heavy-ion collisions to the QCD phase diagram

- Chemical freeze-out curve in $T$-$\mu_B$ plane
  \[ T_{ch}(\mu_B) \approx a - b \mu_B^2 - c \mu_B^4 \]
  J. Cleymans et al., PRC (2006)

- Energy per particle $E/N \approx 1$ GeV

HRG model and lattice QCD equation of state

[HotQCD collaboration, 1407.6387; similar results from Wuppertal-Budapest collab., 1309.5258]
HRG model and lattice QCD equation of state

[HotQCD collaboration, 1407.6387; similar results from Wuppertal-Budapest collab., 1309.5258]

HRG describes quite well LQCD thermodynamic functions below and in the vicinity of the pseudocritical temperature
Thermal model and radial flow

\[ N_i^{\text{hrg}} = V \frac{d_i}{2\pi^2} \int_0^{\infty} p^2 \, dp \left[ \exp\left( \frac{E_i - \mu_i}{T} \right) \pm 1 \right]^{-1} \]

In thermal model yields are computed in **local rest frame**, i.e. no flow. But matter in HIC appears to have a substantial collective flow, so how can the model be applied to data?
Thermal model and radial flow

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**Hydro:**

\[ N_i = \int_{\sigma} d\sigma \mu u^\mu \left. \int \frac{d^3p_i}{p^0} \frac{p_i \mu}{(2\pi)^3} \frac{d_i}{V_{\text{eff}}} \left[ \exp \left( \frac{p_i \mu - \mu_i}{T} \right) \pm 1 \right]^{-1} \right|_{n_i^{\text{hrg}}} \]

“Freeze-out” across space-time hypersurface \( \sigma(x) \) with collective velocity profile \( u^\mu(x) \). If \( T \) and \( \mu_i \) uniform across the hypersurface then

\[ N_i = n_i^{\text{hrg}} \left. \int_{\sigma} d\sigma \mu u^\mu \right|_{V_{\text{eff}}} \quad \text{and} \quad \frac{N_i}{N_j} = \frac{N_i^{\text{hrg}}}{N_j^{\text{hrg}}} \]
Thermal model and radial flow

\[ N_{i}^{\text{hrg}} = V \frac{d_{i}}{2 \pi^{2}} \int_{0}^{\infty} p^{2} \, dp \left[ \exp \left( \frac{E_{i} - \mu_{i}}{T} \right) \pm 1 \right]^{-1} \]

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\[ N_{i} = \int_{\sigma} d\sigma_{\mu} u^{\mu} \int \frac{d^{3}p_{i}}{p^{0}} p_{\mu} u^{\mu} \frac{d_{i}}{(2\pi)^{3}} \left[ \exp \left( \frac{p_{i}^{\mu} u_{\mu} - \mu_{i}}{T} \right) \pm 1 \right]^{-1} \]

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Effects of collective motion largely cancel out in yield ratios.
Many aspects of the thermal model

- particle list and decay properties
- finite resonance widths
- loosely bound states
- chemical non-equilibrium ($\gamma_q, \gamma_s$)
- excluded volume/van der Waals interactions
- exact conservation of conserved charges (canonical ensemble)
- particle number fluctuations
- statistical hadronization of charm
Different particle lists

- Established (*** & ****) hadrons from PDG (the standard option)
- Include unconfirmed (* & **) or theoretical (quark model) states

Evidence for extra strange baryons from lattice QCD

- Exponential Hagedorn mass spectrum \( \rho(m) = A m^{-\alpha} \exp(m/T_H) \)

New phenomena: "limiting" temperature, (phase) transition to QGP etc.

[Alba et al., 1702.01113; see also 1404.6511 (HotQCD)]

[Gallmeister et al., 1712.04018; V.V. et al., 1811.05737]
Decay properties of many resonances are not very well established. This affects determination of feeddown contributions.

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<th>$K_1(1400)$ Decay Modes</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
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<td>$K^*(892)\pi$</td>
<td>(94 $\pm$ 6) %</td>
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<td>$K\rho$</td>
<td>(3.0$\pm$3.0) %</td>
</tr>
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<td>$K\omega$</td>
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Decay properties

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<tr>
<td>$\Xi(1530)\pi$</td>
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"Educated" guesses sometimes needed to calculate feeddown.

Source of a systematic uncertainty ~10%
Finite resonance widths

\[ n_i(T, \mu; m_i) \rightarrow \int_{m_i^{\text{min}}}^{m_i^{\text{max}}} dm \rho_i(m) n_i(T, \mu; m) \]

1) Zero-width approximation
   
   Simplest possibility, used commonly in LQCD comparisons

2) Constant Breit-Wigner (BW) in \( \pm 2\Gamma_i \) interval
   
   Popular choice in thermal fits
   Enhances resonance yields

   \[ \rho_i(m) = A_i \frac{2m m_i \Gamma_i}{(m^2 - m_i^2)^2 + m_i^2 \Gamma_i^2} \]

3) Energy-dependent Breit-Wigner (eBW)
   
   Suppression at the threshold
   Suppresses resonance yields

   \[ \Gamma_{i \rightarrow j}(m) = b_{i \rightarrow j} \Gamma_i \left[ 1 - \left( \frac{m_{i \rightarrow j}^{\text{thr}}}{m} \right)^2 \right]^{l_{ij} + 1/2} \]

4) Phase shifts within the S-matrix approach
   
   Usually based on measured scattering phase shifts

\[ \rho_i(m) \propto \frac{\partial \delta(m)}{\partial m} \]
Finite resonance widths: effect on thermal fits

Energy-dependent Breit-Wigner leads to a 15% suppression of proton yields

This is enough to describe the ‘proton yield anomaly’ at the LHC

[**V.V., Gorenstein, Stoecker, 1807.02079; see also phase shift analysis P. Lo et al., 1808.03102**]
Thermal model and loosely-bound states

Yields of light nuclei at LHC decrease exponentially with mass

![Graph showing exponential decrease in yields of light nuclei with mass. The graph is labeled "0-10% Pb-Pb, $\sqrt{s_{NN}} = 2.76$ TeV".]

ALICE collaboration, 1710.07531
Thermal model and loosely-bound states

Yields of light nuclei at LHC decrease exponentially with mass, agree well with thermal model at $T = 155$ MeV

ALICE collaboration, 1710.07531

Andronic et al., 1710.09425

Yield $dN/dy$

0-10% Pb-Pb, $\sqrt{s_{NN}} = 2.76$ TeV

ALICE
Yields of light nuclei at LHC decrease exponentially with mass, agree well with thermal model at $T = 155$ MeV.

Loosely-bound states (few MeV or less binding energy) expected to be immediately destroyed at $T = 155$ MeV. Why the thermal model works so well for the yields of light nuclei remains not fully understood.
Incomplete chemical equilibrium of strangeness

A reasonable description of strangeness production often requires introduction of *strangeness saturation parameter* $\gamma_S$, which in thermal picture interpreted as an incomplete equilibration of strangeness

\[ N_{i}^{hrg} \rightarrow (\gamma_S) |s_i| N_{i}^{hrg} \]

$|s_i|$ - strange quark content

$\gamma_S < 1$ in p-p & A-A at AGS/SPS

$\gamma_S \approx 1$ in A-A at RHIC and LHC

Figure from Castorina, Plumari, Satz, 1603.06529
Chemical non-equilibrium scenario

In chemical non-equilibrium scenario $N_i^{hrg} \rightarrow (\gamma_q)|q_i| (\gamma_s)|s_i| N_i^{hrg}$ both light ($|q_i|$) and strange ($|s_i|$) quarks out of chemical equilibrium

Scenario: hadronization of chem. non-eq. supercooled QGP [Letessier, Rafelski, ‘99]

- smaller reduced $\chi^2$ compared to chem. equilibrium scenario
- $\gamma_q = 1.63$ $\Rightarrow$ $\mu_\pi \approx 135$ MeV $\approx m_\pi$ $\Rightarrow$ pion BEC? [V. Begun et al., 1503.04040]
- However, $\gamma_q \approx \gamma_s \approx 1$ when light nuclei included in fit [M. Floris, 1408.6403]
Excluded volume corrections

Notion that hadrons have finite eigenvolume suggested awhile ago

Excluded volume model: \( V \rightarrow V - bN \) \( \Rightarrow \ p(T, \mu) = p^{\text{id}}(T, \mu - bp) \)

Recent lattice QCD data favor EV-like effects in baryonic interactions

Evidence for EV effects for mesons is less compelling

\[ 4\text{stout, } N_L = 12 \]

\( b_1 \) \( \text{Im} \chi_1^B = b_1(T)^* \sin(\mu_B/T) \)

\( b_2 \) \( + b_2(T)^* \sin(2\mu_B/T) \)

\( b_3 \) \( + b_3(T)^* \sin(3\mu_B/T) \)

\( b_4 \) \( + b_4(T)^* \sin(4\mu_B/T) \)

\( \cdots \)

EV-HRG, \( b = 1 \text{ fm}^3 \)

\[ \chi_B^2 - \chi_4^B \]

V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

Evidence for EV effects for mesons is less compelling
Excluded volume corrections and thermal fits

Excluded volume effect: $N_i \rightarrow \kappa e^{-\frac{v_i p}{T}} N_i$

This may have an effect on data description if $v_i$ are different

Depending on $v_i$ parameterization effects on fits are between being negligible to strong and controversial ($\chi^2$ minima at very high $T$)
van der Waals interactions in HRG

NN potential: repulsive core and attraction

vdW-HRG: baryons described by the vdW equation

\[ \rho = \rho_M^{id} + \rho_B^{vdW} + \rho_B^{vdW} \quad \rho_B^{vdW} \simeq \frac{T n_B}{1 - b n_B} \quad a n_B^2 \]

\( a = 329 \text{ MeV fm}^3, \quad b = 3.42 \text{ fm}^3 \) from fit to ground state

Critical point at \( T_c = 19.7 \text{ MeV}, \mu_c = 908 \text{ MeV} \)

\[ \frac{\varepsilon}{T^4}, \quad \frac{3p}{T^4} \]

\[ \frac{c_s^2}{T^2} \]

V.V., M.I. Gorenstein, H. Stoecker, PRL 118, 182301 (2017)
Canonical statistical model

Grand-canonical ensemble: configurations with all possible quantum numbers

\[ Z_{gce}(\mu_B, \mu_Q, \mu_S) = \sum_{B=-\infty}^{\infty} \sum_{Q=-\infty}^{\infty} \sum_{S=-\infty}^{\infty} e^{\frac{B\mu_B + Q\mu_Q + S\mu_S}{T}} Z_{ce}(B, Q, S) \]

including those not realized in heavy-ion collisions, e.g. \( S \neq 0 \)

Thermodynamic equivalence of ensembles: \( N_{i}^{gce} = N_{i}^{ce} + O(V^{-1}) \)

GCE justified for large systems, but canonical effects needed for smaller systems

[Rafelski, Danos, et al., PLB ’80; Hagedorn, Redlich, ZPC ‘85]
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[Bafelski, Danos, et al., PLB '80; Hagedorn, Redlich, ZPC '85]

Canonical partition function:

\[ Z(B, Q, S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} e^{-i(B\phi_B + Q\phi_Q + S\phi_S)} \exp \left[ \sum_j z_j^1 e^{i(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)} \right] \]

[Becattini et al., ZPC '95, ZPC '97]

\[ z_j^1 = V_c \int dm \rho_j(m) d_j \frac{m^2 T}{2\pi^2} K_2(m/T) \]

\[ \langle N_j^{\text{prim}} \rangle_{\text{ce}} = \frac{Z(B - B_j, Q - Q_j, S - S_j)}{Z(B, Q, S)} \langle N_j^{\text{prim}} \rangle_{\text{gce}} \]

CE effects typically suppress yields relative to the GCE (canonical suppression)

Strangeness enhancement as a manifestation of an absence of CE suppression

[Hamieh, Redlich, Tounsi, PLB (2000)]
Canonical statistical model and thermal fits

Canonical thermodynamics allows to use thermal model for small systems such as $p-p$, $p-\bar{p}$, $e^+ e^-$

[Image of plots showing multiplicity vs. therm. model multiplicity for $pp \sqrt{s} = 19.4$ GeV, $23.8$ GeV, and $26$ GeV]

More on this on Wednesday, 15:40

[F. Becattini et al., ZPC ‘95, ZPC ‘97]
Thermal model tools

Available thermal model codes:

1) **SHARE 3**  [G. Torrieri, J. Rafelski, M. Petran, et al.]  
   *Since 2003*  
   *Fortran/C++. Chemical (non-)equilibrium, fluctuations, charm, nuclei*  
   **open source:** http://www.physics.arizona.edu/~gtshare/SHARE/share.html

2) **THERMUS 4**  [S. Wheaton, J. Cleymans, B. Hippolyte, et al.]  
   *Since 2004*  
   *C++/ROOT. Canonical ensemble, EV corrections, charm, nuclei*  
   **open source:** https://github.com/thermus-project/THERMUS
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   C++/ROOT. Canonical ensemble, EV corrections, charm, nuclei
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New development:

Thermal-FIST v1.1 (or simply “The FIST”)  [V.V., H. Stoecker]  Since 2018
   C++. Chemical (non-)equilibrium, EV/vdW corrections, Monte Carlo,
   (higher-order) fluctuations, canonical ensemble, combinations of effects
   open source: https://github.com/vlvovch/Thermal-FIST
Graphical user interface for *general-purpose* thermal fits and more
Thermal-FIST

Graphical user interface for general-purpose thermal fits and more

“So that’s how you get your results so quickly!”

J. Cleymans

“Thanks for reproducing my results!”

F. Becattini
Using Thermal-FIST

The package is **cross-platform** (Linux, Mac, Windows, Android)
Installation using **git** and **cmake**

```
# Clone the repository from GitHub
git clone https://github.com/vlvovch/Thermal-FIST.git
cd Thermal-FIST

# Create a build directory, configure the project with cmake
# and build with make
mkdir build
cd build
cmake ..
making

# Run the GUI frontend
./bin/QtThermalFIST

# Run the test calculations from the paper
./bin/examples/cpc1HRGTDep
./bin/examples/cpc2chi2
./bin/examples/cpc3chi2NEQ
./bin/examples/cpc4mcHRG
```

*GUI requires free **Qt5 framework**, the rest of the package has no external dependencies*
Summary

• The statistical thermal model is the “simplest” model for particle production, which describes yields across many collision energies on a 10-15% level

• The model has many ambiguous details – sources of systematic uncertainty in the model – currently under investigation

• Model applications available through a number of open source codes. New **Thermal-FIST** package provides most of the features used in thermal model analysis in a convenient way.

  https://github.com/vlvovch/Thermal-FIST
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Thanks for your attention!