Multiplicity dependence of particle production at the LHC in (canonical) statistical model

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In collaboration with B. Doenigus and H. Stoecker, paper in preparation

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Hadron resonance gas (HRG) at the chemical freeze-out:

\[ N_i^{\text{hrg}} = V \frac{d_i m_i^2 T}{2\pi^2} K_2 \left( \frac{m_i}{T} \right) e^\frac{\mu_i}{T}, \quad N_i^{\text{tot}} = N_i^{\text{hrg}} + \sum_j BR(j \rightarrow i) N_j^{\text{hrg}} \]

Fair description of central Pb-Pb collisions \(\rightarrow\) equilibrated matter formed?
Particle production at the LHC

- Hadron yield ratios exhibit multiplicity dependence
- Grand-canonical picture predicts no multiplicity dependence
- Ratios appear to approach a plateau at high-multiplicities → grand-canonical plateau?
- Can multiplicity-dependence be considered in a macroscopic model?

ALICE collaboration, 1807.11321
Canonical statistical model (CSM)

**Grand-canonical approach:** yield ratios $N_i / N_j$ volume-independent, but conserved charges not conserved exactly. Canonical treatment of conservation laws important for small reaction volumes  

[Becattini et al., ZPC ‘95, ZPC ‘97]

**Canonical partition function:**

$$Z(B, Q, S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} e^{-i(B\phi_B + Q\phi_Q + S\phi_S)} \exp \left[ \sum_j z_j^1 e^{i(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)} \right]$$

$$z_j^1 = V_c \int dm \rho_j(m) d_j \frac{m^2 T}{2\pi^2} K_2(m/T)$$

[Becattini et al., ZPC ‘95, ZPC ‘97]

$$\langle N_{j,\text{prim}} \rangle_{\text{ce}} = \frac{Z(B - B_j, Q - Q_j, S - S_j)}{Z(B, Q, S)} \langle N_{j,\text{prim}} \rangle_{\text{gce}}$$

$\approx 1$ at large volume (GCE), $<1$ for smaller volumes; stronger effect for multi-charged particles; neutral particles unaffected

Can multiplicity dependence be understood as a canonical suppression?
Strangeness-canonical picture: $S$ is canonical, $B$ & $Q$ grand-canonical

- Describes trend for most yield ratios, but not $\phi$

- What is the role of baryon and electric charge conservation?

[CSM at LHC: strangeness-canonical ensemble]

[Vislavicius, Kalweit, 1610.03001]

[ALICE collaboration, 1807.11321]
When is the canonical treatment necessary?

Normally, when the total number of particles carrying a conserved charge is smaller or of the order of unity

The canonical treatment is often restricted to strangeness only (SCE) [STAR collaboration, 1701.07065; ALICE collaboration, 1807.11321]

Along the freeze-out curve

- Strangeness conservation is most important at low energies (HADES, CBM)
- Small systems at RHIC and LHC: exact baryon conservation at least as important as strangeness
The **Thermal-FIST** package is employed in the present analysis by V.V., H. Stoecker, [arXiv:1902.05249](https://arxiv.org/abs/1902.05249)

**Open source:** [https://github.com/vlvovch/Thermal-FIST](https://github.com/vlvovch/Thermal-FIST)

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**Canonical Statistical Model implementation in Thermal-FIST:**

- Selective canonical treatment of charges
- Quantum statistics
- Supports $|B_j| > 1$ (*light nuclei*)
- Particle number fluctuations and correlations

See also talk of A. Motornenko, Thursday 17:05
Canonical statistical model: $T = 155$ MeV, $V_C$ – canonical volume, selective (grand-)canonical treatment of B, Q, S
CSM at LHC: yield ratios to pions

$V_C$ dependence of yield ratios to pions

- SCE appropriate for $K$, $\Omega$, $\Xi$, less so for $\Lambda$, totally off for $\rho$ and $\phi$

- Baryon-strangeness-CE appropriate for most observables, except $\phi/\pi$ and $\pi$

- In general, full canonical treatment of B,Q,S required
Connecting CSM to data

Enforce \textit{local} exact conservation of charges, $B = Q = S = 0$, in a correlation volume $V_C$ around midrapidity.

In general, $V_C \neq dV/dy$

\textbf{Causality argument:} exact conservation across a few units of rapidity?

[Castorina, Satz, 1310.6932]

"Vanilla" CSM:

- $T = 155$ MeV for all multiplicities
- Multiplicity dependence of yield ratios driven by \textit{canonical} suppression only
- $V_C = k \, dV/dy$, where $k$ varied to establish systematics
“Vanilla” CSM at LHC: comparison with data
“Vanilla” CSM at LHC: light nuclei

- **CSM** qualitatively captures the behavior seen in the data
- Data prefers $V_C > dV/dy$ and/or $T_{p+p} > T_{Pb+Pb}$

[V.V., B. Doenigus, H. Stoecker, 1808.05245]
“Vanilla” CSM at LHC: summary

• The CSM captures fairly well multiplicity dependence of hyperon-to-pion and nuclei-to-proton ratios

• Trend in $K/\pi$ captured, but the data are significantly overshooted

• Some tension with the $p/\pi$ data, which shows no clear evidence for canonical suppression

• Behavior of $\phi/\pi$ in the model is opposite to the behavior in the data. Unless production mechanism of $\phi$ is separate from the rest of hadrons, this invalidates “Vanilla” CSM for p-p and p-Pb
Full CSM

• Allow variation of $T$ with multiplicity

• Allow incomplete chemical equilibration of strangeness (as suggested by the behavior of $\phi$):

\[
N_i^{\text{hrg}} \rightarrow (\gamma_S)^{|s_i|} N_i^{\text{hrg}}
\]

$|s_i|$ - strange quark content

• $V_C = 3dV/dy +$ deviations

• $T, \gamma_S, dV/dy$ fitted to data at each centrality

• Data: $\pi, K, K_0^S, \phi, p, \Lambda, \Xi, \Omega$ in p-p 7 TeV, p-Pb 5.02 TeV, Pb-Pb 2.76 TeV
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A similar analysis recently presented in [Sharma et al., 1811.00399], with two important differences:

• There $\phi$ excluded from analysis, here it is included

• There $V_C = dV/dy$ strictly enforced, here not
Full CSM: Extracted parameters

CSM (Thermal-FIST)

- $V_c = 3 \text{ dV/dy}$
- $V_c \to \infty$ (GCE)

**Graphs:***

- Temperature ($T$) vs. $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5}$
- $\gamma_s$ vs. $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5}$
- $dV/dy$ vs. $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5}$
- $\chi^2$/dof vs. $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5}$
CSM at LHC: data description

- \( p/\pi \)
- \( K/\pi \)
- \( \phi/\pi \)
- \( \Lambda/\pi \)
- \( \Xi/\pi \)
- \( \Omega/\pi \)
Remarks:

• Canonical model preferred over GCE in p-p, not in p-Pb and Pb-Pb. Apparent reasons are $\Omega$’s, which are measured with better precision in p-p, and the fact that canonical suppression is partially manifest in the GCE through smaller values of $\gamma_S$. *New/better measurements of $\Omega$’s will be very useful.*

• $V_C = 3dV/dy$ found to be optimal. For $V_C = dV/dy$ CE effects are too strong and in bad agreement with p-p and p-Pb data.

• $T$ decreases with multiplicity in CSM, from $\sim 175$ MeV for the lowest multiplicities in p-p to $\sim 155$ MeV for the highest multiplicities in Pb-Pb. $\gamma_S$ increases with multiplicity, saturates at $\gamma_S \approx 1$ at $dN_{ch}/d\eta \approx 100$

• Canonical effects negligible above $dN_{ch}/d\eta \approx 50$ – effective thermodynamic limit

• Energy-dependent Breit-Wigner widths used. If zero widths used instead, $p/\pi$ pushed up by $\sim 15\%$, further away from the data at all multiplicities.
CSM at LHC: model accuracy

χ² x2 larger in p-p and p-Pb compared to Pb-Pb

Reflects mainly the differences in measurement uncertainties rather than model performance

A measure of model accuracy in describing the data

\[
\text{model accuracy} = \sum_i w_i \left| \frac{N_{i}^{\text{mod}}}{N_{i}^{\text{exp}}} - 1 \right|, \quad w_i \propto \text{contribution to } \chi^2
\]

Relative accuracy of CSM with γₜ is ~15% for all multiplicity bins
Summary

• Exact conservation of baryon number at least as important as strangeness in the canonical picture at the LHC. Strangeness-canonical ensemble only appropriate for multistrange hyperons.

• The “vanilla” CSM captures multiplicity dependence of hyperons and light nuclei, but goes the opposite way when applied to $\phi/\pi$.

• CSM with $\gamma_S \leq 1$ and multiplicity-dependent $T$ describes hadron yield data on a 15% level across all multiplicities considered.

• Canonical effects irrelevant above $dN_{ch}/d\eta \approx 50$.
Summary

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Thanks for your attention!